Dynamic Phasor Simulation of Systems with High Shares of Inverter-Based Resources

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Abstract: Integrating inverter-based resources into power systems increases their complexity, and makes their operation more challenging. The conventional approach for simulating power systems neglects transients of the power grid, using steady-state phasors and considering only fundamental frequency values. However, to accurately predict the dynamic behavior of systems with high shares of inverter based resources, electromagnetic transient simulations (EMTSs) are necessary. The primary limitation of classical time-domain EMTSs is their frequent time-consuming nature, making them impractical for larger systems. In this work, we explore the use of dynamic phasors (DPs) as a technique to accelerate EMTSs. For this purpose, we first adapt the power grid and inverter model to DP simulation. Subsequently, we assess the effectiveness of this approach through three numerical examples, demonstrating a reduction in the simulation times by a factor of 4 to 40.

Keywords: Power systems, simulation, dynamic phasor, electromagnetic transient, inverter based resources

1. INTRODUCTION

Power systems are continuously expanding, with their size increasing daily to meet the surging demands for energy. As this scale continues to broaden, it has substantially heightened the intricacies related to designing, maintaining, and ensuring the long-term stability of these systems, as explained by Demiray (2008). Therefore, it is necessary to attain a certain standard of security and reliability, which entails analyzing and simulating the power system’s behaviour. Conventional power plants still play a key role in electrical systems, and the stability analysis and simulation of slower transients occurring in such systems is largely done without considering power grid dynamics, Fankhauser et al. (1990), Astic et al. (1994). These analyses depend on steady-state phasors and sufficiently large time intervals, which are made feasible by excluding rapid electromagnetic transients.

Integrating a large number of inverter-based resources (IBRs) leads to many changes in power system operation due to their volatility, distribution in the grid, and different dynamic behavior compared to conventional generation. The dynamic response of IBRs is even approaching the time scale of various power grid elements, such as cables and transformers. As a result, electromagnetic transient simulation (EMTS) becomes necessary even for stability evaluations, Markovic et al. (2021). As depicted by Vega-Herrera et al. (2020) simulating the dynamics of systems with large shares of IBRs using classical EMTSs proves to be excessively time-consuming. Conventional EMTSs become slow due to the necessity for small time steps when conducting detailed simulations of power electronic devices and power grid elements. To speed up the simulations, specially designed hardware is used, combined with new digital control algorithms, Mirz et al. (2020). New techniques also include co-simulations, where different subsystems, which form a coupled problem, are modeled and simulated in a distributed manner, as in Lin et al. (2011) and Scheibe et al. (2022). Switching between solvers during co-simulation poses challenges, leading to the development and proposal of new changeover algorithms to facilitate smoother transitions, Rupasinghe et al. (2020), Mudunkotuwa and Filizadeh (2018). Unfortunately, these methods are not particularly convenient, mainly due to the high costs of specialized hardware, and the additional complexity when co-simulating several subsystems. Furthermore, the assessment of power system stability commonly relies on tools that utilize quasi-static phasor calculus to represent voltages and currents. However, the increasing penetration of IBRs in power systems makes the quasi-static phasor calculus only suitable for stability analysis if low bandwidths of converter controls are given, as explained by Vega-Herrera et al. (2020).
One possible way to overcome the problem of simulating electromagnetic transients is the use of dynamic phasors (DPs), which are proposed by Sanders et al. (1991). They are based on a time-dependant Fourier series as a generalized state space averaging method, but can also be derived from the Hilbert transform, Strunz et al. (2006). Since signals in the DP domain oscillate with lower frequencies compared to the time domain, solvers can make larger step sizes without sacrificing accuracy. Furthermore, the elimination of the necessity for proportionality between the simulation time step and the highest frequency in the simulated signals presents an intriguing advantage for power electronics-based equipment, Mirz et al. (2020). Demiray (2008) shows that the DP model of the power grid can be easily derived from the classical EMTS model. Integrating DPs into existing solutions has been done, like modeling of general transmission and distribution networks that uses DPs in the dynamic-quadrature-zero (DQ0) reference frame, Levron and Belikov (2017). Also, methods for extracting DPs from samples of natural waveforms generated during EMTSSs are developed, as well as methods to projects all the frequency contents including the dc component to the frame of the fundamental-frequency, as done by Rupasinghe et al. (2021). They are used to simulate power system behaviours in steady state conditions, as well in transient periods, see Demiray (2008). Also, single-phase induction machine modeling and the analysis of asymmetrical faults in power systems were accomplished through the utilization of DPs, Stankovic et al. (1999), Stankovic and Aydin (2000). Afterwards, Liu et al. (2014) and Stankovic et al. (2000) extend the application of DPs to model high-voltage direct current and flexible alternating current transmission systems. Inverter-based microgrids using DPs have been analysed by Shuai et al. (2019), where up to two three-phase inverters are considered. Modelling techniques, model reduction approaches, and challenges of applying these methods in systems with IBRs are discussed by Lara et al. (2023), while also examining the implications for stability analysis.

Contemporary developments in the field of power systems lean towards the integration of IBRs, such as wind and solar energy, alongside the incorporation of battery energy storage systems. Consequently, deviating from prior research, our emphasis is directed towards employing DPs to simulate IBRs within EMTSSs, with a specific focus on stability analysis. Furthermore, we focus on the simulation of large three-phase systems using DPs, with up to 10 generators and 228 states, and evaluate the reduction of simulation time. Also, simulating power systems with 100% shares of renewable energy sources is accomplished, and the benefits of using DPs in the stability analysis are demonstrated as well.

The paper is structured as follows. Section 2 presents the mathematical model of the system, describing the power grid model and the used inverter model. Afterwards in Section 3, the DP concept is explained, and the necessary modifications are presented, which are needed for modeling the power system in the DP domain. The numerical results are presented in Section 4, where the DP and classical approach are compared, before the conclusions are formulated in Section 5.

2. POWER SYSTEM MODELLING

Classical and DP-based EMTS models consist of the power grid model, and of models for connected components such as generators or loads. The connected components, depending on their control mode, measure and control the voltage or current, as shown in Fig. 1. The power grid model is composed of diverse elements, including cables, power lines, and transformers. We focus on IBRs as grid-connected components. Although the approach is applicable to any component, for simplicity we apply the DPs on a grid-forming (GFM) inverter model.

2.1 Power Grid Modelling

The power grid is typically described using the bus/branch model, as shown in Fig. 2. In this model, the grid’s branches are commonly depicted using the two-port model. The bus/branch model can be succinctly represented as a graph denoted as $G = (N, E)$, where $N = \{1, \ldots, n\}$ denotes the set of buses, while $E \subseteq N \times N$ signifies the set of network branches. For every bus $i \in N$, the following equation can be formed:

$$\mathbf{i}_i(t) = \mathbf{R}_i^{-1} \mathbf{v}_i(t) + \mathbf{C}_i \frac{d\mathbf{v}_i(t)}{dt} + \sum_{(i,j) \in E_i} \mathbf{i}_{ij}(t), \quad (1)$$

where $\mathbf{i}_i(t)$ stands for the three-phase injected current into the bus $i \in N$, and $\mathbf{v}_i(t)$ for the three-phase voltage of the corresponding bus. The matrices $\mathbf{R}_i^{3 \times 3}$ and $\mathbf{C}_i^{3 \times 3}$ represent the shunt resistors and capacitors, respectively. The current $\mathbf{i}_{ij}(t)$ represents the three-phase branch current, and $E_i$ denotes the set of branches incident to the bus $i \in N$. Since we observe instantaneous changes in the voltage and current, the voltage sources cannot be connected in parallel with capacitors. Analogously, current sources cannot be in a series connection with inductors. Note that if there is no capacity on the $i \in N$ bus, equation (1) becomes algebraic and be used to reduce the system size.

A branch in a power grid represents physical connections between two different buses in the network. Branches encompass a range of elements, including transmission

![Fig. 1. Exemplary power system consisting of four IBRs $P_i$ and four conventional plants and consumers. The obtained current/voltage from the grid is taken as the input into the IBRs, and their voltage/current is considered as the input into the grid.](image-url)
The active power is used to control the angular frequency of the inverter \( \omega \). Therefore, the droop equations are utilized, as the most established GFM method. The angular frequency of the inverter \( \omega \) is determined by the \( P - \omega \) droop equation:

\[
\omega = \omega_{ref} - k_P (P - P_{ref}),
\]

where \( \omega_{ref} \) represents the referent angular frequency, \( P \) and \( P_{ref} \) the measured and referent active power, respectively. The droop gain is denoted as \( k_P \). Analogously, the voltage magnitude is determined using the \( Q - V \) droop equation:

\[
V = V_{ref} - k_Q (Q - Q_{ref}),
\]

with \( V_{ref} \) as the referent voltage, \( Q \) measured reactive power, and \( Q_{ref} \) the referent reactive power. The power calculation in this inverter model is done using the amplitude-invariant DQ0 transformation:

\[
P = \frac{3}{2} (vQID - vQDI),
\]

\[
Q = \frac{3}{2} (vQID + vQDI).
\]

Before connecting to the grid, the voltage signal is passed through a low-pass filter, which is used to remove high-frequency components from the signal. The referent values \( P_{ref} \) and \( Q_{ref} \) are determined using the power flow equations, which also describe the whole grid itself. Also, in order to function as an ideal voltage source, it is necessary to maintain the output impedance of the inverter at a value of zero.

3. DYNAMIC PHASOR SYSTEM MODEL

As seen in Bracewell (1986) work, a time-domain periodic waveform \( x(t) \) can be represented on the interval \( t \in [t - T, t] \) using a Fourier series:

\[
x(t) = \sum_{k=-\infty}^{\infty} X_k(t)e^{j k \omega_s t},
\]

where \( \omega_s = 2\pi/T \), and \( X_k(t) \) is the \( k \)-th complex Fourier coefficient, which can be determined by the following averaging operation:

\[
X_k(t) = \frac{1}{T} \int_{t-T}^{t} x(t)e^{-j k \omega_s t} dt = \langle x \rangle_k(t).
\]

The time-varying complex Fourier coefficients in (10) represent dynamic phasors, and they are functions of time since the interval under consideration slides as a function of time. An important property of the DPs is the relation between the derivatives of \( x(t) \) and the derivatives of \( \langle x \rangle_k(t) \):

\[
\left\langle \frac{dx}{dt} \right\rangle_k = \frac{d\langle x \rangle_k}{dt} + jk\omega_s \langle x \rangle_k,
\]

where the time argument \( t \) is omitted for clarity. Due to the real-valued nature of our original variables \( x(t) \), the phasor \( X_{-k}(t) \) is equal to the complex conjugate of \( X_k(t) \).

To adapt the power system model to DPs, the grid equations need to be adjusted first. That is done by using (11), so that the equation (3) and (4) can be rearranged:

\[
\frac{dX_k(t)}{dt} = AX_k(t) + BU_k(t) - jk\omega_s X_k(t),
\]

\[
Y_k(t) = CX_k(t) + DU_k(t),
\]

where the DPs \( X_k(t) \) and \( U_k(t) \) become the new continuous dynamic states, and the new algebraic inputs, respectively. The output DPs are expressed by the vector...
As the DPs of each inverter rotate with the frequency of one inverter in the power system, the frequency and voltage magnitude of each inverter, standing for the amplitude, \(A\), and the phase of the grid, \(\theta\), are neglected, leaving only the fundamental frequency described with the following equation:

\[
v(t) = \begin{bmatrix}
A \sin(\omega t + \phi) \\
A \sin(\omega t + \frac{2\pi}{3}) \\
A \sin(\omega t + \frac{2\pi}{3})
\end{bmatrix},
\]

(14)

with \(A\) standing for the amplitude, \(\omega\) for the rotating frequency, and \(\phi\) for the initial angle. Considering (14), the equation (10) can be rewritten as:

\[
V(t) = \begin{bmatrix}
\frac{A}{2} \sin(\phi - \frac{2\pi}{3}) - j\frac{A}{2} \cos(\phi - \frac{2\pi}{3}) \\
\frac{A}{2} \sin(\phi) - j\frac{A}{2} \cos(\phi) \\
\frac{A}{2} \sin(\phi + \frac{2\pi}{3}) - j\frac{A}{2} \cos(\phi + \frac{2\pi}{3})
\end{bmatrix},
\]

(15)

where \(V(t)\) represents the DPs of the three-phase voltage, when only the fundamental harmonic is used. The frequency of one inverter in the power system is taken as the referent frequency, which is also used as the power grid frequency \(\omega_g\). Before connecting the inverters to the grid, the voltage needs to be passed through a low-pass filter. Therefore, the filter equations also need to be transferred into the DP domain, which is done using (12) and (13). As the DPs of each \(i\)-th inverter rotate with \(\omega_i\), and the filter equations use \(\omega_g\), it is necessary to synchronize these two frequencies. That is done by multiplying the voltage DPs by \(e^{j(\omega_t - \omega_g)}\), where \(\omega_t\) stands for the phase of the \(i\)-th inverter and \(\omega_g\) for the phase of the grid. Further on, to be able to use (7) and (8) for calculating the active and reactive power, the DPs need to be translated into the DQ0 frame. For that, the DPs first need to be transferred back into the time domain using (9). Afterwards, the obtained values are multiplied by:

\[
T(\theta) = \frac{2}{3} \begin{bmatrix}
\cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\
-\sin\theta & -\sin(\theta - \frac{2\pi}{3}) & -\sin(\theta + \frac{2\pi}{3}) \\
\frac{1}{2} & 1/2 & 1/2
\end{bmatrix},
\]

(16)

where \(T(\theta)\) represents the transformation matrix into the rotating DQ0 frame. By this it is possible to calculate the active and reactive power of each inverter. No additional modifications of the inverter need to be done, to be able to operate in the DP domain.

In contrast to conventional methods, the DP approach offers a numbers of advantages. Instead of handling sinusoidal waveforms in the time domain, DPs transform them into a unified complex number in the phasor domain, simplifying calculations and analysis. They are effective in systems where frequencies other than the fundamental occur (e.g. due to FACTS devices), in contrast to the DQ0-transformation based grid simulation. However, the number of variables and equations in the DP domain can increase depending on the number of selected harmonics, and that has an impact on the simulation performances, Mirz et al. (2020).

4. NUMERICAL RESULTS

In this section, we apply the DP approach to simulate transients occurring in power systems. First of all, the accuracy of the DP simulations is verified by comparing the results from the DP simulation with results obtained using Simscape Electrical. After that, to evaluate the execution time of the DP simulations, the dynamics of the IEEE 9-bus system and the IEEE 39-bus system are simulated with a disturbance. Power systems without GFM inverters are taken into account, as well as systems containing up to 10 GFM inverters. Afterwards, asymmetrical systems are examined and the transients occurring in such systems are simulated and analysed in the IEEE 9-bus system. In all test cases, the systems are initialized to start operating from a steady-state condition.

4.1 Comparison of DP simulations and Simscape Electrical simulations

In order to make sure that the DP simulations give the accurate results, a small power system shown in Fig. 4, with 2 inverters and 1 load is analysed. The results from the DP simulation are compared with those obtained from the Simscape Electrical simulation. The power system in Fig. 4 is taken from Rahmoun et al. (2018). The load consumes only active power, and both inverters have the same active power reference, as well as the voltage magnitude reference. During the simulation, a load step change to simulate a disturbance. For that, the DPs first need to be transferred back into the time domain using (9). Afterwards, the obtained values are multiplied by:

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Fig. 4. Power system comprising two generators, each equipped with an inverter, and a single load.

Fig. 5. Comparison of frequencies in the DP simulation and Simscape Electrical simulation.

Fig. 6. Left - comparison of active powers in the DP simulation and Simscape Electrical simulation; right - close-up of the high frequency oscillations.

is performed by increasing the active power demand of the load by a factor of two. The frequency of each GFM inverter obtained from the DP simulation is compared to the corresponding frequency from the Simscape Electrical simulation, as shown in Fig. 5. The absolute error between the frequencies in the DP simulation, and the Simscape Electrical simulation is less than $10^{-3}$. Therefore, we can conclude that both simulations give the same results. This can also be verified by observing the active power of the GFM inverters, as it is presented in Fig. 6.

4.2 Execution time evaluation of DP simulation and classical EMTS

To evaluate the execution time of the DP simulations, the dynamics of the IEEE 9-bus system shown in Fig. 7, and the IEEE 39-bus system shown in Fig. 8 are simulated for 10 seconds with a disturbance. In all tests, the disturbance is performed by increasing the magnitude of load current by a factor of two, on bus 9 for the IEEE 9-bus system, and on bus 26 for the 39-bus system. Table 1 shows the comparison of the DP and classic simulations, where $t_{DP}$ and $t_C$ represent DP and classical EMTS execution times, respectively. In order to present the results in a more general way, they are also showcased as a ratio between the execution times of the classical EMTS, and the DP based simulations, displayed in Table 1 as the normalized time $t_C/t_{DP}$. The results demonstrate that DP-based simulations are significantly faster compared to classic EMTSs. It is noticeable that DP based simulation reduce the execution time in both systems without and with IBRs. Additionally, to verify the accuracy of the simulations, the frequency of each GFM inverter obtained from the DP simulation is compared to the corresponding frequency from the classic EMTS for the IEEE 9-bus system with 3 GFM inverters. As shown in Fig. 9, DP simulation achieves the same results as classical EMTS, and the absolute error between the frequencies obtained by the DP simulation and the classical EMTS is less than $10^{-3}$. Using (9) it
power demands across phases are made, simulating a fault from that phase to zero at of bus 9, setting the amplitude of the injected current but this time the disturbance is only applied to phase A. In the symmetrical case, the system is simulated for 10 seconds, with 3 GFM inverters to introduce asymmetry. As in the previous systems, modifications are made to the IEEE 9-bus system to simulate and analyse transients in asymmetrical systems.

To simulate and analyse transients in asymmetrical systems, modifications are made to the IEEE 9-bus system with 3 GFM inverters to introduce asymmetry. As in the symmetrical case, the system is simulated for 10 seconds, but this time the disturbance is only applied to phase A of bus 9, setting the amplitude of the injected current from that phase to zero at $t = 2s$. In this way, uneven power demands across phases are made, simulating a fault in the system. Comparison of the injected currents from bus 1 obtained from the DP simulation and classical EMTS can be observed in Fig. 12. As seen, both simulations yield identical outcomes, yet once more, the simulation time is reduced within the DP domain. Specifically, the DP simulation requires 36.85 seconds, while the classical EMTS demands 727.7 seconds, resulting in a reduction of roughly 20 times. This approach can be easily extend to other kinds of asymmetry, such as asymmetry occurring in line parameters.

5. CONCLUSIONS

In this paper the DPs are used to simulate electromagnetic transients occurring in systems with high shares of IBRs. The properties of DPs were analysed, and a model of the GFM inverter based on DPs was proposed. In conclusion, DP simulations offer a substantial increase in speed compared to traditional EMTSs, without sacrificing accuracy, even in asymmetric systems. They are able to simulate both the grid itself, and IBRs, using models adapted to the DP domain. The large step sizes that the solver can take, has proven to be very useful in simulations of large networks and IBRs, as well when a slow instability occurs. In our future work, we would like to integrate DP models of GFL inverters, synchronous machines, and FACTS, as well as to investigate the benefits of using DPs in such systems. Additionally, we aim to analyze fault detection in inverter-based systems using DPs. Our field of interest in using the DPs also includes the stability analysis, as well as the inertia estimation of power systems with high amount of IBRs.
REFERENCES


