Multi-objective terminal trajectory optimization based on hybrid genetic algorithm pseudospectral method

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Abstract

During terminal guidance, the attack platform is provided with high-resolution image of the target area through the application of synthetic aperture radar (SAR). Additionally, the stealth trajectory with low observability can significantly impact mission success. This paper considers both missile-borne SAR imaging performance and stealth performance as influencing factors for terminal trajectory optimization, which is modeled as a constrained multi-objective optimization problem. With the successful application of the pseudospectral method in the solution of optimal control problems, the hybrid genetic algorithm pseudospectral optimization (HGAPO) framework is proposed. The problem is decomposed to several single-objective optimal control problems, which can generate a specific initial population for the genetic algorithm to obtain a set of Pareto-optimal solutions. Finally, the numerical simulations demonstrate the effectiveness of the proposed optimization approach compared with the benchmark scheme.

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During terminal guidance, the attack platform is provided with high-resolution image of the target area through the application of synthetic aperture radar (SAR). Additionally, the stealth trajectory with low observability can significantly impact mission success. This paper considers both missile-borne SAR imaging performance and stealth performance as influencing factors for terminal trajectory optimization, which is modeled as a constrained multi-objective optimization problem. With the successful application of the pseudospectral method in the solution of optimal control problems, a hybrid genetic algorithm pseudospectral optimization (HGAPO) framework is proposed. The problem is decomposed into several single-objective optimal control problems, which can generate a specific initial population for the genetic algorithm to obtain a set of Pareto-optimal solutions. Finally, the numerical simulations demonstrate the effectiveness of the proposed optimization approach compared with the benchmark scheme.

Introduction: For an attack platform, such as a missile, the goal during terminal guidance is to generate a trajectory with an end point close to the target position while satisfying various constraints. In order to better complete the mission, various radar technologies are usually applied during the flight, including airborne synthetic aperture radar (SAR), low observability technology [1], and so on. So far, numerous studies have introduced the radar imaging or radar detection performance indicators as objective functions and constraints in the trajectory optimization models, which have also been solved with different methods. In [2], an optimal control model for a missile with a radar imaging seeker using the Dopper beam sharpening was proposed and solved by the so-called direct shooting method. Different from [2], the similar problem was solved based on a finite-dimensional discretization approach and the sequential quadratic programming (SQP) solvers in [3], [4]. Presented a minimax optimal control problem of path planning for unmanned combat aerial vehicles in the presence of radar threat, where the radar cross section (RCS) model was considered to be an ellipsoid. In [5], a trajectory design framework based on the pseudospectral method for coupling the aircraft RCS characteristics, calculated by the physical optics (PO) method, was proposed to improve the aircraft penetration capability. In the developed radar detection model, the RCS is used as one of the influencing factors of radar detection probability to evaluate the radar detection threat performance of the trajectory.

In recent decades, in addition to traditional optimization methods based on the optimal control model, various intelligent optimization algorithms applied in trajectory optimization have also been extensively studied. Among them, the evolutionary algorithms have a great advantage in the global search of optimal trajectories. In [6], a hybrid modified teaching-learning-based particle swarm optimization initialized by the normalized step cost was proposed for solving trajectory optimization. In [7], a biased particle swarm optimization approach was proposed to handle the constrained trajectory design problem. In [8] applied the multi-objective evolutionary algorithm (MOEA) to a path planning scheme for unmanned aerial vehicles (UAVs) to balance the length of path and the margin of safety. The multi-objective UAV path optimization problem was then solved based on the famous NSGA-II [9]. In [10], the path planning for geosynchronous-UAV bistatic SAR was modeled as a constrained multi-objective optimization problem. The optimization method based on a constrained-adaptive-multi-objective-differential-evolution algorithm was proposed. In addition, a special chronological iterative search framework (CISF) was presented in [11], where the terminal trajectory optimization for SAR imaging performance was investigated.

The aim of this paper is to investigate the joint consideration of multiple radar-related properties in the terminal trajectory optimization, specifically the performance of SAR imaging and low radar observability. Such a terminal trajectory optimization is modeled as a multi-objective optimization problem, and the proposed hybrid optimization method is implemented. The hybrid scheme combines the benefits of optimal control model for solving single-objective problems and genetic algorithm for solving multi-objective problems.

Terminal Trajectory Model: Generally, the geosynchronous bistatic SAR (GEO-BiSAR) approach is commonly used in the SAR terminal guidance phase. Electromagnetic wave is transmitted from the geosynchronous satellite and received by the missile-borne SAR after being reflected by the target scene. The coordinate system of terminal trajectory model is given in Figure 1. \( \alpha \)-\( \chi \)-\( \gamma \) is the target scene local coordinate (TSLC) system and \( \alpha_1 \)-\( \chi_1 \)-\( \gamma_1 \) is the missile-body coordinate system. The point mass model with freedom of three degrees is adopted and the kinematical equations of the missile in TSLC are determined as

\[
\begin{align*}
    mV &= T \cos \alpha - X - mg \sin \theta \\
    mV \theta &= T \sin \alpha \cos \gamma + Y \cos \gamma - mg \cos \theta \\
    mV \cos \psi \theta &= -T \sin \alpha \sin \gamma - Y \sin \gamma \\
    \dot{x} &= V \cos \theta \cos \psi \\
    \dot{y} &= V \sin \theta \\
    \dot{z} &= -V \cos \theta \sin \phi
\end{align*}
\]

where \( m \) is the mass of the vehicle, \( g \) is the gravity acceleration, and \( V \) is the velocity. The flight path angle and heading angle are denoted by \( \theta \) and \( \psi \). \( \alpha \) denotes the attack angle. \( \gamma \) denotes the velocity tilt angle, which represents the roll angle based on the velocity vector axis. \( X, Y \) and \( Z \) are the thrust, drag and lift forces, respectively. \( x, y \) and \( z \) are the position coordinate components in TSLC. To simplify the analysis, a special type of geosynchronous satellite in GEO-BiSAR, geostationary satellite, is considered. Therefore, the coordinates of the geostationary satellite are constant in TSLC, represented as \((X_{GEO}, Y_{GEO}, Z_{GEO})\), where \( H_{GEO} = 35786 \) km.

In the genetic algorithm-based trajectory optimization algorithm, each individual in the population can be regarded as a complete discretized trajectory. The trajectory is encoded as a sequence of discretized control variables consisting of the attack angle and the velocity tilt angle. Furthermore, the encoded sequence introduces the discrete time interval variable \( \Delta \tau \). This approach avoids fixing the flight time of the trajectory and allows the genetic algorithm to iterate and obtain a suitable flight time of each individual. The trajectory individual is encoded as

\[
x = [u(0), \ldots, u(i), \ldots, u(N - 1), \Delta \tau],
\]

where \( u(i) = (\alpha(i), \gamma(i)) \) denotes the \( i \)th discrete control variable couple and \( N \) denotes the discrete number of the terminal trajectory.

Problem Formulation: Under the defense scenario based on radar detection, GEO-BiSAR terminal guidance not only needs to consider the SAR imaging performance, but also the stealth performance of the trajectory. Additionally, the off-target performance of the trajectory is introduced as the distance function between the end point of the trajectory and the target position. The above considerations constitute a multi-objective terminal trajectory optimization problem.

The stealth performance metric in a radar threat environment can be described as the instantaneous detection probability, which is determined by the RCS and distance of the vehicle. Low detection probability means low radar observability. Therefore, the first objective function is the cost
of average radar detection probability for terminal trajectory, which is formulated as

\[ f_1(x) = \frac{1}{N} \sum_{i=0}^{N-1} P_d(i), \]

where \( P_d \) represents the instantaneous detection probability of a specific state, which is mainly influenced by the attitude of the aircraft and further influenced by the RCS, given by

\[ P_d = \frac{1}{1 + |c_2 (R/R_{\text{max}})^3 / \sigma|^c}, \]

where \( R \) denotes the slant distance between the radar and the aerial vehicle, \( R_{\text{max}} \) denotes the maximum detection range of the radar, \( \sigma \) is the instantaneous RCS. The parameters \( c_1 \) and \( c_2 \) can be specified in advance as it depends on the specific performance and the type of the radar.

On the other hand, the missile-borne BiSAR imaging performance is subject to variations in imaging geometry and is generally evaluated with respect to the RCS, given by

\[ f_2(x) = \frac{1}{N} \sum_{i=0}^{N-1} S_{\text{cell}}(i). \]

The spatial resolution cell area can be completely determined by range resolution \( \rho_x \), azimuth resolution \( \rho_y \), and resolution direction angle \( \omega \), which is formulated as

\[ S_{\text{cell}} = \frac{\rho_x \rho_y \omega}{\sin \omega}. \]

In addition to the comprehensive consideration of the above two radar-related performance, the off-target performance of the trajectory is also important. The distance between the end point of the trajectory and the target position should be minimized to achieve a better attack effect. The corresponding objective function is expressed as

\[ f_1(x) = \| P_T - P_T^\text{m} \|, \]

where \( \| \cdot \| \) denotes the norm operator. \( P_T = (x_T, y_T, z_T) \) and \( P_T^\text{m} = (x_T, y_T, z_T) \) represent the end position of the terminal trajectory and the target position, respectively.

During the entire terminal control, the control variables should also meet the overload constraints as follows: \( \| \alpha(i) \| \leq \alpha_{\text{max}} \) and \( \| \gamma(i) \| \leq \gamma_{\text{max}} \), where \( \alpha_{\text{max}} \) and \( \gamma_{\text{max}} \) are the maximum overload of the attack angle and the velocity tilt angle, respectively. In order to improve the iterate efficiency of genetic algorithm, the objective function value constraints are introduced, which are represented by

\[ f_2(x) = f_{\text{max},k}, \quad k = 1, 2, 3. \]

Through the above analysis, the terminal trajectory optimization is modeled as a constrained multi-objective optimization problem.

**Proposed Method:** According to the above modeling and problem formulation, it can be seen that among several objective functions, except the off-target objective function, the radar-related objective functions are highly nonlinear. In this paper, NSGA-II, one of MOEAs, is adopted to solve the constructed constrained multi-objective trajectory optimization problems, as MOEAs do not require accurate gradient information and can effectively handle the nonlinear objective functions to obtain Pareto-optimal solutions. However, as shown in Equation 2, the encoding of each trajectory individual is determined by \( 2N + 1 \) variables, and the search space size will exponentially increase with the discrete number \( N \). It leads to the difficulty for genetic algorithms in finding feasible solutions that satisfy constraints in such a large search space at the beginning of the population iteration. On the other hand, it leads to the low search efficiency and bad convergence of the algorithm.

Inspired by traditional gradient-based optimization methods, a hybrid genetic algorithm pseudospectral optimization framework is proposed. The principal concept of HGAPO is to execute the MOEA based on the generated specific high quality initial population. The multi-objective terminal trajectory optimization problem is decomposed to several single-objective optimization problems to generate the initial individuals. Each single-objective trajectory optimization is modeled as an optimal control problem (OCP), which can be solved by the practical pseudospectral method. It should be noted that the off-target objective function is not used as the objective function of the optimal control problem, but is used to assist in generating a sufficient number of initial individuals. The specific algorithm details are presented as Algorithm 1.

**Algorithm 1 Multi-objective terminal trajectory optimization based on HGAPO framework**

**Input:** \( N, \ P_T, \) initial state of the trajectory (\( V_0, \theta_0, \psi_0, x_0, y_0, z_0, \)) \( k \) \( f_k \) \( C_j \), constraints \( j \) \( M \) \( G_{\text{max}} \)

1. set the initial population \( P_1 = \emptyset \)
2. generate \( M \) random positions near the target position as the trajectory end position \( P_T^\text{m} = (m = 1, \ldots, M) \), while subject to the off-target function value constraint, i.e. \( \| P_T^m - P_T^\text{m} \| \leq f_{\text{max},k} \)
3. \% generate the specific initial population using OCP
4. \% for \( m = 1 \) to \( M \) do
5. \% set the objective function of the OCP \( f = f_{m+1} \)
6. \% construct the OCP such that minimize the objective function \( f \) and subject to the kinematical constraints as Equation 1, path constraints \( C_{\text{OCP}} \), end position as \( P_T^m \), and initial state as \( (V_0, \theta_0, \psi_0, x_0, y_0, z_0) \)
7. \% apply pseudospectral method to solve the OCP to obtain the optimal control \( \alpha(t) \) and \( \gamma(t) \), also the flight time \( T \)
8. \% \( \Delta t = T/N \)
9. \% \( u(i) = (\alpha(i\Delta t), \gamma(i\Delta t)), \quad j = 0, \ldots, N - 1 \)
10. \% \( x = [x(0), \ldots, u(i), \ldots, u(N - 1), \Delta t] \)
11. \( P_1 = P_1 \cup \{x\} \)
12. \% end for
13. \% solve the multi-objective optimization using NSGA-II
14. \% for \( G = 1 \) to \( G_{\text{max}} \) do
15. \% generate the offspring population \( Q_G \) with respect to the current population \( P_G \) using the recombination and mutation operators.
16. \% \( R_G = P_G \cup Q_G \)
17. \% generate the trajectory for each individual in \( R_G \) using Runge-Kutta method.
18. \% evaluate the objective functions \( f \) and constraints \( C_j \) for each trajectory.
19. \% select \( M \) individuals from \( R_G \) to form the new population \( G_{\text{opt}} \) based on the non-dominated sorting, i.e. NSGA-II.
20. \% \( P_{G+1} = G_{\text{opt}} \)
21. \% end for

**Simulation Results:** Two test cases of terminal trajectory optimization with different position parameter settings are designed. The simulation results are compared with the solutions solved by NSGA-II only to evaluate the performance of the proposed HGAPO. The simulation parameters are listed in Table 1, mainly including the trajectory parameters, SAR parameters, radar detection parameters and so on. The detection radar position is denoted by \( (x_D, y_D, z_D) \). \( T_c \) denotes the SAR integration time, \( f_c \) and \( B_i \) are the Carrier frequency and bandwidth of the SAR, respectively.

**Table 1. Simulation parameters.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>((x_T, y_T, z_T))</td>
<td>(15, 0, 5) km</td>
<td>(12, 0, 10) km</td>
</tr>
<tr>
<td>((x_R, y_R, z_R))</td>
<td>(10, 0, 5) km</td>
<td>(5, 0, 5) km</td>
</tr>
<tr>
<td>((x_D, y_D, z_D))</td>
<td>(0, 10, 0) km</td>
<td>(2000 m/s, 0°, 0°)</td>
</tr>
<tr>
<td>((R_{\text{max}}, c_1, c_2))</td>
<td>(10 km, 1, 1, 1)</td>
<td>(10 s, 1.25 GHz, 100 MHz)</td>
</tr>
<tr>
<td>((\alpha_{\text{max}}, \gamma_{\text{max}}))</td>
<td>((30°, 30°))</td>
<td>((0.7, 20 m^2, 100 m))</td>
</tr>
</tbody>
</table>

On the other hand, for the genetic algorithm, the trajectory discrete number is set to be \( N = 100 \), population size is \( M = 100 \), and the
maximum generation number is $G_{\text{max}} = 2000$. The simulated binary crossover (SBX) and the polynomial mutation (PM) with the distribution indexes as $\eta_c = 15$ and $\eta_m = 10$, respectively, are adopted. To evaluate the performance of the Pareto front (PF), obtained the hyper-volume (HV) index is generally considered and can be calculated as $HV(S) = \text{Leb} \left( \bigcup_{x \in S} \{ f_1(x), r_2 \} \times \cdots \times \{ f_K(x), r_K \} \right)$, where $S$ is the PF and Leb(-) denotes the Lebesgue measure. $r = (r_1, \cdots, r_K)^T$ is a predefined reference vector in the objective space and the superscript $T$ is the transpose operator. Specifically, the reference vector is set to be $r = (1, 30, 150)^T$ in this work.

The objective space results of Pareto-optimal trajectory solutions obtained by the proposed HGAPO on the two test cases are illustrated in Figure 2 and Figure 3, which are compared with the benchmark scheme NSGA-II-only. The statistical results are listed in Table 2. It can be observed that, compared with NSGA-II-only, the PF obtained by HGAPO framework has a larger HV value on both two cases, which means that it is closer to the real PF and has a better diversity performance. As can be analyzed from the minimum, the radar-related objective functions can be iterated to the smaller values after using the HGAPO method. It reflects that the OCP solutions based on pseudospectral method generates the more excellent initial population, so that the genetic algorithm does not have to spend too much iteration time searching for feasible solutions and optimizing them. Therefore, the convergence with the same $G_{\text{max}}$ is improved greatly after integrating the OCP. Specifically, HGAPO achieves the greatly smaller average radar detection threats on both cases and smaller average SAR spatial resolution cell area on case 2. On the other hand, HGAPO obtains the larger variance of the SAR imaging objective function, which can reflect the diversity of the solutions. It should be noted that the off-target performance is not improved because it is not optimized as an objective function during generating the initial population, but used to generate the individuals with different attack precision.

Conclusion: In this paper, the terminal trajectory optimization jointly considering both the SAR imaging performance and the penetration stealth performance is studied. The terminal trajectory optimization is modeled as a constrained multi-objective optimization problem and a novel HGAPO framework is proposed. The simulation results on the two cases demonstrate the better performance than the benchmark.

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Table 2. Statistics of the objective function values on the two cases.

<table>
<thead>
<tr>
<th>Method</th>
<th>Statistics</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NSGA-II-only (case 1)</td>
<td>Maximum</td>
<td>0.4907</td>
<td>14.1375</td>
<td>97.4767</td>
</tr>
<tr>
<td></td>
<td>Minimum</td>
<td>0.2927</td>
<td>4.9307</td>
<td>0.2570</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>0.3214</td>
<td>6.9130</td>
<td>36.6495</td>
</tr>
<tr>
<td></td>
<td>Variance</td>
<td>0.0019</td>
<td>6.1185</td>
<td>947.3150</td>
</tr>
<tr>
<td>HV</td>
<td></td>
<td>2629.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HGAPO (case 1)</td>
<td>Maximum</td>
<td>0.1682</td>
<td>19.5166</td>
<td>99.7901</td>
</tr>
<tr>
<td></td>
<td>Minimum</td>
<td>0.1464</td>
<td>1.9664</td>
<td>0.8735</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>0.1527</td>
<td>7.1443</td>
<td>50.7088</td>
</tr>
<tr>
<td></td>
<td>Variance</td>
<td>4.0482e-05</td>
<td>30.8568</td>
<td>866.3115</td>
</tr>
<tr>
<td>HV</td>
<td></td>
<td>3549.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NSGA-II-only (case 2)</td>
<td>Maximum</td>
<td>0.1473</td>
<td>18.7049</td>
<td>99.8409</td>
</tr>
<tr>
<td></td>
<td>Minimum</td>
<td>0.1248</td>
<td>3.0718</td>
<td>0.0456</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>0.1305</td>
<td>6.6260</td>
<td>50.6887</td>
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<tr>
<td></td>
<td>Variance</td>
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<td>18.2198</td>
<td>869.7222</td>
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<tr>
<td>HV</td>
<td></td>
<td>3505.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HGAPO (case 2)</td>
<td>Maximum</td>
<td>0.4497</td>
<td>17.9725</td>
<td>99.7958</td>
</tr>
<tr>
<td></td>
<td>Minimum</td>
<td>0.3838</td>
<td>5.8894</td>
<td>0.5543</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>0.4063</td>
<td>10.3901</td>
<td>51.2058</td>
</tr>
<tr>
<td></td>
<td>Variance</td>
<td>0.0004</td>
<td>11.9901</td>
<td>780.8165</td>
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References