MULTIPLICITY OF SOLUTIONS TO CLASS OF NONLOCAL ELLIPTIC PROBLEMS WITH CRITICAL EXPONENTS

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Abstract

In this paper, we establish existence of infinitely many weak solutions for a class of quasilinear stationary Kirchhoff type equations, which involves a general variable exponent elliptic operator with critical growth. Precisely, we study the following nonlocal problem \begin{equation*} \begin{cases} -\displaystyle{M}(\mathscr{A}(u))\operatorname{div}\Bigl(a(|\nabla u|^{p(x)})|\nabla u|^{p(x)-2}\nabla u\Bigl) = \lambda f(x,u)+ |u|^{s(x)-2}u \text{ in }\Omega, \\ u = 0 \text{ on } \partial \Omega, \end{cases} \end{equation*} where Ω is a bounded smooth domain of $\mathbb{R}^N,$ with homogeneous Dirichlet boundary conditions on $\partial \Omega,$ the nonlinearity $f:\overline{\Omega}\times \mathbb{R}\to \mathbb{R}$ is a continuous function, $a:\mathbb{R}^{+}\to\mathbb{R}^{+}$ is a function of the class $C^{1},$ $M:\mathbb{R}^{+}-{0}\to\mathbb{R}^{+}$ is a continuous function, whose properties will be introduced later, λ is a positive parameter and $p,s\in C(\overline{\Omega})$. We assume that $\mathscr{C}=\{x\in \Omega: s(x)=\gamma^{*}(x)\}\neq \emptyset,$ where $\gamma^{*}(x)=N\gamma(x)/(N-\gamma(x))$ is the critical Sobolev exponent. We will prove that the problem has infinitely many solutions and also we obtain the asymptotic behavior of the solution as $\lambda\to 0^{+}$. Furthermore, we emphasize that a difference with previous researches is that the conditions on $a(\cdot)$ are general overall enough to incorporate some interesting differential operators. Our work covers a feature of the Kirchhoff's problems, that is, the fact that the Kirchhoff's function M in zero is different from zero, it also covers a wide class of nonlocal problems for $p(x)>1,$ for all $x\in \overline{\Omega}.$ The main tool to find critical points of the Euler Lagrange functional associated with this problem is through a suitable truncation argument, concentration-compactness principle for variable exponent found in (missing citation), and the genus theory introduced by Krasnoselskii.

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References