Multifractal cross-correlations between green bonds and financial assets

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Abstract

We analyze multifractality for green bonds, stock sector indices, and US economic sector bonds. Green bonds and US bonds show non-linear cross-correlations. We perform Multifractal Detrended Cross-Correlations Analysis (MF-DCCA) to analyze multifractal cross-correlations and the weak version of the Efficient Market Hypothesis (EMH). Our findings are relevant to academics, financial professionals and the general public. Although green bonds are bonds used exclusively to finance sustainable investments, they are still inefficient assets. We find that bond indices for consumer staples and equity indices for information technology and the real state sector can be used to hedge investments in green bonds.

Keywords: COVID-19, Cryptocurrencies, Volatility, Multifractality, Cross-correlation, Complexity
1. Introduction

This article analyzes the multifractal cross-correlation of green bonds and US bond and equity indices. Understanding the interconnection between the green bond market and economic sectors is essential due to several factors. First, it allows the development of investment portfolio-building strategies. Second, it is possible to establish different ways of hedging operations to mitigate risks in the asset market - equity and bonds. Finally, green bonds are especially relevant as they allow the financing of more sustainable operations that are environmentally friendly.

A more detailed analysis of the interrelationship between green bonds and economic sectors helps better understand this market that has developed rapidly in recent years. We evaluate the cross-correlations and the multifractal spectrum behind these interrelationships.

Several articles show that green bonds have exciting hedging properties, enabling risk diversification for investors (Jiang et al. (2022), Arif et al. (2022), Han & Li (2022)) and enhancing the green economic recovery (Zhao et al. (2022), Teti et al. (2022)).

The literature on green bonds shows that there are important connectedness between green bonds and stock market and the bond market. The main findings suggest a stronger connection between green bonds and the bond market (due to exposure to common factors) and a smaller connectivity to the stock market (Reboreda & Ugolini (2020), Reboreda et al. (2020), Reboreda (2018), Mensi et al. (2021), Naeem et al. (2021), Zhuang & Wei (2022), and Naeem et al. (2022)).

Our contribution to the literature is fourfold. First, we show that green bonds, sectoral stock market and bond indices have multifractal properties. Second, we provide evidence of multifractality for pairs of assets using the green bond index as a benchmark. Third, we provide evidence of the rejection of the weak-form efficiency and acceptance of the fractal market hypothesis. Fourth, we also provide evidence that some pairs of assets (including green bonds) can be used in hedging strategies.

The remainder of this paper is organized as follows. Section 2, describes the data and the methodology used in this letter. Section 3 presents our empirical results. Finally, Section 4 formalizes our concluding remarks.

2. Data and methodology

2.1. Data

We have investigated the financial time series of the daily Green bonds price index and US S&P sector bond and equity indices. Our study considers a representative quantity of 23 indices. To the best of your knowledge, it is the first research that considers a representative record of Green bonds.

1See Bariviera (2021), Mensi et al. (2019).

2A recent paper finds that the pandemic of the Covid-19 weakened the hedging property of green bonds (Guo & Zhou (2021)) whereas Pham & Nguyen (2021) finds that the benefits of green bonds as hedging instruments against traditional asset classes vary between severe and typical market circumstances.
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For all datasets, the periods cover more than 13 years, from November 28, 2008, until August 16, 2022, with 3575 observations. We have collected the data at https://www.spglobal.com/en/. Table 1 lists the details of financial assets.

We apply the Box plot to explore the outliers for these Green bonds from November 28, 2008, until August 16, 2022. Fig. 1 shows the Box plot.
Figure 1: The Box plot is a classic non-parametric statistical method that lets us look at the location of these financial records by looking at 50% of the most likely values, the median, and the outliers. The black dots show that there are outliers in the data.

For each financial time series of daily Green bonds price index, US bond and equity indices, we calculate the logarithmic change in price (Return time series) by

\[ R_i(t) \equiv \ln P_i(t + \Delta t) - \ln P_i(t) \]  

where \( \Delta t \) represents one day, \( P_i(t) \) is the daily closing price \( i \) at time \( t \).
2.2. Methodology

2.2.1. Multifractal Detrended Cross-Correlation Analysis (MF-DCCA)

The MF-DCCA proposed by Zhou et al. (2008) is a hybrid model between MF-DFA (Kantelhardt et al. 2002) and DCCA originally proposed by Podobnik & Stanley (2008). It is designated to quantify long term correlations between two simultaneously recorded non-stationary time series. The MF-DCCA method is implemented following these steps:

(i) Consider two time series denoted by \( \{a_i, b_i, i = 1, 2, ..., N\} \), being \( N \) the length of time series. Determine the profile as:

\[
A_t = \sum_{k=1}^{t} (a_k - \bar{a}), \quad t = 1, 2, ..., N
\]

\[
B_t = \sum_{k=1}^{t} (b_k - \bar{b}), \quad t = 1, 2, ..., N
\]

\( \bar{a} \) and \( \bar{b} \) are the average of \( a_t \) and \( b_t \).

(ii) Divide \( A \) and \( B \) into \( N_s \equiv \lfloor N/s \rfloor \) non-overlapping segments of same length \( s \) (time scale). Thus, \( 2N_s \) segments are obtained.

(iii) For each sub-segment \( v \), apply least squares (LS) method to obtain the local trends using a \( k \)-th order polynomial fit.

\[
a_v(i) = c_1 i^k + ... + c_2 i^{k-1} + ... + c_k i + c_{k-1}, \quad i = 1, 2, ..., S; \quad k = 1, 2, ...
\]

\[
b_v(i) = d_1 i^k + ... + d_2 i^{k-1} + ... + d_k i + d_{k-1}, \quad i = 1, 2, ..., S; \quad k = 1, 2, ...
\]

(iv) Calculate the detrended covariance \( F^2(s, v) \). When \( v = 1, 2, ..., N_s \),

\[
F^2(s, v) = \frac{1}{s} \sum_{i=1}^{s} \{|X[(v-1)s+i]|Y[(v-1)s+i] - y_v(i)|\} \quad (6)
\]

When \( v = N_s + 1, N_s + 2, ..., 2N_s \),

\[
F^2(s, v) = \frac{1}{s} \sum_{i=1}^{s} \{|A[N-(v-N_s)s+i] - x_v(i)|B[N-(v-N_s)s+i] - y_v(i)|\} \quad (7)
\]

(v) Average the detrended covariances to obtain the \( q \)-th order wave function as

\[
F_q(s) = \left\{ \frac{1}{2N_s} \sum_{v=1}^{2N_s} [F^2(s, v)]^{\frac{q}{2}} \right\}^{\frac{1}{q}} \quad (8)
\]

(vi) When \( q = 0 \), it reads

\[
F_q(s) = \exp \left( \frac{1}{2N_s} \sum_{v=1}^{2N_s} \ln [F^2(s, v)] \right) \quad (9)
\]
The Power-Law correlations satisfy $F_q(s) \propto s^{h_{xy}(q)}$, where $h_{xy}(q)$ stands for the Generalized Hurst exponent versus $q$. We calculate the range of $h_{xy}(q)$ to derive the degree of multifractality. A larger $\Delta H_{xy} = h_{xy}(q_{\text{min}}) - h_{xy}(q_{\text{max}})$ implies a higher multifractal feature.

In the case of $q = 2$ the MF-DCCA turns into the DCCA. In such case, if $h_{xy}(2) = 0.5$, the two time series exhibit no cross-correlations. However, when $h_{xy}(2) > 0.5$, the cross-correlations are positive (or persistent - possess long memory), and when $h_{xy}(2) < 0.5$, there is anti-persistence in the cross-correlations.

We define the mass exponent spectrum as:

$$\tau_{xy}(q) = qh_{xy}(q) - 1$$

where $h_{xy}(q)$ is computed from MF-DCCA. The singularity strength $\alpha_{xy}$, which displays the singular degree of each segment in a complex system; and the singularity spectrum $f_{xy}(\alpha)$, which reveals the fractal dimension of $\alpha_{xy}$:

$$\alpha = h_{xy}(q) + qh'_{xy}(q)$$

$$f_{xy}(\alpha) = q[\alpha_{xy} - h_{xy}(q)] + 1$$

The spectrum of the singularity strength $\Delta \alpha_{xy} = \alpha_{xymax} - \alpha_{xmin}$ specifies the degree of multifractality. Thus, a larger $\alpha_{xy}$ reveals a heightened fluctuations.

In order to identify the multifractal spectrum $f(\alpha)$ quantitatively, It is also simple to compute the spectrum’s width $W(\alpha_{xymax} - \alpha_{xmin})$ derived by fitting a curve to zero, and the skew parameter $r = (\alpha_{xymax} - \alpha_0)/(\alpha_{xmin} - \alpha_0)$, where $\alpha_0$ represents the overall Hurst exponent. The term $r = 1$ stands for symmetric shapes, $r > 1$ for right-skewed shapes, and $r < 1$ for left-skewed shapes. If the spectrum is right-skewed (depending on the $r$), the scaling behavior of tiny fluctuations dominates the multifractal behavior, but if the spectrum is left-skewed, the scaling behavior of big fluctuations prevails.

Also, we use the multifractal risk cross-correlation (MRCC) measure Fernandes et al. (2022f) to investigate the relation between the $W_{xy}$ (width of the spectrum) and the $\alpha_{xy}(0)$ (persistence or anti-persistence). We can calculate The MRCC as:

$$\Gamma = \frac{W_{xy}}{\Delta \alpha_{xy}(0)}$$

The greater MRCC value indicates that the relationship between the two studied time series is more intricate and enduring. The lowest MRCC score suggests less complexity and less persistence when comparing these two time series. The MRCC is an appropriate method for examining the cross-correlation between two underlying time series.

3. Empirical Results

Return is a traditional financial measure that indicates gains (positive returns) and losses (negative returns) (Fernandes et al., 2020b). Figure 2 displays the return time series of Green bonds and US bond from November 28, 2008, until August 16, 2022, with 3575 observations.
Figure 2: The return time series suffer fluctuations due to endogenous and exogenous variables—these fluctuations impact people’s financial dynamics and the economy.

Using the MF-DCCA method, we evaluate the multifractal cross-correlations between Green bonds and US bond and equities indexes. In addition, we employ a shuffling process that performed $1000 \times N$ transpositions on each series and was repeated 1000 times using distinct random number generator seeds.

In this way, we perform a fourth-order polynomial regression on the singularity spectrum $f(\alpha)$ to obtain the position of $\alpha_{xy}(0)$ and the zeros of the polynomial, $\alpha_{xy}^{max}$ and $\alpha_{xy}^{min}$. These parameters are employed to calculate the complexity parameters. More specifically, the width of spectrum $W_{xy}$, the asymmetry parameter $r_{xy}$, $h(2)_{xy}$ and $\Gamma$. Fig. 3 demonstrates multifractal spectrum charts for the original and shuffled return time series pairs. Table 3 presents complexity parameters’ values.
For all return time series pairs, considering $q = 2$, the majority values inherent to $h_{xy}(2)$ are greater than 0.5 encompassing the original and shuffled time series exclude the original pairs for SP_Green_Bond vs b_const_staples, SP_Green_Bond vs eq_info_tech, and SP_Green_Bond vs eq_real_state the values of $h_{xy}(2) \rightarrow 0.5$. It indicates that these pairs with hedging strategy.

For all return time series pairs fixed SP_Green_Bond, we emphasize that the greater value of $\Delta H_{xy}$ implies in stronger multifractal features. Given this, the pairs SP_Green_Bond vs b_info_tech ($\Delta H_{xy} = 0.60494$) present stronger multifractal feature than other pairs.

Bearing in mind the values exhibited by $\alpha_{xy}(0)$ our empirical results suggest a common

\[
\alpha_{xy}(0)
\]

\[
\Delta h_{xy}
\]
behaviour considering this complexity parameters. We find that all pairs are characterized by long-term memory (correlated) or persistent behaviour \( \alpha_{xy}(0) > 0.5 \). It means that our results are align with the Fractal Market Hypothesis (FMH) and categorically discard the weak form of the Efficiency Market Hypothesis in Fama’s sense.

The examine of the values of \( W_{xy} \) allows us to check that the pairs SP_Green_Bond vs b_telec_svs (\( W_{xy} = 0.790244 \)) presents the greater values of \( W_{xy} \). Looking into the values exhibited by \( r_{xy} \), we observe that the majority of these pairs are marked by \( (r_{xy} < 1) \). It reveals that the multifractality of these return time series pairs is primarily driven by the scaling behavior of big fluctuations and excludes the SP_Green_Bond vs eq_industrials \( (r_{xy} = 1.034075) \), SP_Green_Bond vs eq_info_tech \( (r_{xy} = 1.135435) \) and SP_Green_Bond vs eq_materials \( (r_{xy} = 1.096367) \).

The MRCC values indicate that the pairs SP_Green_Bond vs eq_industrials \( (\Gamma = 1.223373) \) is more complex and persistent than the other pairs. Otherwise, the pairs SP_Green_Bond vs eq_financials \( (\Gamma = 0.929883) \) are less complex and persistent.

### 4. Conclusion

We have investigated the non-linear cross-correlations between Green bonds, stock sector indices, and US economic sector bonds. For each record, we calculate the return time series and apply the Multifractal Detrended Cross-Correlations Analysis (MF-DCCA) to examine the multifractal cross-correlations and the weak form of the Efficient Market Hypothesis (EMH) simultaneously, considering the pairs of these financial records.

In this way, we use a fourth-order polynomial regression on the singularity spectrum \( f(\alpha) \) to obtain the position of \( \alpha_{xy}(0) \) and the zeros of the polynomial, \( \alpha_{xy,max} \) and \( \alpha_{xy,min} \), which allow calculating the complexity parameters.

Our empirical findings reveal that the pairs formed by fixing the SP_Green_Bond and the other records still need to reach a certain maturity to become efficient financial assets,
that is, in compliance with the weak form in Fama’s sense. Also, based on the values inherent to the complexity parameter \( r_{xy} \), we find a common behaviour considering the pairs of these records. All pairs are aligned with the information asymmetries theory Greenwald & Stiglitz (1990). We observe that the majority of these pairs are characterized by \( r_{xy} < 1 \). Thus, it indicates that the multifractality of these return time series pairs is primarily impacted by the scaling behavior of big fluctuations, excluding the SP_Green_Bond vs eq_industrials \( (r_{xy} = 1.034075) \), SP_Green_Bond vs eq_info_tech \( (r_{xy} = 1.135435) \) and SP_Green_Bond vs eq_materials \( (r_{xy} = 1.096367) \).

Our findings are relevant not only to academics and financial professionals but to the general public, given the dramatic increase in environmental issues, sustainability and social impact. Although Green bonds are bonds used exclusively to finance sustainable investments, they are still inefficient assets. However, they are a long-term investment alternative that effectively contributes to the planet’s future, preserving the environment and encouraging sustainable development.

Other articles by the authors, see Fernandes et al. (2022d,b,g,h,a,e,c); Araujo & Fernandes (2022); Fernandes et al. (2021d); De Araujo et al. (2021); Fernandes et al. (2021e); De Araujo & Fernandes (2021); Fernandes et al. (2021c,a,b); de Araujo et al. (2020); Fernandes et al. (2020a); de Araujo et al. (2019)

5. Declaration of Competing Interest

The authors declare that this work has no conflicting personal or financial influences.

References


