Design of Rectangular Waveguide Bandpass Filters with Transmission Zeros Using High-$Q_u$ Complementary Split Ring Resonators with Irises

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Abstract—This paper introduces a novel bandpass filter designed in rectangular waveguide (RW) technology using an inline structure. The main design concept involves inserting a complementary split-ring resonator (CSRR) into the RW cross-section along with an iris to introduce a transmission zero (TZ) near the passband region. The location of the TZ above or below the passband region is achieved by employing either an inductive or capacitive iris. This structure is modeled as a singlet block, simplifying the design process, as higher-order filters can be designed by cascading CSRR-based singlets. Two design methods are presented, each illustrated with a filter example. Additionally, the use of tuning screws is also studied. The proposed bandpass filter has been experimentally validated through the fabrication and measurement of a fourth-order prototype designed in the C-band with TZs at both sides of the passband region. The measurement results demonstrated a high-performance selective bandpass filter with an unloaded quality factor of $Q_u \approx 3000$.

Index Terms—Compact size, complementary split ring resonator (CSRR), singlet, waveguide filters.

I. INTRODUCTION

OVER last decades, some of the main demands placed on filter designers by microwave companies have been to improve stopband rejection, reduce device size, and optimize the electrical performance. To address these challenges, designers have had to apply innovative design and technology strategies for practical implementation [1], [2]. In satellite communications systems, bandpass filters with the aforementioned requirements are essential devices for selecting and managing transmitted and received signals. Rectangular waveguide (RW) technology is commonly used to implement these filters due to its low-loss characteristics and high-power handling capability. The main drawbacks of this technology are its excessive size and mass, resulting in significantly bulkier devices than other technologies, such as microstrip [3] or substrate integrated waveguide (SIW) [4]. Nonetheless, the significant insertion losses of planar and SIW filters, attributed to the dielectric substrate and conductors, and a lower power handling capability in contrast to RW counterparts are their main limitations for certain applications, such as satellite communications. For this reason, considerable research efforts are being dedicated to miniaturizing RW devices while preserving their exceptional electrical performance [1], [2].

Resonators play a major role in the design of bandpass filters, since the dimensions of their physical structures significantly impact on the overall size of these devices. Considering this issue, sub-wavelength resonators based on split ring resonators (SRRs) or complementary SRRs (CSRRs) have been widely studied to achieve more compact RW filters in contrast to conventional cavity- or resonator-based filters [5]–[22]. Moreover, a recent design methodology has been proposed to introduce transmission zeros (TZs) in bandpass filters based on split ring resonators [22]. This strategy improves the rejection near the passband, thus avoiding channel interference without the need of employing high-order filters, which generally present greater manufacturing difficulties and larger volumes and insertion loss. However, all reported RW filters based on ring resonators share a crucial common drawback: despite being implemented in RW, they exhibit unloaded quality factors ($Q_u$) on the order of SIW technology, slightly higher than microstrip, and far from other sub-wavelength resonators in RW, such as metallic posts [23] or dielectric resonators [24].

This paper describes a design methodology for high-performance bandpass filters in RW technology with transmission zeros close to the passband using high-$Q_u$ CSRRs with inductive/capacitive irises. To this purpose, a design approach based on cascaded singlets [25]–[27] has been applied for the first time to RW filters based on ring resonators, analyzing its benefits and limitations. Another crucial contribution of this work lies in demonstrating the remarkable low-loss capabilities of these devices, achieving an experimental quality factor of $Q_u \approx 3000$, significantly exceeding the values of previously reported SRR- and CSRR-based RW filters [5]–[22]. Section II presents the singlet model for the RW loaded with a single CSRR. The insertion of an iris on the same metallic plate that contains the CSRR provides an alternative path for the incoming signal, thus creating a cross coupling that introduces a TZ close to the passband domain. By changing the type of this iris, whether inductive or capacitive, the TZ can be positioned respectively above or below the passband, providing
design flexibility. Section III describes the proposed design methodology for RW bandpass filters with TZs, using CSRRs and inductive/capacitive irises, by means of a synthesis method based on cascaded singlets [25]–[27]. Design strategies that allow adapting this approach to the proposed filter structure are also included, deriving into two design methods that are illustrated through two fourth-order examples, also reported in Section III. Section IV presents a study to compensate for manufacturing tolerances by means of tuning screws. Section V shows the measured results of a fourth-order bandpass filter with a TZ on both sides of the passband region to verify the proposed design procedure, the low-loss capabilities of these filters, and the strategy employed to compensate for manufacturing tolerances. Section V also includes the manufacturing process and a comparison between this prototype and other reported bandpass filters based on SRRs and CSRRs in RW technology. Finally, conclusions are presented in Section VI.

II. ANALYSIS AND DESIGN OF BASIC STRUCTURES

In this section, three basic structures, without and with transmission zero (TZ), are analyzed for the design of the proposed CSRR-based bandpass filters in rectangular waveguide (RW) technology. The WR-159 rectangular waveguide standard (a = 40.86 mm, b = 20.193 mm) was selected to perform these analyses in the frequency range 4.9–7.05 GHz.

The first basic structure, represented in Fig. 1, does not produce TZ. It consists of a RW loaded with a complementary split ring resonator (CSRR). The dimensions of the RW are width a, height b, and length l, while the CSRR is defined by its width W, radio R, split width D, and rotation angle φ. As can be seen in Fig. 1, the CSRR is etched on a metallic plate of thickness t, which is placed on the cross section of the RW. Besides, the CSRR is located at mid-height (b/2) and shifted along the x-axis by a Δx value from the center of the RW (a/2). The results of the electromagnetic (EM) simulations using CST Studio [28] for the structure presented in Fig. 1 are shown in Fig. 2 (dimensions included in the caption). The S-parameters reveal a transmission pole around 6 GHz. The CSRR-loaded RW section depicted at the bottom of Fig. 2 shows the simplified electric \( \vec{E} \) and magnetic \( \vec{H} \) field patterns produced around the CSRR for a better understanding of the resonance. Imposing the boundary conditions on the metallic plate [29], the tangential electric fields vanish whereas the tangential magnetic fields induce surface electric current densities \( \vec{J}_e \). With the TE_{10} fundamental mode of the RW exciting the resonance, the x-component of the magnetic field \( \vec{H}_x \) is perpendicular to the normal vector of the surface \( -\hat{e}_z \), thus the boundary condition \( -\hat{e}_z \times \vec{H}_x = \vec{J}_e \) indicates a surface electric current density oriented along the y-axis. This current flows across the metallic split of the ring, bouncing with the TE_{10} fundamental mode of the RW. Thus, an intense magnetic field \( \vec{H} \) is stored around the metallic split, whereas the electric field \( \vec{E} \) of the resonance is stored in a capacitance distributed along the bottom of the CSRR.

Once the resonance analysis is completed, we study the effects of the design parameters defined in Fig. 1 on the resonant frequency and the coupling to the source/load. Larger ring radii (R) imply resonances at lower frequencies, as this increases the distributed capacitance. The resonant frequency could also be controlled by the split width (D), although the radio (R) has been chosen for this purpose, so the parameter D has simply been fixed to a suitable value for manufacturing: \( D = 2.5 \text{ mm} \). Regarding the coupling control, two alternatives are proposed to reduce the interaction: rotating the ring (increasing φ) or horizontally displacing it from the center of the RW (increasing Δx). Rotating the metallic split of the ring, which provides the path for the current, curtails the vector product \( -\hat{e}_z \times \vec{H}_x = \vec{J}_e \), thus mitigating the coupling mechanism. Similarly, as the \( \vec{H}_x \)-field of the TE_{10} mode is maximum at the center of the RW, the displacement also produces this mitigation. The scenarios where φ or Δx are more convenient will be analyzed later. The remaining design parameters are the width of the ring (W) and the thickness (t) of the CSRR metallic plate. These strongly impact on the unloaded quality factor \( Q_u \) of the CSRR, so they are fixed to improve the \( Q_u \) according
to the curves shown in Fig. 3. The $Q_u$ factor grows linearly as a function of $W$, so a hasty decision would be to select the largest possible value for $W$. The issue is that widening the ring (increasing $W$) also increases the resonant frequency, since the distributed capacitance at the bottom of the ring is reduced. Therefore, the ratio $R$ should be increased to compensate the frequency shift, complicating the filter design with large values of $W$ and $R$. For this study, a compromise value that maximizes the $Q_u$ while facilitating filter design is $W = 3.5$ mm. Regarding the thickness $t$ of the metallic plate, it can be seen in Fig. 3 that $Q_u$ saturates around $t = 3$ mm. This metallic plate thickness was selected, since its value is suitable for the manufacturing of the proposed filters.

Fig. 4 shows the coupling matrix (CM) scheme of a singlet, which consists of one resonator coupled to the source and the load ($M_{S1}$ and $M_{1L}$), and a direct coupling between the source and the load ($M_{SL}$) [25–27]. To complete the singlet, the first basic structure presented in Fig. 1 can be modified by including an iris on the metallic plate to implement the cross coupling $M_{SL}$. According to the type of iris, inductive (second basic structure, see Fig. 5) or capacitive (third basic structure, see Fig. 6), the sign of the cross coupling ($M_{SL}$) will coincide or oppose to the sign of the main couplings ($M_{S1}$ and $M_{1L}$). Thus, the position of the TZ will be above the passband region (Fig. 5) when $M_{S1}$, $M_{1L}$ and $M_{SL}$ have the same sign, while it will be below the passband region (Fig. 6) when $M_{SL}$ has the opposite sign of $M_{S1}$ and $M_{1L}$. The frequency response will not have TZ when the cross coupling $M_{SL}$ is equal to zero. This is the case for the structure without iris shown in Fig. 2. As a result of this analysis, it can be anticipated that the design of filters using these basic structures and cascaded singlets approach will offer some interesting features, such as simplicity of design and manufacturing, high selectivity and low insertion loss.

**III. BANDPASS FILTER DESIGN USING CASCADED SINGLETS**

In this section, two design methods for in-line CSRR-based bandpass filters with irises in RW technology are presented. The first one is more convenient for designing these filters only with capacitive irises, while the second method is useful for both capacitive and inductive irises. These design approaches are described through two fourth-order filter examples. The first one (Filter 1) has a center frequency at $f_0 = 6$ GHz, an equiripple fractional bandwidth FBW = 1.25 %, return loss RL = 20 dB, and two TZs located below the passband region at 5.86 GHz and 5.91 GHz. The second filter (Filter 2) exhibits two TZs below the passband region at 5.81 GHz and 5.9 GHz, and another two ones above at 6.1 GHz and 6.19 GHz for the specifications: $f_0 = 6$ GHz, FBW = 2 %, and RL = 20 dB. Although the proposed design methods are based on lossless simulations, a typical requirement for bandpass filters is the acceptable insertion loss along the passband. This is generally specified as a minimum $Q_u$ value and, as discussed in Section II, the achievable $Q_u$ value for this kind of filters is determined by the width $W$ of the ring and the thickness $t$ of the metallic plate. Thus, these two parameters must be fixed prior to apply the proposed design methods. We impose $Q_u \approx 3400$ to Filter
1 and Filter 2, so $W = 3.5$ mm and $t = 3$ mm for both.

The requirements for Filter 1 prescribe TZs only below the bandpass region. Therefore, according to Section II, CSRR-based structure with capacitive iris should be employed. Fig. 7 shows the coupling matrix (CM) scheme, the physical structure, and the S-parameters for Filter 1. This was designed according to the first design method we propose in this paper, which is illustrated in Fig. 8. Let us recall that design methods based on cascading singlets [25]–[27] start by optimizing separately each singlet, i.e. adjusting the physical parameters of the singlet until the magnitude of the S-parameters fits the expected values from the coupling matrix around $f_0$. Due to the simplified structure of individual singlets compared to the entire filter assembly, optimizing them is relatively easy, even when employing brute force. This first step is represented in Fig. 8(a), which, for brevity, depicts only the S-parameters optimized for singlet 1 (see Fig. 7). After optimizing the S-parameters for each singlet (first step), the couplings between singlets shown in Fig. 7(a) must be implemented with RW sections of lengths $l_{12}, l_{23}$ and $l_{34}$ [see Fig. 7(b)]. These lengths are typically obtained by converting the couplings between singlets into ideal phase shifters [25]–[27] and compensating the phase mismatch between the synthesized CM and the physical realization. However, this step can be challenging for the proposed CSRR-based singlet implementation. Unlike the cavity resonators [25]–[27], the volume of the resonance for the CSRR-based singlets is not as obvious, i.e. the beginning and end of the resonance are not known beforehand. This information is crucial to properly determine the required distance to the following singlet. Therefore, the easiest solution we propose consists in performing EM simulations for each pair of singlets and simply adjust the lengths $l_{12}, l_{23}$ and $l_{34}$ [see Fig. 7(b)] until the S-parameters fit the synthesized CM response [see Fig. 7(a)]. This second step is represented in Fig. 8(b), which, for brevity, depicts only the S-parameters optimized for the pair of singlet 1-2. This alternative second step is very straightforward to apply and, as shown in Fig. 8(c), provides an excellent initial dimensioning of the complete filter. The values of the design parameters for the initial dimensioning [Fig. 8(c)] and the final optimization [Fig. 7(c)] are detailed in Table I.

Please note that, for Filter 1, the displacement $\Delta_2$ from the center of the RW was not used (see Table I). Instead, the couplings to the resonators were adjusted only by rotating the rings ($\phi$). This is because displacing the CSRRs from the center of the RW can notably excite higher-order modes, such as the TE$_{20}$, whose cut-off frequency is not so far from the
Nevertheless, there is a simple way of preserving the design below cut-off modes, as explained in previous paragraph. The requirements for Filter 2 prescribe two TZs below the passband, so the block to optimize is composed of singlets 1, 2 and 3 [see Fig. 10(c)]. Finally, the last singlet is added to the structure forming the complete fourth-order filter. As can be seen in Fig. 10(d), the starting point prior to the whole filter optimization is very satisfactory. The values of the design parameters at each step are detailed in Table II.

<p>| TABLE II |
| DIMENSIONS OF FILTER 2 PRESENTED IN FIG. 9. PARAMETERS ARE DEFINED IN FIG. 1 (UNITS: mm or °). |</p>
<table>
<thead>
<tr>
<th>Singlet 1</th>
<th>Singlet 2</th>
<th>Singlet 3</th>
<th>Singlet 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>t</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>D</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>W</td>
<td>3.5</td>
<td>3.5</td>
<td>3.5</td>
</tr>
<tr>
<td>R</td>
<td>6.142</td>
<td>6.145</td>
<td>6.144</td>
</tr>
<tr>
<td>Δφ</td>
<td>8.8</td>
<td>10.26</td>
<td>11.11</td>
</tr>
<tr>
<td>wcap</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>l_{i,i+1}</td>
<td>16</td>
<td>19.5</td>
<td>19.5</td>
</tr>
</tbody>
</table>

The second design method, although slightly more complex, is more robust because it considers the interactions between CSRRs through evanescent modes. Since both procedures provide accurate initial dimensions for the filters, the first design method, which is simpler and faster, should be prioritized when TZs above the passband region are not required.

IV. TOLERANCE COMPENSATION USING TUNING SCREWS

Tuning screws are a common strategy employed to compensate for manufacturing tolerances of resonator-coupled filters, or even for reconfigurability purposes [2]. The completely tunable configuration with screws for the CSRR-based structure with inductive iris is shown in Fig. 11. As explained in Section II, the coupling to the CSRR is produced by the tangential
component of the magnetic field. Therefore, a tuning screw near the metallic split of the CSRR (see Fig. 11) augments this magnetic field, thus increasing the coupling to the CSRR. The inconvenient of using tuning screws at regions with intense magnetic fields is that electric current surface densities are also induced across the screws themselves, hence increasing the insertion losses of the device. The resonant frequency can be decreased by inserting a tuning screw in the area where the electric field is maximum —at the bottom of the CSRR (see Fig. 11)—, thus enlarging the equivalent capacitance of the resonator. This screw would not introduce significant insertion losses due to the absence of magnetic field in this region.

Finally, a tuning screw inserted in the iris adjusts the cross coupling $M_{SL}$, intensifying the magnetic field.

Regarding the other two basic structures presented in Section II, i.e. CSRR without irises or with capacitive irises, the position of the tuning screws would be identical. This is, near the metallic split of the ring to control the coupling to the resonator, and near the bottom of the ring to control the resonant frequency. For the structure without irises, there is no cross coupling to control, so this screw would be removed. For the structure with capacitive irises, one or two tuning screws should be inserted in the region with more intense electric field around the irises (different to the region of intense electric field at the bottom of the ring). For the sake of brevity, these two structures with tuning screws are not depicted, as they closely resemble the case with inductive iris shown in Fig. 11.
V. EXPERIMENTAL VALIDATION AND DISCUSSION

The potential of the proposed bandpass filters to achieve high selectivity and a high unloaded quality factor ($Q_u$) is demonstrated by designing, manufacturing, and measuring a fourth-order bandpass filter prototype in the WR-159 standard ($a = 40.86 \text{ mm}$, $b = 20.193 \text{ mm}$) with the following characteristics: $f_0 = 6 \text{ GHz}$, equiripple FBW = 2 %, RL = 24 dB, and a TZ at both sides of the passband region at 5.837 GHz and 6.152 GHz. The structure of this filter with tuning screws is depicted in Fig. 12 and its dimensions, which were obtained from the second method described in Section III, are indicated in Table III.

![Fig. 12. 3-D view of the CSRR-based bandpass filter structure for experimental validation. The structure includes capacitive and inductive irises, rounded corners for CNC milling, and tuning screws for tolerance compensation.](image1)

<table>
<thead>
<tr>
<th>TABLE III</th>
<th>DIMENSIONS OF THE PROTOTYPE SHOWN IN Fig. 12. PARAMETERS ARE DEFINED IN Fig. 1 (UNITS: mm OR °).</th>
</tr>
</thead>
<tbody>
<tr>
<td>t</td>
<td>Singlet 1</td>
</tr>
<tr>
<td>D</td>
<td>2.5</td>
</tr>
<tr>
<td>W</td>
<td>3.5</td>
</tr>
<tr>
<td>R</td>
<td>6.17</td>
</tr>
<tr>
<td>$\Delta z$</td>
<td>12.3</td>
</tr>
<tr>
<td>$w_{\text{ind}}$</td>
<td>12.3</td>
</tr>
<tr>
<td>$h_{\text{cap}}$</td>
<td>0</td>
</tr>
<tr>
<td>$\varphi$ (°)</td>
<td>0</td>
</tr>
<tr>
<td>$\ell_{i,i+1}$</td>
<td>16.5</td>
</tr>
</tbody>
</table>

The prototype of this filter was successfully manufactured. For this purpose, the structure was divided into different parts as shown in Fig. 13. A computer numerical control (CNC) milling machine and copper with electrical conductivity $\sigma = 5.8 \cdot 10^7 \text{ S/m}$ and surface roughness $R_a = 0.8 \mu\text{m}$ were used for piece-by-piece manufacturing. The corners were rounded for the CNC drill oriented along the $z$-axis, with radii of 0.75 mm for the rings, 1.45 mm for the capacitive irises, and 4 mm for the inductive iris and the RW sections.

![Fig. 13. 3-D view of the different prototype parts for manufacturing.](image2)

The unloaded quality factor ($Q_u$) of this filter is estimated by introducing resistive elements into the resonators of the CM (Fig. 16). These resistive elements are known as dissipation factor $\delta$ [2], and can be calculated as

$$\delta = \frac{Q_u}{\text{FBW}}. \quad (1)$$

The $Q_u$ value is determined by matching the passband inser-
Fig. 14. Top and front views of the assembled prototype.

Fig. 15. S-parameters for the prototype of Fig. 14 with the tuning screws adjusted to compensate for manufacturing tolerances.

The value of the unloaded quality factor, estimated through the same process above, has now grown to $Q_u = 3000$. This indicates that the tuning screws strongly deteriorate the $Q_u$ (insertion loss) as they penetrate more into the structure. The insertion loss can be reduced, thus improving the performance of the filter, by manufacturing it with tighter tolerances, coating the tuning screws with copper or silver, and avoiding, as much as possible, inserting the tuning screws into regions with intense magnetic fields.

Table IV presents a comparison between the measurements of the proposed bandpass filter and various SRR- and CSRR-based bandpass filters reported in the literature. One of the key differences lies in the technology, since the majority of previously reported filters are based on planar circuits with dielectric substrate inserted into the RW [12], [14]–[16], [22]. The single study that incorporates metal parts [13] uses very thin metal resonators. This and a higher filter order explain why the proposed filter has a longer length. Similarly, for filters based on SRR-loaded evanescent-mode RW sections
With prescribed TZs.

This offers a more comprehensive framework for designing filters with prescribed TZs in rectangular waveguide (RW) technology. This is achieved by loading complementary split-ring resonators (CSRRs) with other structures on the same metallic plate creates cross-coupling that introduces a TZ above or below the resonance singlets and have been analyzed to validate the proposed strategy of combining the CSRRs with other structures on the control the position of TZs in the frequency response of the filters (see Table IV), achieving or even exceeding the values of other state-of-the-art sub-wavelength resonators in RW technology [23, 24]. Finally, the bandpass filter presented in this work exhibits high performance in the passband and TZs at both sides of the passband region. The frequency of these TZs is easily controllable thanks to the coupling-control mechanisms and the insertion of irises discussed in Section II, in addition to the two design methods developed in Section III. With the exception of [22], most previous works [12]–[14] did not provide a design methodology to introduce and control the position of TZs in the frequency response of the filters. Although works such as [15] and [16] employ a similar strategy of combining the CSRRs with other structures on the same plate to create the TZs, the methods presented in Section III offer a more comprehensive framework for designing filters with prescribed TZs.

VI. CONCLUSION

This paper proposes a novel bandpass filter with transmission zeros (TZs) in rectangular waveguide (RW) technology. This is achieved by loading complementary split-ring resonators (CSRRs) combined with irises into a propagating rectangular waveguide. The combination of inductive or capacitive iris with the CSRR on the same metallic plate creates cross-coupling that introduces a TZ above or below the resonance of the CSRR. These basic structures have been modeled as singlets and have been analyzed to validate the proposed coupling-control mechanisms and to improve the unloaded quality factor ($Q_u$) of the resonators. A design methodology based on cascaded singlets, including two examples, has been developed for the proposed filter. This approach has not only demonstrated flexibility in the introduction of TZs but has also shown a high degree of control over them. The use of tuning screws for compensating manufacturing tolerances has been also introduced. The performance of the proposed bandpass filter was confirmed by experimental validation using a fourth-order prototype designed for C-band with narrow-band specification FBW = 2%, return loss RL = 24 dB, TZs around the passband region, and $Q_u \approx 3000$. The measurement results revealed a high-selective bandpass filter, with an impressive $Q_u$ value, thus satisfying the required design specifications. These characteristics make this class of filters highly suitable for communications applications that demand high selectivity and performance using compact devices.

REFERENCES


