Two-Dimensionl Ducting Propagation of Whistler-Mode Chorus Waves in Magnetic Peaks

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Abstract

Through one-dimensional propagation theory, recent studies have demonstrated that chorus waves can be ducted not only by density structures, but also by magnetic peaks. However, such a ducting propagation in the inner magnetosphere has not been verified yet. In this study, we perform ray tracing simulations to investigate the two-dimensional propagation of chorus waves in magnetic peaks. It is found that chorus waves indeed can be ducted by the magnetic peak via repetitive reflections in both sides, similar to that of one-dimensional propagation theory. The physical mechanism for these repetitive reflections has also been discussed. Furthermore, parameter analysis has been implemented to show the effect of wave frequencies and incidence positions on the propagation of chorus waves. Our results will be helpful in improving the understanding of the duct propagation for whistler waves in the magnetosphere.
Figure 1.
Two-Dimensional Ducting Propagation of Whistler-Mode Chorus Waves in Magnetic Peaks

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Key Points:

- Ray tracing simulations have been utilized to investigate the two-dimension ducting propagation of chorus waves observed in magnetic peaks.
- Our result reveals that chorus waves can be ducted by the magnetic peaks via repetitive reflections, especially for upper-band chorus waves.
- Theoretical analysis about ducting chorus waves has been given to explain the repetitive reflections along the wave propagation path.
Abstract

Through one-dimensional propagation theory, recent studies have demonstrated that chorus waves can be ducted not only by density structures, but also by magnetic peaks. However, such a ducting propagation in the inner magnetosphere has not been verified yet. In this study, we perform ray tracing simulations to investigate the two-dimensional propagation of chorus waves in magnetic peaks. It is found that chorus waves indeed can be ducted by the magnetic peak via repetitive reflections in both sides, similar to that of one-dimensional propagation theory. The physical mechanism for these repetitive reflections has also been discussed. Furthermore, parameter analysis has been implemented to show the effect of wave frequencies and incidence positions on the propagation of chorus waves. Our results will be helpful in improving the understanding of the duct propagation for whistler waves in the magnetosphere.

Plain Language Summary

The existence of the duct can change the propagation characteristics of whistler-mode waves in the inner magnetosphere, thus affect wave-particle interaction. Recently one-dimensional theoretical analysis and numerical simulation have exhibited the wave-mode coupling and propagation of chorus waves in the magnetic peaks. In this study, two-dimensional ray tracing simulations are performed. Through parametric analysis and theoretical explanation, our results show the propagation mechanism of ducted waves in dipole magnetic field, which exhibit repetitive reflections in the magnetic duct. Furthermore, it also shows that the waves launched near the center can propagate to higher latitudes, especially for the upper band waves. Our results add new insights into the propagation of whistler-mode waves within the magnetosphere.

1 Introduction

Whistler-mode chorus waves are typically observed in the low-density nightside region outside the plasmapause. These waves often exhibit unique time-frequency spectrograms characterized by discrete rising or occasionally falling tones lasting a few tenths (e.g., Burtis & Helliwell, 1969; Burton & Holzer, 1974; Li et al., 2011). Due to the power gap near $0.5f_{ce}$, chorus waves are often classified into lower band ($0.1-0.5f_{ce}$) and upper band ($0.5-0.8f_{ce}$) components. They can both accelerate and cause loss of energetic electrons within Earth’s inner magnetosphere (Horne & Thorne, 1998; Summers et al., 1998; Tao et al., 2008; Thorne et al., 2013, Wang & Shprits, 2019). They are also believed to be responsible for diffuse and pulsating aurora (Thorne et al., 2010; Ni et al., 2011). Furthermore, chorus waves are considered to be the source of plasmaspheric hiss through propagating into the plasmasphere (Bortnik et al., 2008, 2009, 2011).

In general, chorus waves in the inner magnetosphere are considered to propagate in a dipolar magnetic field, which is named as nonducted wave. During its propagation, the wave normal angle (WNA) becomes increasingly oblique due to the inhomogeneous magnetosphere (Colpitts et al., 2020). On the other hand, ducted chorus waves, as the name suggests, can be guided by fine-scale density irregularities (Ke et al., 2022; Hanzelk & Santoli, 2019) and propagate along the field line over considerable distances with less attenuation. Recent studies have shown that magnetic peaks, which are local magnetic structures characterized by sudden increases in magnetic field strength, can also act as ducts guiding on whistler-mode waves (Yu & Yuan, 2022; Nejad & Streltsov, 2023; Streltsov & Nejad, 2023). However, these studies focus on
one-dimension propagation of chorus waves. The propagation properties of chorus waves under
the effect of a magnetic peak in the inner magnetosphere remains uncovered yet.

In this letter, we report two cases of whistler-mode waves in magnetic peaks observed by
Van Allen Probe A. Through ray tracing simulations and comparison with nonducted waves, we
find the whistler-mode waves can propagate guided by magnetic duct, exhibiting repetitive
reflections within the edges of ducts. However, we also observe differences between upper band
and lower band whistler waves. Our results suggest that magnetic peak duct have potential to
significantly affect the propagation of whistler-mode waves.

2 Observations

In this work, the properties of chorus waves are provided by the waves instrument of the
Electric and Magnetic Field Instrument Suite and Integrated Science suite (EMFISIS) (Kletzing
et al., 2013) onboard Van Allen Probes. The plasma density is obtained from the high-frequency
receiver on EMFISIS. In addition, the AE and SYM-H indices are obtained from the OMNI
database.

Event 1 (Figures 1a-1f) was observed during 20:00-22:00 UT on 8 December 2016. Our
period of interest spans from 20:28 to 21:28 UT, highlighted in gray shading. During this time
frame, the probe was located near the magnetic equator (0° < MLAT < 1°) at midnight (23:30 <
MLT < 00:00) within L shells mainly ranging between 5.35 and 5.75 (~ 0.4 L). Figure 1a
illustrates the AE and SYM-H indexes. The increase in AE index implies a moderate substorm
and enhanced injection of particles. The minimum of SYM-H index decreases to about -30 nT,
indicating enhancement of the ring current. As shown in Figure 1b, the electron number density
is about 2.5 cm⁻³, meaning the probe is in a low-density plasma trough area. As shown in Figure
1c, the Bz component in the GSM coordinate system exhibits a significant enhancement,
meaning a typical magnetic peak. Figures 1d-1f show the characteristics of waves: power
spectral density (PSD) of magnetic field, ellipticity polarizability (Ellip) and WNA. Chorus
waves in the magnetic peak are identified based on several criteria: quasi-parallel propagating
(WNAs < 45°), right-hand polarization (Ellip ~ 1) and intense wave power (PSDs >
10⁻⁶ nT²/Hz) near 0.5 fce (marked by the dashed curves).
Figure 1. Overview of the observations of two events from Van Allen Probes A. Event 1 (left) is during 20:00–22:00 UT on 8 December 2016 and Event 2 (right) is during 17:15–17:20 UT on 25 October 2018. (a) AE and SYM-H indices. (b) Total background electron number density. (c) Total background magnetic field strength (black line) and z component (blue line) in GSM coordinate. (d) PSD of magnetic field component of the waves. (e) Ellipticity. (f) WNA. The gray shadow region in every event indicates the time interval of observation of magnetic peak structure. (g–j) are like panels (a)–(d), (k–m) are like panels (d)–(f).

Event 2 was observed during 17:15-17:20 UT on 25 October 2018. During this period, the satellite was located near the magnetic equator (MLAT ~ -1°) on the nightside (MLT ~ 22:40) within an L shell of approximately 5.77. Our period of interest is also highlighted in gray shading. The AE index during this time is about 450 nT and the SYM-H index is near -6 nT (Figure 1g). At 17:17:30 UT, there is a significant DFB with a sudden enhancement of $B_z$ (Figure 1i), while the electron number density experiences a slight change (Figure 1h). Fortunately, burst mode data near 17:17:27 UT are available. Based on the criteria described earlier, the properties of whistler-waves with a frequency near $0.6f_{ce}$ are shown in Figures 1k-1m.

From both two events, whistler-mode waves are observed within the magnetic peak structure as the electron number density slightly changes on the nightside near the magnetic equator during period of strong geomagnetic activity (AE > 300 nT). In event 1, the magnetic peak crosses over ~ 0.4 L shells (2548 km) and the maximum magnetic field strength increases by around 40 nT. Oppositely, in event 2, the magnetic peak only crosses over about 2.7 km and the maximum magnetic field strength increases about 5 nT. Based on these differences, we...
define event 1 as a large duct case (LD) and event 2 as a small duct case (SD). These two events provide an excellent opportunity to investigate the propagating mechanism of whistler-mode emissions in different size of magnetic peak ducts.

3 Ray Tracing Simulations

To investigate the propagation of whistler-mode waves in two sizes of magnetic peaks, we adopt the method of ray tracing simulation developed by Horne (1989), which is often employed to study the evolution and propagation properties of waves (Chen et al., 2010, 2021, 2022; Xiao et al., 2016; Deng et al., 2022; Feng et al., 2023). In this study, we perform a 2-D ray tracing simulation, where a diffusive equilibrium plasma density model mentioned by Bortnik et al. (2011) as the cold plasma number density. Specifically, we set the nightside parameters \(N_e = 2 \text{ cm}^3 \text{at } L = 6\) mentioned in it and position the plasmapause at \(L = 4\) to align with the observed geomagnetic activity (AE > 300 nT).

To model the magnetic peak structure, we adopt a model of dipolar magnetic field superposed by a field-aligned increase, which is described by a Gaussian-like term in \(B_z\) as shown in Equation (1)-(2).

\[
B'_z = B_{zdip} + B'_d \\
B'_d = B_d \exp \left(-\frac{4(L - L_d)^2}{\delta L^2}\right)
\]

where \(B_{zdip}\) is the \(z\) axis component of the dipolar magnetic field model, \(B_d\) is the magnetic field strength increase at the center of the duct, \(L\) is the \(L\) shell, \(\delta L\) is the half-width of magnetic duct (in units of \(L\) shell), and \(L_d\) is the center of the duct (in units of \(L\) shell). Note that this approach is akin to previous study on density duct (Ke et al., 2022). In our simulations, we set \(L_d = 6\) for both cases. Although there may be weak enhancement in \(B_{xy}\) at higher latitude, this little influence is ignored in this study.

To study the effects of both ducts on the propagation of whistler-mode waves and the differences with nonducted waves, we launch four rays initially in parallel (i.e., WNA = 0°) from the magnetic equator at \(L = 6\) to study the propagation of lower band and upper band chorus waves in LD and SD cases. We choose the frequency of \(0.6 \times f_{ce}\) \((0.3 \times f_{ce})\) for the upper (lower) band chorus wave. The rays are traced until they reach either the plasmapause boundary \((L = 4)\) or MLAT > 40°. Additionally, we use equation 2 from Chen et al. (2010) to calculate the Landau damping. The distribution of electron velocity is fitted to the statistical results presented in Bortnik et al. (2007), which are for hot electron flux (0.21 - 26 keV) within \(4 < L < 7\) on the nightside during strong activity (AE > 300 nT). We also follow the fitting method described in it to obtain a fitted distribution function:

\[
f(v) = 244.2 v^{-2.7} m^{-6} s^3\]

To model the magnetic fields of LD, we use the following parameters for the modified magnetic field: \(\delta L = 0.2\ L, B_d = 40\ \text{nT}\). Our simulated results are shown in Figure 2. Figure 2a shows the magnetic field strength at the magnetic equator with blue and black curves denoting LD and nonduct, respectively. Figures 2b-2c exhibit the ray paths for upper band \((0.6f_{ce})\) and lower band \((0.3f_{ce})\) waves in the meridian plane. The arrows represent the direction of the wave vector, while the gray and green shadows represent the plasmasphere and duct, respectively. For the propagation process of upper band waves (Figure 2b), the ducted wave (blue solid line)
generally propagates along a field line at the center of the duct with slowly growing WNA. However, the nonducted wave (black solid line) tends to cross more L shells inward to the earth, and the WNAs become more oblique as propagating to higher latitudes. For lower band waves (Figure 2c), the paths of both rays seem similar. The ducted wave (blue dashed line) exhibits slight reflections near the center, and the WNAs rapidly increase at the beginning. While the nonducted wave (black dashed line) initially propagates outward (higher L shells) and then returns inward with gradually increasing in WNAs, the behavior is different from the upper band nonducted wave, which may because of the frequency, WNA and Gendrin angle characteristics (Smith et al., 1960).

Figures 2d-2g show the further insights into the behaviors of the refractive index (n), L shell, WNA and wave gain as functions of latitudes during wave propagation. Initially, all four rays exhibit similar features, but the differences become apparent as they propagate away from the equator. For nonducted waves (black solid (upper band) and dashed (lower band) lines), the n (Figure 2d) of upper band wave grows faster, meaning its phase velocity is slower than that of lower band wave. The variation in L shells (Figure 2e) aligns with the ray paths has been discussed earlier, where the upper band wave propagates from L = 6 to L ~ 4.8 and the lower band wave propagates outward firstly at L ~ 6.2 and then inward at L ~ 6. The WNAs (Figure 2f) of both nonducted rays point outward (WNA > 0°) and become more oblique with increasing latitudes, but the upper band wave undergoes a more rapid oblique propagation. This behavior leads to intense Landau damping. As shown in Figure 2g, when the attenuation of both rays reaches -20 dB, the lower band wave can propagate to latitude of ~ 4°, while the upper band wave only propagates to latitude of ~ 4°. For ducted waves as shown by blue solid (upper band) and dashed (lower band) lines, however, the n exhibits different behavior compared with that of nonducted waves. It remains almost constant at n ~ 5 for the upper band wave, whereas the lower band wave exhibits two significant crests in n. The ray paths in Figure 2e show both ducted waves generally propagate along a single magnetic field line from the launch position (L = 6), especially for the upper band wave. In correspondence with the n, the WNAs of lower band wave also exhibit two significant crests, consistently surpassing those of the upper band wave, which propagates with gradually increasing WNAs. Figure 2g illustrates that when the attenuation of both ducted rays reaches -20 dB, the upper (lower) band ducted wave can propagate to latitude of ~ 5° (~ 6°), experiencing smaller (higher) Landau damping compared to nonducted waves with the same normalized frequency.
Figure 2. Comparison of two-band waves in large duct (LD) and nonduct. (a) Magnetic field strength model of duct and nonduct. Ray paths of (b) upper band rays launched and (c) lower band rays in LD and nonduct, arrow represents direction of wave vector. (d)-(g) Refractive index, L shell, WNA and wave gain as functions of latitudes during propagation of rays. (h)-(i)
are partial amplification for (d) and (e), corresponding to magenta box in (d) and (e). (j)-(r) are similar to (a)-(i), but for waves in LD.

To investigate the fine features of two-band waves (blue lines) in LD, such as the variation of n and L shell during wave propagation, we magnify the latitude range from 5° to 20°, corresponding to the magenta boxes in Figure 2d and 2e. As shown in Figure 2h, the n of the lower band wave experiences an initial rise followed by a decline, but it remains approximately constant for upper band wave. In addition, we exhibit the magnetic field gradient with L shells in Figure 2i. It shows that the lower band wave reflects first in the negative gradient region and then in the positive gradient region with the reflection points being about ±0.06 ΔL (30% δL, ΔL is the distance from the center), corresponding to a gradient of approximately ~±500 nT/L. Interestingly, combining Figure 2h and 2i, for lower band wave, it is obvious that when the ray propagates in the negative gradient region, the n increases, and it decreases while the ray is in the positive gradient region. The center the ray passes is associated with the maximum (or minimum) n. But for upper band wave, both n and ray path generally maintain a straight line with slight changes near the center, whereas its reflections are clearer in following SD.

The propagation of waves in SD is depicted in Figure 2j-2r, where we utilize the following parameters for the modified magnetic field: δL = 0.02 (127 km), B_d = 5 nT. It should be noted that we have enlarged the simulated duct width to 50 times of the observation due to the constraints of the WKB condition, which dictates that the variation of space media must be slow within a certain wavelength range. Although the designed model cannot fully simulate the observed fine structure, certain characteristics can still be discerned. The propagation characters of nonducted waves have been introduced above, we only state the ducted waves below. Figures 2j-2r are similar as Figures 2a-2i. Figure 2j shows the magnetic field strength of SD and nonduct. During the propagation of the upper band wave (blue solid line in Figure 2k), the ray path also remains near the center, but unlike in LD, the wave vector aligns the ray path lasting a few times (lifetime ~ 0.25s, latitudes < 10°). The lower band (blue dashed line in Figure 2l) wave may propagate obliquely with slight reflections in the duct, compared to wave in LD, the WNA is larger at beginning. The n (Figure 2m) of the lower band wave consistently exceeds that of the upper band wave with many small fluctuations appearing. The variation of WNAs (Figure 2o) keeps up with n, lower band wave exhibit frequently rapid increase and decline with multiple fluctuations in gradually increasing WNAs, but the upper band wave propagates with small fluctuating WNAs (~ 0°) within latitudes < 10° at the beginning. Associated with wave attenuation (Figure 2p), when the attenuation reaches -20 dB, upper band wave can propagate to latitude = 14°. The lower band wave experiences more Landau damping due to its rapidly variational oblique propagation, which can only propagate to latitude ~ 3.8°.

Like Figures 2h and 2i, we choose the latitude range from 17° to 23° for magnification, as shown in Figures 2q and 2r. The n of the lower band ducted wave experiences a potential process of falling twice and rising once. For upper band ducted wave, the n maintains a slightly fluctuating around n ~ 6 during this period. As shown in Figure 2r, the lower band wave reflects twice in the positive gradient region and once in the negative gradient region, where the reflection points are in about ±0.014 ΔL (70% δL), which corresponds to the gradient is ~±200 nT/L near the edges of the duct. By examining Figures 2q and 2r together, it also exhibits the same relationship between n and gradient as in LD model. But in SD, fluctuations are more
pronounced, and the upper band wave may exhibit the similar characteristic (even though the
gradient is close to zero).

4 Discussions

Since the repetitive reflections (as shown in Figure 2) are the key to the ducting
propagation of chorus waves, we will give some schematic explanation about the physics behind.
For simplicity, the background magnetic field will be assumed to be homogeneous and aligned
along with the z axis, while the magnetic intensity varies only in the x axis, as demonstrated in
Yu & Yuan (2022). Recalling that the dispersion relation for whilster waves can be given by (Yu
et al., 2023)

\[
\left( \frac{ck}{\omega} \right)^2 = \frac{\omega_{pe}^2}{\omega(|\Omega_e| \cos \theta - \omega)}
\]

or say,

\[
D(\omega, k; x) = \frac{\omega_{pe}^2}{\omega(|\Omega_e| \cos \theta - \omega)} - \left( \frac{ck}{\omega} \right)^2
\]

where \( \omega, k, \) and \( \theta \) are the wave frequency, wave number, and wave normal angle,
respectively, \( c \) is the speed of light, while \( \Omega_e \) and \( \omega_{pe} \) denote the electron cyclotron frequency
and electron plasma frequency. From this dispersion relation, we can obtain that

\[
\frac{\partial D}{\partial k_x} = - \left( \frac{c}{\omega} \right)^2 k \sin \theta \frac{|\Omega_e| \cos \theta - 2\omega}{(|\Omega_e| \cos \theta - \omega)}
\]

and

\[
\frac{\partial D}{\partial x} = \left( \frac{ck}{\omega} \right)^2 \frac{|\Omega_e| \cos \theta}{(|\Omega_e| \cos \theta - \omega)} \frac{B_d'}{B} \left( \frac{8}{\delta L^2} \frac{L - L_d}{R_E} \right)
\]

Consequently, the variation of perpendicular wave number can be given by

\[
\frac{dk_x}{dx} = - \frac{\partial D / \partial x}{\partial D / \partial k_x} = k \frac{|\Omega_e| \cos \theta}{\sin \theta (|\Omega_e| \cos \theta - 2\omega)} \frac{B_d'}{B} \left( \frac{8}{\delta L^2} \frac{L - L_d}{R_E} \right)
\]

or in a dimensionless form,

\[
\frac{d(ck_x/\omega)}{d(L/\delta L)} = \frac{ck}{\omega \sin \theta (|\Omega_e| \cos \theta - 2\omega)} \frac{B_d'}{B} \frac{L - L_d}{\delta L}
\]

Obviously, chorus waves would be ducted when their frequency satisfy the following
condition

\[
\frac{\omega}{|\Omega_e|} > \frac{1}{2} \cos \theta
\]

This because that when these waves are located in the right-hand side (\( L > L_d \)), their
perpendicular wave number will become smaller and smaller until the wave reflection occurs
once \( k_x \) reduces to zero (see Yu & Yuan 2022). The reflected waves will pass through the center
of the magnetic peak into the left-hand side, where their perpendicular wave number will also
reduce into zero (since they are propagating along the negative x axis) and the waves are
reflected again. As a result, repetitive reflections occur along the wave propagation path, which result in the ducting propagation.

For upper-band chorus waves, the condition expressed by formula (10) is always satisfied so that these waves are ducted in their whole propagation path. For lower-band chorus waves, however, the condition expressed by formula (10) is often not satisfied near the equator where the waves mostly parallel. Due to the refraction of the background magnetic field, the normal angle of lower-band chorus waves would become larger to match the condition. That is to say, lower-band chorus waves are nonducted around the center of magnetic field flux tube but become ducted somehow far away from the center. Consequently, lower-band chorus waves will sway from side to side, as shown in Figure 2.

Figure 3. Characteristics of waves in two ducts (LD and SD) and nonducts (LND and SND). (a) Final latitudes and (b) lifetimes as functions of launching positions and frequencies of rays in
LD. (c)-(d) are similar as (a)-(b), but in LND. (e)-(f) are the differences of final latitudes and lifetimes between rays in LD and LND. (g)-(l) are similar as (a)-(f), but for SD and SND.

As shown in observed cases (Figure 1), whistler waves are not only found in the central duct region but are also observed within other regions (positive and negative magnetic gradient) across a wide frequency range. To comprehensively investigate the final latitudes and lifetimes ($\tau$) of multifrequency whistler waves within the magnetic peak ducts, and the differences between ducted waves. For LD (SD), 1491 rays are parallelly launched to represent the waves with frequency range from $0.1f_{ce}$ to $0.8f_{ce}$ (with a step of $0.01f_{ce}$) and L range from different position from 5.8 L (5.98 L) to 6.2 L (6.02 L), with steps of 0.02 L (0.002 L) for LD (SD). The same initial frequency and launch position are used in nonduct for comparison. When a ray satisfies one of the following conditions, it is considered to be damped out or arrive at the terminus, then the ray tracing is ceased: 1. Eventually damped to 1% of its initial power, 2. Propagating at Latitude $= 40^\circ$, 3. Arriving at the plasmapause. The simulated results are shown in Figure 3, where the magenta solid line in each panel indicates $0.5f_{ce}$ rays. Due to different widths, we refer to nonduct as small nonduct (SND) and large nonduct (LND).

In addition, Figures 3a (c) and Figure 3b (d) exhibit the final latitudes and lifetimes as functions of the initial position and frequency of rays in LD (LND). Figures 3e and 3f show the differences of these parameters between LD and LND (positive values indicate higher duct parameters). The horizontal axis ($\Delta L$) represents the distance from the center of the duct and the vertical axis represents the normalized frequency of rays. For LND (Figures 3c and 3d) model, it shows that wherever the ray launched, the final latitudes and lifetimes decline as the frequency increases. For LD, as shown in Figures 3a and 3b, compared with rays launched in other position, which launched near the center can propagate to higher latitudes with longer lifetimes. Interestingly, the rays launched at 5.98 L ($\Delta L = -0.02$ L) may exhibit higher final latitudes and lifetimes than those launched at the center, since the gradient at 5.98 L is closer to zero, as a sum result of duct and dipolar magnetic fields. For rays launched here, the ray with $0.8f_{ce}$ can propagate to latitude $\sim 5^\circ$ with $\tau \sim 0.2$ s, while the ray with $0.1f_{ce}$ can propagate to latitude $\sim 16^\circ$ with $\tau \sim 0.15$ s. The differences of waves in LD and LND are clearly seen in Figures 3e and 3f. The final latitudes and lifetimes of ducted waves exceed those of nonducted waves at L = 5.98, covering mostly the whole frequency band. However, as the launch position moves away from the center and the frequency decreases, these positive differences down to zero and eventually becomes negative rapidly.

Figures 3g-3l are similar as Figures 3a-3f. For SND (Figures 3i and 3j), there is a similar frequency distribution pattern as seen in LND. For SD, as shown in Figures 3g and 3h, the gap between two-band waves are evident near the center, where the ray with $0.8f_{ce}$ ($0.1f_{ce}$) can propagate to latitude of $\sim 18^\circ$ ($\sim 5^\circ$) with $\tau \sim 0.4$ s ($\sim 0.05$ s), which is even higher (lower) than those exhibited in LD. Figures 3k and 3l illustrate the differences between SD and SND. It suggests that only upper band ducted waves launched near the center can propagate to higher latitudes with longer lifetimes, final latitudes and the lifetimes of almost all lower band waves in duct are much lower than those in nonduct. Additionally, the relationship between position and frequency observed in LD also occur here.

5 Conclusion

In this letter, chorus waves in two distinct magnetic peaks are observed by Van Allen Probe A. By combining observations and ray tracing simulation, this study reveals the
propagation characteristics of whistler-mode waves in two different magnetic peaks (referred to as LD and SD).

It implies that the whistler waves launched in center of both ducts show similar characteristics: the upper band wave can propagate to higher latitudes with small WNAs, whereas the lower band wave only reaches lower latitudes with large variable WNAs. However, the existence of the SD can more support upper band wave propagation and restricts lower band wave propagation. Furthermore, we find that the increase or decrease of n correspond to the region of negative or positive region, respectively. This is associated with repetitive reflections of waves in the duct.

Moreover, by launching multifrequency rays at various position. For rays launched near the center, compared with rays launched at other position, the ducted waves can propagate to higher latitudes with longer lifetimes. By comparing to nonducted waves, in LD, waves at the region of magnetic gradient near zero (L = 5.98) can propagate to higher latitudes across nearly whole frequency. But for waves in SD, there is a significant gap between two-band waves near the center of duct, upper band waves launched here can propagate to higher latitudes, whereas the propagation of lower band waves are restrained. In addition, with the launch position moves away from the center, the waves in both ducts are restrained, especially for lower band waves.

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Open Research

The Van Allen Probes data used for this study are publicly available at the websites: https://spdf.gsfc.nasa.gov/pub/data/rbsp/, and the OMNI data are publicly available at the websites https://spdf.gsfc.nasa.gov/pub/data/omni/.
References


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- Theoretical analysis about ducting chorus waves has been given to explain the repetitive reflections along the wave propagation path.
Abstract

Through one-dimensional propagation theory, recent studies have demonstrated that chorus waves can be ducted not only by density structures, but also by magnetic peaks. However, such a ducting propagation in the inner magnetosphere has not been verified yet. In this study, we perform ray tracing simulations to investigate the two-dimensional propagation of chorus waves in magnetic peaks. It is found that chorus waves indeed can be ducted by the magnetic peak via repetitive reflections in both sides, similar to that of one-dimensional propagation theory. The physical mechanism for these repetitive reflections has also been discussed. Furthermore, parameter analysis has been implemented to show the effect of wave frequencies and incidence positions on the propagation of chorus waves. Our results will be helpful in improving the understanding of the duct propagation for whistler waves in the magnetosphere.

Plain Language Summary

The existence of the duct can change the propagation characteristics of whistler-mode waves in the inner magnetosphere, thus affect wave-particle interaction. Recently one-dimensional theoretical analysis and numerical simulation have exhibited the wave-mode coupling and propagation of chorus waves in the magnetic peaks. In this study, two-dimensional ray tracing simulations are performed. Through parametric analysis and theoretical explanation, our results show the propagation mechanism of ducted waves in dipole magnetic field, which exhibit repetitive reflections in the magnetic duct. Furthermore, it also shows that the waves launched near the center can propagate to higher latitudes, especially for the upper band waves. Our results add new insights into the propagation of whistler-mode waves within the magnetosphere.

1 Introduction

Whistler-mode chorus waves are typically observed in the low-density nightside region outside the plasmapause. These waves often exhibit unique time-frequency spectrograms characterized by discrete rising or occasionally falling tones lasting a few tenths (e.g., Burtis & Helliwell, 1969; Burton & Holzer, 1974; Li et al., 2011). Due to the power gap near $0.5f_{ce}$, chorus waves are often classified into lower band ($0.1-0.5f_{ce}$) and upper band ($0.5-0.8f_{ce}$) components. They can both accelerate and cause loss of energetic electrons within Earth’s inner magnetosphere (Horne & Thorne, 1998; Summers et al., 1998; Tao et al., 2008; Thorne et al., 2013, Wang & Shprits, 2019). They are also believed to be responsible for diffuse and pulsating aurora (Thorne et al., 2010; Ni et al., 2011). Furthermore, chorus waves are considered to be the source of plasmaspheric hiss through propagating into the plasmasphere (Bortnik et al., 2008, 2009, 2011).

In general, chorus waves in the inner magnetosphere are considered to propagate in a dipolar magnetic field, which is named as nonducted wave. During its propagation, the wave normal angle (WNA) becomes increasingly oblique due to the inhomogeneous magnetosphere (Colpitts et al., 2020). On the other hand, ducted chorus waves, as the name suggests, can be guided by fine-scale density irregularities (Ke et al., 2022; Hanzelk & Santoli, 2019) and propagate along the field line over considerable distances with less attenuation. Recent studies have shown that magnetic peaks, which are local magnetic structures characterized by sudden increases in magnetic field strength, can also act as ducts guiding on whistler-mode waves (Yu & Yuan, 2022; Nejad & Streltsov, 2023; Streltsov & Nejad, 2023). However, these studies focus on
one-dimension propagation of chorus waves. The propagation properties of chorus waves under
the effect of a magnetic peak in the inner magnetosphere remains uncovered yet.

In this letter, we report two cases of whistler-mode waves in magnetic peaks observed by
Van Allen Probe A. Through ray tracing simulations and comparison with nonducted waves, we
find the whistler-mode waves can propagate guided by magnetic duct, exhibiting repetitive
reflections within the edges of ducts. However, we also observe differences between upper band
and lower band whistler waves. Our results suggest that magnetic peak duct have potential to
significantly affect the propagation of whistler-mode waves.

2 Observations

In this work, the properties of chorus waves are provided by the waves instrument of the
Electric and Magnetic Field Instrument Suite and Integrated Science suite (EMFISIS) (Kletzing
et al., 2013) onboard Van Allen Probes. The plasma density is obtained from the high-frequency
receiver on EMFISIS. In addition, the AE and SYM-H indices are obtained from the OMNI
database.

Event 1 (Figures 1a-1f) was observed during 20:00-22:00 UT on 8 December 2016. Our
period of interest spans from 20:28 to 21:28 UT, highlighted in gray shading. During this time
frame, the probe was located near the magnetic equator (0° < MLAT < 1°) at midnight (23:30 <
MLT < 00:00) within L shells mainly ranging between 5.35 and 5.75 (~ 0.4 L). Figure 1a
illustrates the AE and SYM-H indexes. The increase in AE index implies a moderate substorm
and enhanced injection of particles. The minimum of SYM-H index decreases to about -30 nT,
indicating enhancement of the ring current. As shown in Figure 1b, the electron number density
is about 2.5 cm$^{-3}$, meaning the probe is in a low-density plasma trough area. As shown in Figure
1c, the $B_z$ component in the GSM coordinate system exhibits a significant enhancement,
meaning a typical magnetic peak. Figures 1d-1f show the characteristics of waves: power
spectral density (PSD) of magnetic field, ellipticity polarizability (Ellip) and WNA. Chorus
waves in the magnetic peak are identified based on several criteria: quasi-parallel propagating
(WNAs < 45°), right-hand polarization (Ellip ~ 1) and intense wave power (PSDs >
$10^{-6}$nT$^2$/Hz) near 0.5 fce (marked by the dashed curves).
Figure 1. Overview of the observations of two events from Van Allen Probes A. Event 1 (left) is during 20:00–22:00 UT on 8 December 2016 and Event 2 (right) is during 17:15–17:20 UT on 25 October 2018. (a) AE and SYM-H indices. (b) Total background electron number density. (c) Total background magnetic field strength (black line) and z component (blue line) in GSM coordinate. (d) PSD of magnetic field component of the waves. (e) Ellipticity. (f) WNA. The gray shadow region in every event indicates the time interval of observation of magnetic peak structure. (g–j) are like panels (a)–(d), (k–m) are like panels (d)–(f).

Event 2 was observed during 17:15–17:20 UT on 25 October 2018. During this period, the satellite was located near the magnetic equator (MLAT ~ -1°) on the nightside (MLT ~ 22:40) within an L shell of approximately 5.77. Our period of interest is also highlighted in gray shading. The AE index during this time is about 450 nT and the SYM-H index is near -6 nT (Figure 1g). At 17:17:30 UT, there is a significant DFB with a sudden enhancement of Bz (Figure 1i), while the electron number density experiences a slight change (Figure 1h).

Fortunately, burst mode data near 17:17:27 UT are available. Based on the criteria described earlier, the properties of whistler-waves with a frequency near 0.6fce are shown in Figures 1k-1m.

From both two events, whistler-mode waves are observed within the magnetic peak structure as the electron number density slightly changes on the nightside near the magnetic equator during period of strong geomagnetic activity (AE > 300 nT). In event 1, the magnetic peak crosses over ~ 0.4 L shells (2548 km) and the maximum magnetic field strength increases by around 40 nT. Oppositely, in event 2, the magnetic peak only crosses over about 2.7 km and the maximum magnetic field strength increases about 5 nT. Based on these differences, we...
define event 1 as a large duct case (LD) and event 2 as a small duct case (SD). These two events provide an excellent opportunity to investigate the propagating mechanism of whistler-mode emissions in different size of magnetic peak ducts.

3 Ray Tracing Simulations

To investigate the propagation of whistler-mode waves in two sizes of magnetic peaks, we adopt the method of ray tracing simulation developed by Horne (1989), which is often employed to study the evolution and propagation properties of waves (Chen et al., 2010, 2021, 2022; Xiao et al., 2016; Deng et al., 2022; Feng et al., 2023). In this study, we perform a 2-D ray tracing simulation, where a diffusive equilibrium plasma density model mentioned by Bortnik et al. (2011) as the cold plasma number density. Specifically, we set the nightside parameters \( N_e = 2 \text{ cm}^3 \text{ at } L = 6 \) mentioned in it and position the plasmapause at \( L = 4 \) to align with the observed geomagnetic activity (AE > 300 nT).

To model the magnetic peak structure, we adopt a model of dipolar magnetic field superposed by a field-aligned increase, which is described by a Gaussian-like term in \( B_z \) as shown in Equation (1)-(2).

\[
B'_z = B_{zdip} + B'_d \tag{1}
\]

\[
B'_d = B_d \exp \left( -\frac{4(L - L_d)^2}{\delta L^2} \right) \tag{2}
\]

where \( B_{zdip} \) is the z axis component of the dipolar magnetic field model, \( B_d \) is the magnetic field strength increase at the center of the duct, \( L \) is the L shell, \( \delta L \) is the half-width of magnetic duct (in units of L shell), and \( L_d \) is the center of the duct (in units of L shell). Note that this approach is akin to previous study on density duct (Ke et al., 2022). In our simulations, we set \( L_d = 6 \) for both cases. Although there may be weak enhancement in \( B_{XY} \) at higher latitude, this little influence is ignored in this study.

To study the effects of both ducts on the propagation of whistler-mode waves and the differences with nonducted waves, we launch four rays initially in parallel (i.e., WNA = 0°) from the magnetic equator at \( L = 6 \) to study the propagation of lower band and upper band chorus waves in LD and SD cases. We choose the frequency of \( 0.6 f_{ce} \) (0.3 \( f_{ce} \)) for the upper (lower) band chorus wave. The rays are traced until they reach either the plasmapause boundary (\( L = 4 \)) or MLAT > 40°. Additionally, we use equation 2 from Chen et al. (2010) to calculate the Landau damping. The distribution of electron velocity is fitted to the statistical results presented in Bortnik et al. (2007), which are for hot electron flux (0.21 - 26 keV) within \( 4 < L < 7 \) on the nightside during strong activity (AE > 300 nT). We also follow the fitting method described in it to obtain a fitted distribution function:

\[
f(v) = 244.2v^{-2.7} m^{-6}s^3 \tag{3}
\]

To model the magnetic fields of LD, we use the following parameters for the modified magnetic field: \( \delta L = 0.2 \text{ L, } B_d = 40 \text{ nT} \). Our simulated results are shown in Figure 2. Figure 2a shows the magnetic field strength at the magnetic equator with blue and black curves denoting LD and nonduct, respectively. Figures 2b-2c exhibit the ray paths for upper band (0.6 \( f_{ce} \)) and lower band (0.3 \( f_{ce} \)) waves in the meridian plane. The arrows represent the direction of the wave vector, while the gray and green shadows represent the plasmasphere and duct, respectively. For the propagation process of upper band waves (Figure 2b), the ducted wave (blue solid line)
generally propagates along a field line at the center of the duct with slowly growing WNA. However, the nonducted wave (black solid line) tends to cross more L shells inward to the earth, and the WNAs become more oblique as propagating to higher latitudes. For lower band waves (Figure 2c), the paths of both rays seem similar. The ducted wave (blue dashed line) exhibits slight reflections near the center, and the WNAs rapidly increase at the beginning. While the nonducted wave (black dashed line) initially propagates outward (higher L shells) and then returns inward with gradually increasing in WNAs, the behavior is different from the upper band nonducted wave, which may because of the frequency, WNA and Gendrin angle characteristics (Smith et al., 1960).

Figures 2d-2g show the further insights into the behaviors of the refractive index (n), L shell, WNA and wave gain as functions of latitudes during wave propagation. Initially, all four rays exhibit similar features, but the differences become apparent as they propagate away from equator. For nonducted waves (black solid (upper band) and dashed (lower band) lines), the n (Figure 2d) of upper band wave grows faster, meaning its phase velocity is slower than that of lower band wave. The variation in L shells (Figure 2e) aligns with the ray paths has been discussed earlier, where the upper band wave propagates from L = 6 to L ~ 4.8 and the lower band wave propagates outward firstly at L ~ 6.2 and then inward at L ~ 6. The WNAs (Figure 2f) of both nonducted rays point outward (WNA > 0°) and become more oblique with increasing latitudes, but the upper band wave undergoes a more rapid oblique propagation. This behavior leads to intense Landau damping. As shown in Figure 2g, when the attenuation of both rays reaches -20 dB, the lower band wave can propagate to latitude of ~ 8°, while the upper band wave only propagates to latitude of ~ 4°. For ducted waves as shown by blue solid (upper band) and dashed (lower band) lines, however, the n exhibits different behavior compared with that of nonducted waves. It remains almost constant at n ~ 5 for the upper band wave, whereas the lower band wave exhibits two significant crests in n. The ray paths in Figure 2e show both ducted waves generally propagate along a single magnetic field line from the launch position (L = 6), especially for the upper band wave. In correspondence with the n, the WNAs of lower band wave also exhibit two significant crests, consistently surpassing those of the upper band wave, which propagates with gradually increasing WNAs. Figure 2g illustrates that when the attenuation of both ducted rays reaches -20 dB, the upper (lower) band ducted wave can propagate to latitude of ~ 5° (~ 6°), experiencing smaller (higher) Landau damping compared to nonducted waves with the same normalized frequency.
Figure 2. Comparison of two-band waves in large duct (LD) and nonduct. (a) Magnetic field strength model of duct and nonduct. Ray paths of (b) upper band rays launched and (c) lower band rays in LD and nonduct, arrow represents direction of wave vector. (d)-(g) Refractive index, L shell, WNA and wave gain as functions of latitudes during propagation of rays. (h)-(i)
are partial amplification for (d) and (e), corresponding to magenta box in (d) and (e). (j)-(r) are similar to (a)-(i), but for waves in LD.

To investigate the fine features of two-band waves (blue lines) in LD, such as the variation of \( n \) and L shell during wave propagation, we magnify the latitude range from 5° to 20°, corresponding to the magenta boxes in Figure 2d and 2e. As shown in Figure 2h, the \( n \) of the lower band wave experiences an initial rise followed by a decline, but it remains approximately constant for upper band wave. In addition, we exhibit the magnetic field gradient with L shells in Figure 2i. It shows that the lower band wave reflects first in the negative gradient region and then in the positive gradient region with the reflection points being about ±0.06 \( \Delta L \) (30% \( \delta L \), \( \Delta L \) is the distance from the center), corresponding to a gradient of approximately ±500 nT/L. Interestingly, combining Figure 2h and 2i, for lower band wave, it is obvious that when the ray propagates in the negative gradient region, the \( n \) increases, and it decreases while the ray is in the positive gradient region. The center the ray passes is associated with the maximum (or minimum) \( n \). But for upper band wave, both \( n \) and ray path generally maintain a straight line with slight changes near the center, whereas its reflections are clearer in following SD.

The propagation of waves in SD is depicted in Figure 2j-2r, where we utilize the following parameters for the modified magnetic field: \( \delta L = 0.02 \) (127 km), \( B_d = 5 \) nT. It should be noted that we have enlarged the simulated duct width to 50 times of the observation due to the constraints of the WKB condition, which dictates that the variation of space media must be slow within a certain wavelength rang. Although the designed model cannot fully simulate the observed fine structure, certain characteristics can still be discerned. The propagation characters of nonducted waves have been introduced above, we only state the ducted waves below.

During the propagation of the upper band wave (blue solid line in Figure 2k), the ray path also remains near the center, but unlike in LD, the wave vector aligns the ray path lasting a few times (lifetime ~ 0.25s, latitudes < 10°). The lower band (blue dashed line in Figure 2l) wave may propagate obliquely with slight reflections in the duct, compared to wave in LD, the WNA is larger at beginning. The \( n \) (Figure 2m) of the lower band wave consistently exceeds that of the upper band wave with many small fluctuations appearing. The variation of WNAs (Figure 2o) keeps up with \( n \), lower band wave exhibit frequently rapid increase and decline with multiple fluctuations in gradually increasing WNAs, but the upper band wave propagates with small fluctuating WNAs (~ 0°) within latitudes < 10° at the beginning. Associated with wave attenuation (Figure 2p), when the attenuation reaches -20 dB, upper band wave can propagate to latitude = 14°. The lower band wave experiences more Landau damping due to its rapidly variational oblique propagation, which can only propagate to latitude ~ 3.8°.

Like Figures 2h and 2i, we choose the latitude range from 17° to 23° for magnification, as shown in Figures 2q and 2r. The \( n \) of the lower band ducted wave experiences a potential process of falling twice and rising once. For upper band ducted wave, the \( n \) maintains a slightly fluctuating around \( n \sim 6 \) during this period. As shown in Figure 2r, the lower band wave reflects twice in the positive gradient region and once in the negative gradient region, where the reflection points are in about ±0.014 \( \Delta L \) (70% \( \delta L \)), which corresponds to the gradient is ±200 nT/L near the edges of the duct. By examining Figures 2q and 2r together, it also exhibits the same relationship between \( n \) and gradient as in LD model. But in SD, fluctuations are more
pronounced, and the upper band wave may exhibit the similar characteristic (even though the gradient is close to zero).

4 Discussions

Since the repetitive reflections (as shown in Figure 2) are the key to the ducting propagation of chorus waves, we will give some schematic explanation about the physics behind. For simplicity, the background magnetic field will be assumed to be homogeneous and aligned with the $z$ axis, while the magnetic intensity varies only in the $x$ axis, as demonstrated in Yu & Yuan (2022). Recalling that the dispersion relation for whistler waves can be given by (Yu et al., 2023)

$$\left( \frac{ck}{\omega} \right)^2 = \frac{\omega_{pe}^2}{\omega(\Omega_e \cos \theta - \omega)}$$

or say,

$$D(\omega, k; x) = \frac{\omega_{pe}^2}{\omega(\Omega_e \cos \theta - \omega)} - \left( \frac{ck}{\omega} \right)^2$$

where $\omega$, $k$, and $\theta$ are the wave frequency, wave number, and wave normal angle, respectively, $c$ is the speed of light, while $\Omega_e$ and $\omega_{pe}$ denote the electron cyclotron frequency and electron plasma frequency. From this dispersion relation, we can obtain that

$$\frac{\partial D}{\partial k_x} = -\left( \frac{c}{\omega} \right)^2 k \sin \theta \frac{|\Omega_e| \cos \theta - 2\omega}{(|\Omega_e| \cos \theta - \omega)}$$

and

$$\frac{\partial D}{\partial x} = \left( \frac{ck}{\omega} \right)^2 \frac{|\Omega_e| \cos \theta}{(|\Omega_e| \cos \theta - \omega)} \frac{B_d}{B} \left( \frac{8}{\delta L^2} \frac{L - L_d}{R_E} \right)$$

Consequently, the variation of perpendicular wave number can be given by

$$\frac{dk_x}{dx} = -\frac{\partial D}{\partial x} \frac{\partial D}{\partial k_x} = \frac{k}{\sin \theta \left( \frac{|\Omega_e| \cos \theta - 2\omega}{B} \right)} \frac{B_d}{B} \left( \frac{8}{\delta L^2} \frac{L - L_d}{R_E} \right)$$

or in a dimensionless form,

$$\frac{d(ck_x/\omega)}{d(L/\delta L)} = \frac{ck}{\omega \sin \theta \left( \frac{|\Omega_e| \cos \theta - 2\omega}{B} \right)} \left( \frac{B_d}{B} \frac{L - L_d}{\delta L} \right)$$

Obviously, chorus waves would be ducted when their frequency satisfy the following condition

$$\frac{\omega}{|\Omega_e|} > \frac{1}{2} \cos \theta$$

This because that when these waves are located in the right-hand side ($L > L_d$), their perpendicular wave number will become smaller and smaller until the wave reflection occurs once $k_x$ reduces to zero (see Yu & Yuan 2022). The reflected waves will pass through the center of the magnetic peak into the left-hand side, where their perpendicular wave number will also reduce into zero (since they are propagating along the negative $x$ axis) and the waves are
 reflected again. As a result, repetitive reflections occur along the wave propagation path, which result in the ducting propagation.

For upper-band chorus waves, the condition expressed by formula (10) is always satisfied so that these waves are ducted in their whole propagation path. For lower-band chorus waves, however, the condition expressed by formula (10) is often not satisfied near the equator where the waves mostly parallel. Due to the refraction of the background magnetic field, the normal angle of lower-band chorus waves would become larger to match the condition. That is to say, lower-band chorus waves are nonducted around the center of magnetic field flux tube but become ducted somehow far away from the center. Consequently, lower-band chorus waves will sway from side to side, as shown in Figure 2.

Figure 3. Characteristics of waves in two ducts (LD and SD) and nonducts (LND and SND). (a) Final latitudes and (b) lifetimes as functions of launching positions and frequencies of rays in
LD. (c)-(d) are similar as (a)-(b), but in LND. (e)-(f) are the differences of final latitudes and lifetimes between rays in LD and LND. (g)-(l) are similar as (a)-(f), but for SD and SND.

As shown in observed cases (Figure 1), whistler waves are not only found in the central duct region but are also observed within other regions (positive and negative magnetic gradient) across a wide frequency range. To comprehensively investigate the final latitudes and lifetimes ($\tau$) of multifrequency whistler waves within the magnetic peak ducts, and the differences between ducted waves. For LD (SD), 1491 rays are parallelly launched to represent the waves with frequency range from $0.1 f_{ce}$ to $0.8 f_{ce}$ (with a step of $0.01 f_{ce}$) and L range from different position from 5.8 L (5.98 L) to 6.2 L (6.02 L), with steps of 0.02 L (0.002 L) for LD (SD). The same initial frequency and launch position are used in nonduct for comparison. When a ray satisfies one of the following conditions, it is considered to be damped out or arrive at the terminus, then the ray tracing is ceased: 1. Eventually damped to 1% of its initial power, 2. Propagating at Latitude = 40°. 3. Arriving at the plasmapause. The simulated results are shown in Figure 3, where the magenta solid line in each panel indicates $0.5 f_{ce}$ rays. Due to different widths, we refer to nonduct as small nonduct (SND) and large nonduct (LND).

In addition, Figures 3a (c) and Figure 3b (d) exhibit the final latitudes and lifetimes as functions of the initial position and frequency of rays in LD (LND). Figures 3e and 3f show the differences of these parameters between LD and LND (positive values indicate higher duct parameters). The horizontal axis ($\Delta L$) represents the distance from the center of the duct and the vertical axis represents the normalized frequency of rays. For LND (Figures 3c and 3d) model, it shows that wherever the ray launched, the final latitudes and lifetimes decline as the frequency increases. For LD, as shown in Figures 3a and 3b, compared with rays launched in other position, which launched near the center can propagate to higher latitudes with longer lifetimes. Interestingly, the rays launched at 5.98 L ($\Delta L = -0.02$ L) may exhibit higher final latitudes and lifetimes than those launched at the center, since the gradient at 5.98 L is closer to zero, as a sum result of duct and dipolar magnetic fields. For rays launched here, the ray with $0.8 f_{ce}$ can propagate to latitude $\sim 5^\circ$ with $\tau \sim 0.2$ s, while the ray with $0.1 f_{ce}$ can propagate to latitude $\sim 16^\circ$ with $\tau \sim 0.15$ s. The differences of waves in LD and LND are clearly seen in Figures 3e and 3f. The final latitudes and lifetimes of ducted waves exceed those of nonducted waves at L = 5.98, covering mostly the whole frequency band. However, as the launch position moves away from the center and the frequency decreases, these positive differences down to zero and eventually becomes negative rapidly.

Figures 3g-3l are similar as Figures 3a-3f. For SND (Figures 3i and 3j), there is a similar frequency distribution pattern as seen in LND. For SD, as shown in Figures 3g and 3h, the gap between two-band waves are evident near the center, where the ray with $0.8 f_{ce}$ ($0.1 f_{ce}$) can propagate to latitude of $\sim 18^\circ$ ($\sim 5^\circ$) with $\tau \sim 0.4$ s ($\sim 0.05$ s), which is even higher (lower) than those exhibited in LD. Figures 3k and 3l illustrate the differences between SD and SND. It suggests that only upper band ducted waves launched near the center can propagate to higher latitudes with longer lifetimes, final latitudes and the lifetimes of almost all lower band waves in duct are much lower than those in nonduct. Additionally, the relationship between position and frequency observed in LD also occur here.

5 Conclusion

In this letter, chorus waves in two distinct magnetic peaks are observed by Van Allen Probe A. By combining observations and ray tracing simulation, this study reveals the
It implies that the whistler waves launched in center of both ducts show similar characteristics: the upper band wave can propagate to higher latitudes with small WNAs, whereas the lower band wave only reaches lower latitudes with large variable WNAs. However, the existence of the SD can more support upper band wave propagation and restricts lower band wave propagation. Furthermore, we find that the increase or decrease of $n$ correspond to the region of negative or positive region, respectively. This is associated with repetitive reflections of waves in the duct.

Moreover, by launching multifrequency rays at various position. For rays launched near the center, compared with rays launched at other position, the ducted waves can propagate to higher latitudes with longer lifetimes. By comparing to nonducted waves, in LD, waves at the region of magnetic gradient near zero ($L = 5.98$) can propagate to higher latitudes across nearly whole frequency. But for waves in SD, there is a significant gap between two-band waves near the center of duct, upper band waves launched here can propagate to higher latitudes, whereas the propagation of lower band waves are restrained. In addition, with the launch position moves away from the center, the waves in both ducts are restrained, especially for lower band waves.

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Open Research

The Van Allen Probes data used for this study are publicly available at the websites: https://spdf.gsfc.nasa.gov/pub/data/rbsp/, and the OMNI data are publicly available at the websites https://spdf.gsfc.nasa.gov/pub/data/omni/.
References


