# Compton Effect

Forrest Bullard[1]

[1]California State University, Chico

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## Introduction

In this experiment we will explore the particle nature of light. In particular we will see that scattered photons have less energy than unscattered photons in accordance with the same equations that can be used in classical elastic collisions. In general we will show that this collision conserves both energy and momentum.

## Math

To derive the equation necessary to model Compton scattering we will need to use both conservation of energy and momentum. Conservation of momentum shows us that the initial momentum of the incoming photon $\vec{p_1}$ is equal to the final momentum of the photon $\vec{p_2}$ plus the momentum of the electron $\vec{p_e}$ .

$$\vec{p_1} = \vec{p_2} + \vec{p_e} \tag{1}$$

Now we will need to find the magnitude of the momentum of the electron for later use. A simple dot product will do.

$${p_e}^2 = \vec{p_e} \cdot \vec{p_e} = {p_1}^2 + {p_2}^2 - 2p_1p_2\cos(\theta) \tag{2}$$

Conservation of energy gives us

$$E_1 + E_0 = E_2 + \sqrt{{E_0}^2 + {p_e}^2c^2} \tag{3}$$

where $E_0$ is the rest energy of the electron and the notations 1 and 2 are used for first the initial energy of the incoming photon and then the scattered photon. Rearranging and squaring both sides gives us

$$E_0{}^2 + 2E_0(E_1 - E_2) + (E_1 + E_2)^2 = E_0{}^2 + p_e{}^2c^2. \tag{4}$$

Now substitute in equation 2 to obtain

$$E_0{}^2 + 2E_0(E_1 - E_2) + (E_1 + E_2)^2 = E_0{}^2 + c^2[p_1{}^2 + p_2{}^2 - 2p_1p_2\cos(\theta)], \tag{5}$$

with the substitution $E = pc$ and a bit more algebra we will get Compton's equation in a more reasonable format for energy measurements:

$$\frac{1}{E_2} = \frac{1}{E_0}(1 - \cos(\theta)) + \frac{1}{E_1}. \tag{6}$$

Which we have used in the form

$$\frac{1}{E_2} = \frac{2}{E_0}\sin(\theta)^2 + \frac{1}{E_1}. \tag{7}$$

# Procedure

We will be using a NaI detector on a goniometer arm so that more precise angles can be made with correlation to the direction of incoming photon's incidence with the aluminum target. For this experiment we use as a target an aluminum sphere around one inch in diameter. Aluminum is a good target as it maintains a large number of valence electrons with small ionization energies so that we can treat these electrons as being free. The goniometer arm is able to be rotated from zero degrees (directly in line with the direction of the incoming photons) to 90 degrees (perpendicular to the direction of the incoming photons with target as our axis). Scattered photons from the target will then incidence with our detector with about 30% efficiency. This incidence will be measured and amplified by a photomultiplier tube (PMT) which will then be amplified further before reaching our multi-channel analyzer (MCA). The size of the electrical pulse arriving at the MCA will be proportional to the energy of the photon incident with the detector so after some calibration we will be able to measure the energy of incoming scattered photons. To calibrate the MCA we used a source with known gamma emission, namely , directly in line with our detector then set the two channel calibration on the MCA program to be in line with the two known gamma emissions in the sodium-22 spectrum. After everything is aligned we specify a time interval to take data over then collect data for multiple angles to check that the energy of the photons arriving at the detector are in line with what is believed from Compton's equation. Here we used once more a sodium-22 source and looked for scattered 511 MeV photons. Angles close to zero degrees are left out of the data collection as these angles receive a large number of photons directly from the source.

## Data

Data was taken at multiple angles all over intervals of two hours then regions of interest (ROI) were issued to the area about where the peak occurred. Our MCA program was then able to give an approximation of the center of our peak along with the uncertainty in that peak which I have taken as the full width half max (FWHM) of each given peak.

| Angles (degrees) | Net (Counts) | Centroid (MeV) | FWHM (MeV) |
|---|---|---|---|
| 90 | 1102 | 0.25 | 0.01 |
| 80 | 1687 | 0.27 | 0.01 |
| 70 | 2171 | 0.3 | 0.01 |
| 60 | 1581 | 0.33 | 0.01 |
| 50 | 2346 | 0.37 | 0.01 |
| 40 | 2924 | 0.41 | 0.02 |
| 30 | 2838 | 0.44 | 0.02 |

Table 1: Angular Correlation data

This data was then used to create a weighted best fit with Compton's equation in it's linear form with $E_0$ and $E_1$ as our fit parameters as shown below.

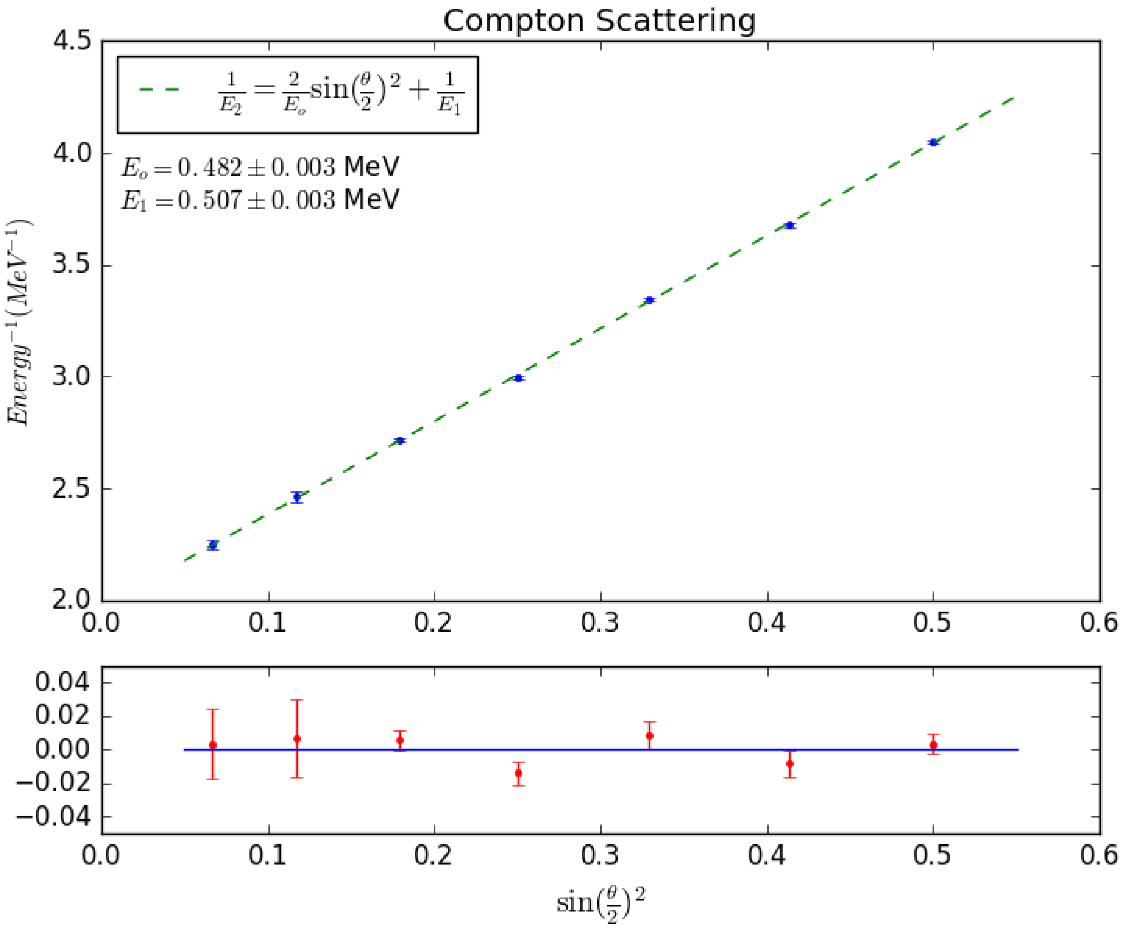


Figure 1: Best Fit and Residuals

It can be seen that this is a reasonable fit for the data however estimation of the rest energy of the electron is largely outside of the expected value for the rest energy of an electron given our small uncertainty in this value. This I believe can be explained by the increasing amount of noise that we received near the low energy side of the window set for our MCA. An attempt was made to limit the effect of this noise by making runs of 2 hour intervals without a target to check radiation coming from unintentional scattering, then subtracting this background noise from runs made with the target in. This however did not drastically reduce the shift in energies believe to be caused by the background noise. A second attempt could be made to reduce this noise by increasing the amount of shielding between possible unnecessary scattering alignments from the source to the detector.

# Conclusion

Given the information obtained from our residuals we find a reduced chi-squared value of 1.5 which allows us to be 20% confident in our fit. It is possible however that error here has been underestimated by our inability to properly account for error in our independent variable, namely the angle at which we were collecting data, which if accounted for is likely to improve our confidence. We conclude that treating photon-electron interactions as elastic collisions is a proper way to model this interaction.