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1. Introduction

The two key components constituting the representation of moist convection in GCMs are the cloud models and approximations for the interactions of clouds with the surrounding environment (Arakawa and Schubert 1974). For cloud models, variants of steady plumes (or parcels) have been adapted for conventional parameterizations and to aid process-level understanding of convection (Simpson and Wiggert 1969; Zhang and McFarlane 1995; Bretherton et al. 2004; Siebesma et al. 2007; Peters et al. 2021). These models are limited by the simplifying assumptions under which they are constructed, including (i) omitting time dependence; (ii) parameterizing the dynamically interactive inflow and outflow as prescribed entrainment mixing; and (iii) substituting drag for the nonlocal effects of nonhydrostatic perturbation pressure. Furthermore, when formulating an ensemble of clouds or updrafts through a bulk representation, the individual constituents are often treated as non-interacting. Such cloud representations have long-standing issues, e.g., failure to simultaneously predict realistic cloud height, water content, convective onset and other relevant variables (Warner 1970; Sherwood et al. 2013); overestimated updraft strength (Donner et al. 2016); and difficulty launching land nighttime convection when a surface inversion or strong CIN occurs (Lee et al. 2008; Wang et al. 2015).

Useful elements in representing the interactions between clouds and the surroundings include warming of the large-scale environment through subsidence compensating the convective updraft (Yanai et al. 1973; Emanuel et al. 1994) and adjustment toward a weak temperature gradient (WTG) state (Neelin and Held 1987; Raymond 2000; Sobel and Bretherton 2000; Raymond and Zeng 2005; Kuang 2008; Singh and O’Gorman 2013; Biagioli and Tompkins 2023). However, adjustment processes involving higher gravity-wave modes (Bretherton and Smolarkiewicz 1989; Tulich et al. 2007; Tulich and Mapes 2008) dynamically interacting with stratiform cloud evolution are typically sidestepped in the parameterization effort—plume-based cloud models omit or have difficulty representing pathways through which these interactions occur such as time dependence, interactive inflow and outflow, and nonlocal pressure effects (Tarshish et al. 2018; Kuo and Neelin 2022).

Naturally, these processes are comprehensively treated in large-eddy simulations (LES) and cloud-resolving models (CRMs) sitting at the other end of the complexity spectrum (Bryan and Fritsch 2002; Khairoutdinov and Randall 2003; Jung and Arakawa 2008). Superparameterization embeds a limited-domain CRM within each grid cell of a GCM (Grabowski 2001; Khairoutdinov and Randall 2001); recent global storm-resolving simulations have also shown progress (Stevens et al. 2019; Wing et al. 2020)—though these remain computationally costly, challenging to analyze and do not always overcome biases with respect to observations (Ma et al. 2022; Su et al. 2022).

Seeking a class of solutions filling the model hierarchy between simple plumes and CRMs, in Part 1 (Kuo and
Neelin 2024) of this work, a time-dependent formulation termed anelastic convective entities (ACEs) is presented. The ACE construct captures aspects of the time evolution of convection, and provides self-consistent nonlocal solutions including the nonhydrostatic perturbation pressure associated with the convective flow. Illustrations with land nighttime soundings in Part 1 have simulated time-varying convective updrafts, downdrafts and other aspects of convection for a single column interacting with a fixed environment, showing promise as a substitute for traditional plume-based cloud models.

Through studying the behaviour of systems consisting of multiple interacting entities built upon the single ACE formulated in Part 1, this manuscript begins to examine large-scale multiple interacting entities built upon the single ACE formulation, and briefly describes the experiment implementation. Using an oceanic sounding (to contrast with the land nighttime cases in Part 1), section 4 shows a single-ACE example that admits multiple steady states, and section 5 presents multi-ACE instances illustrating adjustment processes and remote initiation of convection. Coupling strategies to large-scale models are touched upon in section 6, followed by summary and discussion in section 7 covering caveats, wishlist and promising features.

2. Summary of equations for interacting ACEs

This section summarizes the equations governing a multi-ACE system so that the reader wishing to skip technical details can go directly to the results in sections 4-5. A brief summary of implementation can be found in section 3.c (see also section 4 of Part 1).

For simplicity, this manuscript focuses on a configuration, schematized in Fig. 1a, consisting of \( n \) nested ACEs constituting a rectangular domain resembling a typical GCM grid cell. A special concentric example in Fig. 1b is used for the multi-ACE illustrations in section 5 (while not covered here, equations for different geometry, e.g., an ensemble of entities separated from each other, can be written down following the procedure outlined in section 3).

Indexing these ACEs by \( i = 1, 2, \ldots, n \) (ascending outward). We adopt the following notation and postulates, elaborating on assumptions (A1-A2) and notation of Part 1:

(P1) Denoting by \( A_i \) the horizontal cross section of ACE-\( i \); and \( \ell_i \) the outer perimeter of \( A_i \) (i.e., the lateral boundary between ACE-\( i \) and \( i + 1 \)). Both \( A_i \) and \( \ell_i \) do not vary notably with height and time. For each \( i > 1 \), \( A_i \) has a ring-like shape that is disjoint from \( A_{i-1} \). For notational simplicity, the same \( A_i \) and \( \ell_i \) are used for the cross section area \( |A_i| \) and perimeter length \( |\ell_i| \), where the meaning is clear from the context.

(P2) The multi-ACE system is embedded in a larger-scale environment where the tracer \( C \approx C_{\text{env}}(z) \) and vertical velocity \( w \approx w_{\text{env}}(z) \). For convenience, \( i = n + 1 \) is used referring to the environment.

(P3) Flux exchanges of air mass and tracers are permitted across the lateral boundaries between adjacent entities. Moreover, these entities can interact nonlocally via the nonhydrostatic perturbation pressure arising from buoyancy and momentum flux convergence.

(P4) For variables of interest, \( \bar{\cdot} \) denotes the mean over \( A_i \), \( \bar{\cdot} \) the mean over \( \ell_i \), and \( \cdot' \) the deviations from the means.

For single-ACE examples in section 4, the ACE index \( i \) is omitted from the notations to be consistent with Part 1.

a. Multi-ACE tracer equation

The multi-ACE case corresponding to Eq. (1) of Part 1 for tracers \( C \) (\( z, t \)) is (for \( 1 \leq i \leq n; z \geq 0 \))

\[
\frac{\partial}{\partial t} \left( \rho_0 C^i \right) = \rho_0 \nabla C^i - \nabla \cdot \left( \rho w C^i \right) - \nabla \cdot \left( C \rho w' C' \right) - \nabla \cdot \left[ \gamma_i^{-1} \left( \nabla \cdot \left( \rho w' C^i \right) \right) \right] - \nabla \cdot \left[ \gamma_i^{-1} \left( \nabla \cdot \left( \rho w' C^i \right) \right) \right] C^i
\]

where

\[
\text{Outflow from ACE-}i \to i-1 \quad \text{Inflow from ACE-}i \to i
\]

\[
\frac{1}{\sigma_i^e} \left[ \nabla \cdot \left( \rho w C^i \right) \right] C^i + \frac{1}{\sigma_i^e} \left[ \nabla \cdot \left( \rho w C^i \right) \right] C^i
\]

\[
\text{Inflow from ACE-}i \to i+1 \quad \text{Outflow from ACE-}i \to i
\]

\[
\frac{g_i^i \rho_0 u' \nabla C^i \cdot \nabla \cdot \left( \rho w C^i \right) \right] - g_i^i \rho_0 u' \nabla C^i \cdot \nabla \cdot \left( \rho w C^i \right)
\]

Here \( \rho_0(z) \) denotes the environmental density under the anelastic framework; \( S_C \) nonconservative processes; \( u_n \) the horizontal flow across the ACE-i lateral boundaries; \( \sigma_i^e, \gamma_i^e, \) and \( g_i^i \) constants specified in Eq. (10) determined by the multi-ACE geometry; and

\[
W = \sum_{j=1}^{i} \sigma_j^i \bar{w}^j
\]

is the area-weighted average vertical velocity within the compound entity combining ACE-1 through ACE-i.

b. Multi-ACE mass flux equation

\footnote{Following Part 1, the equivalent potential temperature \( \theta_e \) (units: K) and total water specific mass \( q_t \) (units: kg/kg) are used as the thermodynamic tracers; see Part 1 Appendix A.}
The multi-ACE case corresponding to Eq. (2) of Part 1 for the mean vertical mass flux \( \rho_0w \) is

\[
\partial_t (\rho_0w) = \sum_{j=1}^{n} \mathcal{N}_{ij}(\bar{F}^j) + \nabla^\perp \cdot \mathcal{D}^j + \mathcal{N}_{ext}^j. \tag{3}
\]

where the nonlocal operator \( \mathcal{N}_{ij}() \) is defined via Eq. (13); \( \bar{F}^j \) by Eq. (14) with an expression similar to the righthand side of Eq. (1); \( \mathcal{N}_{ext}^j \) for external forcing on ACE-\( i \) arising from sources outside the multi-ACE cell; and the second term on the right-hand side is a set of dynamical terms (not normally included in plume models) described in Appendix C of Part 1, crudely substituted here using a pragmatic vertical momentum diffusion following Part 1.

3. Multi-ACE extension of single-ACE formulation

The multi-ACE equations are a simple extension of the single-ACE case in Part 1: the results for the single-ACE case in Part 1: the results for the single-ACE case in Part 1—Eq. (5) for \( i = 1 \) becomes [under the anelastic approximation and upwind condition; see Eqs. (B1) and (14) of Part 1]

\[
\partial_t (\rho_0C^i) = \rho_0S_C - \partial_z (\rho_0wC^i) - \nabla_h \cdot (\rho_0u_h C^i). \tag{5}
\]

where \( u \) is the 3D velocity field; and subscript \( h \) denoting the horizontal component. Horizontally integrating Eq. (4) over \( A_i \) yields the mean budget within ACE-\( i \)

\[
\partial_t (\rho_0\bar{C}^i) = \rho_0\bar{S}_C - \partial_z (\rho_0\bar{w}C^i) - \nabla_h \cdot (\rho_0\bar{u}_h C^i). \tag{6}
\]

The second and third terms on the right-hand side represent the vertical and horizontal flux convergence, with the latter term yielding boundary transfers as treated in Part 1.

1) SINGLE-ACE TRACER EQUATION APPLIES TO \( i = 1 \)

We note that the single-ACE equation applies to the innermost ACE-1 with the adjacent ACE-2 being its immediate surroundings. Thus by repeating the steps outlined in section 3b of Part 1—or simply by adapting Eq. (19) therein—Eq. (5) for \( i = 1 \) becomes [under the anelastic approximation and upwind condition; see Eqs. (B1) and (14) of Part 1]

\[
\partial_t (\rho_0C^1) = \rho_0S_C - \partial_z (\rho_0wC^1) - \nabla_h \cdot (\rho_0u_h C^1).
\]

The expression here is equivalent to the form in Part 1 but is phrased in terms of inflow and outflow for subse-
quent applications. On the right-hand side of Eq. (6), the vertical mean-field flux convergence includes the vertical advection contribution, leading to the expression of dynamic entrainment—for a single entity—when combined with the horizontal inflow/outflow

$$
-\partial_z (\rho_0 \bar{w}^1 \bar{C}^1) + \left[ \partial_z (\rho_0 \bar{w}^1) \right] \bar{C}^2 + \left[ \partial_z (\rho_0 \bar{w}^1) \right]^{-1} \bar{C}^2 \\
\frac{\partial_z (\rho_0 \bar{w}^1)}{\partial z} \bar{C}^1 - \left[ \partial_z (\rho_0 \bar{w}^1) \right]^{-1} (\bar{C}^1 - \bar{C}^2).
$$

(7)

As the expression suggests, dynamic entrainment has a net effect on ACE-1 when inflow occurs, i.e., $\partial_z (\rho_0 \bar{w}^1) > 0$.

The dynamic detrainment $-\left[ \partial_z (\rho_0 \bar{w}^1) \right]^{-1} (\bar{C}^1 - \bar{C}^2)$, on the other hand, would have a net effect on ACE-2—but see caveats below if ACE-2 is interacting with other entities or the environment.

2) Tracer equation for $i = 2$

In Eq. (6), the flow across the lateral boundary is derived from the anelastic continuity given $\rho_0 \bar{w}^1$. By treating ACE-1 plus ACE-2 as one compound entity, we have another single-ACE case in which the flow across the boundary $\ell_2$ can be determined knowing the mean vertical mass flux within the compound entity. Moreover, the evolution of the mean tracer profile—$(A_1 \bar{C}^1 + A_2 \bar{C}^2)/(A_1 + A_2)$—within this compound entity has to obey the same Eq. (6).

With the area fractions and mean vertical velocity

$$
\sigma_i \equiv A_i/(A_1 + A_2), \quad i = 1, 2,
$$

$$
\bar{W}^2 \equiv \sigma_1 \bar{w}^1 + \sigma_2 \bar{w}^2,
$$

we can substitute $\sigma_1 \bar{C}^1 + \sigma_2 \bar{C}^2$ and $\bar{W}^2$ into Eq. (6) in lieu of $\bar{C}^1$ and $\bar{W}^1$. After simplifying, again using Eq. (6), this results in the ACE-2 tracer budget

$$
\partial_t (\rho_0 \bar{C}^2) = \rho_0 \bar{S}^2 - \partial_z (\rho_0 \bar{w}^2 \bar{C}^2) - \partial_z (\rho_0 \bar{w}^1 \bar{C}^2) \\
\left[ \frac{A_1}{A_2} \right] \partial_z (\rho_0 \bar{w}^1) \bar{C}^2 - \left[ \frac{A_1}{A_2} \right] \partial_z (\rho_0 \bar{w}^1) \bar{C}^1 \\
\frac{1}{\sigma_1} \left[ \partial_z (\rho_0 \bar{w}^1) \bar{C}^2 \right] - \frac{1}{\sigma_2} \left[ \partial_z (\rho_0 \bar{w}^1) \bar{C}^2 \right] - \bar{C}^2.
$$

(9)

The lateral exchange terms (including turbulent entrainment) in Eq. (9) between ACE-2 and 1 across $\ell_1$ (i.e., the inner lateral boundary of ACE-2) are identical to those in Eq. (6) with a $-A_1/A_2$ factor to correct for the differences in the flow direction as well as the cross section area. On the other side of ACE-2, the flows across the outer boundary $\ell_2$ are inferred from the nonlinear terms $[\partial_z (\rho_0 \bar{w}^1)]^n$, scaled by $1/\sigma_2$ due to these terms contributing to the tendency over an area fraction $\sigma_2 < 1$. Note that the concise expression for dynamic entrainment (and detrainment) in Eq. (7) is no longer feasible for $\bar{C}^2$. The conventional assertion that dynamic entrainment (detrainment) only affects the entity (surroundings) would apply to the equation for the combined evolution of $\sigma_1 \bar{C}^1 + \sigma_2 \bar{C}^2$, but not to $\bar{C}^2$ alone. Thus for the multi-ACE case, it is simpler to track fluxes across boundaries with inflow and outflow.

3) Tracer equation for general $i$

The derivation for ACE-2 can be repeated by considering the compound entity combining ACE-1 through $i$. Straightforward induction yields Eq. (1) for $1 \leq i \leq n$ with

$$
\sigma_i \equiv A_i/(A_1 + A_2), \quad 1 \leq j \leq i \leq n,
$$

$$
\gamma_{i-1} \equiv \gamma_{i-1} \left[ \left[ \frac{A_i}{A_{i-1}} \right] \frac{1}{\sigma_{i-1}} \right], \quad 1 < i \leq n.
$$

(10)

For $i = 1$, we set $\gamma_0 = g_1 \equiv 0$ so that Eq. (6) becomes a special case of Eq. (1). As for $i = n$, we have $\bar{C}_n \equiv C_{env}$ for the outermost ACE-$n$ following the postulate (P2).

To help understand Eq. (1), we briefly clarify the notations introduced in Eq. (10), in which all terms are constants prescribed by the ACE-system geometry. The index $i$ refers to the compound entity combining ACE-1 through $i$ during the derivation for the ACE-$i$ budget; $\gamma_i$ denotes the ACE-$j$ area fraction within the compound entity [and $\bar{W}$ in Eq. (2) the mean vertical velocity]. For instance, $\sigma_1$ and $\sigma_2$ as defined via Eq. (10) are identical to $\sigma_1$ and $\sigma_2$ in Eq. (9).

We see in Eq. (1) that the horizontal inflow and outflow across the lateral boundary $\ell_i$ of the compound entity includes a $1/\sigma_i^2$ factor to account for the ACE-$i$ area fraction. Similarly, the contribution by the horizontal flow across $\ell_{i-1}$, the compound boundary of the previous stage—would have a $1/\sigma_{i-1}^2$ factor. Indeed, the expression of $\gamma_{i-1}$ in Eq. (10) includes this factor as well as $A_{i-1}/A_i$ for the difference in cross section area between the adjacent ACE-($i-1$) and $i$ (e.g., $\gamma_2 \equiv A_1/A_2$ in Eq. (9)). Finally, other things being equal, turbulent entrainment is expected to be proportional to the perimeter-to-area ratio $g_i$ ($j = i-1, i$), which is a generalization of the shape factor $1/G$ in Part 1; see Eq. (12) therein.
Before turning to the mass flux budget, note that the terms on the right-hand side of Eq. (1), omitting nonconservative processes, represent the mean tracer flux convergence within ACE-i [recall Eq. (5)]. The same expression applies to vertical momentum by substituting \( w \) in lieu of \( C \), as used next for the mass flux equation.

b. Multi-ACE mass flux equations from single-ACE case

For the vertical mass flux, we start with Eq. (8) of Part 1 (for which no approximations have been made beyond the anelastic continuity equation)

\[
\frac{\partial (\rho_0 w)}{\partial t} = \nabla^{-2} \nabla_h^2 \left[ \rho_0 B - \nabla \cdot (\rho_0 u w) \right] + \nabla^{-2} \mathcal{D},
\]

(11)

where \( B \) denotes the buoyancy; \( \mathcal{D} \) a quadratic function of spatial derivatives of \( u \) [see Eq. (C2) of Part 1]; and \( \nabla^{-2} \) solving the 3D Poisson equation with boundary conditions imposed on \( \partial_t (\rho_0 w) \). Averaging Eq. (11) over \( A_i \) gives

\[
\partial_t (\rho_0 \bar{w}^j) = \nabla^{-2} \nabla_h^2 \left[ \rho_0 B - \nabla \cdot (\rho_0 u w) \right] + \nabla^{-2} \mathcal{D}^j.
\]

(12)

Next, we seek for the first term on the right-hand side an expression in terms of \( \bar{w}^j \).

1) Forcing partition and separable approximation

Postulate (P2) asserts that \( F \equiv \rho_0 B - \nabla \cdot (\rho_0 u w) = 0 \) outside the compound entity combining ACE-1 through \( n \). We can partition \( F \) into contributions from individual ACEs by introducing for each \( 1 \leq i \leq n \), \( \mathcal{H}_i(x,y) = 1 \) for \( (x,y) \in A_i \) and vanishes elsewhere, so that \( F = \sum_{i=1}^n F \cdot \mathcal{H}_i \) and

\[
\nabla^{-2} \nabla_h^2 \left[ \rho_0 B - \nabla \cdot (\rho_0 u w) \right] = \sum_{j=1}^n \nabla^{-2} \nabla_h^2 (F \cdot \mathcal{H}_j).\]

(13)

The second equality (marked \( s \)) here follows that the separable approximation \( F \cdot \mathcal{H}_j \approx \bar{F}^j(z) \cdot \mathcal{H}_j(x,y) \) is sufficient for recovering the mean acceleration profile due to the robustness of the nonlocal dynamics to fine-scale forcing variations; see section 3.d.1 of Part 1. The resulting expression is linear in \( \bar{F}^j \) by which the nonlocal operators \( \mathcal{N}_i(j) \) are defined.

The expression for \( \bar{F}^j \) can be identified through replacing \( C \) by \( w \) in Eq. (1), yielding

\[
\bar{F}^j(z) \approx \rho_0 \bar{B}^j - \partial_z (\rho_0 \bar{w}^j \bar{w}^j) - \partial_z (\rho_0 \bar{w}^j \bar{w}^j) - \bar{w}^j - \gamma_j^{-1} \mathcal{W}^{-j-1} \mathcal{W}^{-j-1} - \bar{w}^j - \gamma_j^{-1} \mathcal{W}^{-j-1} \mathcal{W}^{-j-1} + \frac{1}{\sigma_j} \left[ \partial_z (\rho_0 \mathcal{W}) \right]^{-j+1} + \frac{1}{\sigma_j} \left[ \partial_z (\rho_0 \mathcal{W}) \right]^{-j+1} \mathcal{W}^{-j} + g_j^{-1} \rho_0 u_{n} \bar{w}^{-j-1} - g_j^{-1} \rho_0 u_{n} \bar{w}^{-j}.\]

(14)

Equation (3) for the ACE-i mass flux then follows substituting Eq. (13) into Eq. (12).

Note that Eq. (3) is equivalent to Eq. (30) of Part 1 except we have not approximated the second term on the right-hand side here. An external tendency \( \mathcal{N}_\text{ext} \) on ACE-i arising from source outside the multi-ACE system is also included.

2) Consideration for general forcing distribution

In Eq. (13), the partition step (marked \( p \)) isolates forcing within each ACE that gives rise to the nonlocal acceleration field; the separable approximation step (marked \( s \)) then condenses the acceleration field down to a 1-D representation relying on the robustness of the nonlocal dynamics. Although the same indicator functions \( \mathcal{H}_j \)’s are used here for both steps, more sophisticated distributions—inaugured by observations or cloud-resolving simulations—can be substituted in the latter step in lieu of \( \mathcal{H}_j \)’s to include sub-volume variations within the entities. A single-ACE example illustrating such treatment is provided in Fig. 4d of Part 1 (see also Fig. F1 therein). We further note that the sub-volume structures of buoyancy and the convergence of vertical momentum flux need not be identical.

c. Spatial and numerical implementation

For the large-scale environment outside the multi-ACE system, we use the ARM Best Estimate (ARMBE) Atmospheric Measurements (ARMBEATM) data from Manus Island (Tropical Western Pacific; 2.06°S, 147.43°E; Xie et al. 2010). The environmental sounding is held constant during all of the simulations. The NOAA NCEI GIBBS imagery for GridSat B1 cloud-top temperature (Knapp 2008) supplements information on convective activities around the time/location of the sounding.

The example solutions presented in this manuscript are integrated following the procedure detailed in section 4 of Part 1, with modifications to accommodate for multi-ACE solutions. Specifically, the key Eqs. (1-2) of Part 1 are replaced here by Eqs. (1-3), (10), and (14). The same parameterized condensate loss, vertical eddy transport (set \( \nu_e = 0 \) given that lateral entrainment dominates.
mixing; Boing et al. 2014) and momentum diffusion4 apply separately within individual ACEs. The lateral turbulent entrainment is similarly treated with the mixing between ACE-i and i ± 1, across ℓi and ℓi−1, being (with i = j = −1, i)
\[
\frac{\overline{u_i'C_i'}}{\overline{u_i'w_i'}} \approx \mu \left( C_i' - C_i'^{i+1} \right),
\]
\[
\frac{\overline{u_i'w_i'}}{\overline{u_i'w_i'}} \approx \mu \left( w_i' - w_i'^{i+1} \right),
\]
where µ = 0.09 (units: m s−1)—consistent with Eq. (1) of Part 1 in which the 1/G factor is replaced by g_i and the turbulent entrainment rate ε_tur is now given by μg_i (see Appendix for time-marching).

Finally, while solutions are not presented here, one can easily implement a radiative-convective equilibrium by imposing \( S_0 = \left( \partial \theta_e / \partial T \right)_R \), where \( R \) is the radiative cooling, and by including parameterized surface fluxes in the lowest layer, e.g., with a simple bulk formula.

4 Oceanic deep-convective single-ACE case

Having seen the land nighttime single-ACE cases in Part 1, we examine an oceanic example in this manuscript for contrast, using the 0700 (LT) ARM-site sounding on 10 August 1998 from the tropical western Pacific island of Manus for solutions shown below. This section covers single-ACE instances serving as an opportunity to (i) demonstrate an example having multiple steady-state solutions; (ii) revisit the traditional-plume approximation to the ACE solution; and (iii) test the sensitivity of the ACE model to the parameterized component of dynamic perturbation pressure. A few multi-ACE illustrations using the same sounding for adjustment processes are included in the next section.

At the time of the Manus sounding, the environment is saturated between \( z = 0.3 \) and 2.2 km with the data recording 0.06-mm h−1 precipitation/drizzle. A close examination of the GridSat-B1 brightness temperature shows the island covered by low clouds, and isolated congestus clouds in surrounding areas around and after the time of the sounding; ∼ 9 h later an isolated deep-convective cloud can be seen near the island.

a. ACE with multiple steady states

The GridSat-B1 data indicates that deep convection did not occur until ∼ 9 h after the Manus sounding. While a steady updraft model indicates conditional instability and would convect immediately, the ACE admits another steady state. If initialized with extremely weak forcing or low-level downdraft, the ACE evolves to a weak shallow downdraft—with \( (\rho_0 \overline{w})_{\text{min}} \approx -0.25 \) kg m−2s−1—that can be maintained for hours. This is depicted as thick dotted magenta lines in Fig. 2a-d, shown for horizontal ACE diameter \( d = 5 \) km, and \( q_{\text{c ramp}} = 0.5 \) g kg−1 for the parameterized condensate loss. While the amplitude is small, this is a consistent solution dominated by negative buoyancy, reduced humidity, and downward motion in the lowest kilometer. Surface fluxes and ongoing ABL turbulence would likely cause the eventual exit from this state, but the fact that such a self-sustaining stable solution exists can yield differences in the timing of convection.

b. Land-ocean contrast of deep-convective ACE

The growth of the ACE into a deep-convective structure in Fig. 2a-d is illustrated for the same parameters \( d \) and \( q_{\text{c ramp}} \). The solution takes ∼ 1.5 h to reach the upper troposphere after being initiated by a weak updraft (1-km deep, \( \overline{w}_{\text{max}} \approx 9 \) mm s−1; with \( m_0 = 0.01 \) kg m−2s−1, \( z_d = 1 \) km following Appendix G of Part 1). The saturated low-level environment minimizes the CIN, yielding a deep layer of positive buoyancy. The vertical extent and magnitude of the transient cold-top negative buoyancy are moderate, consistent with the horizontal ACE diameter \( d = 5 \) km. But as the solution converges toward the steady state (thick green lines), the positive buoyancy yields a cold top that is substantially stronger than the transients as well as the land nighttime cases of the same and even larger horizontal sizes in Figs. 5-6 of Part 1.

Figure 2a-d also includes the deep-convective steady state for \( d = 20 \) km (thick dotted blue lines) showing the expected dependence on size, but the overall contrast between the small and large oceanic single-ACEs tend to be less dramatic than in the land instances.

c. DIB plume approximation to ACE buoyancy

Comparison of steady-plume solutions in the land case (Fig. 5 of Part 1) asked the question of whether common mixing assumptions could mimic ACE solutions (see section 5 of Part 1 for plume computation procedure). The results indicate that the deep-inflow B (DIB) plume provides a reasonable approximation to the ACE steady-state buoyancy, albeit with deviations in buoyancy in the middle and upper troposphere, while exhibiting substantial errors in mass flux. DIB mixing decreases to small values in the mid-upper troposphere, so here we consider whether addition of a modest vertically constant turbulent mixing in the steady plume would be sufficient to overcome this difference. In Fig. 2, a comparison similar to that in Fig. 5 of Part 1 is conducted but adding a vertically constant turbulent component to the DIB entrainment. As demonstrated by the DIB plume profiles (thin magenta lines) in Fig. 2a-d computed with and without this turbulent mixing \( \varepsilon_{\text{tur}} \), including \( \varepsilon_{\text{tur}} \) can improve the approximation to the ACE buoyancy in the middle to upper troposphere, although compensating errors are still visible in \( \overline{\theta_e} \) and \( \overline{q_l} \).

4Following Morrison (2017), we set \( \varepsilon_{\text{tur}} = 4 \mu / d \) with a dimensionless \( \mu = 0.09 \) and \( d = 5 \) km.
Fig. 2. Growth of the oceanic deep-convective single-ACE using the 0700 (local time) 10 August 1998 sounding from Manus Island (tropical western Pacific). (a-d) Profiles for horizontal ACE diameter $d = 5$ km, and $q_{c,\text{cmp}} = 0.5$ g kg$^{-1}$ for condensate loss, initiated with a 1-km deep weak updraft with maximum vertical velocity $w_{\text{max}} \approx 9$ mm s$^{-1}$ ($z_d = 1$ km and $m_0 = 0.01$ kg m$^{-2}$s$^{-1}$ following Appendix G of Part 1); the thick green lines represent the eventual steady state. An alternative steady state of the single-ACE solution—started with a weak initial downdraft—is included as thick dotted magenta lines showing a weak shallow downdraft. For comparison, the thick dotted blue lines show the corresponding steady solution of a deep-convective single-ACE for $d = 20$ km. The thin magenta lines display traditional plume computations with the deep-inflow B (DIB) entrainment assumption, with and without an additional turbulent mixing $\varepsilon_{\text{tur}}$, using the same $q_{c,\text{cmp}} = 0.5$ g kg$^{-1}$. The thick gray and purple lines in (a-b) indicate environmental $\theta_e$, $q_v$ and saturation values $\theta_{e,s}, q_{s}$. DIB plume mass flux in (d) is scaled by a factor of 2/3 to fit. (e-f) Same as the case in (a-d) but without the momentum diffusion $\mu D \partial^2 (\rho_0 w)$ [see Eq. (2) of Part 1], showing a growing deep convection subsequently weakened by the environmental air inflow of dynamic entrainment. The color shading is saturated for $\theta_e$ over 355 K, and for the values of dynamic entrainment/detrainment $\partial \xi (\rho_0 w)$ below $-4 \cdot 10^{-3}$ or exceeding $2.5\cdot 10^{-3}$ kg m$^{-3}$s$^{-1}$. White contour marks the cloud boundary as measured by $q_v = 0.01$ g kg$^{-1}$.

However, as in the land case, the mass flux from the steady-plume model remains a poor approximation since it does not properly include the nonlocal effects. Such crude representation of mass flux strength implies that the com-
pensating error in \((\overline{\theta_e}, \overline{q}_t)\) cannot be remedied simply by tuning parameters in the entraining plume models. Recall that in a steady state [Eq. (33) of Part 1], the removal of condensate \(\overline{\rho}_c \equiv \overline{S}_c / \overline{w}\) depends on the vertical velocity, which even the tuned plume models poorly capture.

d. Mixing enhancement by dynamic-induced pressure perturbation

One key simplification for the current ACE implementation is the use of mass flux diffusion in lieu of the term \(\nabla^2 D\) as part of the dynamic-induced PGF [recall Eq. (28) of Part 1]. This and the examples presented are in part based on their tendency to yield steady-state deep-convective solutions suitable for comparison with traditional updraft plumes. Here, we test the sensitivity of the model by switching off the vertical diffusion of mass flux, i.e., setting \(\mu_D = 0\) in Eq. (2) (of Part 1; note that \(v_e = 0\) for all runs), and repeating the single-ACE oceanic run for \(d = 5\) km, illustrated in Fig. 2e-f.

After the initial chain of thermals seen in \(\overline{\theta}_e\), the cloud top reaches a maximum height of \(\sim 13.5\) km, then drops gradually with the in-cloud \(\overline{S}_c\) falling simultaneously. The cloud-top height here is notably lower than the steady-state counterpart. Without the mass flux diffusion, the representation of dynamic-induced pressure perturbation—\(a_{DV}\) in Eq. (7) of Part 1—tends to spawn small-scale bubble-like structures in the vertical, therefore enhancing the overall mixing via dynamic inflow as illustrated by \(\partial_x (\rho_0 \overline{w})\) (see also Morrison et al. 2020a; Peters et al. 2020; Gu et al. 2020b). The strong lateral mixing in turn results in a lower cloud top and weakening of the deep-convective ACE.

5. System of interacting ACEs

Examples in the last section (and those in Part 1) are for single ACE. Here, following the approach outlined earlier for multiple interacting ACEs, a case is illustrated for a multi-ACE system resembling a typical GCM grid cell. To aid discussion of the solution behavior, we briefly review the adjustment process associated with convection before turning to the results.

a. The convective adjustment problem

Much work on convective adjustment focuses on the subsidence compensating the convective updraft (Yanai et al. 1973; Arakawa and Schubert 1974) and the associated warming feedback on convection to yield quasi-equilibrium (e.g., Xu and Emanuel 1989; Emanuel et al. 1994; Raymond 2000; Raymond and Sessions 2007; Tulich et al. 2007). As noted in Holloway and Neelin (2007) and Li et al. (2022), the typical profile of convective heating includes a cold-top feature, i.e., cold anomalies in the layer encompassing the top of convection. This convective cold top is a large-scale phenomenon—not to be confused with the overshooting top commonly seen in pictures of cumulonimbus clouds. The cold-top signal emerging in, e.g., the southeast Pacific region where deep convection rarely occurs (see Fig. 7 of Li 2022) points to adjustment involving the large-scale wave dynamics.

Examples in Part 1 have demonstrated that the cold top emerges naturally in the anelastic solution from the vertically nonlocal dynamics. How this convective-scale phenomenon interacts with the large-scale environment warrants further clarification—since the subsidence compensating the convective updraft cannot produce a cooling tendency in the stably-stratified upper troposphere.

To this end, Figs. 2 and 4 of Liu and Moncrieff (2004) provide guidance by displaying in a 2D CRM (i) the cold anomaly above 12 km driven by the imposed heating below; (ii) the propagating wavefront of the deepest mode of speed \(\sim 50\) m s\(^{-1}\) around \(X = 1,500\) km; and (iii) a slower wavefront near \(X = 500\) km of high-vertical-mode gravity waves (\(\sim 17\) m s\(^{-1}\)) spreading the cold anomalies in the upper and lower troposphere. The deepest mode—associated with the compensating subsidence—can quickly impact a broad area, while the slower, higher gravity-wave modes yielding upward motion at locations in the upper and lower troposphere drive the large-scale environment adjusting towards a weak temperature gradient (WTG) state. Including the Coriolis force does not fundamentally alter the picture; see Figs. 3 and 5 of Liu and Moncrieff (2004).

Knowing from the CRMs what to expect, we now turn to the multi-ACE example and examine the extent to which the ACE solutions can represent the adjustment process while simultaneously simulating cloud formation.

b. A concentric 8-ACE cell

The multi-ACE system of interest consists of 8 concentrically nested ACEs separated by diameters \(d = 5, 10, 15,\) 25, 35, 45, and 55 km as illustrated in Fig. 1b. Together, these ACEs constitute a grid cell of 64 km \(\times\) 64 km—the same horizontal domain size used for the computation of the nonlocal basis (Appendix F of Part 1). The outermost ACE-8 can be viewed as a GCM-grid-cell environment in which the inner ACE-1 through 7 are embedded. This 8-ACE cell is assumed to be in a larger-scale environment where \(C_{env}\) profiles are held constant and \(w_{env} = 0\). The solution displayed in Figs. 3-4 are from the same simulation which uses \(q_{c, ramp} = 0.5\) g kg\(^{-1}\) for the parameterized condensate loss.

We choose an example that illustrates both remote forcing and the potential time lag of convective development. This case is initiated by a weak, shallow updraft in the outermost ACE-8, effectively from the grid-cell environment \(c_d = 1\) km; \(m_0 = 0.03\) kg m\(^{-2}\) s\(^{-1}\) or \(w_{max} \approx 0.03\) m s\(^{-1}\) causing deep convection in the innermost ACE over 3 h later.
Fig. 3. Evolution of the multi-ACE system with the geometry in Fig. 1b, showing the mean vertical mass flux \( \rho_0 w' \) (units: \( \text{kg m}^{-2}\text{s}^{-1} \)) within individual ACEs. The 0700 (LT) 10 August 1998 Manus sounding is used for the large-scale environment, held constant during the run. The solution is initiated by imposing a 1-km-deep updraft on the outermost ACE-8 (\( z_d = 1 \text{ km}, n_0 = 0.03 \text{ kg m}^{-2}\text{s}^{-1} \)) as indicated by the orange arrow in (h). The colorbar ranges are chosen to highlight the inward and outward propagation of the signal, with values exceeding the ranges saturated.

Figure 3 shows the evolution of the mean vertical mass flux within individual ACEs; the corresponding buoyancy (color shading) and cloud boundary (blue contours) are in Fig. 4. Initially, a very shallow signal propagates inward from the outer ACE. We start with Fig. 3h in which an orange arrow marks the initial updraft in ACE-8. The updraft induces an inflow mixing from the large-scale environment, yielding a shallow layer of cloud and positive buoyancy below 2 km (Fig. 4h), which gradually enhances the shallow updraft. The buoyancy subsequently drives a
Fig. 4. As in Fig. 3, but showing the buoyancy $\mathbf{B}'$ (color shading; units: m s$^{-2}$) and cloud boundary (blue contours for $\bar{q}_c = 0.01$ g kg$^{-1}$).

downdraft in the neighboring ACE-7 (Fig. 3g) and a heating tendency (Fig. 4g), starting a slow inward propagation ($\sim 8$ m s$^{-1}$) of the shallow layer of downdraft (Fig. 3a-f) and buoyancy (Fig. 4a-f), followed by shallow cloud formation in ACE-6 and inner ACEs.

Around 80 min into the simulation, the signal originating from ACE-8 has reached the inner ACE-1 with the buoyancy (i.e., temperature deviation from the large-scale sounding) below 2 km almost homogenized across individual ACEs. It takes more than 100 min afterwards for shallow cloud to emerge in ACE-1 (Fig. 4a), and another 40 min to transition to deep convection. We have noted in Part 1 that buoyant layers at low levels do not induce strong vertical acceleration because of the nonlocal dynamics in-
teracting with the surface boundary condition. This and the weak horizontal buoyancy gradient—recall the effective buoyancy due to horizontal variation of Archimedean buoyancy (Kuo and Neelin 2022)—result in the slow transition to deep convection seen here, despite the persistent low-level positive buoyancy. During this period, vertically propagating gravity-wave packets are visible in the mass flux evolution (e.g., Fig. 3a-c).

Starting around 220 min, ACE-1 goes through a rapid (~45 min) development of a deep-convective updraft. With ACE-1, the evolution is similar to the single-ACE case in Fig. 2a-d. This deep convection initiates an outward propagating adjustment process that shuts down the deep updraft about 30 min after its maximum.

c. Combined evolution of stratiform cloud and grid-cell adjustment

The adjustment process combines features of classic horizontal gravity-wave adjustment and dynamical interactions with the stratiform cloud layer. The buoyancy signal, showing positive values below ~11 km followed by negative values above (Fig. 4; see also Fig. 5 for a quantitative depiction of the profiles), propagates outward with an estimated speed of ~20-30 m s⁻¹. The stratiform-cloud formation lags the buoyancy since the moist air outflow from the inner ACEs is slower than gravity waves. In the outer ACEs 6 through 8, the buoyancy structure, including the cold-top feature, arises from the descending and ascending motions (Fig. 3f-h) that would be characteristic of adjustment by the deepest and higher gravity-wave modes. For ACEs in between, e.g., ACE-2 adjacent to the deep-convective ACE-1, the formation of the cold top involves the outflow of saturated air carrying condensate from ACE-1 (Fig. 4b) which leads to latent cooling and hydrometeor loading offsetting the adiabatic heating induced by compensating subsidence (Fig. 3b). The resulting mass flux exhibits a core-cloak structure as noted in Gu et al. (2020a). Overall, the adjustment constrains the multi-ACE cell towards a state of weak buoyancy gradient (WBG) in the horizontal, as expected from the wave dynamics seen in Liu and Moncrieff (2004).

By holding the large-scale sounding constant, the environment is treated as an external reservoir of conditional instability from which the ACE cell can draw energy indefinitely. For the single-ACE cases in Fig. 2a-d, the temperature difference between the ACE and environment (i.e., buoyancy) permits the ACE to convert the environmental instability to high-θe/MSE for the upper tropospheric outflow while maintaining the deep-convective updraft. In the multi-ACE example here, the updraft in ACE-1 shuts down around 280-300 min (Fig. 3a) as soon as the cell interior reaches a WBG state (Fig. 4).

Figure 5 provides three more multi-ACE examples with 8, 4, and 2 entities, respectively labeled as 8-ACE, 4-ACE, and 2-ACE in the legend. The near-equilibrated buoyancy profiles are shown for each ACE (profiles closely overlap). All three instances are initiated by imposing on the innermost ACE-1 a weak shallow updraft of 1-km depth and \( \bar{w}^1_{\max} \approx 9 \text{ mm s}^{-1} \) together with a compensating substance in the outermost ACE. The 8-ACE configuration is from Fig. 1b, in which the lateral boundaries of diameters \( d = 5, 10, 25 \) km are retained for the 4-ACE; the 2-ACE cell consists of an ACE of \( d = 5 \) km and the rest of the grid cell. The buoyancy profiles are snapshots at 200 min into the simulations for all 3 instances, ~140 min after starting transitioning to deep convection. The vertical line segments (indexed i) in the magenta box mark the anvil-layer depths within individual ACE-i at the same time. Single-ACE (green curve) denotes the 5-km ACE steady state in Fig. 2c. The buoyancy snapshot at 360 min (~140 min after the transition starts) from Fig. 4a (brown curve) shows the effect of remote initialization.
layers are shallower in the 2-ACE cell than in the other two cases.

As noted in the last subsection, the single-ACE setup by construction maintains a horizontal temperature gradient relative to the fixed large-scale environment. In contrast, a multi-ACE cell tends to evolve to homogenize the temperature. Thus the near-equilibrated multi-ACE buoyancy profiles in Fig. 5 exhibit lower LNBs and weaker cold tops compared with the single-ACE buoyancy (green curve)—which is steady-state but without any equilibration of its surroundings—despite the same environmental sounding and initial updraft. The impact of horizontal buoyancy gradient also introduces a dependence on nonlocal initiation. In the 8-ACE case in Figs. 3-4 in which the evolution is initiated from the grid-cell environment ACE-8, the inward propagation of the shallow signal adjusts the lower troposphere toward a WBG state before the onset of deep convection (Fig. 4). This yields a weaker deep-convective state \( \sim 140 \text{ min afterwards} \) (Fig. 5: brown curve) showing a lower LNB and a reduced cold top compared with the other multi-ACE instances.

While interactive embedded ACE systems and careful evaluation of the optimal approach for grid-cell adjustment must be left for further work, Figs. 3-5 suggest the following. The horizontal grid-scale adjustment process in even a simple multi-ACE setting can capture the convective cold top and gravity-wave behavior—albeit discretized to simplify horizontal structure. Furthermore, the co-evolution of stratiform-cloud formation and the adjustment process differs significantly from the typical assumptions in convective parameterizations, pointing to potential advantages of a more self-consistent treatment.

6. Considerations for coupling to a large-scale model

While the results presented here are limited to evolution of ACE(s) with a fixed environment, it is important to provide context for how this can be used as a parameterization, and as a diagnostic in support of parameterization. In principle, the ACE approach is similar to the multiscale modeling framework (MMF, a.k.a. superparameterization; Khairoutdinov and Randall 2001; Randall et al. 2003) in running a higher-resolution, time-dependent solution for subgrid convective processes embedded in each grid cell of the large-scale model. As such, the most straightforward use of ACE is to build on the formulation (and potentially on existing code) of the MMF following the presentation of Grabowski (2001) and Randall et al. (2016). Consider a small number of ACEs integrated under a GCM grid cell. These can be connected as demonstrated in previous sections or assumed to be sufficiently far apart that they evolve independently. Presumably, at least two should be coupled in a manner resembling the 2-ACE cell in Fig. 5. Including stochastic forcings via \( \mathcal{N}_{\text{ext}}^i \) in Eq. (3) for influences of boundary layer turbulence and other unrepresented processes is likely useful, although this could also be supplied from the PBL and/or shallow-convective schemes. The cell-average tendency from the ACE solution would be supplied to the corresponding tendencies in the large-scale model as in MMF.

Potential benefits of ACE in this context include (i) postulated lower computational cost—Fig. 5 has demonstrated that relatively few instances per grid cell can capture the effects of relatively rare deep-convective elements; (ii) the possibility of leveraging the partially analytic approach for understanding and parameterization diagnostics; (iii) helping to fill a gap in the parameterization hierarchy between traditional parameterizations and superparameterization; and (iv) the possibility that machine learning efforts (Brenowitz and Bretherton 2018; Rasp et al. 2018; Beucler et al. 2021; Lopez-Gomez et al. 2022) can leverage the explicit physics, causal pathways and conservation properties inherent in the ACE while empirically determining, e.g., properties of stochastic terms or probability distribution of assumed geometry as elaborated in the Discussion. We further postulate that a nested ACE system of grid-cell size like those examined earlier might reduce the sensitivity exhibited by MMF to the size of the CRM subdomain (Kuo et al. 2023), or at least, could provide useful ways of diagnosing this issue.

Alternately, for a GCM that is set up with a mass-flux based parameterization, an ensemble of ACEs can be used for the computation of overall mass flux (presumably in addition to that from a shallow-convective parameterization). The eddy-diffusivity mass-flux (EDMF) scheme (Siebesma et al. 2007), the multi-plume analytic model (MAP; Peters et al. 2021), the bin-macrophysics method (Neggers 2015) or the deep-convective element of the current GFDL two-plume parameterization (Zhao et al. 2018) are apparent targets for which ACE can substitute for the mass-flux component. Added value would likely be provided by (i) process-oriented approximation to dynamic inflow and outflow for postulated cloud-size distribution; (ii) explicit time dependence with memory of subgrid-scale processes (Tan et al. 2018; Schneider et al. 2021); (iii) consistent closure for perturbation pressure (He et al. 2022). The multi-ACE formulation introduced here should also provide a framework for assisting scale-aware implementation (Arakawa and Wu 2013; Zheng et al. 2016; Su et al. 2021). But even without fully implementing ACE in GCMs, results in Part I have highlighted an empirical remedy that permits convection under surface inversion with simplified nonlocal representation (Lee et al. 2008; Wang et al. 2015).

7. Summary and Discussion

a. ACE properties and behaviors


This manuscript presents solutions for multiple interacting entities constituting a typical GCM grid cell by iteratively applying the single-column ACE formulated in Part 1 while keeping track of transfers between adjacent entities—expressed as dynamic inflow and outflow in a form that is slightly different from the traditional notions of entrainment and detrainment. To illustrate the multi-ACE behavior and to contrast with land nighttime cases in Part 1, simulations using an oceanic sounding are included here. In a single-ACE solution with a fixed environment, this can yield two distinct steady states (or near-steady), respectively representing a deep-convective updraft and a weak subsidence. The convective cold top again emerges as part of the solution but with depth and strength being less sensitive to feature size than the Amazon cases in Part 1. The deep-convective solutions need not be steady—for instance, turning off the parameterized momentum diffusion yields a thermal-chain succession of buoyant bubbles (Morrison et al. 2020a; Peters et al. 2020) that enhance the overall mixing via dynamic inflow (see also Peters 2016; Gu et al. 2020b).

To demonstrate multi-ACE interaction, an ACE system resembling a typical GCM grid cell is considered, with 2-, 4-, and 8-ACE instances, to examine grid-cell environmental adjustment in response to deep convection. As expected, the evolution after onset of convection is consistent with gravity-wave adjustment towards a WTG/WBG state (Neelin and Held 1987; Sobel and Bretherton 2000; Singh and O’Gorman 2013). However, it includes detailed vertical structure associated with higher gravity-wave mode adjustment as in Liu and Moncrieff (2004) that is more complex than the deepest-mode adjustment by compensating subsidence (Yanai et al. 1973; Arakawa and Schubert 1974). This carries the cold-top structure from the embedded convection into the grid-scale domain. Intriguingly, the adjustment interacts strongly with stratiform-cloud formation. Outflow of saturated air carrying condensate does not simply spread passively into the larger domain, but are altered by dynamical interactions such as vertical adjustment within the stratiform layer during formation. We also test remotely initiating convection in the concentric 8-ACE system with small vertical velocities in the grid-cell environment. Higher gravity-wave modes, seen in the inward propagation of a shallow perturbation, initiate deep convection after a multi-hour delay. This illustrates the potential role of time dependence in development of convection.

b. ACE geometry, feature size, and spatial grid spacing

The ACE formulation effectively makes the following bet: that the combination of partially analytic pre-solution and judicious choice of geometry can yield solutions that are useful both for understanding convection and to capture features omitted in steady-plume models at less cost than superparameterization. It also aims at situations where relatively few convective entities occur within a grid cell, such that an assumption of a large ensemble would not work well and investing in more detailed solutions is worthwhile. While optimizing the choice of geometry (including the number of ACEs) in comparison with observations will be an endeavor for future work, considerations for the single- and multi-ACE cases include the following.

As noted in Kuo and Neelin (2022), at the larger scales involved in convection (≥ 5 km) the nonlocal dynamics helps simplify the representation of convective cloud/storm without fully resolving the fine-scale turbulence. Radar wind profiler observations typically having a horizontal resolution of ∼ 2.5 to 5 km indicate that the area fraction of convective updraft is not a strong function of height (Schiro et al. 2018; Savazzi et al. 2021)—at least through the lower to middle troposphere where substantial entrainment occurs. These motivated the single-ACE formulated in Part 1 where each deep-convective ACE represents a cloud column, contrasting with traditional plume models in which one tends to have in mind (an ensemble of) smaller features (LeMone and Zipser 1980; Lucas et al. 1994; Peters et al. 2023).

The multi-ACE solutions in this manuscript reduce dependence on column assumptions, albeit in a discretized way. In the concentric scenario schematized in Fig. 1b, if one were to increase the number of entities while reducing the thickness of individual cylindrical shells, the solution would converge toward a system similar to an axisymmetric LES/CRM. Viewed in this way, multi-ACE instances covered here have time-dependent geometry—see Figs. 3-4 in which the convective area varies with both time and height, be it defined by positive vertical velocity or high condensate value. Moreover, results here suggest these instances yield useful approximation to the adjustment even with only a small number of entities.

Trade-offs between running multiple interacting ACEs versus including a prognostic component for the feature size—akin to tracking the horizontal inflow and outflow via a Lagrangian approach—should be considered. For example, analytic estimates for the growth rates of a convective system like Seeley et al. (2019) and Elsaesser et al. (2022) for which dynamic outflow from the current ACE implementation could be used as an input. Large-domain LES/CRM—aided by machine-learning approaches—can provide additional constraints on choices for ACE structure to best match CRM results as well as the interaction with the background flow. However, while we have shown that a few ACEs per GCM grid cell can capture behavior missing from conventional parameterizations, the extent to which this is a useful alternative to superparameterization is yet to be determined.
c. Caveats, wishlist and promising features

A number of caveats missing features should be underlined. While the inherent cold top in ACE solutions seems promising, coordinated LES/CRM experiments (Bretherton and Smolarkiewicz 1989; Alexander and Holton 2004; Morrison and Peters 2018; Tarshish et al. 2018) would be crucial in clarifying the relative contributions of processes to cold-top formation and the large-scale environment responding to convection. While it is in principle easier to include state-of-the-art microphysics schemes (e.g., Grabowski et al. 2019; Morrison et al. 2020b; Chandrakar et al. 2021) in the ACE model than in steady plumes, this has not yet been demonstrated. Though ACE columns can handle updrafts or downdrafts, including interactive precipitation flux that can generate downdrafts and cold pools through evaporative cooling is likewise still a wishlist item. The current implementation neglects part of the dynamic perturbation pressure—the expression for $D$ [Eq. (C2) of Part 1] suggests links to storm-relative flow, windshear, vorticity, and propagation of the convective systems that could be included, albeit with some effort.

Besides deep-convective parameterizations in GCMs, conventional plume models have been adapted for a range of applications. We envision that ACE solutions can be a potentially useful substitute for plume models for these, including PBL/shallow convection (Suselj et al. 2019; Shen et al. 2022), wildfire/biomass-burning plumes (Paugam et al. 2016; Brockway 2021; Tuite 2022), pyrocumulus clouds (Tarshish and Romps 2022), as well as subglacial discharge plume driving melting in fjord (Jackson et al. 2017; Hewitt 2020; Zhao et al. 2021, 2024).

Even though the current version of the ACE model is not yet ready for representing complex behaviors such as MCS circulation, propagation, and interaction with the large-scale flow, the examples covered in this work demonstrate promising features, such as self-consistent dynamic inflow and outflow, convective cold top, less sensitivity to CIN in nighttime convection, and more consistent interactions with the grid-scale environment. Overall, this class of model offers potential advantages over steady-plume models both as process models and to experimentally replace the conventional parameterization approach.

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Data availability statement. DOE’s ARM Data Center maintains and provides the ARMBEATM data accessible via http://dx.doi.org/10.5439/1333748. The GridSat-B1 dataset is maintained and provided by NOAA NCEI available at https://www.ncei.noaa.gov/products/climate-data-records/geostationary-IR-channel-brightness-temperature; the Global ISCCP B1 Browse System (GIBBS) provides an online interface for visualizing the GridSat-B1 (formerly ISCCP-B1) data at https://www.ncdc.noaa.gov/gibbs/. A MATLAB implementation of the ACE model is available at https://github.com/yihungkuo/ACE_MATLAB_v0.1/.

APPENDIX

Time marching for multiple interacting ACEs

While the ACE formulation permits multiple numerical discretizations in the vertical and time, for solutions here we note the following. The parameterized diffusion terms would require a small time step $\Delta t \sim (\Delta z)^2$ using an explicit method. For multi-ACE simulations, we adapt an operator-splitting approach (Hundsdoerfer and Verwer 2003) with an implicit method for the diffusive tendencies (permitting $\Delta t \sim \Delta z$). Specifically, at each time step $t_k$, state variables $\overline{C}_k$, $\overline{w}_k$ are first updated by all non-diffusive tendencies using an explicit Bogacki-Shampine scheme (Bogacki and Shampine 1989), yielding an intermediate state denoted by $(\overline{C}_{k+1/2}, \overline{w}_{k+1/2})$. The intermediate values are then updated by the diffusive tendencies using a 1st-order backward Euler method for $\overline{C}_{k+1}$, $\overline{w}_{k+1}$ at time $t_{k+1}$. In this work, the momentum diffusion $\mu D \frac{\partial^2 (\overline{p_0 w})}{\partial x^2}$ with $\mu D$ defined via the nonlinear Eqs. (C3-C4) of Part 1 would complicate the backward Euler iterations. We thus approximate the $(\mu D)_{k+1}$ values at $t_{k+1}$ using the values evaluated based on the intermediate state.

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