

Mathematica Pseudocodes for Implementing Block Adams Family

jimevwo Oghonyon¹, Sheila Bishop², Kanayo Eke², Temitope Abodunrin¹, and Ogbu Imaga¹

¹Covenant University College of Science and Technology

²Covenant University

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Abstract

This study is considered to formulate Mathematica pseudocodes for implementing block Adams family (MPIBAF). An idea multinomial basis function approximant will be utilized to process the interpolation and collocation methods. A special block Adams family in form of block Adams-Bashforth and Block Adams-Moulton methods will be developed via interpolation and collocation method to foster the principal local truncation error thereby bringing into existence the convergency limits. The application of Mathematica pseudocodes will be processed on some applied math problems in a parallel manner with each processor depending on one another. The computable results will be produced via a compiled Mathematica pseudocode in a gradual manner. In addition, technical computation supersedes manual computation as demonstrated by the results of the numerous gains such as ease of computable processes and structures, better accuracy and quicker convergency.

Introduction

According to [1, 28], Mathematica is the universe solely incorporated environs for proficient computation. Since issued in 1988, it has had unfathomed outcome on the manner electronic computers are applied in various proficiencies and other areas.

Technically, Mathematica is seen to a great degree viewed as a principal achievement of software system technology. It is observed as the most prominent individual applications of all time formulated. Thus, it comprises of a huge lay out of new algorithmics and prominent proficiency inventions. Amongst its most essential inventions are its interlinked algorithmic noesis cornerstone and its conceptions of symbolical computer programming and of document-centered interfaces. See [1, 24, 28].

Mathematica as a math software package is split up into two component parts: the Mathematica forepart (notebook) and Mathematica kernel (core). The forepart takes stimulus and manifests the end product. The forepart is the most significant that grants the user to act together with the schemes for the objective of performing infinitesimal calculus and to uphold them recycle or for point of reference. The Mathematica kernel is the unseeable component part of the ciphering computer program that put in effect completely the calculations. This aspect is very essential to this research study because all codes written on MPIBAF and executed is implemented on this platform. See [1, 12, 24, 28].

Prompted by the need to design a Mathematica pseudocodes of block Adams family under the Mathematica kernel platform to estimate engineering science problems, biologic and natural phenomenon. A number of bookmen started to invent efficient and real accurate techniques to solve ODEs (1). Among them admits; [13] developed hybrid block predictor-hybrid block corrector for the solution of first-order ordinary differential equations. This approach involves tedious manual computation and fixed step size strategy. [29] devised the predictor-corrector block iteration method for solving ODEs. This technic is coded in C language and executed via variable step size strategy. [17] implemented a seventh-order linear multistep method

in a predictor-corrector mode or block mode: which is more efficient for the general second order initial value problem. Again, the implementation is a combination carried out on a composed cipher in PERL computer programming terminology and Mathematica as well. This combination is manually executed on fasten step size strategy. [2] considers a new block-predictor corrector algorithm for the solution of $y'''' = f(x, y, y', y'')$ and whose execution takes a difficult task with fixed step size scheme. All of these techniques and implementations involves utilizing different fixed length to arrive at the any max-errors. This eventually leads to increase human effort and large space size used. Hence, the need to design firstly, Mathematica pseudocodes for implementing block Adams family as discussed in the abstract. Secondly, a suitable length that will ensure efficiency, accelerate the mode of convergence and stabilize the loop via the introduction of the convergency limits.

Consider the first-order ordinary differential equation of the form

$$y' = f(x, y), y(x_0) = y_0, \text{ for } x \in [a, b], \quad (1)$$

where f meets the Lipschitz precondition as acknowledged in [7, 8, 10, 17].

Hence, the primary objective of this study is geared towards the development of Mathematica pseudocodes for implementing block Adams family. This build up exist in literatures as discussed in [19-23]. Other superior qualities of (1) necessitate varying the step size, determining worth step size and checking max ciphered computer errors. Scientifically, for computation involving the use of Mathematica pseudocodes to demonstrate the superiority of the study and engender a new breed of computational mathematics. See [1, 12, 24, 28].

The motivation behind this study stem from the fact that, computational mathematics without the use of Mathematical packages carries a difficult task during looping which leads to low accuracy and large global errors inherent due to approximation via manual computation. Thus, there is a need to build an iterative code for looping with better accuracy and low computer error.

The remain of this study is examined as succeeds: Section 2 Materials and Methods. Section 3 Results. Section 4 Discussions as stated in [3, 4, 20].

Materials and Methods

This aspect contains materials and methods of the Mathematica pseudocodes of block Adams family. Block Adams family is divided into two parts; block Adams-Bashforth method and block Adams-Moulton method. Both pairs will be formulated using interpolation and collocation technique via a well coded multinomial and executed under the platform of Mathematica kernel to produce the desired results. The unification of block Adams family is described as

$$y(x) = \sum_{i=0}^1 \alpha_i y_{n-i} + h \left[\sum_{i=0}^3 \beta_i^* (u) f_{n-i+1} \right] \quad (2)$$

$$y(x) = \sum_{i=2}^1 \alpha_i y_{n-i} + h \left[\sum_{i=0}^1 \beta_i^* (u) f_{n+i+1} \right] \quad (3)$$

Equations (2) and (3) forms the block Adams-Bashforth method and block Adams-Moulton method of the block Adams family. Citing that y_{n+i} represents the mathematical estimate to the analytic effects $y(x_{n+i})$ i.e. $y(x_{n+i}) \approx y_{n+i}, f(x_{n-i}, y_{n-i}) \approx f_{n-i}$ and $f(x_{n+i}, y_{n+i}) \approx f_{n+i}$ owing $i = 0, 1, 2, 3$. To reach equations (2) and (3), the mathematical multinomial is composed in the format of Mathematica pseudocodes that are required to estimate the analytic effect $y(x)$ on trenchant definite length of time, $[x_n]$ via the interpolating subroutine of the class (4) that is clearly characterized as

$$\cdot \quad (4)$$

Transforming (4) will bring-forth the Mathematica pseudocodes which can be constituted under the platform of Mathematica Kernel as

$$y[x_-] = w[0] + w[1] \frac{(x-x[n])}{h_1} + w[2] \frac{(s-x[n])^3}{h^2} + w[3] \frac{(x-x[n])^3}{h}, \quad (5)$$

where $w[0]$, $w[1]$, $w[2]$ and $w[3]$ are unknown parametric quantity anticipated to be resolve in a unique way. Take for granted that equations (5) equates with the analytic effect at an approximate time-interval $[x_n]$, $[x_{n-j}]$ to suit the estimates of

$$y(x_n) \approx y_n, y(x_{n-j}) \approx y_{n-j}. \quad (6)$$

Demanding that the mathematical function (6) meet the requirements for (1) at the points $\setminus nx_{n+j}$, $j = 1, 2, 3$ to formulate the next estimate as

$$y'(x_{n+j}) \approx f_{n+j}, j = 0, 1, 2, 3. \quad (7)$$

To achieve this formulation of the block family, (5) is encrypted into Mathematica pseudocodes to carry out the process of interpolation and collocation.

A combination of (6) and (7) will give-rise to quartet structure which constitute $Ax = d$.

matrixa =

$$\left\{ \begin{array}{l} \{1, 0, 0, 0\}, \\ \{0, 1, -2, 3\}, \\ \{0, 1, -4, 12\}, \\ \{0, 1, -6, 27\}, \end{array} \right\};$$

$$d = \{y[n], f[n-1], f[n-2], f[n-3]\};$$

$$\{b, c, l, q\} = \text{Inverse}[\text{matrixa}].d, \quad (8)$$

matrixa =

$$\left\{ \begin{array}{l} \{1, -2, 4, -8\}, \\ \{0, 1, 2, 3\}, \\ \{0, 1, 4, 12\}, \\ \{0, 1, 6, 27\} \end{array} \right\};$$

$$d = \{y[n-2], f[n+1], f[n+2], f[n+3]\};$$

$$\{b, c, l, q\} = \text{Inverse}[\text{matrixa}].d. \quad (9)$$

This moves on to evaluate at certain fixed time-interval of x_{n+j} , $j = 1, 2, 3$ and generates the encrypted continuous Mathematica pseudocodes for block Adams-Bashforth method of the form

$$y[x_-] = (1)y[n] + \left(\frac{3(x-x[n])^1}{h} + \frac{5(x-x[n])^2}{4h^2} + \frac{(x-x[n])^3}{6h^3} \right) f[n-1]h + \left(-3\frac{(x-x[n])^1}{h} - 2\frac{(x-x[n])^2}{4h^2} - \frac{1}{3}\frac{(x-x[n])^3}{6h^3} \right) f[n-2]h + \left(\frac{(x-x[n])^1}{h} + \frac{3}{4}\frac{(x-x[n])^2}{4h^2} + \frac{(x-x[n])^3}{6h^3} \right) f[n-3]h, \quad (10)$$

while the continuous Mathematica pseudocodes for block Adams-Moulton method is seen as

$$y[x_-] = (1)y[n-2] + \left(\frac{37}{3} + \frac{3(x-x[n])^1}{h} - \frac{5(x-x[n])^2}{4h^2} + \frac{(x-x[n])^3}{6h^3} \right) f[n+1]h + \left(-\frac{50}{3} - \frac{3(x-x[n])^1}{h} + \frac{2(x-x[n])^2}{h^2} - \frac{(x-x[n])^3}{3h^3} \right) f[n+2] + \left(\frac{19}{3} + \frac{(x-x[n])^1}{h} - \frac{3(x-x[n])^2}{4h^2} + \frac{(x-x[n])^3}{6h^3} \right) f[n+3]. \quad (11)$$

Equations (10) and (11) are called the Mathematica pseudocodes of block Adams family designed under the platform of Mathematical Kernel. This turns out to produce the block Adams-Bashforth method of order three expressed below:

$$y[n+1] = y[n] + h \left(\frac{53}{12}f[n-1] - \frac{16}{3}f[n-2] + \frac{23}{12}f[n-3] \right),$$

$$y[n+2] = y[n] + h \left(\frac{37}{3}f[n-1] - \frac{50}{3}f[n-2] + \frac{19}{3}f[n-3] \right), \quad (12)$$

$$y[n+3] = y[n] + h \left(\frac{99}{4}f[n-1] - 36f[n-2] + \frac{57}{4}f[n-3] \right).$$

Similar manner, the block Adams-Moulton method of order three is given below:

$$\begin{aligned}
 y[n+1] &= y[n-2] + h \left(\frac{57}{4} f[n+1] - 18f[n+2] + \frac{27}{4} f[n+3] \right), \\
 y[n+2] &= y[n-2] + h \left(\frac{44}{3} f[n+1] - \frac{52}{3} f[n+2] + \frac{20}{3} f[n+3] \right), \quad (13) \\
 y[n+3] &= y[n-2] + h \left(\frac{175}{12} f[n+1] - \frac{50}{3} f[n+2] + \frac{85}{12} f[n+3] \right).
 \end{aligned}$$

Equations (12) and (13) represent the block Adams family of order three. Study [1, 12-14, 19-23, 28] for more info.

Constituting the convergency limits for block Adams family

To plunge the Mathematica pseudocodes of block Adams family, block Adams-Bashforth $j - step$ method and block Adams-Moulton $j - 1 - step$ are known to own ilk order. The combination of [5-6, 9, 15-16, 19-23] reveals that it is more beneficial to determine the approximant of the principal local truncation error of the block Adams-Bashforth method-block Adams-Moulton method without the presence of loftier differential coefficients of $y(x)$. Accepting without verification that $p = \bar{p}$, where p and \bar{p} lays down the order of the block Adams-Bashforth method and block Adams-Moulton method. Without delay, for a method of order p , the systematic analysis of the block Adams-Bashforth $j - step$ will generate a block principal local truncation errors as

$$\begin{aligned}
 C_{p+1}^{[1]} h^{\bar{p}+1} y^{(\bar{p}+1)}(x_n) &= y(x_{n+1}) - y_{n+1}^{[q_1]} + O(h^{\bar{p}+2}), \\
 C_{p+1}^{[2]} h^{\bar{p}+1} y^{(\bar{p}+1)}(x_n) &= y(x_{n+2}) - y_{n+2}^{[q_2]} + O(h^{\bar{p}+2}), \quad (14) \\
 C_{\bar{p}+1}^{[3]} h^{\bar{p}+1} y^{(\bar{p}+1)}(x_n) &= y(x_{n+3}) - y_{n+3}^{[q_3]} + O(h^{\bar{p}+2}).
 \end{aligned}$$

Similar inquiry of the block Adams-Moulton $j - 1 - step$ will work out the block principal local truncation errors as

$$\begin{aligned}
 \tilde{C}_{\bar{p}+1}^{[1]} h^{\bar{p}+1} y^{(\bar{p}+1)}(\tilde{x}_n) &= y(x_{n+1}) - y_{n+1}^{[l_1]} + O(h^{\bar{p}+2}), \\
 \tilde{C}_{\bar{p}+1}^{[2]} h^{\bar{p}+1} y^{(\bar{p}+1)}(\tilde{x}_n) &= y(x_{n+2}) - y_{n+2}^{[l_2]} + O(h^{\bar{p}+2}), \quad (15) \\
 \tilde{C}_{\bar{p}+1}^{[3]} h^{\bar{p}+1} y^{(\bar{p}+1)}(\tilde{x}_n) &= y(x_{n+3}) - y_{n+3}^{[l_3]} + O(h^{\bar{p}+2}).
 \end{aligned}$$

Where, $C_{p+1}^{[1]}$, $C_{p+1}^{[2]}$, $C_{p+1}^{[3]}$, $\tilde{C}_{p+1}^{[1]}$, $\tilde{C}_{p+1}^{[2]}$ and $\tilde{C}_{p+1}^{[3]}$ are currently in existence as an autonomous physical entity of the step size h and $y(x)$ makes up the computed final result to the differential constant coping with the foremost pre-condition $y(x_n) \approx y_n$. See [5-7, 13-14, 17-21] for more info.

In moving forward, the presumptuousness for a very small assess of h is achieved as

$$y^{(4)}(x_n) \approx y^{(4)}(\tilde{x}_n),$$

and the effectuation of the Mathematica pseudocodes of block Adams family hopes on this pre-condition.

Further step-down of the principal local truncation errors of (14) and (15) over and similar way dismissing terms of degree $O(h^{\bar{p}+2})$, it goes very well-situated to find the mathematical computation of the principal local truncation errors of block Adams family as

$$\begin{aligned}
 \tilde{C}_{\bar{p}+1}^{[1]} h^{\bar{p}+1} y^{(\bar{p}+1)}(\tilde{x}_n) &\approx \frac{C_{p+1}^{[1]}}{\tilde{C}_{p+1}^{[1]} - C_{p+1}^{[1]}} \left[y_{n+1}^{[l_1]} - y_{n+1}^{[q_1]} \right] < \varphi_1, \\
 \tilde{C}_{\bar{p}+1}^{[2]} h^{\bar{p}+1} y^{(\bar{p}+1)}(\tilde{x}_n) &\approx \frac{C_{p+6}^{[2]}}{\tilde{C}_{p+1}^{[2]} - C_{p+1}^{[2]}} \left[y_{n+2}^{[l_2]} - y_{n+2}^{[q_2]} \right] < \varphi_2, \quad (16) \\
 \tilde{C}_{\bar{p}+1}^{[3]} h^{\bar{p}+1} y^{(\bar{p}+1)}(\tilde{x}_n) &\approx \frac{C_{p+6}^{[3]}}{\tilde{C}_{p+1}^{[3]} - C_{p+1}^{[3]}} \left[y_{n+3}^{[l_3]} - y_{n+3}^{[q_3]} \right] < \varphi_3.
 \end{aligned}$$

Showing quick the arguments that $\tilde{C}_{p+1}^{[1]} \neq C_{p+1}^{[1]}, \tilde{C}_{p+1}^{[2]} \neq C_{p+1}^{[2]}, \tilde{C}_{p+1}^{[3]} \neq C_{p+1}^{[3]}$ are not equal as defined by (16), while others are mentioned as block Adams-Bashforth and block Adams-Moulton calculates which are products of the block Adams family of order p and at the same time, $\tilde{C}_{\bar{p}+1}^{[1]} h^{\bar{p}+1} y^{(\bar{p}+1)}(\tilde{x}_n), \tilde{C}_{\bar{p}+1}^{[2]} h^{\bar{p}+1} y^{(\bar{p}+1)}(\tilde{x}_n)$ and $\tilde{C}_{\bar{p}+1}^{[3]} h^{\bar{p}+1} y^{(\bar{p}+1)}(\tilde{x}_n)$ in a distinguishable manner are addressed as the principal local truncation errors. Thus, φ_1, φ_2 and φ_3 are the terminal points of the convergency limits extracted from the block Adams family.

All the same, the calculates of the block principal local truncation errors (16) is capable of being applied to reach alternatives of taking the effectuates of the co-occurrent iterative aspect or make new the co-occurrent iterative aspect with an ever-changing step size. This iterative aspect is veritably founded on iterative calculation blueprinted by (16). See [5-6, 9, 15-16, 19-23] for more info. The principal local truncation errors (16) is the convergency limits of block Adams family other than the block Milne's device (calculate) for adjusting to convergency.

Results

This aspect shows the computed effects of block Adams family yielded through Mathematica pseudocodes (MP) for the aim of demonstrating proficiency, high level of accuracy and skillfulness in avoiding wasted time and effort.

Two practical illustrations were considered and enforced employing MP at different convergency limits of $10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}, 10^{-8}, 10^{-9}$ and 10^{-13} . See [11, 17, 18, 25, 26, 27] for details. The Mathematica pseudocodes of block Adams Family is coded utilizing Mathematica Kernel in concert with an algorithmic rule. This Mathematica pseudocodes is executed side by side as developed by the block Adams family. See appendix.

Time-tested problem 1. Consider the inhomogeneous IVP:

$$y'(x) = -10(y - 1)^2, y(0) = 2,$$

having the exact result $y(x) = 1 + \frac{1}{1+10x}$. This time-tested problem 1 has been worked-out and published by [25, 26, 27] displaying that many predictor-corrector method, block method and block hybrid method suffers setbacks in the course of implementation such as lack of stability, fixity or firmness. Again, large number of predictor-corrector method [25, 26, 27] ends up as predicting-correcting technique without any concern for determining a suitable step size thereby result to developing the convergency limits for the loop. This time-tested problem 1 has been incorporated in $[0,0.1]$ assuming length $h = 0.01$ and published in [25, 26, 27]. Table 1 is comprising of method employed, max-errors and convergency limits at various levels that displays the technique introduced in this study. MP executed more better as a results of the special quality of desire suitable step size leading to the discovery of the convergency limits of the loop.

Time-tested problem 2. Consider the initial value ODE:

$$y' = -10xy, y(0) = 1$$

of which the exact result is

$$y(x) = \exp(-5x^2)$$

and has shown up in [11, 17, 18] employing the time-interval $[0, 10]$ having various lengths. Table 2 presents the method employed, max-errors and convergency limits of [11, 17, 18] and MP as the technique of this study as slated in (18).

The language applied are enlisted beneath. Table I and Table II displays the computed results of time-tested problems 1 and 2 employing MP equated with subsisting techniques. The typifies make reference to on Table I and Table II are showed beneath.

Table I

Time-tested problem 1

Method Employed	Max-Errors	Convergency Limits
SS-S-F-F-O	$4.880978E - 05$	10^{-5}
MP	$2.40971E-08$	10^{-5}
MP	$2.44765E - 08$	
MP	$2.48602E - 08$	
BI	$1.159479E - 08$	10^{-8}
MP	$6.70575E - 14$	10^{-8}
MP	$6.99441E - 14$	
MP	$9.85878E - 14$	
BHM	$1.159479E - 08$	10^{-9}
MP	$4.44089E - 16$	10^{-9}
MP	$2.22045E - 16$	
MP	$2.22045E - 16$	

Table II

Time-tested problem 2

Method Employed	Max-Errors	Convergency Limits
2BBDF	$2.47600E - 02$	10^{-2}
3BEBDF	$1.24084E-02$	
MP	$8.70261E - 03$	10^{-2}
MP	$9.28415E - 03$	
MP	$9.8681E - 03$	
2BBDF	$2.86614E - 03$	10^{-3}
2IBBDF	$1.49805E - 03$	
MP	$7.25539E - 05$	10^{-3}
MP	$1.02834E - 04$	
MP	$1.06259E - 04$	
3BEBDF	$7.36421E-04$	10^{-4}
MP	$6.73522E - 07$	10^{-4}
MP	$7.67231E - 07$	
MP	$7.77108E - 07$	
2IBBDF	$1.51149E-04$	10^{-5}
MP	$8.41274E - 09$	10^{-5}
MP	$1.23133E - 08$	
MP	$1.27668E - 08$	
BHM	$7.196978E-13$	10^{-13}
MP	0.	10^{-13}
MP	$1.11022E - 16$	
MP	$1.11022E - 16$	

MP: errors in MP (Mathematica pseudocodes for implementing block Adams family) for time-tested problem 1 and 2.

Method Employed: method employed.

Max-Errors: the magnitude of the maximum errors of MP.

Convergency Limits: convergency Limits of MP

BHM: errors in BHM (an optimized two-step hybrid block method for solving first-order initial-value problem in ODEs) for time-tested problem 1 and 2. See [25].

SS-S-F-F-O: errors in SS-S-F-F-O (a self-starting four-step fifth-order block integrator for stiff and oscillatory differential equations) for time tested problem 1. See [27]

BI: errors in BI (numerical solution of stiff and oscillatory differential equations using a block integrator) for time-tested problem 1. See [26].

2BBDF: errors in 2BBDF (2-point block backward differentiation formula) for time-tested problem 2. See [11].

2IBBDF: errors in 2IBBDF (2-point improved block backward differentiation formula) for time-tested problem 2. See [18].

3BEBDF: errors in 3BEBDF (3-point block extended backward differentiation formula) for time tested problem 2. See [17].

Algorithmic Rule

A step by step process that specify the implementation of MP is stated beneath as follows:

Step 1: Select the length for h

Step 2: The rank of the block predictor-bloc corrector pair must be the similar.

Step 3: The number of step for block predictor method and block corrector must not be equal.

Step 4: Select the convergency limits from the max-errors of the author's paper.

Step 5: Write and implement the block predictor- block corrector pair in Mathematica Kernel 9.

Step 6: Combine the single step code and step 5 during the implementation.

Step 7: Execute Step 6, if Step 7 fails to converge return to step 1, if not, proceed to step 8. Utilize this formula to generate a new length for h if necessary.

$$qh = \left| \frac{0.5\varphi_i}{\left(\hat{C}_{p+1}^{[i]} - C_{p+1}^{[i]}\right)} \right|^{\frac{1000}{4000}}, \text{ for } i = 1, 2, 3.$$

Step 8: Assess the max-errors when the convergency limits is achieved.

Step 9: Write out the max-errors. See [19, 20 ,21].

Discussion

A Mathematica pseudocodes for implementing block Adams family has been suggested and executed using a suitable length for computing first-order initial value problems of ordinary differential equations. This study considers the block Adams family which possess the ability to stabilize the loop via the method of designing a suitable length and as a result generate the convergency limits. The numeric results prove its skillfulness in avoiding wasted time and effort yielding a more better precision liken with other techniques published by authors earlier mentioned in time-tested problems 1 and 2.

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Declaration of Interest Statement

There are no conflict of interest among the authors.

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Appendix.

The Mathematica pseudocodes of block Adams family for computing time-tested problem 1 and time-tested problem 2 is given beneath.

The block Adams-Bashforth of the block Adams family.

$g[t_]=$ exact result

$h=x[n]=$ given taking off parameters

$t =$ the result of the above parameters

$$g[1]=g[0]+h(g'[0])+(h^2/2)g''[0]+(h^3/6)g'''[0]$$

$$g[2]=g[1]+h(g'[x[n]])+(h^2/2)g''[x[n]]+(h^3/6)g'''[x[n]]$$

$$g[3]=g[2]+h(g'[x[n]+h])+(h^2/2)g''[x[n]+h]+(h^3/6)g'''[x[n]+h]$$

$$g[4]=g[3]+h(g'[x[n]+2h])+(h^2/2)g''[x[n]+2h]+(h^3/6)g'''[x[n]+2h]$$

$$g[5]=g[4]+h(g'[x[n]+3h])+(h^2/2)g''[x[n]+3h]+(h^3/6)g'''[x[n]+3h]$$

$$t=x[n]+2h$$

$$g[4]=g[3]+h((53/12)g'[t-x[n]]-(16/3)g'[t-(x[n]+h)]+(23/12)g'[t-(x[n]+2h)])$$

$$t=x[n]+4h$$

$$g[6]=g[4]+h((37/3)g'[t-x[n]]-(50/3)g'[t-(x[n]+h)]+(19/3)g'[t-(x[n]+2h)])$$

$$t=x[n]+6h$$

$$g[8]=g[5]+h((99/4)g'[t-x[n]]-(36)g'[t-(x[n]+h)]+(57/4)g'[t-(x[n]+2h)])$$

$$t=x[n]+5h$$

$$g[7]=g[6]+h((53/12)g'[t-x[n]]-(16/3)g'[t-(x[n]+h)]+(23/12)g'[t-(x[n]+2h)])$$

$$t=x[n]+7h$$

$$g[9]=g[7]+h((37/3)g'[t-x[n]]-(50/3)g'[t-(x[n]+h)]+(19/3)g'[t-(x[n]+2h)])$$

$$t=x[n]+9h$$

$$g[11]=g[8]+h((99/4)g'[t-x[n]]-(36)g'[t-(x[n]+h)]+(57/4)g'[t-(x[n]+2h)])$$

$$t=x[n]+8h$$

$$g[10]=g[9]+h((53/12)g'[t-x[n]]-(16/3)g'[t-(x[n]+h)]+(23/12)g'[t-(x[n]+2h)])$$

$$t=x[n]+10h$$

$$g[12]=g[10]+h((37/3)g'[t-x[n]]-(50/3)g'[t-(x[n]+h)]+(19/3)g'[t-(x[n]+2h)])$$

$$t=x[n]+12h$$

$$g[14]=g[11]+h((99/4)g'[t-x[n]]-(36)g'[t-(x[n]+h)]+(57/4)g'[t-(x[n]+2h)])$$

The block Adams-Moulton method of the block Adams family.

$$y[u.] = \text{exact result}$$

$$h=x[n] = \text{given taking off parameters}$$

$u =$ the result of the above parameters

$$y[1]=y[0]+h(y'[0])+(h^2/2)y''[0]+(h^3/6)y'''[0]$$

$$y[2]=y[1]+h(y'[x[n]])+(h^2/2)y''[x[n]]+(h^3/6)y'''[x[n]]$$

$$y[3]=y[2]+h(y'[x[n]+h])+(h^2/2)y''[x[n]+h]+(h^3/6)y'''[x[n]+h]$$

$$y[4]=y[3]+h(y'[x[n]+2h])+(h^2/2)y''[x[n]+2h]+(h^3/6)y'''[x[n]+2h]$$

$$y[5]=y[4]+h(y'[x[n]+3h])+(h^2/2)y''[x[n]+3h]+(h^3/6)y'''[x[n]+3h]$$

$$u=x[n]+2h$$

$$y[4]=y[1]+h((57/4)y'[u+x[n]]-(18)y'[u+x[n]+h]+(27/4)y'[u+x[n]+2h])$$
$$u=x[n]+4h$$

$$y[6]=y[2]+h((44/3)y'[u+x[n]]-(52/3)y'[u+x[n]+h]+(20/3)y'[u+x[n]+2h])$$
$$u=x[n]+6h$$

$$y[8]=y[3]+h((175/12)y'[u+x[n]]-(50/3)y'[u+x[n]+h]+(85/12)y'[u+x[n]+2h])$$
$$u=x[n]+5h$$

$$y[7]=y[4]+h((57/4)y'[u+x[n]]-(18)y'[u+x[n]+h]+(27/4)y'[u+x[n]+2h])$$
$$u=x[n]+7h$$

$$y[9]=y[5]+h((44/3)y'[u+x[n]]-(52/3)y'[u+x[n]+h]+(20/3)y'[u+x[n]+2h])$$
$$u=x[n]+9h$$

$$y[11]=y[6]+h((175/12)y'[u+x[n]]-(50/3)y'[u+x[n]+h]+(85/12)y'[u+x[n]+2h])$$
$$u=x[n]+8h$$

$$y[10]=y[7]+h((57/4)y'[u+x[n]]-(18)y'[u+x[n]+h]+(27/4)y'[u+x[n]+2h])$$
$$u=x[n]+10h$$

$$y[12]=y[8]+h((44/3)y'[u+x[n]]-(52/3)y'[u+x[n]+h]+(20/3)y'[u+x[n]+2h])$$
$$u=x[n]+12h$$

$$y[14]=y[9]+h((175/12)y'[u+x[n]]-(50/3)y'[u+x[n]+h]+(85/12)y'[u+x[n]+2h])$$