Hertzsprung-Russell Diagrams: Nuclear Fusion Reactions in the Main Sequence

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December 7, 2020

Introduction

During the nineteenth-century, the Harvard College Observatory performed many photographic spectroscopic surveys of stars. This produced spectral classifications for roughly 225,000 stars, which created the Henry Draper Catalogue.

Ejnar Hertzsprung realised that narrow lined stars had smaller proper motions compared to other stars of the same spectral classification. He was then able to estimate the stars' absolute magnitudes[("1", n.d.)] by computing their secular parallaxes as a result of the difference of proper motions.

Definitions

Before we start constructing and analysing a Hertzsprung-Russell Diagram, we should familiarise ourselves with some of the words used in this paper.

Firstly, Absolute Magnitude (M) is a measure of the luminosity of a celestial object, on an inverse logarithmic astronomical magnitude scale. It is further defined to be equal to the apparent magnitude that the object would have if it were viewed from a distance of exactly 10 parsecs without extinction of its light.

Apparent Magnitude (m) is a measure of the brightness of a celestial object observed from Earth. This value is dependent on factors such as the extinction of the body's light, its intrinsic luminosity and its distance from Earth. We state a relationship, the brighter an object, the lower its magnitude value.

Luminosity ($L_\circ$ or $L*$) refers to the absolute measure of radiated electromagnetic power emitted by a light-emitting object. We are interested in both the solar luminosity, denoted by $L_\circ$ and the stellar luminosity, denoted by $L*$ of a particular star.

Lastly, we define the Hertzsprung-Russell Diagram as a scatter plot of stars which shows the relationship between the stars' absolute magnitudes or luminosities versus their stellar classifications or effective temperatures.

Absolute Magnitude

We know that absolute magnitude is measured by a body's luminosity, which provides us with the relationship between Absolute Magnitude and Luminosity, which states that the more luminous an object is, the smaller the numerical value of its absolute magnitude. In terms of variables, we can express this relationship by the following:

A difference of n magnitudes (in absolute magnitude) corresponds to a luminosity ratio of $100^{\frac{n}{5}}$.

In a general sense, subscripts are used alongside M to represent the filter band used for the specific measurement, e.g. M_V for measurements in the V-Band. We generalise this over all wavelengths with an object's bolometric magnitude. By applying a bolometric correction[“2”, n.d.], we can convert absolute magnitudes in specific filter bands to its absolute bolometric magnitude.

$$M_{bol} = M_v + BC$$

Where BC is the Bolometric Correction, needed to factor in specific types of radiation by celestial bodies.

Apparent Magnitude

A numerical scale by Hipparchus describes the brightness of stars that appear in the night sky, where $m = 1$ is assigned to the brightest stars and $m = 6$[“3”, n.d.] assigned to the dimmest stars.

The equation,

$$100^{\frac{mM}{5}} = \frac{F}{F_{10}} = \left(\frac{d}{10pc}\right)^2$$

relates objects within the neighbourhood of the Sun, where their brightness differs by a factor of 100 for m and M from any distance d.

The following equation is derived, given that d is measured in parsecs,

$$M = m - 5\log_{10}(d_{pc}) + 5$$

Which when simplified produces,

$$M = m - 5(\log_{10} d_{pc} - 1)$$

This equation can also be written in two other forms,

In terms of stellar parallax.

(1)

$$M = m + 5(\log_{10} p + 1)$$

Where a distance modulus is known:

(2)

$$M = m - \mu$$

For this paper, we will only be concerned with equation (1).

Gathering Data

The following table was generated by using data available from VizieR. The Vmag column represents the H5 V Johnson magnitudes, Plx the Trigonometric Parallax, B-V the Johnson B-V Colours and finally SpType to represent all the Spectral Types.

We start by creating an additional column, M_v, to represent the absolute magnitudes in the V-Filter.

This is done by using equation (1) in such form,

HIP Identification	Magnitude in Johnson V H5 (Vmag)	Trigonometric Parallax H11 (Plx)	Johnson B-V Colour
1	9.10	3.54	0.482
2	9.27	21.90	0.999
3	6.61	2.81	-0.019
4	8.06	7.75	0.370
5	8.55	2.87	0.902
...	...	...	...
10003	8.45	-0.93	1.404
10004	7.84	4.26	1.14
10005	9.38	3.61	0.507
10006	7.64	4.75	0.075
10007	9.00	10.61	0.566

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$$M_V = V_{mag} + 5\left(\log_{10}\ \frac{plx}{100}\right)$$

This produces:

By plotting an Absolute Magnitude versus Spectral Class diagram, we obtain a simplified Hertzsprung-Russel Diagram as shown below.

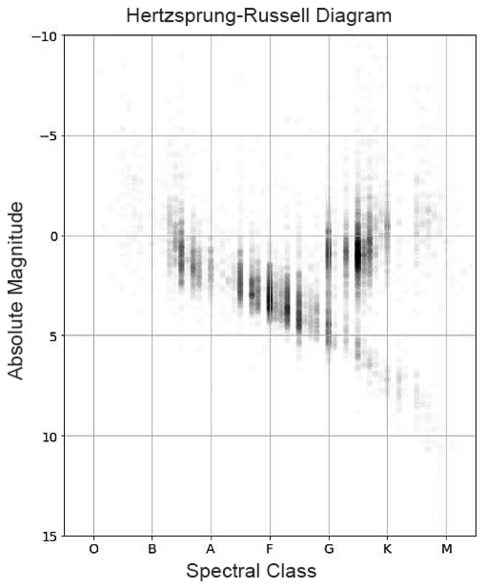

Figure 1: Simplified Hertzsprung-Russell Diagram.

Although the plotted values do not fill much space, it starts to represent the basic pattern you would expect from a Hertzsprung-Russell Diagram, especially the formation of the Main-Sequence.

Now, by plotting Absolute Magnitude values against Colour Index.

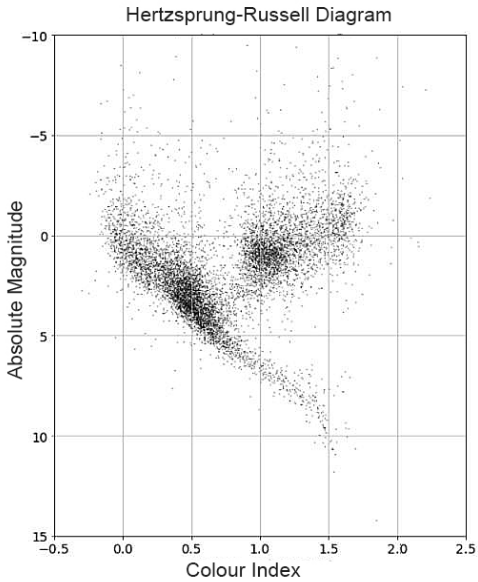

Figure 2: Improved Hertzsprung-Russell Diagram constructed with data from 9000+ stars/

Finally, we can create a coloured diagram to represent different star types to explain the Hertzsprung-Russell Diagram("4", n.d.).

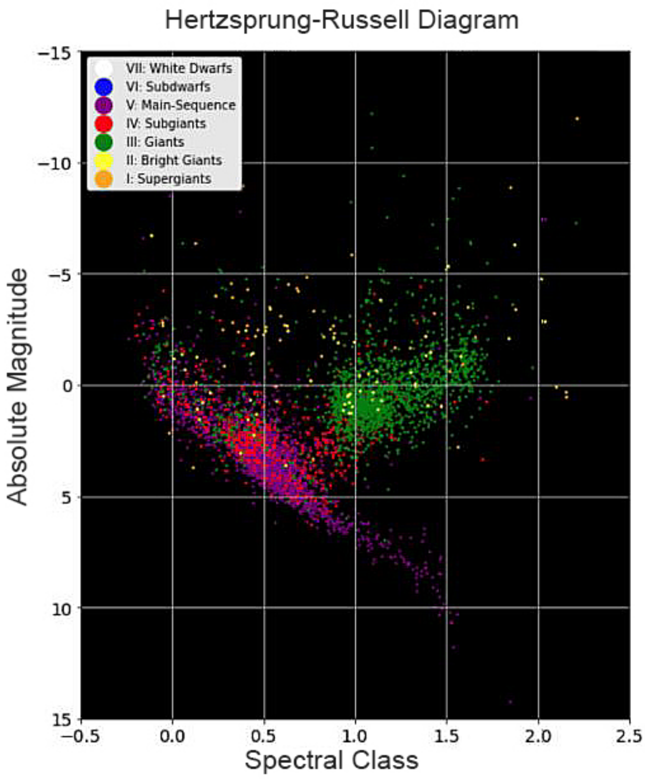

Figure 3: Generated Hertzsprung-Russell Diagram

Interpreting the Hertzsprung-Russell Diagram

The purple area shows the main sequence, where stars spend most of their lives. Stars located here undergo nuclear fusion, where they fuse Hydrogen into their cores while also remaining stable. During nuclear fusion, Hydrogen is converted into Helium.

This reaction can very easily be described by the Proton-Proton Chain Reaction where two protons fuse together to produce Deuterium. Deuterium is a stable isotope of Hydrogen and is given by the following chemical equation,

$$p \ + \ p \ \rightarrow \ {}^{2}_{1}D \ + \ e^{+} + v_{e} + 1.442 \ MeV$$

Upon completion, the Deuterium can fuse with another proton to produce the light isotope of Hydrogen. This reaction is given by,

$${}^{2}_{1}D \ + \ {}^{1}_{1}H \ \rightarrow^{3}_{2} He + \gamma + 5.49 \ MeV$$

To the right of the diagram, we see the Red Giants whose cores' Hydrogen have been exhausted. The Red Giants have lower temperatures than the stars located in the main sequence but have higher stellar luminosity values.

The final stage presented by a Hertzsprung-Russell Diagram is the formation of White Dwarfs, but since our data did not include White Dwarfs we cannot explain them.

The Proton-Proton Cycle

We already discussed the basic chemical equation by which two protons produce Deuterium. Now we will study this process in greater detail.

Our two protons can be seen as two ${}^{1}_{1}H$ atoms, and we can rewrite the chemical equation as,

$${}^{1}_{1}H + \ {}^{1}_{1}H \rightarrow \ {}^{2}_{1}D \ + e^{+} + v_{e}$$

Where e^{+} is the Positron and v_{e} the Neutrino.

Due to the collision between electrons and positrons, annihilation occurs whereby two other particles are produced.

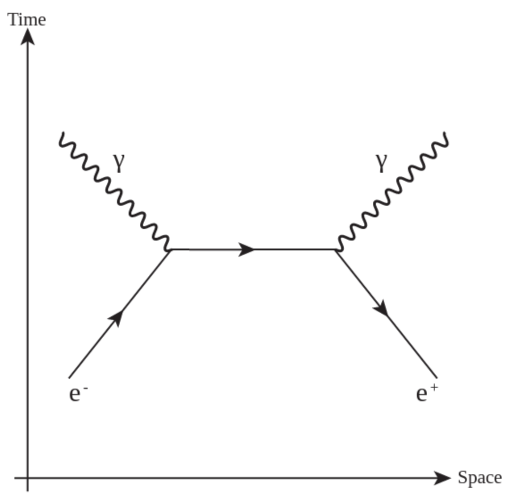

Figure 4: Annihilation of an electron-positron pair given by a Feynman Diagram("5", n.d.).

Electron-Positron Annihilation

Positrons produced from equation (10) do not last very long as they collide with electrons. The electron annihilates the positron to produce two photons or more specifically two Gamma Rays.

This reaction is given by,

$$e^{+} + e^{-} \;\rightarrow 2\gamma$$

Energy is also produced in Megaelectronvolts, and amounts to 0.511 MeV. This energy contributes to the total value of released energy from equation (10).

The total energy released sums to 1.442 MeV such that Q=1.442 MeV

Deuterium-Proton Fusion

The second part of the products also undergo a separate reaction whereby the Deuterium fuses with another proton in order to produce $^{3}_{2}He$ and is given by,

$$^{2}_{1}H \;+^{1}_{1}H \rightarrow^{3}_{2}He + \gamma$$

We also notice a change in energy where the total energy released increases due to the loss in mass which obeys the mass-energy equivalence[("6", n.d.)] equation given by $DE = Dmc^{2}$.

The binding energy equation follows as,

$$2.22452\;MeV \;+ 0.0000136\;MeV \;\rightarrow 7.7181\;MeV$$

We see that mass is converted into energy.

The Proton-Proton 1 Branch

Our final fusion reaction includes two $^{3}_{2}He$ atoms fusing together to produce $^{4}_{2}He$. The chemical equation is given by,

$$^{3}_{2}He \;+\; ^{3}_{2}He \rightarrow^{4}_{2}He \;+^{1}_{1}H + \;^{1}_{1}H + \gamma$$

The total energy produced by the Proton-Proton Chain equates to 26.7 MeV. Some energy is also lost to neutrinos.

Conclusion

The Hertzsprung-Russell Diagram can be used as an accurate method to approximate the distance between different Galaxies and Star Clusters from the Earth. We are also further able to explain the nuclear physics behind the stars found in the main sequence and find that conservation of mass is at play during fusion.

We are also able to get a better understanding of the Proton-Proton Chain.

References

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