On Optimization of Multi-RIS enabled SWIPT-IoT with Non-linear Energy Harvesting

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Abstract—Reconfigurable Intelligent Surfaces (RIS) stand as a groundbreaking technology within the wireless communication domain, comprising passive reflective elements capable of manipulating electromagnetic waves to improve signal quality and network performance. When combined with Simultaneous Wireless Information and Power Transfer (SWIPT), RIS-SWIPT addresses the energy constraints of Internet-of-Things (IoT) devices, potentially extending their operational life, making it particularly beneficial for hard-to-reach IoT devices. The network coverage may be expanded with the inclusion of multiple RISs into SWIPT-IoT systems. This study explores the integration of a practical energy harvesting (EH) mechanism and multiple RIS within the framework of Time-Switching (TS) and Power-Splitting (PS) protocols of SWIPT-IoT. The paper presents and resolves distinct optimization problems for information rate and EH maximization under TS/PS protocols, obtaining closed-form solutions using Karush-Kuhn-Tucker (KKT) conditions, respectively. To address the complex task of jointly optimizing the TS/PS ratio and transmit power, an alternating optimization-based algorithm is proposed. Numerical interpretations and comparisons are provided for TS and PS scenarios, taking into account factors such as the number of RIS surfaces, RIS element count, separation distance, transmit power constraints, desired EH, and angular placement. The insights and findings derived from this research contribute to the practical application and understanding of SWIPT in diverse wireless communication scenarios.

Index Terms—Reconfigurable Intelligent Surfaces (RIS), Multi-RIS, Simultaneous Wireless Information and Power Transfer (SWIPT), Energy-Harvesting (EH), Internet-of-Things (IoT), Power-splitting (PS), Time-switching (TS)

I. INTRODUCTION

The burgeoning field of the Internet-of-Things (IoT) has ushered in an era of unprecedented connectivity and automation, with billions of devices poised to reshape the way we interact with our environment. The need for advanced wireless networks that can sustain these energy-constrained, low-power sensors and actuators is paramount. Sixth Generation (6G)’s ultra-reliable, low-latency communication capabilities, coupled with its energy-efficient design, hold the key to unlocking the true potential of energy harvesting (EH) in IoT, enabling applications ranging from remote environmental monitoring to precision agriculture. A transformative technology in 6G that addresses the bottle-neck of the massive and seamless IoT connectivity with battery-saving capabilities, is Reconfigurable Intelligent Surface (RIS) $^1$ It is comprised of a multitude of tiny passive reflecting elements made up of meta-materials, that allows for dynamic control of electromagnetic waves’ properties, including amplitude, phase, and direction $^2$. RIS holds the promise of significantly improving spectrum, coverage, and energy efficiency. This introductory glimpse into RIS sets the stage for exploring its diverse applications in evolving future communication systems $^3$. The self-sufficiency of IoT devices required for medical applications requires EH solutions that are convenient, safe, uninterruptible and flexible. The conventional EH options like wind, solar etc. are highly unpredictable, thus harnessing electromagnetic waves and convert them to usable direct current (DC) serves our purpose. A green communication solution is Simultaneous Wireless Information and Power Transfer (SWIPT) capable of data communication and battery recharging to occur concurrently $^4$. SWIPT has the potential to extend the operational lifespan of battery-powered devices, reduce the need for frequent battery replacement or recharging, an ideal choice for wireless body area networks (WBAN).

A. Related work

SWIPT is a well explored area with two major protocols namely Time-Switching (TS) and Power-Splitting (PS). TS characterises for simpler receiver architecture while PS rules with higher data rate and EH even in highly constrained environment $^5$. The essential EH part in SWIPT is modeled as linear or linear-variants i.e., constant-linear, piece-wise linear and constant-linear-constant $^6$, $^7$. In practice the RF-to-DC conversion process has rectenna (rectifier-antenna combination), a non-linear device, thus practical EH model is explored by authors in $^8$.

When RIS is integrated with SWIPT, they offer the advantages of improved energy efficiency by lowering power consumption while simultaneously enhancing data rates $^9$. The amalgamation of RIS-SWIPT has been explored in literature with the adopted EH model as linear or linear-like $^{10}$, $^{11}$. There are few works which considers accurate non-linear EH (NL-EH) but divide single RIS’s elements into unit segments performing either EH or reflection operation $^{12}$, hence making an autonomous RIS without the need of external power supply. With the similar context in $^{13}$, RIS unit cells are divided into two working modes namely S mode

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$^1$This work is the extension of [1] in which a simple single RIS TS-SWIPT model engaging NL-EH with a primary EH maximization problem along with iterative algorithmic solution is considered.

$^2$References are included for more details.

$^3$Note the importance of RIS in future wireless communication systems.


(simultaneous reflection and amplification) and H mode (only EH process). A step ahead is the multiple RIS contributing to pass the information to the user node, enhancing the coverage. This invokes the selection strategy of the RISs. Authors in [14] coined two terms as exhaustive RIS approach (ERA) and opportunistic RIS approach (ORA), where ERA involves all RIS with all its elements to be the part of the communication [15], while for ORA the best RIS giving highest Signal-to-Noise ratio (SNR) is selected out of all RISs [16]. To save the resources, selection planning of the RIS in case of distributed RIS scenarios becomes an essential parameter, authors in [17] worked on RIS selection for vehicular communication based on the highest SNR in the first hop from vehicle to the best RIS, also authors in [18] gives selection approach based on the location providing highest end-to-end SNR with product-scaling and sum-scaling path loss models. The energy efficiency improvement by dynamically turning ON-OFF multiple RISs along with jointly optimizing phase shift and beamforming matrix is reported in [19]. A wireless powered communication network (WPCN) adopted to the distributed RISs is reported in [20] which involves EH at the device from access point (AP) via first RIS then device uses harvested energy to transmit to AP via another RIS. The EH model used here is again linear and only performance related analysis has been carried out.

### B. Motivation and contribution

The literature suggests a scarcity of studies exploring accurate non-linear EH models in SWIPT communication with RIS. Limited literature addresses the SWIPT protocol (encompassing TS and PS), involving a multi-RIS configuration with an accurate EH model for optimizing EH and data rate. This paper aims to bridge this research gap, outlining its main contributions as listed below.

1) **Practical EH model**: The IoT node utilizes the SWIPT protocol based on considered TS/PS architecture, incorporating a logistic NL-EH model. This model effectively captures the dynamic, non-linear characteristics of RF energy conversion circuits, which differs from the typically assumed constant behavior of various linear EH models, including linear, piece-wise, or CLC models.

2) **Multi-RIS selection**: To illustrate the adaptability of positioning within multi-RIS setups, we examine the use of non-identical and independently distributed (n.i.i.d.) RIS channels as in practical scenarios. The RIS surface selection scheme is based on the strongest channel implying to the RIS providing best SNR among others.

3) **Closed form solution**: Two distinct problem statements for the rate and EH maximization considering TS or PS protocol at a time, are collectively devised with the set of constraints for optimized transmit power and TS/PS ratio. A closed form solution for optimization variables are obtained by invoking Karush-Kuhn-Tucker (KKT) conditions.

4) **Algorithmic insights**: To optimize the TS/PS ratio and transmit power for the same maximization problem of rate and EH, we devise an algorithm based on alternating optimization algorithm (AOA) which uses the divide-and-conquer approach to simplify the formulated challenging problem.

5) **Numeric interpretation**: The TS and PS scenarios are compared for rate and EH maximization along with the KKT and the proposed algorithmic results. For the distributed RIS, a circular emulation environment is considered with the angular placements of RIS surfaces. Inferences are drawn based on the variation of number of RIS surfaces, number of RIS elements in each surface, separation distance, maximum allowed transmit power, desired EH, and angular placement.

The paper is organized as follows. Section II gives system model encapsulating signal modelling, RIS selection and SWIPT. Section III and Section IV presents maximization problem of data rate and EH respectively. Section V provides algorithmic solution. The results and related discussions are scripted in Section VI. The paper finally rests with concluding remarks in Section VII.

### II. SYSTEM MODEL

We investigate a communication system that utilizes a single source base station (BS), a single IoT user equipment (UE) node, and a set of $N$ RISs positioned at random intervals from each other. Each of these RISs, denoted as $\{R_n\}_{n=1}^{N}$, has a total of $\{M_n\}_{n=1}^{N}$ reflecting elements. Both the BS and the UE are equipped with a single antenna, making this system operate in a half-duplex (HD) mode. It is important to note that there is no direct communication possible between the BS and UE due to the presence of physical obstacles. The Fig. 1 depicts communication path starting from the BS and then being directed towards the $i^{th}$ element of the $n^{th}$ RIS ($R_n$), where $i$ falls within the range of $[1, M_n]$, and ultimately reaching the UE. We are examining a scenario of low mobility, where RISs are strategically placed on building walls and billboards, ensuring reliable services to areas that would otherwise be unserved. This arrangement ensures that there is no interference among the RISs. As a result, the reflections from the RISs do not interfere when they reach the UE, as discussed in [14]. The IoT UE node possesses
the ability to engage in concurrent operations, encompassing SWIPT, enabling it to concurrently harvest energy and information decoding (ID) in accordance to either TS or PS receiver architecture.

We examine the block Rayleigh fading channels, where the channel coefficients remain unchanged within a given block with duration $T = N_s T_s$ seconds. Here, $N_s$ is the number of signal symbols sent by BS with each symbol having $T_s$ duration. Considering $\alpha$ and $\beta$ as TS and PS ratios respectively. For TS protocol, EH is performed for $\alpha T_s$ duration followed by ID for the remaining time. While for PS protocol, received power is split for EH and ID, performing EH for $\sqrt{\beta}$ times received power and rest of the power for ID. An important thing to note here is that the EH model used in both architectures will be sigmoid/logistic function based non-linear EH model due to its mathematically practical replication of the RF-to-direct current energy conversion. A lot of work in literature has proved its nearness to the practical model and reasons for failure of other linear models. The mathematical in literature has proved its nearness to the practical model and reasons for failure of other linear models. The mathematical model followed by various signals will be described in the next section.

A. Signal modeling

We define the signal received by the UE via $M_n$ elements of $R_n$ RIS from the BS. Let $\hat{h}_{R_n(i)}$ and $\hat{g}_{R_n(i)}$ are assumed to be the identical and independently distributed (i.i.d.) complex channels from BS $\rightarrow i^{th}$ element of $R_n$ RIS, and $j^{th}$ element of $R_n$ RIS $\rightarrow$ UE, where $i = 1, 2, \ldots M_n$ These channels have magnitude of $\mid \hat{h}_{R_n(i)} \mid$ and $\mid \hat{g}_{R_n(i)} \mid$, with $e^{j \phi_{R_n(i)}}$ and $e^{j \psi_{R_n(i)}}$ as phase angles respectively. The channel magnitudes follow Rayleigh distribution, while phase angles following uniform distribution of $[-\pi, \pi]$ [1], [21], [22]. Different RIS surfaces are assumed to be well distant apart, consequently the channels associated with them are i.i.d in nature [14]. For instance, $\hat{h}_{R_n(i)}$ and $\hat{g}_{R_n(i)} \forall n = 2, \ldots, N$ are i.i.d. The received signal at U via combined reflected signals from all $M_n$ elements of $n^{th}$ RIS ($R_n$) is given as

$$y_n = \sqrt{P_b} \left( \sum_{i=1}^{M_n} \hat{g}_{R_n(i)} \eta_{R_n(i)} e^{j \phi_{R_n(i)}} \hat{h}_{R_n(i)} \right) s + w_n, \quad (1)$$

where $\eta_{R_n(i)}$ is the magnitude of the reflection coefficient ranging from 0 to unity, and $\phi_{R_n(i)} \in [-\pi, \pi]$ is the provided phase shift by the $i^{th}$ element of $R_n$ RIS. This phase shift plays essential role in directing incoming signal to the desired user and makes it a distinguishing feature of RIS. The transmitting symbol is given by $s$ with $\mathbb{E}[|s|^2] = 1$, $P_b$ power available at BS for transmission and associated i.i.d. additive white Gaussian noise (AWGN) at the receiver node UE is $w_n \sim CN(0, W_0^2)$ with zero mean and $W_0^2$ variance. The Signal-to-Noise (SNR) at UE is given by

$$\Gamma_n = \frac{P_b \left( \sum_{i=1}^{M_n} \eta_{R_n(i)} \hat{g}_{R_n(i)} \eta_{R_n(i)} e^{j \phi_{R_n(i)}} \hat{h}_{R_n(i)} \right)^2}{W_0^2} \quad (2)$$

Additionally, perfect Channel State Information (CSI) is present globally at the BS as well as at RISs controllers. To acquire this CSI, we employ channel estimation techniques suggested for multiple RIS systems, as outlined in [23], [24]. Another crucial step to enhance signal reception by maximizing end-to-end SNR is by generating a phase adjustment by the respective RIS controller, in order to eliminate phase errors, expressed as $(\psi_{R_n(i)} + \phi_{R_n(i)} + \psi_{R_n(i)} = 0)$ [10], [21], [22]. The resulting maximized SNR is then reduced to

$$\Gamma_n = \frac{P_b}{W_0^2} \left( \sum_{i=1}^{M_n} \eta_{R_n(i)} \hat{g}_{R_n(i)} \eta_{R_n(i)} \hat{h}_{R_n(i)} \right)^2 = \frac{P_b Z_n}{W_0^2}. \quad (3)$$

Rephrasing the equation in [3], the summation part consisting of collective channel coefficients of all $M_n$ RIS elements termed as ‘channel term’ is written as $Z_n = \left( \sum_{i=1}^{M_n} \eta_{R_n(i)} \hat{g}_{R_n(i)} \eta_{R_n(i)} \hat{h}_{R_n(i)} \right)$. Now the question is, which RIS surface is to be selected and how the selection is being processed. It will be answered in the next section.

B. RIS surface selection strategy

The selection strategy becomes critical for optimal performance, specially when numerous RIS surfaces are in vicinity. In order to achieve optimum performance, the RIS surface providing highest SNR at the UE will be the wise decision. By [3], the SNR depends majorly on the ‘channel term’ ($Z_n$) involving $n^{th}$ RIS surface for a fixed transmit power. Thus, the selection criteria being highest SNR, in fact implies stronger channel condition for the ‘suitable’ RIS with maximum ‘channel term’. To achieve said criteria, initially ‘channel term’ for all RIS surfaces are assumed to be known at the UE by the channel estimation techniques (perfect CSI). The selection process undertaken by UE is mathematically given by

$$Z_{n^*} = \max_{n=1, \ldots, N} \left\{ \left( \sum_{i=1}^{M_n} \eta_{R_n(i)} \hat{g}_{R_n(i)} \eta_{R_n(i)} \hat{h}_{R_n(i)} \right)^2 \right\} \quad (4)$$

The received signal and SNR with the selected RIS ($n^* = arg(Z_{n^*})$), is modified to

$$y_{n^*} = \sqrt{P_b} \left( \sum_{i=1}^{M_{n^*}} \hat{g}_{R_n(i)} \hat{h}_{R_n(i)} \right) s + w, \quad (5)$$

$$\Gamma_{n^*} = \frac{P_b Z_{n^*}}{W_0^2}. \quad (6)$$

The selected RIS surface is used for SWIPT operation involving EH and ID concurrently, which will be discussed in next subsection.

C. SWIPT

The UE is capable of SWIPT operation being equipped with either TS or PS based receiver. We shall compare and contrast both the architectures and later concludes the best among them for our considered multi-RIS scenario. The information rate for $(1 - \alpha)$ time duration in the TS scenario, and for PS it is $\sqrt{\beta}$ times of received power, mathematically given in [7] and [8] respectively [1], [25].

$$R_u^T = (1 - \alpha) \log_2(1 + \Gamma_n) \quad (7)$$

$$R_u^P = \log_2(1 + (1 - \beta) \Gamma_n) \quad (8)$$

3The selected RIS changes with the change in UE position. This holds good even for multiple UE scenarios, where any one UE is active at a time in random manner.
The sigmoid/logistic function based practical EH model capturing the non-linear energy conversion efficiency (RF to battery current) represents the TS/PS EH part [1], [8], [25], mathematically given as

\[ E_h^T = \frac{(\alpha N_s T_s) E'}{1 - \chi} \left( \frac{1}{1 + \exp(-aP_h Z_{n*} + ab)} - \chi \right), \]  

\[ E_h^P = \frac{(N_s T_s) E'}{1 - \chi} \left( \frac{1}{1 + \exp(-\beta a P_h Z_{n*} + ab)} - \chi \right), \]

where \( \chi \) depends on electronic circuit elements \( a \) and \( b \), associated with the capacitor and diode turn-ON voltage respectively. It is defined as \( \chi = (1 + \exp(ab))^{-1} \). The constant \( E' \) represents the maximum harvested energy when the EH circuit is operating in saturation. The values of the constant parameters \( E', a, b \) are typically determined through the utilization of a curve-fitting tool applied to the analytical data, as discussed in [26].

To improve the clarity of notation, we present the following triad (rate, EH, and TS/PS ratio) \((R, E_h, \Phi)\) in the rest of the discussions.

\[ (R_u, E_h, \Phi) := \begin{cases} (R_u^T, E_h^T, \alpha) : \text{For TS} \\ (R_u^P, E_h^P, \beta) : \text{For PS} \end{cases} \]  

(11)

In the subsequent section, we articulate the primary problem concerning the maximization of rate and EH distinctly and present their proposed solutions, respectively.

III. DATA RATE MAXIMIZATION

We formulate the rate maximization problem while optimizing transmit power and TS/PS factor along with constraints on rate, EH, maximum transmit power \( P_{\text{max}} \) and available range of TS/PS ratio. It is mathematically expressed as

\[ \text{(P1): maximize } R_u(\Phi, P_b) \]  

subject to:

\[ R_u \geq R_{th}, \]  

\[ E_h \geq \xi_{th}, \]  

\[ 0 < P_b \leq P_{\text{max}}, \quad 0 < \Phi < 1, \]

(12) \hspace{1cm} (13) \hspace{1cm} (14) \hspace{1cm} (15)

where the demanded rate and EH are \( R_{th} \) and \( \xi_{th} \) respectively. The choice of TS/PS is taken care of by the notations and the expressions related to (11).

A. Optimized solution for TS

Herein, we address the problem (P1) for the TS receiver invoking (11)-(15). This problem is a non-linear programming challenge that entails simultaneous computation of \( \alpha \) and \( P_b \), making it difficult to obtain an exact solution. Given that the constraints exhibit partial convexity when the other variables are held constant, we consider employing the KKT conditions to address this issue [27]. For KKT, the initial step is to find the Lagrangian dual of (P1) for TS case, given by

\[ L(\alpha, P_b; \Lambda) = R_u^T + \lambda_1(R_u^T - R_{th}) + \lambda_2(E_h^T - \xi_{th}) + \lambda_3(P_{\text{max}} - P_b) + \lambda_4(1 - \alpha). \]  

(16)

To achieve (local or global) optimality, it is necessary for \( \Delta L(\alpha, P_b; \Lambda) = 0 \) to be satisfied. Consequently, we can express the equations that ensure the fulfillment of the optimal conditions as follows

\[ \frac{\partial L(\alpha, P_b; \Lambda)}{\partial \alpha} = \frac{\partial L(\alpha, P_b; \Lambda)}{\partial P_b} = 0, \]  

(17)

\[ (1 - \alpha) Z_{n*} (1 + \lambda_1) + \lambda_3(\theta P_b Z_{n*} \exp(\Sigma)) - \lambda_3 = 0, \]  

(18)

\[ \log_2\left( 1 + \frac{P_b Z_{n*}}{W_0^2} \right) (1 + \lambda_4) + \lambda_3 (\theta P_b Z_{n*} \exp(\Sigma)) - \lambda_4 = 0. \]  

(19)

Here \( \Sigma = -a P_b Z_{n*} + ab \). The feasible conditions include, \( R_u^T - R_{th} \geq 0, E_h^T - \xi_{th} \geq 0, P_{\text{max}} - P_b \geq 0, \) and \( 1 - \alpha > 0 \). The complimentary slackness conditions are stated as \( \lambda_1(R_u^T - R_{th}) = 0, \lambda_2(E_h^T - \xi_{th}) = 0, \lambda_3(P_{\text{max}} - P_b) = 0, \) and \( \lambda_4(1 - \alpha) = 0 \). The Lagrangian variables along with optimizing variables must be a positive quantity i.e., \( \Lambda, \alpha, P_b > 0 \). From all the KKT complementary slackness condition, it is obvious that \( \lambda_4 \) must be zero, implying \( \alpha \neq 1 \). This leaves us with eight different possibilities (see Appendix A) for optimal solution, out of which the final (potential) solutions are provided in Theorem 1 and Theorem 2.

Theorem 1: If \( \lambda_1 = 0 \Rightarrow (R_u^T - R_{th}) \neq 0; \lambda_2 \neq 0 \Rightarrow (E_h^T - \xi_{th}) = 0; \lambda_3 = 0 \Rightarrow (P_{\text{max}} - P_b) = 0, \) and the optimal values are

\[ P_b^* = P_{\text{max}}, \]  

\[ \alpha^* = \frac{\xi_{th}(1 - \chi)}{TE'\left(1 + \exp(-aP_b Z_{n*} + ab) - \chi\right)}. \]

(20) \hspace{1cm} (21)

Proof: See Appendix A.

Theorem 2: If \( \lambda_1 \neq 0 \Rightarrow (R_u^T - R_{th}) = 0; \lambda_2 \neq 0 \Rightarrow (E_h^T - \xi_{th}) = 0; \lambda_3 \neq 0 \Rightarrow (P_{\text{max}} - P_b) = 0, \) and the optimal values are

\[ P_b^\dagger = P_{\text{max}}, \]  

\[ \alpha^\dagger = \min(\alpha_1, \alpha_2). \]

(22) \hspace{1cm} (23)

where

\[ \alpha_1 = 1 - \frac{R_{th}}{\log_2\left(1 + \frac{P_b Z_{n*}}{W_0^2}\right)}, \]

\[ \alpha_2 = \frac{\xi_{th}(1 - \chi)}{TE'\left(1 + \exp(-aP_b Z_{n*} + ab) - \chi\right)}. \]

(24)

Proof: See Appendix A.

It is worth noting that a KKT point may not only represent a local optimum but could also denote a saddle point or even a point of maximum value.

The following section provides optimized solution for considered PS case for rate maximization.

B. Optimized solution for PS

We tackle the problem (P1) for the PS mechanism, referring to equations (11) through (15). To address the complex non-convex problem, we propose employing the KKT based solutions for determination of optimized \( \beta \) and \( P_b \). The Lagrangian dual for problem (P1) in the PS case is

\[ L(\beta, P_b; \nu) = R_u^P + \mu_1(R_u^P - R_{th}) + \mu_2(E_h^P - \xi_{th}) + \mu_3(P_{\text{max}} - P_b) + \mu_4(1 - \beta). \]

(25)

The local optimality is satisfied by \( \Delta L(\beta, P_b; \nu) = 0 \) given by

\[ \frac{\partial L(\beta, P_b; \nu)}{\partial P_b} = \frac{\partial L(\beta, P_b; \nu)}{\partial \beta} = 0, \]

(26)
\[
\frac{(1 - \beta)\Sigma'(1 + \mu_1)}{\ln(2)} + \left(\frac{\mu_2\beta T E'\Sigma''}{(1 - \chi)}\right) - \mu_3 = 0, \tag{27}
\]
\[
-W^2_P\frac{\Sigma'}{\ln(2)}(1 + \mu_1) + \left(\frac{\mu_2 T E' P_b \Sigma''}{(1 - \chi)}\right) - \mu_4 = 0. \tag{28}
\]
Here \(\Sigma' = \frac{Z_{E'}}{W^2_D + (1 - \beta)P_b Z_{n^*}}\) and \(\Sigma'' = \frac{a Z_{E'} \exp(-a \beta P_b Z_{n^*} + ab)}{(1 + \exp(-a \beta P_b Z_{n^*} + ab))^2}\).

The feasible conditions include, \(R_u^P - R_{th} \geq 0, E_b^P - \xi_{th} \geq 0, P_{max} - P_b \geq 0\), and \(1 - \beta > 0\). The complimentary slackness conditions are \(\mu_1(R_u^P - R_{th}) = 0, \mu_2(E_b^P - \xi_{th}) = 0,\)
\(\mu_3(P_{max} - P_b) = 0\), and \(\mu_4(1 - \beta) = 0\). The variables \(\nu, \beta, P_b\) must be positive, considering all KKT conditions gives \(\mu_4 = 0\) and \(\beta \neq 1\). All the possibilities are given in Appendix A, the concise solution is given by Theorems 3 and 4 as

\textbf{Theorem 3}: If \(\mu_1 = 0 \Rightarrow (R_u^P - R_{th}) \neq 0; \mu_2 \neq 0 \Rightarrow (E_b^P - \xi_{th}) = 0; \mu_3 \neq 0 \Rightarrow (P_{max} - P_b) = 0\), and the optimal values are
\[
P_b^* = P_{max}, \tag{29}
\]
\[
\beta^* = \frac{1}{P_{max}Z_{n^*}} \left( b - \frac{1}{\alpha} \ln \left( \frac{(1 - \chi)(TE' - \xi_{th})}{(1 - \chi + \chi TE')} \right) \right). \tag{30}
\]
\textbf{Proof}: See Appendix A.

\textbf{Theorem 4}: If \(\mu_1 \neq 0 \Rightarrow (R_u^P - R_{th}) = 0; \mu_2 \neq 0 \Rightarrow (E_b^P - \xi_{th}) = 0; \mu_3 \neq 0 \Rightarrow (P_{max} - P_b) = 0\), and the optimal values are
\[
P_b^* = P_{max}, \tag{31}
\]
\[
\beta^* = \min(\beta_1, \beta_2). \tag{32}
\]
where
\[
\beta_1 = 1 - \frac{W^2_D(R_{th} - 1)}{P_{max} Z_{n^*}},
\]
\[
\beta_2 = \frac{1}{P_{max} Z_{n^*}} \left( b - \frac{1}{\alpha} \ln \left( \frac{(1 - \chi)(TE' - \xi_{th})}{\xi_{th}(1 - \chi + \chi TE')} \right) \right). \tag{33}
\]
\textbf{Proof}: See Appendix A.

Considering the solutions for problem (P1) provided by theorem 1 and 2 for TS and theorem 3 and 4 for PS, the maximized information rate is given by optimized variables \(P_b^*\) and \(\Phi^*\) as \(R_u(\Phi^*, P_b^*)\). The classic trade-off of rate and EH for SWIPT will be explained in next section when maximizing EH for both TS and PS protocols.

IV. EH MAXIMIZATION

We define the problem of maximizing EH while optimizing transmit power and TS/PS ratio. This is done under various constraints, including those related to rate, EH, \(P_{max}\) and the permissible ratio of TS/PS ratio. This formulation is expressed as

\textbf{(P2): maximize} \(\Phi, P_b\) \quad \begin{align*}
E_h(\Phi, P_b) \tag{34}
\end{align*}
\textbf{subject to : } \begin{align*}
(13) - (15) \tag{35}
\end{align*}

The constraints for (P2) remain the same as for Problem (P1). The determination of the TS/PS is managed through the notations and expressions associated with (11).

A. Optimized solution for TS

This section addresses (P2) for the TS receiver considering (11), (34)- (35). The problem is non-convex in nature due to the non-linear sigmoid objective function and finding an exact solution to it, is quite cumbersome. KKT has proven to provide near-optimal solutions for such non-convex problems, hence we utilize it for (P2). As an initial step, we need to find the Lagrangian dual for TS case in (P2), given by
\[
\mathcal{L}(\alpha, P_b; \lambda) = E_b^T + \lambda_1(R_u^T - R_{th}) + \lambda_2(E_b^T - \xi_{th}) + \lambda_3(P_{max} - P_b) + \lambda_4(1 - \alpha). \tag{36}
\]

The optimality condition, whether local or global, is fulfilled by \(\Delta \mathcal{L}(\alpha, P_b; \lambda) = 0 \Rightarrow \frac{\partial \mathcal{L}(\alpha, P_b; \lambda)}{\partial P_b} = \frac{\partial \mathcal{L}(\alpha, P_b; \lambda)}{\partial \alpha} = 0\). Solving further
\[
\frac{e^{\Sigma}(1 + \lambda_1)\alpha TE' a Z_{n^*} \exp(-\alpha Z_{n^*} \exp(-\alpha Z_{n^*}))}{(1 - \chi)(1 + e^{\Sigma})^2} + \frac{\lambda_2(1 - \alpha) Z_{n^*}}{W^2_D + P_b Z_{n^*}} \ln(2) - \lambda_3 = 0, \tag{37}
\]
\[
\frac{(1 + \lambda_1) TE' (1 - \alpha)}{(1 - \chi)} - \lambda_2 \log_2 \left( \frac{1 + P_b Z_{n^*} \ln(2)}{W^2_D} \right) = \lambda_4 = 0. \tag{38}
\]

The feasibility and complementary conditions are: \(R_u^T - R_{th} \geq 0, E_b^T - \xi_{th} \geq 0, \lambda_1 = 0 \Rightarrow \alpha < 1\). Thus, \(\lambda_1 = 0 \Rightarrow \alpha = 1\). The Theorems 5 and 6 represent the cases which satisfies above conditions and provide a sub-optimal (or optimal) solution.

\textbf{Theorem 5}: If \(\lambda_1 \neq 0 \Rightarrow (R_u^T - R_{th}) = 0; \lambda_2 = 0 \Rightarrow (E_b^T - \xi_{th}) = 0; \lambda_3 = 0 \Rightarrow (P_{max} - P_b) = 0\), the the optimal values are
\[
P_b^* = P_{max}, \tag{39}
\]
\[
\alpha^* = 1 - \frac{R_{th}}{\log_2 \left( 1 + \frac{P_b Z_{n^*}}{W^2_D} \right)}, \tag{40}
\]
\textbf{Proof}: See Appendix B.

\textbf{Theorem 6}: If \(\lambda_1 = 0 \Rightarrow (R_u^T - R_{th}) = 0; \lambda_2 \neq 0 \Rightarrow (E_b^T - \xi_{th}) = 0; \lambda_3 \neq 0 \Rightarrow (P_{max} - P_b) = 0\). Optimal values are
\[
P_b^* = P_{max}, \tag{41}
\]
\[
\alpha^* = \max(\alpha_1, \alpha_2) \tag{42}
\]
where \(\alpha_1\) and \(\alpha_2\) have values from (24).

\textbf{Proof}: See Appendix B.

In the next section, we provide optimized solution for considered PS case for EH maximization.

B. Optimized solution for PS

Herein, we handle (P2) for the PS receiver by solving it using KKT, as (P2) is inherently non-convex in nature. As a preliminary step, we derive the Lagrangian dual for the PS case of (P2).
\[
\mathcal{L}(\beta, P_b; \nu) = E_b^P + \mu_1(R_u^P - R_{th}) + \mu_2(E_b^P - \xi_{th}) + \mu_3(P_{max} - P_b) + \mu_4(1 - \beta), \tag{43}
\]

For Step 2, following condition should be meet, \(\Delta \mathcal{L}(\beta, P_b; \nu) = 0 \Rightarrow \frac{\partial \mathcal{L}(\beta, P_b; \nu)}{\partial P_b} = \frac{\partial \mathcal{L}(\beta, P_b; \nu)}{\partial \beta} = 0\), which further solves to
where values \( \xi \) and \( \mu \) for PS, the maximized EH is provided by optimum values \( Z \), and \( \mu_3 \) have the same set of constraints, we will define a new function at a time. Thus, (50) facilitates four cases, i.e., EH maximization with TS or, TS for rate maximization or, PS for EH maximization for TS/PS (\( E^P_\xi, (\alpha, P) \)) resembling (P1) \( (12) \), and EH maximization for TS/PS (\( E^P_\xi, (\alpha, P) \)) akin to (P2) \( (54) \). Due to presence of \( \Phi \) and \( P \) in both constraints and objective function(s), (P3) becomes non-convex problem. Here, we will adopt divide-and-conquer (DAC) approach by breaking primal problem (P3) involving \( Z \) into sub-problems (P3)\( _\alpha \) and (P3)\( _\beta \) for TS. It is mathematically given as follows

\[
(P3)\_\alpha : \text{maximize} \quad Z_\alpha(\alpha, P) \\
\text{subject to} : \quad (53)
\]

\[
(P3)\_\beta : \text{maximize} \quad Z_\beta(\beta, P) \\
\text{subject to} : \quad (57)
\]

Similar DAC approach gives sub-problems concerning PS and \( Z_p \) as follows

\[
(P3)\_c : \text{maximize} \quad Z_p(\beta, P) \\
\text{subject to} : \quad (61)
\]

\[
(P3)\_d : \text{maximize} \quad Z_p(\beta, P) \\
\text{subject to} : \quad (65)
\]

A commonly developed primal problem that takes into account both (P1) and (P2) is given as

\[
(P3) : \text{maximize} \quad Z_x(\Phi, P) \quad \forall x \in [t, p]; \Phi \in [\alpha, \beta] \\
\text{subject to} : \quad (51 - 55)
\]

Expressions for \( \Sigma' \) and \( \Sigma'' \) are the same as given in Section III-B. As a part of KKT, the feasible conditions and complimentary slackness conditions include, \( R^P_\xi - R_{th} \geq 0, E^P_\xi - \xi_{th} \geq 0, P_{max} - P_\xi \geq 0, 1 - \beta > 0; \mu_1(R^P_\xi - R_{th})=0, \mu_2(E^P_\xi - \xi_{th})=0; \mu_3(P_{max} - P_\xi)=0, \) and \( \mu_4(1 - \beta)=0. \) The always positive quantities are \( \nu, \beta, P. \) The above conditions are satisfied only when \( \mu_4 = 0 \) and \( \beta \neq 1. \) The rest cases count to eight (Appendix B), out of which, theorems 7 and 8 represent the satisfied cases leading to an optimal solution.

**Theorem 7:** If \( \mu_1 \neq 0 \) \( \Rightarrow (R^P_\xi - R_{th})=0; \mu_2 \neq 0 \) \( \Rightarrow (E^P_\xi - \xi_{th})=0; \mu_3 \neq 0 \) \( \Rightarrow (P_{max} - P_\xi)=0. \) Following give optimal values

\[
P^\ast_\xi = P_{max}, \\
\beta^\ast = 1 - \frac{W_0(2R_{th} - 1)}{P_{max}Z_n^\ast}. \\
\]

**Proof:** See Appendix B.

**Theorem 8:** If \( \mu_1 \neq 0 \) \( \Rightarrow (R^P_\xi - R_{th})=0; \mu_2 \neq 0 \) \( \Rightarrow (E^P_\xi - \xi_{th})=0; \mu_3 \neq 0 \) \( \Rightarrow (P_{max} - P_\xi)=0. \) Optimal values are

\[
P^\ast_\xi = P_{max}, \\
\beta^\ast = \max(\beta_1, \beta_2). \\
\]

where \( \beta_1 \) and \( \beta_2 \) is similar to \( (53) \).

**Proof:** See Appendix B.

Taking into account the solutions for combined Problem (P2) furnished by Theorems 5 and 6 for TS; and Theorems 7 and 8 for PS, the maximized EH is provided by optimized variables \( P^\ast_\xi \) and \( \Phi^\ast \in [\alpha^\ast, \beta^\ast] \) as \( E_h(\Phi^\ast, P^\ast_\xi) \in [E^P_\xi(\alpha^\ast, P^\ast_\xi), E^P_\xi(\beta^\ast, P^\ast_\xi)] \) In the next section we will look into the algorithmic aspect for providing solution to Problems (P1) and (P2).

**V. ALGORITHMIC SOLUTION**

As per the aforementioned analysis, we found the solutions obtained via KKT methodology. This section provides algorithmic solution to the inter-twinning optimizing variables \( \Phi \) \( (\alpha/\beta) \) and \( P_\xi \) of the primal problems (P1) for rate maximization and (P2) for EH maximization. Since both problems have the same set of constraints, we will define a new notation \( Z_\xi \forall x \in [t, p] \) that takes either TS \( (Z_\xi(\alpha, P_\xi)) \) or PS \( (Z_\xi(\beta, P_\xi)) \) case at a time. The case for TS/PS further divided into two cases of rate or EH maximization objective function at a time. Thus, \( (50) \) facilitates four cases, i.e., EH maximization with TS or, TS for rate maximization or, PS for EH maximization, or rate maximization with PS, out of which only one will be considered at a time. It is defined as follows

\[
Z_\xi(\Phi, P_\xi) = \begin{cases} 
Z_\xi(\alpha, P_\xi) : & \text{Rate for TS} \\
& (67)
\end{cases}
\]

\[
Z_p(\beta, P_\xi) = \begin{cases} 
Z_p(\beta, P_\xi) : & \text{Rate for PS} \\
& (68)
\end{cases}
\]

\[
(51) - (55)
\]

where \( \Sigma'' = \frac{(1 - \chi)(TE' - \xi_{th})}{\xi_{th}(1 - \chi) + \chi TE'} \). Inspired by the AOA [28], Algorithm 1 regulates the above stated sub-problems \( (P3)\_\alpha \) and \( (P3)\_\beta \) for TS, and \( (P3)\_c \), \( (P3)\_d \) for PS respectively. We shall be writing Algorithm 1 for equivalent mother problem \( (P3) \), which takes care of all the associated four child cases \( (50) \). We commence by initializing an array \( Z_\xi(\Phi, P_\xi) \forall x \in [t, p], \Phi \in [\alpha, \beta] \), along with the variable \( P_\xi \), which is assigned two feasible values. The procedure
unfolds as follows: First, we set $P_b$ at a constant value and calculates ($\Phi = \alpha$) as part of sub-problem $(P3)_a$ for TS, on the other hand for PS evaluates ($\Phi = \beta$) using $(P3)_c$ respectively for both TS and PS cases will be sketched in next section of this paper.

The comparison of Algorithm 1 solutions with the KKT results for two distinct objectives of rate and EH maximization, respectively for both TS and PS cases will be sketched in next section of this paper.
Fig. 5: Comparative Rate and EH maximization plots for TS and PS with the variation of (a) Number of RIS surfaces (b) No. of RIS elements (assumed to be same for all RIS surfaces) (c) BS-$R_m$=$R_n$-UE=$d$ distance.

Fig. 6: Emulation environment for multi-RIS placement.

Fig. 7: Study of multiple RIS circular placement with comparative study of TS/PS for maximized data rate verses demanded harvested energy.

Fig. 8: Study of multiple RIS circular placement with comparative study of TS/PS for maximized EH with $P_{\text{max}}$. maximization of rate and EH respectively. First one belongs to KKT solutions provided in section [III] and [V]. Second solution corresponds to the proposed Algorithm 1 stated in section [V] for solving Problem (P3), depicted as ‘Algo’ in plots. Third solution belongs to the solution by the joint optimization (JO) methodology, in which multiple optimizing variables are optimized simultaneously. Consequently, for Problem (P3) the dependent variables $\Phi$ and $P_b$ are optimized together for selected objective at a time out of four cases of $Z_x \forall x \in [t, p], \Phi \in [\alpha, \beta]$ [50]. This process is complex and provides sub-optimal solution as adjustment to one variable leads to the compensatory fixtures in other, so as to maximize the objective function. The outcome derived from the joint optimization of parameters could exhibit sub-optimal conditions and is highly reliant on the selection of a feasible initial point. The argument of sub-optimal solution difference is supported by Fig. [4] where Algorithm 1 and KKT solutions are better than JO solutions. While all three solutions have very negligible difference for rate case, which implies that all three solutions reach its optimality or the problem becomes convex for the selected constant parameters. Also, TS performs inferior to the PS protocol for both rate and EH. As for PS, SWIPT operations are performed for the complete time period with the split in transmit power. For information rate, this difference in performance for both the protocols becomes more pronounced at the higher demanded EH. On the other hand, for EH this difference is not huge. Fig. [3] shows rate and EH trade-off. Fig. [4] depicts increasing EH with the maximum allowed transmit power ($P_{\text{max}}$). Furthermore, the study involves comparison of the outcomes for a scenario denoted as ‘No RIS,’ which represents the absence of RIS. In simulations for rate and EH maximization, we mimic the absence of RIS by simulating reflections from a concrete wall with a reflectivity of 0.45 [1], [31]. It is worth noting that Algorithm 1 is applied with comparable channel coefficients to optimize parameters in the ‘No RIS’ scenario for both TS and PS. The necessity of intelligent reflections by RIS is highlighted in Fig. [5] and [4] where ‘No RIS’ yields least performance.

B. Variation of RIS surfaces, elements and the distance

Fig. 5(a) highlights the requirement of more number of RIS surfaces for a higher rate and EH. More surfaces give more choice for best RIS selection with strongest channel condition (highest SNR). Fig. 5(b) shows higher rate and EH with the in-
increasing number of elements, provided all RIS surfaces (N=3) have equal number of elements (M=50/100). The increasing M affects the summation in ‘channel term’ (Ze) and hence increment in rate and EH. The same condition stands true for lower BS-RIS/RIS-UE distance as depicted by Fig. 5(c). The performance of PS superior to TS is supported by all three plots Fig. 5(c). Variations of distance and elements is huge for TS rate maximization with demanded EH than for EH maximization.

### C. Multi-RIS circular placement

In this section we consider an emulation with multiple RIS surfaces distributed circularly as shown in Fig. 6. The BS (S) is placed in the center surrounded by the inner concentric disc of radius 5m and 1m width, and outer concentric disc of radius 8m and width 1m. Two RISs (R1, R2) are placed somewhere in the inner disc making an isosceles triangle with the center (S) such that the base is the straight line connecting both RISs (R1 – R2). The UE (U) is placed on the outer disc. The angle \( \Theta \) (value in degree) is between the line joining S-U and S-R1 with available range from around 0\(^\circ\) to 180\(^\circ\). The effect of \( \Theta \) is featured in Fig. 7 for rate (TS/PS) and maximized EH (TS/PS) in Fig. 8. Results show that best performance can be obtained for \( \Theta \approx 0^\circ \) for 2 RIS surfaces with PS. The effect of increased surfaces from 1 to 2 cross-validates the trend with Fig. 5(a). The next best option is \( \Theta \approx (0^\circ, 90^\circ) \) followed by \( \Theta \approx 90^\circ \). For angles more than 90\(^\circ\), the performance reduces due to increased distance of \( d_{1,2} \) with angle as depicted by \( \Theta \approx 120^\circ \) case. The PS protocol undoubtedly underscores the TS, for both data rate and EH.

The results differentiate the three different solution types, namely KKT, Algorithmic and JO for rate and EH maximization. These solutions are almost co-linear for rate, while KKT gives best solution for EH case with a slight variation from Algorithmic and JO parametric solution. The rate and EH both increases with increasing number of RIS surfaces, RIS elements in each surface, maximum allowed transmit power and lesser separation distance. A circular emulation environment is considered suggesting for optimal BS-RIS-UE angular placement. It is obvious that PS works better than TS irrespective of the parameter variation. Along with this the classic rate-EH trade-off has been well underlined too.

### VII. Conclusion

The paper considers the HD SWIPT system with multi-RIS setting employing NL-EH at the IoT UE. Distinct Maximization problems for data rate and EH has been formulated for
TS/PS SWIPT protocols, for optimal TS/PS ratio and transmit power with a set of constraints. Closed form solution using KKT was obtained. Additionally, AOA based algorithm was proposed and its solutions are compared with the KKT, along with the joint parameter optimization based solutions. Rate gives almost similar optimized values for all the three types of solutions, while KKT solution wins for EH maximization with a slight difference than others. It is concluded that more throughput and EH yeams for higher RIS surfaces, RIS elements, upper limit of transmit power, lower separation distance. For circular placement scenarios, it is favored for BS-$R_\text{u}$-UE to be almost in line $\Theta \approx 0^\circ$. An extension of present system model can be made complex by uncovering the ideal condition of perfect CSI and perfect phase-shift, instead discrete phase shift can be incorporated. Multiple user case with their scheduling can be investigated further.

**APPENDIX A**

Analyzing the Rate maximization considering TS protocol together with $\Theta$ with PS respectively, all the feasible solution are tabulated in Table I. It takes total 8 cases out of which 2 possible cases exists and its solution is provided in main text.

**APPENDIX B**

For EH maximization involving TS/PS protocol, the eight available KKT conditions are cataloged in Table II. It considers involving TS protocol and equations for PS case. The possible two cases for individual TS and PS cases, respectively has been marked up too with equations. Their complete solution which maximizes the EH is given in the main text.

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