Localization of an Electromagnetic Source with Magnitude Only Measurements in a 2D Metallic Cavity with Scatterers

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Abstract—A time harmonic electromagnetic (EM) source inside a 2D metallic cavity is considered. Scatterers of known geometry and dielectric constants are present inside the cavity. These scatterers alter the fields in the cavity. The coordinates of the source are unknown, while those of the scatterers are given. The magnitude of the resulting electric field is measured at certain locations in the cavity. The goal is to locate the source from these magnitude-only measurements.

As there is only one source, its localization is formulated as a sparse source reconstruction problem. The 2D metallic cavity is discretized using a Finite Difference scheme. Determination of the source location is posed as a constrained minimization problem with the cost being the norm of the error between the measured and the simulated signal obtained using the discretized Maxwell’s equation. An additional $L_1$ regularization is added to the cost to impose sparsity. Linear constraints based on the principle of Electromagnetic Time Reversal (EMTR) are derived and enforced. Numerical results are presented for random choices of measurement locations. Numerical experiments suggest that the source can be localised with measurements taken at $5\%$ of the total points on the search grid.

Index Terms—Electromagnetic Source localization, Finite Difference Frequency Domain, Computational Electromagnetics, Electromagnetic time reversal

I. INTRODUCTION

Localization of electromagnetic (EM) sources finds applications in several domains due to the evolution of the Internet of Things, Mobile Sensor Networks, etc [1]. Conventional localization methods are based on Time of Arrival, Time difference of Arrival, Direction of Arrival and Received Signal Strength (RSS) [2] and Proximity based method [3]. Methods based on Time of Arrival require synchronization of the sensor clocks and are dependent on the velocity of the field. If there is no direct line of sight (between the source and the sensor) and/or if there are many scatterers, time of arrival methods do not give reliable estimates of the source location due to reflections and multiple paths [4]. RSS-based methods rely on path loss formulas and/or propagation models to find the distance between the source and the sensors. Since the received power is the measurement in RSS, time synchronization between the sensors is not required. If there is no line of sight, parametrization and estimation of the propagation models become crucial [5]. The presence of scatterers can significantly alter the nature of wave propagation. Furthermore, if the region of interest is a closed room/cavity with obstacles, then there will be multiple reflections, and hence, the estimated propagation models become less reliable. Proximity based methods requires large number of sensors such that the source is always in proximity with any one of sensors.

Solving Maxwell’s equation using computational tools is a common practice in many engineering problems in the fields of medicine, defence, nondestructive testing, etc. The solution to Maxwell’s equation is unique if the source, material characteristics, domain and boundary conditions are precisely specified [6]. The Finite Difference Frequency Domain (FDFD) is a full wave numerical solver for Maxwell’s equations. Spatially discretizing Maxwell’s Equations in the frequency domain leads to linear equations. FDFD method has been used successfully for analyzing photonic crystals [7].

Electromagnetic Time Reversal (EMTR) is the process of time reversing and retransmitting the recorded EM signal from the receivers. These retransmitted signals converge at the source locations [8]. Using numerical solutions of Maxwell’s equations for time reversal has generated significant interest recently. Localization with minimal sensors in a cavity was proposed in [9]. It consisted of two stages. In the first stage, called the forward stage, sensors recorded the electric field (in the time domain) at a few locations. In the second stage, i.e., the time reversal stage, the recorded time domain data was time-reversed and retransmitted as sources in a finite difference domain (FDTD) simulation. At each time step of the FDTD simulation, a measure called entropy, an indicator of sparsity, was calculated. The time step corresponding to the last local minima of entropy was noted. Locations corresponding to peak values of the electric fields at that time step indicated the actual source locations. Recent works on EMTR had shown promising results for source localization in applications like lightning localization [10], transformer partial discharge localization [11]–[13], and to find breakpoints in grounding electrodes [14]. In [15]–[17] entropy was replaced by electromagnetic kurtosis. Reconstruction of an impulse source in an ergodic time reversal cavity was proposed in [18]. Amplitudes of the impulse sources were obtained by minimizing the LASSO-type objective function. A correlation based localization using EMTR was proposed for indoor positioning in [19] where localization is achieved using just...
two measurements. Reconstruction of the transmitted time harmonic signal in a cavity using EMTR was discussed in [20]. All of the above-mentioned time-reversal methods require time domain measurements. In practice, the magnitude and phase of fields are measured at a range of frequencies, and time domain data is reconstructed by summing the sinusoids as experimentally validated in [21]. This paper is motivated by the idea of utilizing Computational Electromagnetics tools for source localization in an indoor positioning environment. A closed room with metallic walls is abstracted as a cavity. Objects in the rooms are modelled as scatterers. In the context of indoor positioning, it is easier to measure the magnitude. This paper proposes a method for localizing EM sources using magnitude-only measurements.

A 2D metallic cavity with scatterers inside it resembling the scenario described above is considered. The space in the cavity is discretized using Yee’s grid [6]. The phasor form of Maxwell’s equation is discretized on Yee’s grid using the finite difference scheme. Finite difference approximations at the grid points lead to linear equations with variables being the Electric fields. The absolute value of the electric field at certain known positions in the grid are measured. The sources are modelled as a finite sum of cylindrical wavefronts parameterized by their amplitudes and their centres. Hence, the electric field at any point in the cavity is linearly related to the amplitudes of the sources.

The source localization problem is formulated as a source amplitude reconstruction problem. As a first step, a set of grid points where the source can be expected is defined. This definition of expected source location can be based on apriori information. If there is no apriori information, all the grid points in the cavity, excluding that of boundary and/or scatterers, can be defined as possible source locations. Thus, these sources are now parameterized only by their amplitudes. Ideally, these amplitudes equal the actual value at the true source location. At other locations, these amplitudes should be zero. Since the electric field is measured at a limited set of points, source amplitudes are obtained by solving an optimization problem. The objective function of this optimization problem includes the norm of the difference between the measurement and the solution of discretized Maxwell’s equation. An additional $\mathcal{L}_1$ regularization is added to the objective function to enforce sparsity. As the amplitudes are positive, the search space is restricted to the positive quadrant. Furthermore, linear constraints based on EMTR are derived and enforced to reduce the search space further.

The main novelties of this paper are,

1) Localization of a time harmonic source is proposed, where frequency domain numerical solution of Maxwell’s equations is used in the localization process. To the best of the author’s knowledge, all the existing works use time domain methods which need phase measurements [20], [21].

2) Localization is based only on the measurement of magnitude. To the best of the author’s knowledge, among the works utilizing numerical solutions of Maxwell’s equation for localization, this has not been attempted.

3) Constraints based on EMTR are derived and enforced. Even though EMTR in the frequency domain requires phase conjugation during time reversal, constraints derived here depend only on the magnitude of the measurements.

The rest of this paper is organized as follows

- Section II describes the problem of source localization and the corresponding assumptions involved.
- Section III describes the discretization of Maxwell’s Equation and derivation of linear equations using the FDFD method. Formulation of the optimization problem and the constraints based on the EMTR are also derived therein.
- Section IV presents the numerical results obtained using the above formulation.
- This paper concludes in section V with a detailed discussion of the proposed method and possible future extensions.

II. PROBLEM DESCRIPTION

Consider a 2D cavity with arbitrarily shaped obstacles, as illustrated in Figure 1. A source is emitting a cylindrical EM field at a wavelength $\lambda_0$. The location of the source is assumed to be unknown. Magnitude of the electric field are measured at specific known locations. Measurement points may or may not be in direct line of sight with the source. The objective is to localize the source (determine the position of the source) from the magnitude-only measurements. It is assumed that

- The source is static throughout.
- The surface of the cavity is a Perfect Electric Conductor (PEC).
- The complex permittivity $\epsilon$ and permeability $\mu$ of the obstacles corresponding to the wavelength $\lambda_0$ are known.

![Source](https://via.placeholder.com/150)

Source

Measurements

PEC Boundary

Fig. 1. Metallic cavity with scatterers

IZ. METHODOLOGY

A. Maxwell’s Equations and FDFD solution

Since the problem is in 2D Transverse Magnetic (TM) polarization ($E_z$ polarization) is assumed without loss of gen-
erality. EM field in the cavity satisfies the following Maxwell’s equations:

\[
\begin{align*}
\frac{\partial E_z(r)}{\partial y} &= -j\omega\mu(r)H_x(r) \\
\frac{\partial E_z(r)}{\partial x} &= j\omega\mu(r)H_y(r) \\
\frac{\partial H_y(r)}{\partial x} - \frac{\partial H_x(r)}{\partial y} &= j\omega\epsilon(r)E_z(r)
\end{align*}
\] 

where

- \( \mathbf{r} \) is the Position vector \((m)\)
- \( E_z \) is the electric field along the \( z \) direction \((V/m)\)
- \( H_x \) & \( H_y \) are magnetic field \((A/m)\) along \( x \) and \( y \) directions respectively
- \( \omega = 2\pi c/\lambda_0 \) is the angular frequency with \( c = 3 \times 10^8 m/s \) being the velocity of light in the free space
- \( \epsilon(r) \) and \( \mu(r) \) are complex permittivity \((F/m)\) and permeability \((H/m)\) at location \( r \)

Due to the PEC boundaries, while solving Maxwell’s equation’s boundary conditions are \( E_z(\mathbf{r}) = 0 \forall \mathbf{r} \) on the boundary. As the obstacles are irregular in shape an analytic solution is not determinable. Therefore, numerical methods (approximations) are the only alternative.

Here, the cavity is partitioned into a square grid called Yee’s grid, in which grid points for electric field and magnetic fields are staggered as illustrated in Figure 2. The black dots are the Electric fields grid points while the blue dots are magnetic field grid points. The grid lengths along the \( x \) and \( y \) axes are \( \Delta_x \) and \( \Delta_y \) respectively.

\[
\begin{align*}
\frac{1}{\Delta_x}[E_z(m+1, n) - E_z(m, n)] &= -j\omega\mu(m, n)H_x(m, n) \\
\frac{1}{\Delta_y}[E_z(m, n+1) - E_z(m, n)] &= j\omega\mu(m, n)H_y(m, n) \\
\frac{1}{\Delta_x}[H_y(m+1, n) - H_y(m, n)] - \frac{1}{\Delta_y}[H_x(m, n+1) - H_x(m, n)] &= j\omega\epsilon(m, n)E_z(m, n)
\end{align*}
\] 

with \( m = 1, 2, \cdots, N_x \) and \( n = 1, 2, \cdots, N_y \). The discretized equations (2) corresponding to all grid locations can be represented in a matrix form

\[
\mathbf{A}\mathbf{E}_z = 0,
\]

where \( \mathbf{A} \in \mathbb{C}^{N_x N_y \times N_x N_y} \) (referred to as the wave matrix) is full-rank matrix and sparse. The vector \( \mathbf{E}_z \in \mathbb{C}^{N_x N_y} \) is formed by a column-wise stacking of the electric fields \( E_z \) in the grid. A detailed description of FDFD can be found in [7].

Sources can be incorporated in FDFD simulation as current density or as incident fields. If current density form is used, in equation (1), the curl equation of the magnetic fields will be

\[
\frac{\partial H_y(r)}{\partial x} - \frac{\partial H_x(r)}{\partial y} = j\omega\epsilon(r)E_z(r) + \mathbf{J}_c(r),
\]

where \( \mathbf{J}_c \) is the current density \((A/m^2)\). In this case, during the time reversal phase, calculating the equivalent current density of a point requires knowledge of the electric fields at all its neighbouring points. Further, if the magnitude of the electric field alone is measured, calculating equivalent current density at any point is very difficult. Hence, the incident field approach (mentioned below) is used to incorporate the sources.

If there are \( J \) number of sources producing cylindrical waves with centres \( \mathbf{r}_l, l = 1, 2, \cdots, J \), the total incident electric field at any location \( \mathbf{r} \) is

\[
E_{z(inc)}(\mathbf{r}) = \sum_{l=1}^{J} \frac{a_l}{\| \mathbf{r} - \mathbf{r}_l \|^{2}} e^{j k \| \mathbf{r} - \mathbf{r}_l \|}
\]

\[
= \left[ \begin{array}{c}
\frac{1}{\| \mathbf{r} - \mathbf{r}_1 \|^{2}} e^{j k \| \mathbf{r} - \mathbf{r}_1 \|} \\
\frac{1}{\| \mathbf{r} - \mathbf{r}_2 \|^{2}} e^{j k \| \mathbf{r} - \mathbf{r}_2 \|} \\
\vdots \\
\frac{1}{\| \mathbf{r} - \mathbf{r}_J \|^{2}} e^{j k \| \mathbf{r} - \mathbf{r}_J \|}
\end{array} \right]
\begin{bmatrix}
a_1 \\
a_2 \\
\vdots \\
a_J
\end{bmatrix}
\]

\[
E_{z(inc)}(\mathbf{r}) := g(\mathbf{r})\mathbf{a}.
\]

As before, by stacking the incident fields at the grid points the source vector

\[
\mathbf{f}_{src} = \begin{bmatrix}
E_{z(inc)}(1, 1) \\
E_{z(inc)}(2, 1) \\
\vdots \\
E_{z(inc)}(N_x, 1)
\end{bmatrix} = \begin{bmatrix}
g(1, 1) \\
g(2, 1) \\
\vdots \\
g(N_x, 1)
\end{bmatrix} \mathbf{a}
\]

\[
f_{src} := G\mathbf{a},
\]

where \( G \in \mathbb{C}^{N_x N_y \times J} \) with each row being \( g(\mathbf{r}) \). Columns of \( G \) represent the field due to a point source at each grid point and \( \mathbf{a} = [a_1 \ a_2 \ \cdots \ a_J]^T \) is the vector of source amplitudes.

In equation (3), \( \mathbf{E}_z \) denotes the total field in the cavity. This total field is the sum of the incident field (produced by the sources) and the scattered field. Normally, in order to calculate the total field, the simulation domain is segregated in to regions of two types: the Scattered Field Region (SFR)
and the Total Field Region (TFR) [22]. Segregation is done such that all sources are enclosed in the SFRs while the rest are the TFRs. The boundary between a TFR and a SFR acts as a virtual Huygen’s surface enclosing the source. Due to Huygen’s principle, the surface current induced on the Huygen’s surface produces the incident field in the TFR and cancels out the field due to the source enclosed with in them [23]. Conversely, if the incident field is known, any region in the simulation domain consisting of a source can be designated as SFR enclosed by a Huygen’s surface with no incident field in the interior. It should be noted that Huygen’s surface acts as a virtual source and does not reflect the scattered fields back in the interior. The choice of SFR depends on the nature of the application. In this work, the sources are assumed to produce cylindrical wavefronts. Therefore the SFR is chosen to be the points immediately surrounding the sources. An illustration is given in Figure 3. The source location \((\times)\), along with its immediate neighbouring grid points \((\Box)\) constitute the SFR. The grid points can be chosen small enough such that the measurement locations are not in the SFR. A sparse diagonal masking matrix \(Q\) of dimension \(N_x N_y \times N_x N_y\) with the diagonal elements \(Q_{ii} \in \{0,1\}\) is used for indicating if the grid points belong to SFR or TFR. If a grid point \((m, n)\) belongs to the SFR, then the corresponding diagonal element \(Q(N_x(n-1) + m, N_x(n-1) + m)\) is one. Otherwise, it is zero.

In the following, without loss of generality, \(Q\) is assumed to have ones along the diagonal in the first \(N_y\) rows, i.e.,

\[
Q = \begin{bmatrix}
I_{N_x \times N_x} & 0 \\
0 & 0
\end{bmatrix}.
\]

Using the above \(Q\), the source field vector \(f_{src}\), (6), can be partitioned such that

\[
f_{src} = \begin{bmatrix}
f_s \\
f_t
\end{bmatrix},
\]

where, \(f_s\) and \(f_t\) are elements of \(f_{src}\) in SFR and TFR respectively. Similarly, the total field can be partitioned as

\[
E_z = \begin{bmatrix}
E_{ts} \\
E_{tt}
\end{bmatrix}.
\]

where, \(E_{ts}\) and \(E_{tt}\) are total fields in SFR and TFR respectively. Corresponding scattered fields in SFR \((E_{ss})\) and TFR \((E_{st})\) are related with incident field as,

\[
\begin{align*}
E_{ss} &= E_{ts} - f_s \\
E_{st} &= E_{tt} - f_t
\end{align*}
\]

To capture the behaviour of Huygen’s surface cancelling the incident field in SFR, define a vector \(\bar{E}_z\) such that it has scattered fields in the SFR and total fields in the TFR, i.e.,

\[
\bar{E}_z = \begin{bmatrix}
E_{ss} \\
E_{tt}
\end{bmatrix}
\]

As \(\bar{E}_z\) has combination of scattered and total fields,

\[
A\bar{E}_z \neq 0
\]

Based on the definition of \(Q\), \(A\bar{E}_z\) can be partitioned as,

\[
A\bar{E}_z = \begin{bmatrix}
A_{ss} & A_{st} \\
A_{ts} & A_{tt}
\end{bmatrix} \begin{bmatrix}
E_{ss} \\
E_{tt}
\end{bmatrix}
\]

In equation (12), equality is achieved when all fields in a row are defined either only by scattered fields or only by total fields [22]. i.e.,

\[
\begin{align*}
A_{ss} E_{ss} + A_{st} E_{st} &= 0 \\
A_{ts} E_{ts} + A_{tt} E_{tt} &= 0,
\end{align*}
\]

Substituting the values of \(E_{st}\) and \(E_{ts}\) in equation (14) results in,

\[
\begin{align*}
A_{ss} E_{ss} + A_{st} E_{ts} &= A_{st} f_t \\
A_{ts} E_{ss} + A_{tt} E_{tt} &= -A_{ts} f_s
\end{align*}
\]

In matrix form, it can be represented as

\[
\begin{bmatrix}
A_{ss} & A_{st} \\
A_{ts} & A_{tt}
\end{bmatrix} \begin{bmatrix}
E_{ss} \\
E_{tt}
\end{bmatrix} = \begin{bmatrix}
0 \\
-A_{ts}
\end{bmatrix} \begin{bmatrix}
f_s \\
f_t
\end{bmatrix}
\]

\[
= (QA - AQ)f_{src}
\]

A detailed discussion on the TFR/SFR interface can be found in [22] and [24].

Assume that \(I\) number of measurements are made in the TF region. Each measurement corresponds to absolute value of electric field \(E_z\) at the grid location \((m,n)\). The measurement vector \(Y_m \in \mathbb{R}^I\) can be represented as

\[
Y_m = |H E_z|
\]

where, \(|(\cdot)|\) is element wise absolute value of complex vector and \(H\) is \(I \times N_x N_y\) matrix with elements either 0 or 1. If the magnitude of electric field at grid location \((m,n)\) is the \(i^{th}\) measurement, then, in the \(i^{th}\) row of measurement matrix, the element \(H(i, N_x \times (n-1) + m)\) is 1, others are 0.

From (6) and (16),

\[
Y_m = |HA^{-1}(QA - AQ)G f_{src}|
\]

Denoting \(\bar{A} = HA^{-1}(QA - AQ)G\), the measurement vector becomes

\[
Y_m = |\bar{A} a|
\]
B. Electromagnetic Time Reversal based constraint

Assume that there are $J$ number of sources injecting sinusoidal Electric fields within the cavity at the frequency $\omega_0$ rad/s. Let the input electric fields be given by

$$x_j(t) = a_j \sin(\omega_0 t + \phi_j), \quad j = 1, 2, \ldots, J,$$

(20)

where, $a_j$ is the magnitude and $\phi_j$ is the phase of the source $j$. There are $I$ number of measurements. If $h_{ij}(t)$ is the impulse response for the source at $i^{th}$ location and detector at $j^{th}$ location, the output electric field received by the detector at $i^{th}$ location in frequency domain is given by

$$y_i(\omega) = \sum_{j=1}^{J} H_{ij}(\omega) \cdot x_j(\omega), \quad i = 1, 2, \ldots, I.$$

(21)

However, the measured signal is the magnitude

$$|y_i(\omega)| = \left| \sum_{j=1}^{J} H_{ij}(\omega) \cdot x_j(\omega) \right|.$$

(22)

and not (21).

In order to simulate time reversal, all the measured signals must be re-injected at $j$. However, $y_i(\omega)$ is a complex quantity requiring amplitude and phase measurement. Measurement of phase requires the knowledge of the source signal, which by problem definition is not known. Hence, only the magnitude of the measured field is re-injected at the $j^{th}$ location (in a FDFD simulation), i.e., phase is assumed to be zero. The corresponding electric field at source location $i$ is given by

$$S_i^j(\omega) = H_{ji}(\omega) \cdot |y_i(\omega)|,$$

(23)

which implies

$$|S_i^j(\omega)| = |H_{ji}(\omega) \cdot |y_i(\omega)| |L0^0\rangle,$$

(24)

and

$$= |H_{ji}(\omega)| \cdot |y_i(\omega)|. \quad (25)$$

At the frequency $\omega_0$, the following holds for the magnitudes

$$|y_i(\omega_0)|^2 = |y_i(\omega_0)y_i^*(\omega_0)|$$

$$= \left| \sum_{j=1}^{J} H_{ij}(\omega_0) \cdot x_j(\omega_0)y_j^*(\omega_0) \right|$$

$$\leq \sum_{j=1}^{J} |H_{ij}(\omega_0)| \cdot |x_j(\omega_0)||y_j^*(\omega_0)|$$

$$\leq \sum_{j=1}^{J} |H_{ij}(\omega_0)||x_j(\omega_0)||y_i(\omega_0)||$$

$$\leq \sum_{j=1}^{J} \pi a_j |H_{ij}(\omega_0)||y_i(\omega_0)||.$$  

(26)

Since the cavity considered has linear and isotropic scatterers, it is reasonable to assume that the impulse responses are symmetric in space. Hence, $H_{ij}(\omega) = H_{ji}(\omega)$. Then,

$$|y_i(\omega_0)|^2 \leq \pi \sum_{j=1}^{J} a_j |H_{ji}(\omega_0)||y_i(\omega_0)| = \pi \sum_{j=1}^{J} a_j |S_i^j(\omega_0)|,$$

or,

$$\sum_{j=1}^{J} a_j |S_i^j(\omega_0)| \geq \frac{1}{\pi} |y_i(\omega_0)|^2 \quad (27)$$

For all the $I$ measurements, the condition in equation (27) can be stacked as

$$\begin{bmatrix}
|S_i^1(\omega_0)| & |S_i^2(\omega_0)| & \cdots & |S_i^J(\omega_0)| \\
|S_i^1(\omega_0)| & |S_i^2(\omega_0)| & \cdots & |S_i^J(\omega_0)| \\
\vdots & \vdots & \ddots & \vdots \\
|S_i^1(\omega_0)| & |S_i^2(\omega_0)| & \cdots & |S_i^J(\omega_0)|
\end{bmatrix}_{I \times J} \begin{bmatrix}
a_1 \\
a_2 \\
\vdots \\
a_J
\end{bmatrix}_{J \times 1} \geq \begin{bmatrix}
\frac{1}{\pi} |y_1(\omega_0)|^2 \\
\frac{1}{\pi} |y_2(\omega_0)|^2 \\
\vdots \\
\frac{1}{\pi} |y_I(\omega_0)|^2
\end{bmatrix}_{I \times 1} \quad (28)$$

It should be noted that, the terms $|y_i(\omega_0)|$ and $S_i^j(\omega_0)$, $i = 1, 2, \cdots, I$ and $j = 1, 2, \cdots, J$ are known. Unknown variables are the amplitude of the sources. Appropriately defining $S$ and $d$ the constraint (28) can be compactly written as

$$Sa \geq d. \quad (29)$$

C. Source Localization using Sparse Optimization

Source localization is posed as a problem of amplitude reconstruction. Note that (19) represents an over-determined non-linear system of equations. The amplitudes are positive and must also satisfy the EMTR-based constraint (29). An ideal choice to estimate the source location would be to solve the optimization problem:

$$\min_a \left[ |\tilde{A}a - Y_m| \right]^T [\tilde{A}a - Y_m]$$

$$Sa \geq d \quad (30)$$

$$a \geq 0,$$

where $T$ is the transpose. However

$$[|\tilde{A}a - Y_m|]^T [\tilde{A}a - Y_m] = a^T \tilde{A}^H \tilde{A}a + Y_m^T Y_m - 2Y_m^T \tilde{A}a \quad (31)$$

where $H$ denotes the Hermitian of a matrix. Note that (31) is difference of convex functions which is not convex. Instead, $\| \tilde{A}a - Y_m \|_2^2$ which expands as,

$$\| \tilde{A}a - Y_m \|_2^2 = [\tilde{A}a - Y_m]^H [\tilde{A}a - Y_m]$$

$$= a^T \tilde{A}^H \tilde{A}a + Y_m^T Y_m - 2Re(Y_m^T \tilde{A}a), \quad (32)$$

and has the same quadratic terms as that of (31), is convex (and differentiable). Therefore it is preferred as the objective function.

Furthermore, as there is only one source the vector $a$ should be sparse. An $L_1$ regularization with a weighting factor $\alpha$ is
added to enforce sparsity. Thus the optimal amplitude \( \mathbf{a}^* \) is obtained by solving

\[
\arg \min_{\mathbf{a}} \| \mathbf{Aa} - \mathbf{Y}_m \|_2 + \alpha \| \mathbf{a} \|_1 \\
\text{subject to: } \mathbf{Sa} \geq \mathbf{d} \\
\mathbf{a} \geq 0.
\]

(33)

The choice of \( J \) depends on the apriori information. If there is no apriori information about the source location, a finer search grid spanning the entire cavity can be allocated as possible source locations.

The steps for source localization are summarized below.

1) Define \( J \) number of points (search grid) corresponding to expected source locations
2) Measure the magnitude of the electric fields at locations
3) for each measurement do the following (in FDFD simulation)
   a) Re-inject the received field at the detector location (with phase as zero).
   b) Calculate the field \( S_j^2 \) at all the \( J \) points in search grid
4) Solve the optimization problem in equation (33) to obtain the optimum value of \( \mathbf{a}^* \). Locations corresponding to the element with maximum value \( a_j^* \), is the location of the source.

Initially, a search grid spanning the entire cavity, with a grid-length of 0.1m excluding the boundary is chosen. It has 9801 points. It should be noted that out of 250,000 points in the Yee’s grid, only 9801 points are chosen for search grid. But, 0.1m grid length produces a finer grid in the context of localization. 400 measurement locations are randomly generated such that they are at least 1m away from actual source location and are not on the scatterers. The optimal values of the expected source amplitudes are obtained as in Figure 6. Only the non-zero components of optimal source amplitude are plotted for image clarity. The
maximum value of the optimal input amplitude is observed at \( \mathbf{r} = [9.1 \ 6.9]^T \), which is very close to the actual source location \([9 \ 7]^T\). This slight deviation may be due to inherent numerical dispersion in the FDFD method. Further, non-zero amplitudes are grouped around the actual source location.

In order to check the effect of measurement locations, the numerical experiments are repeated 30 times. A new set of 400 random measurement locations is chosen at every run. As in the previous case, these locations are at least 1\( \text{m}\) away from the actual source location and are not on the scatterers. For the optimal source locations obtained from 30 runs, 2D histogram is shown in the Figure 7. For 13 out of 30 runs, the source is localized at \( \mathbf{r} = [9.1 \ 6.9]^T \). For 11 out of 30 runs, the source is localized at \( \mathbf{r} = [9.4 \ 7.5]^T \). It corresponds to a distance error of 0.69\( \text{m}\).

To test the scenario of No Line of Sight (NLOS), source is placed at \( \mathbf{r} = [9.5 \ 9.5]^T \). 40 measurement locations are randomly chosen in the rectangular region \((x, y) \in [0, 6] \times [0, 7]\) so that there are scatterers between the measurement locations and the source. The search grid is chosen to cover the rectangular region \((x, y) \in [7, 9.9] \times [7, 9.9]\) with spacing of 0.1\( \text{m}\). So there are 900 points in the search grid. Idea of guessing a subdomain of simulation for search grid is not new. It has already been used in \([19]\). The optimal amplitudes are obtained as given in Figure 8 where, the maximum value is observed at the actual source location \(i.e., \mathbf{r} = [9.5 \ 9.5]^T\).

Numerical results indicates that, for a finer search grid, to localize the source, it is sufficient to have number of measurements equals 5% of number of search grid points. While compared with time domain-based time reversal methods, the number of measurements is higher. As discussed in Section 1, time domain based time-reversal methods utilize magnitude and phase information at a range of frequencies. Here, as the time-harmonic sources are considered, the information available is limited to only one frequency. Further in that frequency, phase is not considered for measurement. Loss of phase is compensated by a relatively large number of magnitude-only measurements. It should be noted that if there is no information about the amplitude and phase of the source, measurement of phase difference with respect to that of the source is very difficult. Even in such a scenario, with a relatively large number of measurements, the proposed method can locate the source in the presence of scatterers, even when there is no direct line of sight.

V. CONCLUSION

EM source localization using frequency domain magnitude-only measurement is proposed. The sparse optimization problem is constructed to reconstruct input amplitudes with EMTR-based constraints such that the optimal solution reveals the source location. Numerical simulations indicate a high chance of localization, even if the number of measurements is very small compared to the number of grid points used in solving Maxwell’s equations. The source is localised for a finer grid with measurements in less than 5% of grid points. As Maxwell’s equations are directly solved during the optimization, the proposed method can work even in the presence of scatterers.

Despite the advantages, the proposed method has limitations as given below.

- Computational effort for FDFD simulation while evaluating the matrix \(\mathbf{S}\).
- Need for the knowledge of \(\epsilon_r\) for the scatterers.
- Need for many measurements compared to time domain based time reversal methods. Even though it is a limitation of the proposed method, in many applications, magnitude measurement at many points is easier and cheaper than phase measurement.

A. Scope for future works

- Current work is focused on source localization in a metallic cavity. But localization in non-metallic cavity can have widespread applications.
- Few works in literature based on Surface Integral equations discuss input reconstruction \([27], [28]\). These works...
do not require knowledge of the relative permittivity of scatters. But phase measurement is necessary.

- In this work, measurement locations are randomly chosen. Choice of measurement locations significantly affect the localization accuracy. Proper choice of measurement locations can drastically improve the localization accuracy with fewer measurements. Compressed sensing literature provides a detailed framework for the effective choice of the measurement matrix [29].

Future work will focus on adapting these methods for magnitude-only measurement and extending this work to 3D and evaluation in an experimental setup.

REFERENCES


