Sparse Representations of Random Signals

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Abstract

Sparse (fast) representations of deterministic signals have been well studied. Among other types there exists one called adaptive Fourier decomposition (AFD) for functions in analytic Hardy spaces. Through the Hardy space decomposition of the L^2 space the AFD algorithm also gives rise to sparse representations of signals of finite energy. To deal with multivariate signals the general Hilbert space context comes into play. The multivariate counterpart of AFD in general Hilbert spaces with a dictionary has been named pre-orthogonal AFD (POAFD). In the present study we generalize AFD and POAFD to random analytic signals through formulating stochastic analytic Hardy spaces and stochastic Hilbert spaces. To analyze random analytic signals we work on two models, both being called stochastic AFD, or SAFD in brief. The two models are respectively made for (i) those expressible as the sum of a deterministic signal and an error term (SAFDI); and for (ii) those from different sources obeying certain distributive law (SAFDII). In the later part of the paper we drop off the analyticity assumption and generalize the SAFDI and SAFDII to what we call stochastic Hilbert spaces with a dictionary. The generalized methods are named as stochastic pre-orthogonal adaptive Fourier decompositions, SPOAFDI and SPOAFDII. Like AFDs and POAFDs for deterministic signals, the developed stochastic POAFD algorithms offer powerful tools to approximate and thus to analyze random signals.

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