Real-Time Thermospheric Density Estimation Via Radar And GPS Tracking Data Assimilation

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Abstract

As the number of man-made Earth-orbiting objects increases, satellite operators need enhanced space traffic management capabilities to ensure safe space operations. For objects in Low-Earth orbit, orbit determination and prediction require accurate estimates of the local thermospheric density. In previous work, the estimation of thermospheric densities using two-line element data and a reduced-order model for the upper atmosphere was demonstrated. In this paper we demonstrate an approach for density estimation using radar and GPS tracking data. For this, we assimilate the tracking data in a dynamic reduced-order density model using a Kalman filter by simultaneously estimating the orbits and global density. We used the radar range and range-rate measurements of 20 objects and the GPS position measurements of 10 commercial satellites. The estimated density was validated against accurate SWARM density data and compared with NRLMSISE-00, JB2008, and TLE-estimated densities. We found that the estimated densities are significantly more accurate than NRLMSISE-00 and JB2008 densities. In particular, using the GPS data of 10 satellites, we obtain density estimates with a daily 1-$\sigma$ error of only 5% compared to 14% and 22% for empirical models and 10% for TLE-estimated density. These accurate density estimates can be used to improve orbit determination and the use of real-time tracking data would enable real-time density estimation.
Real-Time Thermospheric Density Estimation Via Radar And GPS Tracking Data Assimilation

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Key Points:

• Thermospheric density is estimated using a reduced-order density model and radar range and range-rate tracking data and GPS measurements.
• The estimated densities are validated against accurate SWARM density data.
• The obtained densities are significantly more accurate than NRLMSISE-00 and JB2008 modelled densities and TLE-estimated densities.

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Abstract

As the number of man-made Earth-orbiting objects increases, satellite operators need enhanced space traffic management capabilities to ensure safe space operations. For objects in Low-Earth orbit, orbit determination and prediction require accurate estimates of the local thermospheric density. In previous work, the estimation of thermospheric densities using two-line element data and a reduced-order model for the upper atmosphere was demonstrated. In this paper we demonstrate an approach for density estimation using radar and GPS tracking data. For this, we assimilate the tracking data in a dynamic reduced-order density model using a Kalman filter by simultaneously estimating the orbits and global density. We used the radar range and range-rate measurements of 20 objects and the GPS position measurements of 10 commercial satellites. The estimated density was validated against accurate SWARM density data and compared with NRLMSISE-00, JB2008, and TLE-estimated densities. We found that the estimated densities are significantly more accurate than NRLMSISE-00 and JB2008 densities. In particular, using the GPS data of 10 satellites, we obtain density estimates with a daily 1-σ error of only 5% compared to 14% and 22% for empirical models and 10% for TLE-estimated density. These accurate density estimates can be used to improve orbit determination and the use of real-time tracking data would enable real-time density estimation.

1 Introduction

For reliable conjunction assessment and collision avoidance, accurate orbit prediction is essential. In Low Earth Orbit, the main cause of error in orbit prediction is the inaccurate modeling of atmospheric drag. Drag modeling is complicated because it requires knowledge of the atmospheric density, which is highly dynamic and depends on external drivers, such as solar and geomagnetic activity (Emmert, 2015). Generally, empirical atmosphere models, such as NRLMSISE-00, JB2008 and DTM-2013 (Picone et al., 2002; Bowman et al., 2008; Bruinsma, 2015), are employed for orbit propagation, because they are fast to evaluate. However, because empirical models describe the average behavior of measurements and observations using a fixed mathematical representation, their accuracy is limited. Improved thermospheric densities can be computed through calibration of these models using recent data. In particular, the dynamic calibration atmosphere (DCA) in the high-accuracy satellite drag model (HASDM), which uses Space Surveillance Network (SSN) observations of tens of objects for calibration, is known to significantly improve thermospheric density estimates (Storz et al., 2005; Casali & Barker, 2002). Still, the calibration of empirical models is limited to nowcasts, since empirical models do not model the dynamics of the thermosphere and therefore forecasts quickly degrade with time. Physics-based models, on the other hand, have most potential for predicting the future thermospheric state, in particular during storm conditions (Sutton, 2018). Examples are the Global Ionosphere-Thermosphere Model (GITM) (Ridley et al., 2006) and the Thermosphere-Ionosphere-Electrodynamics General Circulation Model (TIE-GCM) (Qian et al., 2014). These models solve the full Navier-Stokes equations for density, velocity, and temperature for a number of neutral and charged particles on a discretized grid. This involves $10^4$-$10^6$ state variables which demands considerable computational effort. Moreover, the high number of variables makes the estimation of the atmospheric state through data assimilation very complex and computationally expensive (Matsuo et al., 2012; Sutton, 2018).

Recently, reduced-order models were developed to represent a physics-based atmospheric density model using a smaller number of parameters. The dimensionality reduction was achieved via Proper Orthogonal Decomposition (Mehta & Linares, 2017) and Dynamic Mode Decomposition with control (DMDc) was used to model the dynamics of the atmosphere using the reduced-order state and solar and geomagnetic inputs (Mehta et al., 2018). These developments resulted in dynamic reduced-order models for the thermospheric density that have the computational efficiency of empirical models. Through
simulation, it was shown that the reduced-order models can provide accurate density forecasts.

Moreover, the reduced-order models have been applied to accurately estimate the global thermospheric density through data assimilation of density and orbital data using Kalman filters (Mehta & Linares, 2018b, 2018a, 2020; Gondelach & Linares, 2020c). Density estimation was demonstrated by assimilating accelerometer-derived density measurements (Mehta & Linares, 2018b), simulated GPS position measurements (Mehta & Linares, 2020) and real two-line element (TLE) data (Gondelach & Linares, 2020c). In particular, it was shown that density estimates based on TLE data are more accurate than the JB2008 and NRLMSISE-00 models when compared with CHAMP and GRACE accelerometer-derived densities (Gondelach & Linares, 2020c). The use of publicly available TLE data together with the open-source publication of the reduced-order density models and code (Gondelach & Linares, 2020b) enables thermospheric density estimation for a large community. A drawback of the use of TLE data is, however, that the accuracy of the data is strongly limited due to the simplified dynamical model on which the TLEs are based. The use of more accurate orbital data would enable improved global density estimates.

In this work, we present initial results for employing satellite radar and GPS tracking data to estimate the global thermospheric density. Nowadays, radar tracking data is commercially available and various agencies and institutions possess their own radar tracking stations. The accuracy and wide availability of radar measurements make these observations a promising data source for density estimation. In addition, the number of spacecraft equipped with GPS receivers is increasing and these GPS measurements may be made publicly available, which enables their use.

Radar and GPS data have been used by various authors in the past to estimate atmospheric densities. HASDM uses radar tracking data of around 80-90 objects and some high precision data to compute corrections to the JB2008 model every 3 hours (Storz et al., 2005; Bowman & Tobiska, 2014). Wright (Wright, 2003) used radar tracking data to estimate the local atmospheric density by simultaneously estimating the orbit and local corrections to a global atmospheric density model using sequential filtering and smoothing. GPS data has been used to derive thermospheric densities along satellite orbits. McLaughlin et al. (McLaughlin et al., 2012) used GPS-derived precision orbit ephemeris data to estimate density by computing corrections to a baseline density model using a sequential filtering scheme, while the ballistic coefficient was estimated simultaneously. Kuang et al. (Kuang et al., 2014) used centimeter-level precise orbit determination using GPS tracking data to derive densities by determining accelerations due to drag. Calabia and Jin (Calabia & Jin, 2017) employed numerically differentiated precise orbit ephemeris to derive thermospheric densities. Van den IJssel et al. (van den IJssel et al., 2020) applied precise orbit determination using GPS data of Swarm satellites to estimate non-gravitational accelerations and derive thermospheric densities. The drawback of these approaches using GPS data is that they require sophisticated precise orbit determination techniques or precision orbit ephemeris. Finally, two-line element data, which is generated using radar and optical tracking data from the United States Space Surveillance Network, has been used by various authors to estimate densities (Cefola et al., 2004; Yurasov et al., 2005; Picone et al., 2005; Doornbos et al., 2008; Gondelach & Linares, 2020c). However, due to the limited accuracy and frequency of TLE data, the TLE data of many objects is needed to obtain densities with good accuracy and temporal resolution.

In this paper, we use real range and range-rate measurements and GPS position measurements to estimate the global density. This is achieved by data assimilation of radar and GPS tracking data in reduced-order density models by simultaneously estimating the global density and the orbits and ballistic coefficients of multiple objects. To determine the accuracy of the estimated densities they are validated against accurate GPS-derived thermospheric densities from the Swarm satellites (van den IJssel et al., 2020).
In addition, the densities are compared with NRLMSISE-00, JB2008, and TLE-estimated densities. The main contributions of this paper are:

1. Global thermospheric densities are estimated using data assimilation of the radar and GPS tracking data of multiple objects in a reduced-order density model.
2. The tracking data are processed using a Kalman filter to enable real-time density estimation.
3. The tracking data of debris and operational satellites are used. No attitude data or high-fidelity geometry models of the tracked objects are employed.
4. The estimated densities are validated against SWARM density data.
5. Accurate densities with a daily 1-σ error of only 5% (compared to 22%, 14% and 10% for NRLMSISE-00, JB2008 and TLE-estimated densities, respectively) were obtained using the GPS data of 10 satellites.

The obtained accurate density estimates can be used for improving orbit determination, prediction and uncertainty quantification for space traffic management (Gondelach & Linares, 2020a). In addition, access to real-time radar and GPS observations would enable real-time density estimation.

In the following, first the reduced-order modeling of the thermospheric density is described. After that the radar and GPS tracking data used in this work are discussed and data assimilation approach to estimate the density is described. Then the density estimation results are presented and assessed by comparison against SWARM densities, and NRLMSISE-00, JB2008 and TLE-estimated densities. Finally, the results are discussed and conclusions are drawn.

2 Methodology

For estimation of the thermospheric densities we use a reduced-order model (ROM) for the thermosphere and estimate the state of the ROM through assimilation of orbital tracking data. The low-dimensional state of the ROM captures the main variations in the density on a high-dimensional global grid, such that we obtain an estimate for the global thermospheric density. In this work we use the ROMs developed in previous work (Gondelach & Linares, 2020c). In the following, a brief overview of the reduced-order modeling technique and model characteristics is provided. More information can be found in Gondelach and Linares (2020c).

2.1 Reduced-order modeling

The main idea of reduced-order modeling is to reduce the dimensionality of the state space while retaining maximum information. In our case, the full state space consists of the neutral mass density values on a dense uniform grid in latitude, local solar time and altitude with dimensions 20 by 24 by 36 (in total 17280 points). The goal is to develop an efficient and accurate model for the evolution of the density over time. First, to reduce the dimension of the state space we employed proper orthogonal decomposition (POD). Second, we derived a linear dynamic model that accounts for the effect of space weather by applying Dynamic Mode Decomposition with control (DMDc).

2.1.1 Proper orthogonal decomposition

Order reduction using POD is achieved by projecting the high-dimensional system onto a set of a small number of basis functions or spatial modes. These spatial modes are computed such that the dominant characteristics of the system are captured by the first $r$ modes. Consider the variation $\tilde{x}$ of the neutral mass density $x$ with respect to the
mean value $\bar{x}$:

$$\bar{x}(s, t) = x(s, t) - \bar{x}(s)$$  \hspace{1cm} (1)$$

where $s$ is the spatial grid. A significant fraction of the variance $\tilde{x}$ can be captured by the first $r$ principal spatial modes:

$$\tilde{x}(s, t) \approx \sum_{i=1}^{r} c_i(t) \Phi_i(s)$$  \hspace{1cm} (2)$$

where $\Phi_i$ are the spatial modes and $c_i$ are the corresponding time-dependent coefficients. The spatial modes $\Phi$ are computed using a SVD of the snapshot matrix $X$ that contains $\tilde{x}$ for different times:

$$X = \begin{bmatrix} \tilde{x}_1 & \tilde{x}_2 & \cdots & \tilde{x}_m \end{bmatrix} = U \Sigma V^T$$  \hspace{1cm} (3)$$

where $m$ is the number of snapshots. The state reduction is achieved using a similarity transform:

$$z = U^{-1}_r \bar{x} = U^\top \tilde{x}$$  \hspace{1cm} (4)$$

where $U_r$ is a matrix with the first $r$ POD modes and $z$ is our reduced-order state that contains the corresponding time-dependent coefficients. Projecting $z$ back to the full space gives approximately $\bar{x}$ that allows us to compute the density:

$$x(s, t) \approx U_r(s) z(t) + \bar{x}(s)$$  \hspace{1cm} (5)$$

More details on POD can be found in Mehta and Linares (2017).

### 2.1.2 Dynamic Mode Decomposition with control

The atmospheric density depends strongly on the space weather conditions. Therefore, to predict the future density, we look for a function that takes the current state $z_k$ and space weather inputs $u_k$ and returns the future state:

$$z_{k+1} = f(z_k, u_k)$$  \hspace{1cm} (6)$$

where $z_k$ is the reduced-state at epoch $k$: $z_k = U^\top \bar{x}_k$.

Dynamic Mode Decomposition with control (DMDc) enables us to derive a linear dynamical system that considers exogenous inputs:

$$z_{k+1} = Az_k + Bu_k$$  \hspace{1cm} (7)$$

The dynamic matrix $A$ and input matrix $B$ can be estimated from output data, or snapshots, $z_k$, rearranged into time-shifted data matrices. Let $Z_1$ and $Z_2$ be the time-shifted matrix of snapshots such that:

$$Z_1 = \begin{bmatrix} z_1 & z_2 & \cdots & z_{m-1} \end{bmatrix}, \quad Z_2 = \begin{bmatrix} z_2 & z_3 & \cdots & z_m \end{bmatrix}, \quad Y = \begin{bmatrix} u_1 & u_2 & \cdots & u_{m-1} \end{bmatrix}$$  \hspace{1cm} (8)$$

where $m$ is the number of snapshots and $Y$ contains the inputs corresponding to $Z_1$. Since $Z_2$ is the time evolution of $Z_1$, they are related through Eq. (7) such that:

$$Z_2 = AZ_1 + BY$$  \hspace{1cm} (9)$$

Given $Z_1$ and $Z_2$, we can estimate matrices $A$ and $B$ in least-squares sense and obtain a linear reduced-order model (Eq. 7) that corresponds to the fixed timestep $T$ used for the snapshots. For estimation we require continuous information about the density and therefore need a continuous dynamical model:

$$\dot{z} = Az + Bu$$  \hspace{1cm} (10)$$
where $A_c$ and $B_c$ are the continuous-time dynamic and input matrices, respectively. The continuous-time matrices are obtained by converting the discrete-time matrices using the following relation (DeCarlo, 1989):

$$
\begin{bmatrix}
A_c & B_c \\
0 & 0
\end{bmatrix} = \log \left( 
\begin{bmatrix}
A & B \\
0 & I
\end{bmatrix}
\right) / T
$$

(11)

where $T$ is the sample time, i.e. the snapshot resolution. Now using Eqs. (4) and (5) we can map between the full and reduced space and Eq. (10) allows us to predict the density.

### 2.1.3 ROM density models

The ROM models used in this work were developed using 12-year density data from the NRLMSISE-00 (Picone et al., 2002) and Jacchia-Bowman 2008 (JB2008) models (Bowman et al., 2008). Details on the spatial grid and 12 years periods applied for generating the ROM models can be found in Table 1. More details on the models can be found in Gondelach and Linares (2020c). In the remainder of this paper the two ROMs based on NRLMSISE-00 and JB2008 data will be called ROM-MS and ROM-JB, respectively. In addition, throughout this work we used a reduced dimension of $r = 10$ if not stated otherwise.

<table>
<thead>
<tr>
<th>Base model</th>
<th>Local solar time [hr] Domain</th>
<th>Resolution</th>
<th>Latitude [deg] Domain</th>
<th>Resolution</th>
<th>Altitude [km] Domain</th>
<th>Resolution</th>
<th>Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>NRLMSISE-00</td>
<td>[0, 24]</td>
<td>1.04</td>
<td>[-87.5, 87.5]</td>
<td>9.2</td>
<td>[100, 800]</td>
<td>20</td>
<td>1997-2008</td>
</tr>
</tbody>
</table>

### 2.2 Tracking data

The main innovation of this work is the use of radar and GPS tracking data to estimate the thermospheric density using a ROM. For comparison we also used TLE data to estimate densities. An overview of the characteristics of the tracking data is provided in Table 2. In the following, the data is described in more detail.

#### 2.2.1 Radar tracking data

The radar tracking data used in this work was kindly provided to us by Leolabs (Nicolls et al., 2017; Griffith et al., 2019). Leolabs currently operates three radars, namely the Poker Flat Incoherent Scatter Radar (PFISR) located near Fairbanks, Alaska; the Midland Space Radar (MSR) located near Midland, Texas; and the Kiwi Space Radar (KSR) located in the Central Otago region of New Zealand. On July 22, 2020, Leolabs announced to construct a new phased-array radar in Costa Rica.

For calibration and uncertainty quantification, Leolabs makes use of data from the International Laser Ranging Service. Leolabs reports that the range uncertainties for the PFISR and MSR radars are typically near 15 meters and the doppler uncertainty is near 3 meters per second on PFISR, and 25 centimeters per second on MSR (Griffith et al., 2019). The uncertainty varies per observation and was provided with each measurement.

We used Leolabs’ calibrated range and range-rate measurements and uncertainty estimates (Nicolls et al., 2017) of 20 objects as measurements and measurement noise for estimation. We only used objects with known ballistic coefficients (BCs) to avoid errors due to BC uncertainties, see Table 3. For these objects we had radar measurements for...
the full month of January 2020. To compute measurement residuals for estimation, the
range $\rho$ and range rate $\dot{\rho}$ were computed from the predicted object position and veloc-
ity as follows (Vallado, 2013):

$$
\vec{\rho}_{ECI} = \vec{r}_{ECI} - \vec{r}_{SiteECI} \tag{12}
$$

$$
\rho = |\vec{\rho}_{ECI}| \tag{13}
$$

$$
\vec{\dot{\rho}}_{ECI} = \vec{v}_{ECI} - \vec{v}_{SiteECI} \tag{14}
$$

$$
\dot{\rho} = \frac{\vec{\rho}_{ECI} \cdot \vec{\dot{\rho}}_{ECI}}{\rho} \tag{15}
$$

where $\vec{r}_{ECI}$ and $\vec{v}_{ECI}$ are the position and velocity of the object in Earth-centered in-
ertial (ECI) coordinates and $\vec{r}_{SiteECI}$ and $\vec{v}_{SiteECI}$ the ECI position and velocity of the
radar at the measurement epoch.

### 2.2.2 GPS tracking data

Planet Labs owns Earth observation satellites named SkySat that are equipped with
GPS trackers (Foster et al., 2015, 2018). The SkySat GPS position measurements have
a good accuracy and temporal resolution. We used the GPS data of 10 Skysats (SkySat
C2-C4, C6 and C8-C13), which are Generation-C SkySats that were launched in Sun-
synchronous orbits with a mean local time of descending node of either 10:30am or 1:30pm,
see Table 4. The satellite buses have dimensions of 60 x 60 x 95 cm and weigh 110 kg
at launch (Longanbach & McGill, 2018).

For each Skysat, we have a GPS position measurement on average every 80 sec-
onds. The measurement error is unknown and therefore we tested 1-σ position errors of
20, 10, 5 and 2 m. Based on post-fit position residuals and orbit and density estimates
accuracy we selected a measurement error of 5 m in each direction. We did not consider
GPS receiver clock off-sets although they may be present in the data. More information
on the Planet Labs ephemeris can be found in Foster et al. (2015) and Foster et al. (2018)

It should be noted that SkySats are operational satellites with maneuvering capa-
bilities. Therefore, we only considered Skysats that were not maneuvering during the den-
sity estimation window. In addition, the BC of the SkySat satellite was unknown. To ob-
tain an initial BC estimate, we assumed that the satellites are nadir pointing (i.e. Earth
observing), such that the 60 x 90 cm surfaces are facing the atmospheric flow. Depend-
ing on the yaw angle, frontal area can then vary between 0.5700 and 0.8061 m$^2$. The drag
coefficient $C_d$ is also unknown, but is expected to be in the range of 2.2 to 2.8 for comp-
pact satellites (whose ratio of maximum to minimum diameter is less than 1.5) depend-
ing on the shape, temperature and accommodation coefficient (Moe et al., 1995). As ini-
tial guess for the ballistic coefficient we assume a mean area of 0.6881 m$^2$, a mass of 110
kg and a $C_d$ of 2.2, such that the BC is 0.01376 m$^2$/kg. In addition, we assume that all
Skysats have the same BC, although this may not be true as each satellite may have a
different attitude and fuel expenditure. Since the BC is an estimated parameter, the BC
of different satellites can take different values during density estimation.

### 2.2.3 Two-line element data

For comparison, we also performed density estimation using TLE data. For this,
the measurements are the osculating orbital states extracted from TLE data. At one hour
intervals the osculating state of each object is computed using the nearest newer TLE
by propagating the TLE backward to the measurement epoch using SGP4. These states
are then converted to MEE and used as measurements. The following eccentricity-dependent
measurement noise was used:

$$
[R_p, R_f, R_y, R_h, R_k, R_L] = [c_1 \cdot 10^{-8}, c_2 \cdot 10^{-10}, c_2 \cdot 10^{-10}, 10^{-9}, 10^{-9}, 10^{-8}] \tag{16}
$$
Table 2. Tracking data characteristics.

<table>
<thead>
<tr>
<th>Tracking data state</th>
<th>Measurement</th>
<th>Measurement</th>
<th>Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>TLE</td>
<td>MEE: p, f, g, h, k, L</td>
<td>See Eq. (27)</td>
<td>Every 1 hour</td>
</tr>
<tr>
<td>Radar</td>
<td>Range and range-rate: ρ, ˙ρ</td>
<td>σ_ρ ≈ 15 m (\dagger)</td>
<td>Two passes per day (on average)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>σ_\dot{\rho} ≈ 0.25 - 3 m/s (\dagger)</td>
<td>5-200 measurements per pass</td>
</tr>
<tr>
<td>GPS</td>
<td>Position: x, y, z</td>
<td>σ_x = σ_y = σ_z = 5 m</td>
<td>Every 80s</td>
</tr>
</tbody>
</table>

\(\dagger\) See Griffith et al. (2019)

Table 3. Radar-tracked objects used for density estimation in January 2020. The BC values were taken from Bowman et al. (2004), Emmert et al. (2004), Yurasov et al. (2005), Emmert et al. (2006) and Marcos et al. (2006).

<table>
<thead>
<tr>
<th>NORAD Catalog ID</th>
<th>Object</th>
<th>BC [m²/kg]</th>
<th>Perigee height [km]</th>
<th>Apogee height [km]</th>
<th>Inclination [deg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>Explorer 7</td>
<td>0.02927</td>
<td>486 - 486</td>
<td>680 - 680</td>
<td>50.3</td>
</tr>
<tr>
<td>614</td>
<td>Hitchhiker 1</td>
<td>0.01463</td>
<td>300 - 299</td>
<td>1096 - 1092</td>
<td>82.0</td>
</tr>
<tr>
<td>932</td>
<td>Explorer 25</td>
<td>0.02118</td>
<td>523 - 522</td>
<td>2285 - 2285</td>
<td>81.3</td>
</tr>
<tr>
<td>1807</td>
<td>Thor-Agena B R/B</td>
<td>0.0255</td>
<td>500 - 500</td>
<td>2648 - 2647</td>
<td>79.8</td>
</tr>
<tr>
<td>2153</td>
<td>Thor-Agena B DEB</td>
<td>0.03292</td>
<td>499 - 498</td>
<td>2563 - 2562</td>
<td>79.7</td>
</tr>
<tr>
<td>2389</td>
<td>OV3-3</td>
<td>0.01810</td>
<td>337 - 337</td>
<td>2230 - 2229</td>
<td>81.4</td>
</tr>
<tr>
<td>4221</td>
<td>Azur</td>
<td>0.02201</td>
<td>353 - 353</td>
<td>1242 - 1241</td>
<td>102.7</td>
</tr>
<tr>
<td>4382</td>
<td>DFH-1</td>
<td>0.01105</td>
<td>428 - 428</td>
<td>2032 - 2031</td>
<td>68.4</td>
</tr>
<tr>
<td>7337</td>
<td>Vektor</td>
<td>0.01120</td>
<td>373 - 372</td>
<td>1147 - 1146</td>
<td>82.9</td>
</tr>
<tr>
<td>8744</td>
<td>Vektor</td>
<td>0.01117</td>
<td>368 - 367</td>
<td>1187 - 1186</td>
<td>82.9</td>
</tr>
<tr>
<td>12138</td>
<td>Vektor</td>
<td>0.01115</td>
<td>388 - 387</td>
<td>1425 - 1424</td>
<td>83.0</td>
</tr>
<tr>
<td>12388</td>
<td>Vektor</td>
<td>0.01121</td>
<td>384 - 383</td>
<td>1338 - 1337</td>
<td>82.9</td>
</tr>
<tr>
<td>14483</td>
<td>Vektor</td>
<td>0.01130</td>
<td>383 - 382</td>
<td>1476 - 1476</td>
<td>82.9</td>
</tr>
<tr>
<td>20774</td>
<td>Vektor</td>
<td>0.01168</td>
<td>385 - 384</td>
<td>1587 - 1586</td>
<td>82.9</td>
</tr>
<tr>
<td>23278</td>
<td>Vektor</td>
<td>0.01168</td>
<td>393 - 392</td>
<td>1698 - 1698</td>
<td>83.0</td>
</tr>
<tr>
<td>41771</td>
<td>SkySat C4</td>
<td>0.01626</td>
<td>491 - 491</td>
<td>493 - 493</td>
<td>97.4</td>
</tr>
<tr>
<td>41772</td>
<td>SkySat C5</td>
<td>0.01626</td>
<td>490 - 490</td>
<td>493 - 493</td>
<td>97.4</td>
</tr>
<tr>
<td>41773</td>
<td>SkySat C2</td>
<td>0.01626</td>
<td>491 - 491</td>
<td>493 - 493</td>
<td>97.4</td>
</tr>
<tr>
<td>42989</td>
<td>SkySat C9</td>
<td>0.01626</td>
<td>448 - 447</td>
<td>452 - 451</td>
<td>97.2</td>
</tr>
<tr>
<td>43797</td>
<td>SkySat C12</td>
<td>0.01626</td>
<td>491 - 490</td>
<td>495 - 494</td>
<td>97.4</td>
</tr>
</tbody>
</table>

where \(c_1 = 1.5 \cdot \max(4e,0.0023)\) and \(c_2 = 3 \cdot \max(e/0.004,1)\). More details can be found in Gondelach and Linares (2020c).

2.2.4 Data preprocessing

Both for the radar and GPS measurements, it is paramount to remove any outliers from the measurement data before estimation, because outliers can result in inaccurate orbit and density estimates. Especially since we assume that the radar and GPS data are very accurate (i.e. the measurement noise is very small), the estimates will tend to follow the measurements closely and as a result an outlier can quickly cause erroneous results. Outliers can be detected using sliding window techniques or by comparing against other orbital data, such as TLE data, or against numerical orbit predictions. In this work, to remove outliers we first detrended the radar and GPS measurement data using TLE data and subsequently detected outliers in the detrended data using a sliding window approach.
Table 4. GPS-tracked Skysats used for density estimation in May 2020. The BC was estimated in this work.

<table>
<thead>
<tr>
<th>NORAD Catalog ID</th>
<th>Name</th>
<th>BC [m²/kg]</th>
<th>Perigee height [km]</th>
<th>Apogee height [km]</th>
<th>Inclination [deg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>41771</td>
<td>SkySat C4</td>
<td>0.01626</td>
<td>452</td>
<td>454</td>
<td>97.21</td>
</tr>
<tr>
<td>41773</td>
<td>SkySat C2</td>
<td>0.01626</td>
<td>451</td>
<td>454</td>
<td>97.21</td>
</tr>
<tr>
<td>42987</td>
<td>SkySat C11</td>
<td>0.01626</td>
<td>445</td>
<td>458</td>
<td>97.23</td>
</tr>
<tr>
<td>42988</td>
<td>SkySat C10</td>
<td>0.01626</td>
<td>448</td>
<td>455</td>
<td>97.24</td>
</tr>
<tr>
<td>42989</td>
<td>SkySat C9</td>
<td>0.01626</td>
<td>451</td>
<td>452</td>
<td>97.23</td>
</tr>
<tr>
<td>42990</td>
<td>SkySat C8</td>
<td>0.01626</td>
<td>449</td>
<td>453</td>
<td>97.24</td>
</tr>
<tr>
<td>42992</td>
<td>SkySat C6</td>
<td>0.01626</td>
<td>451</td>
<td>452</td>
<td>97.24</td>
</tr>
<tr>
<td>43797</td>
<td>SkySat C12</td>
<td>0.01626</td>
<td>452</td>
<td>454</td>
<td>97.21</td>
</tr>
<tr>
<td>43802</td>
<td>SkySat C13</td>
<td>0.01626</td>
<td>465</td>
<td>470</td>
<td>97.25</td>
</tr>
</tbody>
</table>

Table 5. SWARM A and B’s orbits in January and May 2020. SWARM density data is used for validation.

<table>
<thead>
<tr>
<th>Name</th>
<th>NORAD Catalog ID</th>
<th>Perigee height [km]</th>
<th>Apogee height [km]</th>
<th>Inclination [deg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>SWARM A</td>
<td>39452</td>
<td>428</td>
<td>432</td>
<td>87.4</td>
</tr>
<tr>
<td>SWARM B</td>
<td>39451</td>
<td>496</td>
<td>500</td>
<td>87.8</td>
</tr>
</tbody>
</table>

Figure 1. Flowchart of density estimation via tracking data assimilation.

2.3 Density estimation

The neutral mass density is estimated through the assimilation of the radar and GPS tracking data in the dynamic ROM model. This is achieved by simultaneously estimating the ROM state and the orbit and BC of objects using an unscented Kalman filter (UKF) and with the tracking data as measurements, see Figure 1. For this, we took a similar approach as for assimilating two-line element data (Gondelach & Linares, 2020c). The key differences with the TLE data assimilation are the type of measurements, measurement frequency, measurement noise and process noise. In addition, the orbits of the tracked objects are different and for the Skysats the ballistic coefficient was unknown. Details of the UKF estimation approach are described in the following.
2.3.1 Unscented Kalman filter

To fuse the model and measurement data, we use the square-root UKF. This filter uses an unscented transform (UT) to compute the mean and covariance of a nonlinearly propagated probability distribution by calculating the nonlinear transform of carefully selected sample points, called sigma points. The UKF is a popular algorithm that is well documented in literature; therefore, for details about the square-root UKF the reader is referred to Wan and Van Der Merwe (2001).

2.3.2 State

The state \( x \) that is estimated in the UKF consists of the osculating orbital states (expressed in modified equinoctial elements) and BCs of the tracked objects plus the reduced-order density state \( z \):

\[
\begin{bmatrix}
x = [p_1, f_1, g_1, h_1, k_1, L_1, BC_1, \ldots, p_n, f_n, g_n, h_n, k_n, L_n, BC_n, z^T]
\end{bmatrix}^T
\]

(17)

where \( n \) is the number of objects and the modified equinoctial elements (MEE) are defined as (Walker et al., 1985):

\[
p = a (1 - e^2), \quad f = e \cos (\omega + \Omega), \quad g = e \sin (\omega + \Omega),
\]

\[
h = \tan (i/2) \cos \Omega, \quad k = \tan (i/2) \sin \Omega, \quad L = \Omega + \omega + \nu.
\]

(18)

where \( a, e, i, \Omega, \omega \) and \( \nu \) are the classical Keplerian orbital elements. MEE are used because they are nonsingular and tend to behave less nonlinear than the Cartesian coordinates, which benefits the Kalman filter estimation by mitigating departure from “Gaussianity” of the propagated state probability density function (PDF) (see Ref. (Poore, 2015, Section 7.2)).

2.3.3 Dynamic model

The dynamic model \( f(x, t) \) for evolving the state \( x \) consists of propagating the orbital states using orbital dynamics and evolving the ROM state using the continuous-time DMDc model:

\[
\dot{x} = f(x, t) = \begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z} \\
\dot{v}_x \\
\dot{v}_y \\
\dot{v}_z \\
BC \\
\dot{z}
\end{bmatrix} = \begin{bmatrix}
v_x \\
v_y \\
v_z \\
\alpha_x \\
\alpha_y \\
\alpha_z \\
A \cdot z + B \cdot u
\end{bmatrix}
\]

(19)

where \([x, y, z], [v_x, v_y, v_z]\) and \([\alpha_x, \alpha_y, \alpha_z]\) are the inertial position, velocity and acceleration of the objects, respectively, and \(BC = \frac{C_D A}{m}\) is the ballistic coefficient. The ROM state \( z \) is used to compute the atmospheric density by converting \( z \) to the full space (see Eq. 5) and interpolating the density grid. The orbital dynamics are expressed in inertial Cartesian coordinates whereas the orbital states estimated in the UKF are expressed in MEE to retain “Gaussianity” of the state PDF. For sigma point propagation in the UKF, the sigma points and weights are first selected in MEE space. Each sigma point is then transformed to Cartesian space and propagated to the next epoch. Finally, each sigma point is transformed back to MEE space and the state and covariance are computed based on the MEE sigma points and weights.

2.3.4 Orbital dynamics

The orbital dynamics used in this work considers:
• Geopotential acceleration computed using the EGM2008 model, up to degree and order 48 for the harmonics;
• Solar radiation pressure with cylindrical shadow model;
• Third body perturbations by Sun and Moon;
• Atmospheric drag considering a rotating atmosphere for computing the velocity relative to the atmosphere. The atmospheric density is computed using the ROM density model.

The spherical harmonics used to model geopotential field was truncated at 48x48, because this provides acceptable accuracy for density estimation according to Bowman et al. (Bowman et al., 2004). Higher-order gravitational terms and solid Earth and ocean tides were not included, but can be considered in future work. For solar radiation pressure, we assumed a reflectivity coefficient of 1.2 and extracted the area-to-mass ratio from the object’s BC assuming a drag coefficient of 2.2. The orbits are propagated using Cartesian position and velocity in the inertial J2000 reference frame while the geopotential and drag accelerations are computed in the Earth-fixed ITRF93 frame. NASA’s SPICE toolbox is used both for Moon and Sun ephemerides (DE430 kernels) and for reference and drag accelerations are computed in the Earth-fixed ITRF93 frame. NASA’s SPICE manifold (Bowman et al., 2004). Higher-order gravitational terms and solid Earth and ocean
dermotions. For this, we assume that our modeling errors can be modeled as continuous

2.3.5 Process noise

In previous work on density estimation using two-line element data, we assumed a discrete 1-hour process noise variance \( Q \) for the orbital state (Gondelach & Linares, 2020c). Here, for radar and GPS-based estimation, the 1-hour process noise variance was slightly changed to:

\[
[Q_p, Q_f, Q_g, Q_h, Q_k, Q_L]_{1hr} = [1.5 \times 10^{-8}, 3.2 \times 10^{-13}, 3.2 \times 10^{-13}, 4.0 \times 10^{-14}, 4.0 \times 10^{-14}, 4.0 \times 10^{-14}]
\] (20)

In addition, in this work the time between measurement updates varies. Therefore, we use a discrete-time process noise that depends on the time interval \( \Delta t \) between two measurements. For this, we assume that our modeling errors can be modeled as continuous time zero-mean white noise on the acceleration. For a constant velocity particle, the position error variance then grows proportionally to the third power of time (\( \sim \Delta t^3 \)) and the velocity error variance grows linearly with time (\( \sim \Delta t \)) (Yaakov Bar-Shalom, 2002). Converting these variance grow rates to modified equinoctial elements, we find that the error variance growth for all elements is cubic with time (\( \sim \Delta t^3 \)). Therefore, we write:

\[
Q_\alpha = q_{\alpha} \frac{1}{3} \Delta t^3
\] (21)

where \( q \) is the power spectral density of the white noise and \( \alpha \in [p, f, g, h, k, L] \). Given the 1-hour process noise variance in Eq. (20), we get:

\[
[q_p, q_f, q_g, q_h, q_k, q_L] = [9.6 \times 10^{-19}, 2.0 \times 10^{-23}, 2.0 \times 10^{-23}, 2.6 \times 10^{-24}, 2.6 \times 10^{-24}, 2.6 \times 10^{-24}]
\] (22)

For GPS tracking data, the average interval between two measurements for one object is only 80s and the power spectral density \( q \) given in Eq. (22) was found to underestimate the actual errors. To obtain higher process noise, the 1-hour process noise variance was scaled linearly from 1 hour to 80 seconds:

\[
[Q_p, Q_f, Q_g, Q_h, Q_k, Q_L]_{80sec} = \frac{80}{3600} [Q_p, Q_f, Q_g, Q_h, Q_k, Q_L]_{1hr}
\] (23)

\[
= [3.3 \times 10^{-10}, 7.1 \times 10^{-15}, 7.1 \times 10^{-15}, 2.2 \times 10^{-16}, 2.2 \times 10^{-16}, 2.2 \times 10^{-16}]
\] (24)

Using this 80-seconds process noise variance, we obtain a power spectral density that is \((\frac{80}{3600})^2\) times higher than Eq. (22), which is used for GPS-based estimation:

\[
[q_p, q_f, q_g, q_h, q_k, q_L] = [2.0 \times 10^{-15}, 4.2 \times 10^{-20}, 4.2 \times 10^{-20}, 5.2 \times 10^{-21}, 5.2 \times 10^{-21}, 5.2 \times 10^{-21}]
\] (25)
The process noise for the ROM state $Q_z$ was computed using the 1-hour ROM prediction error on the training data (Gondelach & Linares, 2020c):

$$
\text{diag}(Q_z) = \text{diag} (\text{Cov}[Z_2 - (A_r Z_1 + B_r Y)])
$$

The values for the process noise reported here were found to give good estimated density results (compared with SWARM densities) and good estimates of the uncertainty in the estimated orbits and density.

### 2.3.6 Initial state and covariance

For the radar tracking case, we use a TLE-derived orbital state as initial guess. For TLE-derived states we assume a MEE covariance of (Gondelach & Linares, 2020c):

$$
[P_p, P_f, P_g, P_h, P_k, P_L]_{\text{TLE}} = [c_1 \cdot 10^{-8}, c_2 \cdot 10^{-10}, 10^{-9}, 10^{-9}, 10^{-8}]
$$

where $c_1 = 1.5 \cdot \max(4e, 0.0023)$ and $c_2 = 3 \cdot \max(e/0.004, 1)$.

In the GPS case, we use a GPS position and velocity measurement as initial state and assume a 1-σ position and velocity error of 40m and 0.2 m/s, respectively. This gives an initial covariance of:

$$
[P_x, P_y, P_z, P_v, P_v, P_v]_{\text{GPS}} = [0.0016, 0.0016, 0.0016, 4 \cdot 10^{-8}, 4 \cdot 10^{-8}, 4 \cdot 10^{-8}]
$$

The GPS position and velocity and corresponding covariance are then converted to MEE using an unscented transform.

For the ballistic coefficients we assume an initial 1-σ error of 1% for objects with known BC (Table 3) and an error of 2% for Skysats (Table 4) whose BC is estimated in the Results section. The ROM state is initialized using densities from the JB2008 model. For the first ROM mode, which has the largest effect on the density magnitude, the initial covariance is set to 20. For the other modes the covariance is set to 5.

### 2.4 Validation

To assess the accuracy of the estimated densities, we compare them with densities derived from Swarm GPS data by Van den IJssel et al. (van den IJssel et al., 2020). Van den IJssel et al. developed a precise orbit determination approach to estimate non-gravitational and aerodynamic-only accelerations from the high-quality Swarm GPS data using a high-fidelity satellite geometry model and improved aerodynamic and radiation pressure models. The derived aerodynamic accelerations were then converted directly into thermospheric densities for all Swarm satellites. The densities are available from the Swarm Data Access website (http://swarm-diss.eo-int).

In this work, we use the orbit-average density values derived by Van den IJssel et al., which are more reliable than the local SWARM densities (van den IJssel et al., 2020). The orbits of the Swarm A and B satellites during the density estimation windows can be found in Table 5. For comparison against SWARM densities, we consider the mean, standard deviation and root-mean-square of the orbit-average and daily-average density error. For this, we compute the difference in density and then divide by the SWARM density to obtain a relative density error $\epsilon$:

$$
\epsilon = \frac{\bar{\rho}_{est} - \bar{\rho}_{swarm}}{\bar{\rho}_{swarm}} \times 100\%
$$

where $\bar{\rho}_{est}$ and $\bar{\rho}_{swarm}$ indicate the orbit- or daily-average estimated and SWARM density, respectively. The mean $\mu$ and standard deviation $\sigma$ of a time series of relative density errors is computed to assess the accuracy of the estimated densities. The mean errors represent scale differences that can be caused by errors in both the estimates and SWARM data, e.g. due to different assumed drag coefficients. The $\sigma$ values are a better indicator of quality of the density estimates (Doornbos, 2012).
Figure 2. Orbit-averaged density along SWARM A’s orbit estimated using radar and TLE data and according to SWARM A data and JB2008 and NRLMSISE-00 models from January 3 to 28, 2020.

3 Results

3.1 Radar

Using the radar tracking data of 20 objects (see Table 3) we estimated the density from January 3 to 28, 2020. The radar density estimates are significantly more accurate than NRLMSISE-00 and JB2008 modelled densities with the orbit-average $\sigma$ reduced by 9 to 13%, see Figure 2 and Table 6. On the other hand, we found that the density estimates obtained using TLE data have a similar accuracy as those obtained using radar data. Both the radar and TLE-estimated density have a bias of only 1 to 3% and a daily-averaged $\sigma$ of 8 to 11%. The radar tracking data that we used is very accurate compared to TLE data. However, there can be long intervals between two passes over a radar (on average 12 hours), whereas in the TLE case we provide a TLE-derived measurement for each object every hour. The continuous availability of the TLE-derived measurements may compensate here for the lack of accuracy of the data. Finally, using the radar data of only 12 objects we obtained significantly less accurate density estimates with daily-average $\mu$ of 13% and $\sigma$ of 11 to 14% (not shown here). This suggests that using radar tracking data of more objects can further improve the density estimates (e.g. HASDM reportedly uses the tracking data of around 80-90 objects to calibrate the JB2008 model). These results show that using range and range-rate measurements we can obtain improved density estimates. In future work, using the tracking data of more objects over a longer period the performance compared to using TLE data can be further assessed.

3.2 GPS

We estimated the density using the GPS data of 5 Skysats (C2, C4, C10, C11 and C13) and assuming a BC of 0.01376 m$^2$/kg (with 2% uncertainty) from May 1 to 30, 2020. Figure 3 (left plot) shows that the resulting estimated density follows the SWARM density trend but with a bias. The standard deviation of the orbit-average density error is 12-13% lower than for JB2008 and NRLMSISE-00 densities, see Table 7 (BC = 0.01376). Also, the $\sigma$ is lower than the $\sigma$ of the radar-estimated density in January 2020; however, the bias of 19% with respect to the SWARM densities is much higher. This bias is expected to be due to the use of the assumed BC of 0.01376 m$^2$/kg for the Skysats.

To obtain an improved BC estimate for the Skysats, we ran 10-day density estimations starting with perturbed BC values and a high 1-$\sigma$ BC uncertainty of 20%. Based on the resulting BC and density estimates we determined a new BC estimate for all Skysats.
Table 6. Accuracy of radar and TLE estimated and modelled densities along SWARM A and B’s orbits in January 2020. The numbers show the mean $\mu$, standard deviation $\sigma$ and root-mean-square (RMS) of the error in orbit-averaged and daily-average density as percentage of true densities.

<table>
<thead>
<tr>
<th>Satellite</th>
<th>Model</th>
<th>Density error [%]</th>
<th>Orbit-average</th>
<th>Daily-average</th>
<th>Orbit-average</th>
<th>Daily-average</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\mu$</td>
<td>$\sigma$</td>
<td>RMS</td>
<td>$\mu$</td>
<td>$\sigma$</td>
<td>RMS</td>
</tr>
<tr>
<td>SWARM-A</td>
<td>NRLMSISE-00</td>
<td>80.5</td>
<td>24.1</td>
<td>84.0</td>
<td>79.8</td>
<td>21.8</td>
<td>82.6</td>
</tr>
<tr>
<td></td>
<td>JB2008</td>
<td>18.8</td>
<td>20.3</td>
<td>27.6</td>
<td>18.6</td>
<td>12.3</td>
<td>22.1</td>
</tr>
<tr>
<td></td>
<td>TLE-ROM-JB</td>
<td>2.3</td>
<td>11.3</td>
<td>11.5</td>
<td>2.1</td>
<td>8.6</td>
<td>8.7</td>
</tr>
<tr>
<td></td>
<td>Radar-ROM-JB</td>
<td>1.4</td>
<td>11.0</td>
<td>11.1</td>
<td>1.2</td>
<td>8.3</td>
<td>8.2</td>
</tr>
<tr>
<td>SWARM-B</td>
<td>NRLMSISE-00</td>
<td>85.3</td>
<td>25.2</td>
<td>88.9</td>
<td>84.8</td>
<td>18.7</td>
<td>86.7</td>
</tr>
<tr>
<td></td>
<td>JB2008</td>
<td>22.2</td>
<td>22.9</td>
<td>31.9</td>
<td>22.2</td>
<td>12.0</td>
<td>25.1</td>
</tr>
<tr>
<td></td>
<td>TLE-ROM-JB</td>
<td>-1.2</td>
<td>15.1</td>
<td>15.1</td>
<td>-1.6</td>
<td>9.8</td>
<td>9.7</td>
</tr>
<tr>
<td></td>
<td>Radar-ROM-JB</td>
<td>-2.2</td>
<td>14.9</td>
<td>15.1</td>
<td>-2.7</td>
<td>10.7</td>
<td>10.8</td>
</tr>
</tbody>
</table>

Figure 3. Orbit-averaged density along SWARM A’s orbit estimated using the GPS data of 5 Skysats with an assumed BC of 0.01376 (left) and a debiased BC of 0.01626 m$^2$/kg (right) and according to SWARM A data, and JB2008 and NRLMSISE-00 models from May 1 to 31, 2020.

Table 7. Accuracy of GPS estimated densities along SWARM A and B’s orbits in May 2020 using 5 Skysats with an assumed BC of 0.01376 and a debiased BC of 0.01626 m$^2$/kg. The numbers show the mean $\mu$ and standard deviation $\sigma$ of the error in orbit-averaged and daily-average density as percentage of true densities.

<table>
<thead>
<tr>
<th>Satellite</th>
<th>Model</th>
<th>BC [m$^2$/kg]</th>
<th>Density error [%]</th>
<th>Orbit-average</th>
<th>Daily-average</th>
<th>Orbit-average</th>
<th>Daily-average</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\mu$</td>
<td>$\sigma$</td>
<td>$\mu$</td>
<td>$\sigma$</td>
<td>$\mu$</td>
<td>$\sigma$</td>
<td>RMS</td>
</tr>
<tr>
<td>SWARM-A</td>
<td>NRLMSISE-00</td>
<td>-</td>
<td>118.3</td>
<td>23.8</td>
<td>117.6</td>
<td>20.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>JB2008</td>
<td>-</td>
<td>39.5</td>
<td>22.5</td>
<td>39.2</td>
<td>13.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>GPS-ROM-JB</td>
<td>0.01376</td>
<td>18.7</td>
<td>10.4</td>
<td>18.4</td>
<td>5.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.01626</td>
<td>2.7</td>
<td>8.9</td>
<td>2.5</td>
<td>4.8</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 4. BC estimate for 5 Skysats during estimation using GPS data and ROM-JB model from May 1 to 10, 2020 starting with perturbed BCs and $\sigma_{BC} = 20\%$ (left) and starting with new debiased BC and $\sigma_{BC} = 2\%$ (right).

Table 8. Debiased BC estimates computed by correcting the median estimated BC for the density bias w.r.t. SWARM densities for six density estimation runs. Reported BC values are in m$^2$/kg. Last column shows estimated bias in the initially assumed BC of 0.01376 m$^2$/kg.

<table>
<thead>
<tr>
<th>ROM model</th>
<th>Dates</th>
<th>Median estimated BC</th>
<th>Density bias [%]</th>
<th>Debiased BC</th>
<th>BC bias [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>SWARM A</td>
<td>SWARM B</td>
<td></td>
</tr>
<tr>
<td>GPS-ROM-MS</td>
<td>1-10 May</td>
<td>0.01335</td>
<td>29.1</td>
<td>31.5</td>
<td>0.01739</td>
</tr>
<tr>
<td>GPS-ROM-MS</td>
<td>11-20 May</td>
<td>0.01154</td>
<td>51.4</td>
<td>48.5</td>
<td>0.01730</td>
</tr>
<tr>
<td>GPS-ROM-MS</td>
<td>21-30 May</td>
<td>0.01174</td>
<td>55.5</td>
<td>45.2</td>
<td>0.01765</td>
</tr>
<tr>
<td>GPS-ROM-JB</td>
<td>1-10 May</td>
<td>0.01219</td>
<td>25.4</td>
<td>29.6</td>
<td>0.01555</td>
</tr>
<tr>
<td>GPS-ROM-JB</td>
<td>11-20 May</td>
<td>0.01216</td>
<td>27.3</td>
<td>32.4</td>
<td>0.01579</td>
</tr>
<tr>
<td>GPS-ROM-JB</td>
<td>21-30 May</td>
<td>0.01173</td>
<td>39.5</td>
<td>40.8</td>
<td>0.01645</td>
</tr>
</tbody>
</table>

We considered five Skysats and applied both the ROM-JB and ROM-MS models to prevent a model-dependent bias in our new BC estimate. Figure 4 (left plot) shows the variation of estimated BCs for 5 Skysats from May 1 to 10 using the ROM-JB model. The initial BC values have a large spread from 0.0097 to 0.0168 m$^2$/kg. After several days the BC range reduces to 0.011 to 0.013 m$^2$/kg and BCs for objects in similar objects converge (41771 and 41773 are in near-identical orbits and 42987 and 42988 are in close orbits, see Table 4). Figure 4 shows the median estimated BC for the five Skysats and the mean error in the density estimates for six 10-day estimation runs. Overall, the average bias in the estimated density is 38% which is close to the bias in the JB2008 density (see Table 7), which was used to initialize the ROM density state. We computed debiased BC estimates by correcting the median BC estimates for the density biases. The debiased BCs are between 13% and 28% higher than the assumed BC of 0.01376 m$^2$/kg. Based on the results, we decided to increase the assumed drag coefficient by 18% from 2.2 to 2.6 to obtain a new BC estimate of 0.01626 m$^2$/kg. (For simplicity we increased the assumed $C_d$ but the BC may also be higher than initially assumed because we neglected protrusions, such as the aperture cover that extends out, or due to fuel expenditure.) In addition, we use a 1-$\sigma$ uncertainty of 2% of the new BC. Using these new values, the estimated BC still changes during estimation, but only by 1 or 2 percent, see right plot in Figure 4. The density estimation results using 5 Skysats and the new BC are shown in Figure 3 (right plot) and Table 7 (BC = 0.01626). The bias in the density reduced from 19% to 3%. In addition, the $\sigma$ improved a little as the orbit-average $\sigma$ decreased by 1.5% and the daily-average $\sigma$ by 0.9%. So changing the BC mainly affects the bias in the estimated density.
Table 9. Accuracy of GPS and TLE estimated densities along SWARM A and B’s orbits in May 2020 using 10 Skysats. The numbers show the mean $\mu$, standard deviation $\sigma$ and root mean square (RMS) of the error in orbit-average and daily-average density as percentage of true densities.

<table>
<thead>
<tr>
<th>Satellite</th>
<th>Model</th>
<th>Density error [%]</th>
<th>Orbit-average</th>
<th>Daily-average</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\mu$</td>
<td>$\sigma$</td>
<td>RMS</td>
</tr>
<tr>
<td>SWARM-A</td>
<td>NRLMSISE-00</td>
<td>118.2</td>
<td>23.9</td>
<td>120.6</td>
</tr>
<tr>
<td></td>
<td>JB2008</td>
<td>39.5</td>
<td>22.5</td>
<td>45.4</td>
</tr>
<tr>
<td></td>
<td>TLE - ROM-JB2008</td>
<td>-4.2</td>
<td>13.2</td>
<td>13.8</td>
</tr>
<tr>
<td></td>
<td>GPS - ROM-JB2008</td>
<td>-1.6</td>
<td>8.3</td>
<td>8.4</td>
</tr>
<tr>
<td></td>
<td>GPS - ROM-MSISE</td>
<td>7.8</td>
<td>8.4</td>
<td>11.4</td>
</tr>
<tr>
<td>SWARM-B</td>
<td>NRLMSISE-00</td>
<td>135.0</td>
<td>32.7</td>
<td>138.9</td>
</tr>
<tr>
<td></td>
<td>JB2008</td>
<td>44.1</td>
<td>25.4</td>
<td>50.9</td>
</tr>
<tr>
<td></td>
<td>TLE - ROM-JB2008</td>
<td>-4.9</td>
<td>19.5</td>
<td>20.0</td>
</tr>
<tr>
<td></td>
<td>GPS - ROM-JB2008</td>
<td>-2.7</td>
<td>11.6</td>
<td>11.9</td>
</tr>
<tr>
<td></td>
<td>GPS - ROM-MSISE</td>
<td>2.8</td>
<td>10.8</td>
<td>11.2</td>
</tr>
</tbody>
</table>

Using the new Skysat BC estimate, we also performed density estimation using the GPS data of 10 Skysats from May 1 to 30, 2020 using both the ROM-JB and ROM-MS models. The resulting estimated density closely follows the SWARM density data trend, see Figure 5, and the bias is less than 3%, except for the ROM-MS estimated density along SWARM A’s orbit that has a bias of 8%. The density estimated using the ROM-JB model shows fluctuations that follow the JB2008 density variations and therefore likely stem from the ROM-JB model dynamics.

The use of the GPS data of 10 instead of 5 Skysats provides more accurate densities (compare Tables 7 and 9). Using the ROM-JB and ROM-MS model, we obtain densities with orbit-average 1-$\sigma$ errors between 8 and 12%, which is 2 to 3 times smaller than for JB2008 and NRLMSISE-00 densities, see Table 9. Considering daily-averaged densities, the 1-$\sigma$ error is about 5%. This is a significant improvement with respect to the daily 1-$\sigma$ error of 22%, 14% and 10% for NRLMSISE-00, JB2008 and TLE-estimated densities, respectively, see Figure 6.

3.2.1 Minor storm

In the first half year of 2020, the highest geomagnetic activity was observed on April 20th when the Kp and Ap indices reached 5- and 39, respectively, i.e. a minor geomagnetic storm. As a result of the storm, within half a day the orbit-average density along SWARM A and B’s orbits increased by more than 100%. To assess the performance of our estimation technique during such an event, we estimated the density from April 17 to 24 using the GPS data of 10 Skysats. (Here used Skysats C5 and C7 instead of C6 and C10 for estimation due to maneuvers.) Figure 7 and Table 10 show that the ROM-JB model was able to estimate the density during the minor storm well as we obtain similar accuracies as in May 2020 (except for the bias in SWARM-B density). On the other hand, using the ROM-MS model there is delay in the density peak compared to the true SWARM density. The ROM-JB outperforms the ROM-MS due to its better predictive capabilities as a result of the superior space weather proxies used by the ROM-JB model.

4 Discussion

The presented results show that radar and GPS tracking data can be used to accurately estimate thermospheric densities. The global character of the density estimates...
Figure 5. Orbit-averaged density along SWARM A and B’s orbits estimated using the GPS and TLE data of 10 Skysats with a debiased BC of 0.01626 m$^2$/kg and using the ROM-JB or ROM-MS model, plus densities according to SWARM A and B data from May 1 to 31, 2020.

Figure 6. 1-σ error in daily-averaged density as percentage of true density along SWARM A and B’s orbits estimated using the GPS and TLE data of 10 Skysats and according to JB2008 and NRLMSISE-00 models from May 1 to 31, 2020.
Figure 7. Orbit-averaged density along SWARM A’s orbit estimated using the GPS data of 10 Skysats with a BC of 0.01626 m$^2$/kg and according to SWARM A data, and JB2008 and NRLMSISE-00 models during minor storm from April 17 to 24, 2020. Right plot shows daily $F_{10.7}$ and 3-hourly Ap.

Table 10. Accuracy of GPS estimated densities along SWARM A and B’s orbits during minor storm in April 2020 using 10 Skysats. The numbers show the mean $\mu$, standard deviation $\sigma$ and root mean square (RMS) of the error in orbit-average and daily-average density as percentage of true densities.

<table>
<thead>
<tr>
<th>Satellite</th>
<th>Model</th>
<th>Density error [%]</th>
<th>Orbit-average</th>
<th>Daily-average</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\mu$  $\sigma$  $\text{RMS}$</td>
<td>$\mu$  $\sigma$  $\text{RMS}$</td>
<td></td>
</tr>
<tr>
<td>SWARM-A</td>
<td>NRLMSISE-00</td>
<td>118.3  23.8  120.7</td>
<td>117.6  20.5  119.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>JB2008</td>
<td>39.5  22.5  45.4</td>
<td>39.2  13.6  41.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>GPS-ROM-MS</td>
<td>6.8  13.1  14.7</td>
<td>5.9  11.4  12.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>GPS-ROM-JB</td>
<td>-1.1  10.0  10.0</td>
<td>-1.4  3.9  3.9</td>
<td></td>
</tr>
<tr>
<td>SWARM-B</td>
<td>NRLMSISE-00</td>
<td>134.6  31.7  138.3</td>
<td>133.0  23.4  134.9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>JB2008</td>
<td>44.1  25.3  50.8</td>
<td>43.3  14.7  45.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>GPS-ROM-MS</td>
<td>6.4  13.8  15.2</td>
<td>5.7  11.1  11.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>GPS-ROM-JB</td>
<td>-8.2  11.7  14.2</td>
<td>-8.1  3.8  8.8</td>
<td></td>
</tr>
</tbody>
</table>

is indicated by the fact that we compared the estimates with densities from SWARM A and B which have orbits that differ from the orbits of the tracked objects. The most accurate densities were obtained using GPS data, whereas we obtained similar accuracies using radar and TLE data. Further studies need to be carried out to determine the achievable accuracy using radar data.

The accuracy of the density estimates are determined by both the accuracy of the ROM model predictions and the accuracy of the measurement data. The ROM-JB model has better predictive capabilities than the ROM-MS model, therefore the densities estimates using ROM-JB are generally more accurate. The ROM-JB model however also tends to follow the JB2008 modelled density, on which it was trained, which causes variations in the ROM-JB estimated density that are not visible in the true density but are visible in the JB2008 density, see e.g. Figure 2 and 7. On the other hand, the ROM-MS estimated densities show a small time lag with respect to the true density, see Figure 5. Wright (Wright, 2003) suggested that this could be because drag accelerations have a delayed effect on position, because the acceleration needs to be integrated over time twice to obtain the change in position. The accuracy of ROMs can be enhanced by develop-
ing a nonlinear dynamical model to replace the linear dynamics and by using more ac-

Notably, only 10 and 20 objects were used for density estimation. Moreover, the
tracked objects were commercial satellites or inactive payloads and debris and no ded-
icated measurements from missions such as CHAMP, GRACE and SWARM were used.
Furthermore, the densities were estimated using a cannonball model for the drag and SRP
perturbations. In future work, the use of attitude information and satellite geometry mod-
els can improve the drag and SRP modelling and result in more accurate density esti-
mates. Alternatively, the drag coefficient and reflectivity coefficient can be estimated sep-
parated, as currently only the BC is an estimated parameter. In addition, considering pertu-
bations due to higher-order gravity terms and solid and ocean tides, which were ne-
glected in this work, could improve the densities.

In this work, the densities were estimated in January and May 2020 during deep
solar minimum. In previous work using two-line element data we found more accurate
density estimates during high solar activity. This suggests that also using radar and GPS
data the relative density error may improve when the solar activity increases. However,
this needs to be tested when tracking data during active solar conditions are available.
Compared with Mehta et al. (Mehta & Linares, 2020) who used simulated GPS measure-
ments, we obtained less accurate density estimates. This was expected as Mehta et al.
assumed that all dynamics models were exactly known except for the density and the
constant ballistic coefficients, whereas here we have errors in our dynamics and the bal-
listic coefficients are not constant. In addition, the coverage of orbits is important. For
the GPS case, all Skysats were located in just two different orbital planes and all at sim-
ilar altitude. Using objects in more diverse objects will help improve the global density
estimate.

Finally, the use of radar and GPS tracking data for density estimation has certain
benefits and drawbacks. Radar data is commercially available and a large number of ob-
jects can be tracked using a network of sensors. On the other hand, the number and fre-
cquency of the measurements depend on the radar network. Due to geometry, radar mea-
surements may not be available for an object for one day or longer. Therefore, to ob-
tain density estimates with a good accuracy and temporal resolution, the use of radar
tracking data of many objects is preferred. GPS tracking data can be obtained continu-
ously. However, to acquire the GPS data, one requires an active satellite equipped with
a GPS tracker. Not all satellites have GPS and this data may only be available to op-
erators and not to the public. Moreover, for active satellites, maneuvers and attitude changes
need to be modelled or the maneuvering satellite needs to be excluded temporarily for
 estimation.

5 Conclusions

In this paper we estimated the global thermospheric density through data assim-
ilation of radar and GPS tracking data of satellites into a reduced-order density model.
The resulting radar- and GPS-estimated densities were found to be significantly more
accurate than JB2008 and NRLMSISE-00 modelled densities and equally or more ac-
curate than TLE-estimated densities. In particular, using GPS data we obtained accu-
rate densities that have a daily 1-σ error of only 5% compared to 22%, 14% and 10% for
NRLMSISE-00, JB2008 and TLE-estimated densities, respectively. Moreover, these re-
results were obtained without knowledge of the tracked satellites’ attitude and precise ge-
omeetry. In future work, the ballistic coefficient and solar radiation pressure models can
be improved to obtain more accurate density estimates. In addition, the use of track-
ing data of more objects will be explored to improve the estimated densities. Finally, the
use of real-time measurements would enable real-time density estimation.
6 Acknowledgments

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References


