Classical Mechanics: Fundamentals

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Introduction

The main objective of these notes is the construction of a point of view of classical mechanics, a point of view that allows a closer connection with modern and contemporary physics. The two main theoretical approaches for the understanding of nature since the beginning of the XX century are General Relativity and Quantum Mechanics. The first is the set of theories for high speeds and massive structures of the universe. The later is the set of theories that allows us to approach beyond the microscopic phenomena. Both worlds are not disconnected, since high energy physics, as the physics done by the Large Hadron Collider, rellies on relativistic quantum field theory, which is a quantum mechanical theory set up by special relativity.

Rich media available at https://youtu.be/0lgs1vTzS2E

Classical mechanics, of course, is a full discipline by itself. It can be learned and developed without reference to any other physical theory. Dynamical systems, orbital dynamics, chaos, and fractal dynamics are examples of modern developments. On the other hand, we live in a world that has molecular and atomic physics, quantum mechanics, quantum field theory, general and special relativity, quantum information. Our fundamental understanding of nature is coded in the standard model of elementary particles, and the theory of the Big Bang. It is not only a waste to study classical mechanics as a separate subject, but it is imperative to understand how all other physical theories rely on classical mechanics in fundamentals and techniques.

Here we explore two main ideas. The first one is that physics is the science that maps natural phenomena into mathematical structures. The other idea is that a physical system is only as definable as it can be measured, and the information that defines a system is the one all observers in a specified class of observers agree.

The particle

Rich media available at https://youtu.be/pelK0Nn-RZc

Some concepts that gave origin to classical mechanics are just too classical. We must have them here even if we came to suffer significantly to get rid of them when necessary. And the most basic of all classical concepts is **the particle.**

In classical mechanics, a particle is an eternal smallest element. It is a brick that builds everything that exists. Particles are the building blocks of any physical system. A particle has no size, no internal structure, but has some information attached. Typically, the information that comes with the particle is the **mass**, but it can also include other observables, as the **electric charge**, the **spin**, or additional internal charges. By eternal, we mean that a particle cannot be created or destroyed. A particle exists.

Of course, this concept of a particle does not have any real correspondence in nature. The real world has extended bodies, formed by molecules, atoms of many types, which are formed by electrons, protons, and neutrons. Protons and neutrons are then formed by other basic structures known as quarks. Electrons are elementary by themselves. However, quarks and electrons cannot be described as classical particles, because eventually, this concept will not be sufficient to specify the way they exist. To understand if we are found in the domain of classical physics, we should learn if the above concept of a particle is, at least, approximately accurate to experiment. If this is the case, we may address the electron as a classical particle. Sometimes the classical particle is a sufficient concept.

The set of information that comes with the particle depends on the specific physical system of interest. Mass is always one of the quantities, and it is related to the concept of inertia, which we will explore in moments. An electron, for example, has other defining quantities; the electric charge, and the spin. If we are treating an electron as a classical particle, these measures must be the defining properties of the electron. In this case, we have the values $m_e \approx 9,109 \cdot 10^{-31}\ Kg$, about 1836 times lighter than the proton, $q_e \approx -1.602 \cdot 10^{-19}$ coulomb, which is the elementary electric charge, and an intrinsic angular momentum, or spin, of $1/2$. No other particle has the same characteristics, and all observers must agree with them.

The origin of mass, charge, and spin can only be explained by the relativistic quantum field theory. Therefore, in classical mechanics, these values must be postulated. But they are the first examples of what is called **dynamical invariants**. The values of (m_e, q_e, s_e) are always the same for the electron, as for any other particle, and they never change.

Another fundamental particle in nature is the photon, the particle of light and electromagnetic radiation. For centuries the debate about the nature of light opposed the particle and the wave points of view. Today, we understand the light fundamentally as a field, which can be made a particle when it reaches a detector. Still, it is also a wave when interacting with slits to form interference phenomena. The photon is of little use in classical mechanics since it has zero mass and zero electric charge. The photon does not have a spin value, but it has, on the other hand, a value called **helicity**, which gives rise to its polarization properties.

Interaction and movement

We know movement should be part of the classical mechanical description because changes in the movement state of objects are part of our everyday lives. How can we accommodate the concept in our theory?

First, we recognize that a universe with a single particle cannot present movement states for the particle. Therefore, **the movement** must be a property of a system of two or more particles. If we have a universe with two particles, we must allow the particles to interact, in this case, to change their respective states of movement. In the real world, we know by experiment that two particles with values of mass do interact by gravitation. We also know that two particles with electric charge interact by electric and magnetic fields.

A single particle that does not interact with any other particle is a **free particle**. Ideally, we may have a system of two or more free particles, i.e., particles that form a mechanical system but do not interact with each other, but this would be a very uninteresting system. We may have a system with several particles that interact among themselves but do not interact with other particles or systems; in this case, we call this a **closed system**, or an **isolated system**.

We may always separate a system of particles, and form sub-systems, by using a definite criterium. For example, a system with $n \in \mathbb{N}$ particles, each with a mass m_1, and another system with $k \in \mathbb{N}$ particles, each with a mass m_2, may be seen as two sub-systems of a larger system with $n + k$ particles with distinct masses. In this case, a system with n particles may always be seen as n systems, each with a single particle.

The observer, the measurement, and the observable

The **observer** is a physical system by itself, which possesses rulers, clocks, or any other measurement apparatus. The job of the observer and its tools is to collect information about other physical systems. We call each information possible to be collected from a system a **measure**. We also use the word **measurement** for the act of obtaining a measure.

A measure must refer to a specific characteristic of the system. For example, some curious mind could wonder about the distribution of eye colors in a system of n human beings. The colors could be brown, blue, and green, and these are the possible measures of the measurement. At the end of all measurements, m humans will have brown eyes, k humans will have blue eyes, and $n - m - k$ humans will present green eyes. The eye color is the characteristic that has been measured by the observer, and it is called the **observable**. This is not the kind of example we will deal with in the classical mechanical theory but serves to illustrate the point.

We say that a measure belongs to an observable in the sense that an observer may perform a measurement on the observable, therefore collecting that measure. The set of all possible measures of a single observable is called the **spectrum** of the observable. In this case, a measurement is the selection of a member of the spectrum.

Here we actually find our first mathematical structure, the **set theory**. The spectrum of an observable is a set in the mathematical sense, and a measure is a member of the set. A measurement, therefore, is also the assignment of a member of the spectrum to a characteristic of the system. The spectrum may be limited or unlimited, countable or non-countable.