On the Shielding Effect of Wire Arrays

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Abstract

A simple two-dimensional model of an infinite array of circular conductors is presented. The model is developed in two variants, a thin model and a thick model. It allows the evaluation of the shielding effectiveness of the array when the wires distances their and radius changes. The shielding effect is expressed through the transmission coefficient of a plane wave normally impinging on the screen. The model is limited to a plane wave impinging normally on the wire’s plane and linearly polarized along the wires.
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Index Terms—electromagnetic shielding, metal wire array

I. INTRODUCTION

The shielding effect of wire arrays and grids is a well known phenomenon observed both for electrostatic fields, i.e. the Faraday effect, and electromagnetic waves. Various theoretical models developed to determine the transmission and reflection characteristics for planar wire arrays are reviewed in [1]. One of the most successful model is presented in [2]. Unfortunately, these models are limited to thin wires, i.e. to cases where the wire radius is much smaller than the wavelength and the wire spacing. More recent papers overcome such limitations using the Green-function method [3] and the lattice-sum method [4].

A common issue in all of the above mentioned models is the presence of some very slow converging series, that hinders their numerical application. This problem is mostly overcome by using a Fourier transform of the Hankel functions.

The model presented in this paper takes a different, simpler approach. It is assumed that the wires are Perfect Electrical Conductors (PEC), that the wave impinges normally to the wire’s plane and is linearly polarized along the wires. The problem of a slow converging series is solved using an acceleration scheme based on the Hankel’s function approximation for large arguments. Also, the case of thick conductors is dealt with by enforcing the null field condition on the wire’s surface in a weak sense.

II. THE MODEL

Let us suppose that an infinite array of circular, $\hat{z}$ directed PEC wires with radius $a$ is placed along the $\hat{y}$ axis and let $d$ be the regular distance between the center of two adjacent wires, as shown in Fig. 1.

A $\hat{z}$ polarized, $\hat{x}$ directed plane wave

$$\vec{E}_i = \hat{z} E_0 e^{-j k_0 x}$$

where $k_0$ is the wavenumber, impinges upon the array, inducing currents along the wires that hopefully will dampen the field beyond the array, i.e. for $x > 0$. As is well known, this setup reduces the problem to a two-dimensional scalar one, since the electric fields depends only on $x$ and $y$ and have only a $\hat{z}$ component. Let us denote with

$$\vec{E}_s = \hat{z} E_s (x, y)$$

the scattered field generated by the induced currents. The scattered field will be an even function of $y$, and is also periodic along $y$, with period $d$. Due to the symmetry of the scattered field with respect to the lines $y = \pm d/2$

$$\left. \frac{\partial E_s (x, y)}{\partial y} \right|_{\pm d/2} = 0.$$  \hspace{1cm} (3)

From the periodicity and (3) it follows that for $x > 0$ $E_s$ can be written as a combination of the following modes

$$E^{(n)}_s (x, y) = \cos \left( \frac{2 \pi n}{d} y \right) e^{-j k_n x}$$

where $n = 0, 1, \ldots$ and

$$k_n^2 = k_0^2 - \left( \frac{2 \pi n}{d} \right)^2.$$  \hspace{1cm} (5)

The fundamental mode for $n = 0$ is an $\hat{x}$ directed plane wave for $x > 0$, hence it can dampen the incident field and so it is the desired mode. The other spurious modes for $n \geq 1$
are unwanted and these modes will be evanescent if \( k_n^2 < 0 \). This condition implies \( k_0 < 2\pi/d \) and in turn \( d < \lambda_0 \), where \( \lambda_0 \) is the wavelength.

The condition \( d < \lambda_0 \) is a necessary but not sufficient condition to achieve a shielding effect. Ideally, the coefficient of the fundamental mode should be \(-E_0\). With this coefficient the magnetic field of the fundamental mode is

\[
\vec{H}(x, y) = \frac{E_0}{Z_0} \begin{cases} 
+ \hat{y} e^{-j k_0 x} & \text{for } x > 0 \\
- \hat{y} e^{j k_0 x} & \text{for } x < 0 
\end{cases}.
\]

Using Ampère-Maxwell law on the surface \(|x| \leq \epsilon, |y| \leq d/2\) shown in Fig. 1 and letting \( \epsilon \to 0 \) the \( + \hat{z} \) current that sustains the fundamental mode with coefficient \(-E_0\) turns out to be

\[
I_0 = \frac{2E_0}{Z_0} d.
\]

Unsurprisingly, the density \( I_0/d \) is the same current density induced on a PEC screen put at \( x = 0 \) by the incident field (1). Similar integrals on the spurious modes for \( n \geq 1 \) will result in a zero current, because they involve an integral of \( \cos(2\pi ny/d) \) over the interval \([-d/2, d/2]\).

The actual wire current \( I_s \) allows the computation of the transmission coefficient as

\[
\tau = \frac{-I_0 + I_s}{-I_0} = 1 - \frac{I_s}{I_0},
\]
due to the linear dependence of the fields from the current. It follows that the reflection coefficient is

\[
\rho = \tau - 1 = -\frac{I_s}{I_0}.
\]

Assuming an uniform current density inside the wires, the electric field of a single wire is

\[
\vec{E}_{wire} = -\frac{k_0}{4} Z_0 I_s H_0^{(2)}(k_0 r) \hat{z}
\]

where \( Z_0 \) is the free-space characteristic impedance. \( H_0^{(2)} \) is the Hankel function of the second type and \( r \) is the distance from the wire [5]. Hence, on points \( \vec{p} = (x, y) \) outside conductors the \( \hat{z} \) directed scattered field is

\[
E_s(\vec{p}) = -\frac{k_0}{4} Z_0 I_s \sum_{n=-\infty}^{n=+\infty} H_0^{(2)}(k_0 \|\vec{p} - nd\hat{y}\|) .
\]

The induced wire current \( I_s \) can be determined enforcing a null total tangential electric field on points \( \vec{p} = (x, y) \) on the wire’s surface

\[
E_0 e^{-j k_0 x} + E_s(\vec{p}) = 0.
\]

For very thin wires (\( a < \ll \lambda_0 \)) the incident field can be approximated with \( E_0 \) and the series in (11) with

\[
S_{thin} = H_0^{(2)}(k_0 a) + 2 \sum_{n=1}^{n=+\infty} H_0^{(2)}(n k_0 d) ,
\]

so that \( I_s \) is computed as

\[
I_s = \frac{4E_0}{k_0 Z_0 S_{thin}}.
\]

This is the thin variant of the model.

On the other hand, for thick wires (12) must be used, although it can not be strictly enforced, since (11) is an even function of \( x \), unlike the incident field (1).

Therefore, the Dirichlet condition (12) is enforced in a weaker sense: the incident field (1) and the scattered field (11) with \( I_s = 1 \) are computed on a finite set of \( N_p \) equispaced points \( p_i = (x_i, y_i) \) on the central wire, forming the arrays

\[
u_i = -\frac{k_0}{4} Z_0 \sum_{n=-\infty}^{n=+\infty} H_0^{(2)}(k_0 \|\vec{p}_i - nd\hat{y}\|),
\]

then \( I_s \) is determined as the value that minimizes \( ||u - I_s v|| \), i.e.

\[
I_s = -\sum_{i=1}^{N_p} u_i^* v_i \sum_{i=1}^{N_p} |v_i|^2
\]

where the asterisk denotes complex conjugation. Note that the values \( v_i \) in (15) are actually computed only for the points with \( x \geq 0, y \geq 0 \), the other values are obtained by symmetry. In other words, only around \( N_p/4 \) points require the evaluation of the series.

This way of computing \( I_s \) is the thick variant of the model.

Obviously, the series appearing in (11) and (13) must be truncated in the actual computations. Regrettably, the convergence of these series is quite problematic. In fact, the asymptotic expression of \( H_0^{(2)} \) for large arguments is [5]

\[
H_0^{(2)}(x) \simeq \sqrt{\frac{2j}{\pi x}} e^{-j x},
\]

hence for large \( n \) the general term resembles

\[
a_n = \sqrt{\frac{2j}{\pi \delta}} \frac{e^{-j \delta n}}{\sqrt{n}}
\]

with \( \delta = k_0 d < 2\pi \). A series whose general term is (18) is convergent, in fact it passes the Dirichlet test, but due to the \( 1/\sqrt{n} \) factor it is very slow to converge and the real and imaginary parts of each term often change sign. For example, for \( d = \lambda_0/2 \) the series is an alternating sign one. This can lead to significant cancellation errors during the computation.

Fortunately, there is a simple way to accelerate the series. Let

\[
S = \sum_{n=1}^{+\infty} a_n,
\]

multiply \( S \) by \( e^{j \delta} \), rewrite it as

\[
e^{j \delta} S = \sum_{n=1}^{+\infty} e^{j \delta} a_n = e^{j \delta} a_1 + \sum_{n=1}^{+\infty} e^{j \delta} a_{n+1}
\]

and subtract the two series

\[
S - e^{j \delta} S = -e^{j \delta} a_1 + \sum_{n=1}^{+\infty} (a_n - e^{j \delta} a_{n+1}) .
\]

Dividing by \( 1 - e^{j \delta} \) an accelerated series whose general term is

\[
b_n = \frac{a_n - e^{j \delta} a_{n+1}}{1 - e^{j \delta}} = \sqrt{\frac{2j}{\pi \delta}} \frac{e^{-j \delta n}}{\sqrt{n}} \left( \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n} + 1} \right)
\]
is obtained.

Since $|b_n| = O(n^{-3/2})$, the transformed series has a faster convergence, and due to the fact that it retains the term $e^{-j \delta a}$ the same scheme can be re-applied to it, thus forming an even faster converging series. Summarizing, the acceleration scheme is as follows

$$b_1 = a_1 - \frac{e^{j \delta} a_2}{1 - e^{j \delta}}$$

$$b_n = \frac{a_n - e^{j \delta} a_{n+1}}{1 - e^{j \delta}} = \frac{a_n^2 - e^{2j \delta} a_{n+1}^2}{(1 - e^{j \delta}) (a_n + e^{j \delta} a_{n+1})},$$

where the last substitution, i.e. multiplying and dividing by $a_n + e^{j \delta} a_{n+1}$, is useful in mitigating the cancellation error that occurs in computing $a_n - e^{j \delta} a_{n+1}$. Obviouly, when $e^{j \delta} \approx 1$ the acceleration scheme breaks down, because the series approaches the divergent series $\sum_{n=1}^{\infty} n^{-1/2}$.

III. NUMERICAL RESULTS

Given the fundamental role played by series of Hankel functions in the wire screen model, the effectiveness of the acceleration scheme is tested on a series whose general term is $H_0^{(2)} (k_0 ||\vec{p} - \hat{y} n d||) + H_1^{(2)} (k_0 ||\vec{p} + \hat{y} n d||)$ with $d = 0.75 \lambda_0$ and $p$ a random point on the surface of the central conductor with radius $a = 0.075 \lambda_0$.

Fig. 2 shows the relative error of the $n$-th sum $S^{(n)}$ for $n = 1 \ldots 400$ with respect to the value $S^{(1000)}$, i.e. $[1 - S^{(n)})/S^{(1000)}]$. This is shown for the original series and for the accelerated series up to the fifth order.

As can be seen, already at $n = 200$ the error for the fourth order accelerated series is $10^{-10}$.

The applicability limit of the thin model is tested using a comparison with the thick model, for the distances $d = 0.25 \lambda_0, 0.5 \lambda_0, 0.75 \lambda_0$ in Fig. 3. For this curves the series are truncated at $n = 200$ and accelerated to the fourth order. For the thick model the value $N_p = 8$ is used in (15,16). Raising these values for $n$ and $N_p$ does not significantly affect the results.

As can be seen, the thin model deviates significantly from the thick one when $a > 0.09 \lambda_0$, giving the physically impossible results of $|\tau| > 1$ for $a > 0.15 \lambda_0$. It is also interesting to note that the optimal screening radius for both $d = 0.25 \lambda_0$ and $d = 0.25 \lambda_0$ is very roughly around one sixth of $d$. This corresponds with the approximate formula for the transmission coefficient in [6], [7], i.e.

$$\tau = \frac{1}{1 - e^{2j \delta} \frac{\lambda_0}{2\pi a}}.$$ (24)

In fact, the coefficient computed according to (24) will be negligible when $a \approx d/2 \pi \approx d/6$.

Intriguingly, the approximate formula (24) explains the "Equal Area Rule" [8], which is a rule of thumb used to calculate the radii of wire-grid models representing metallic surfaces in numerical electromagnetic modeling. According to this rule, the surface area of the wires parallel to one linear polarization must equal the corresponding solid surface that is being simulated.

Unfortunately, such approximation does not give any information about the optimal distance $d$, only about the ratio $a/d$. Intuitively, it may seem that the screening effect increases when $d$ decreases.

This is what actually happens with the thin model variant, but surprisingly the thick model gives a very different result. Both models are used to determine the optimal radius that gives the minimum transmission coefficient $|\tau|$ for a given distance $d$ between the wires, with $d \in [\lambda_0/16, 7\lambda_0/8]$. The results, obtained using a sequential quadratic programming method to optimize $a$ are shown in Fig. 4.

The thin model gives a monotonically increasing transmission coefficient $|\tau|$ for increasing distance $d$, while the thick model produces a very different and unexpected result. Initially, and up to $d \approx 0.328 \lambda_0$, $|\tau|$ increases, then decreases getting at a minimum of $|\tau| = 1.63e - 6$ at $d \approx 0.523 \lambda_0$. At this minimum the optimal ratio is $a/d = 0.178$, that is to
say $a \simeq 0.093\lambda_0$. After this optimal distance, $|\tau|$ increases monotonically approaching 1 towards the end of the range.

Although the values for the transmission coefficient computed using the thick ($\tau_k$) and thin ($\tau_n$) models are quite different, the corresponding optimal radius $a_k, a_n$ are pretty close up to $d \leq 0.6\lambda_0$ after which they diverge. Moreover, in this interval the ratio $a/d$ is close to $1/6$.

IV. CONCLUSION

The simple model for the shielding effect of wire arrays has been presented in two variants. The thin variant is of limited usefulness, due to its limited applicability, and does not introduce any advantage over existing models.

On the other hand the thick variant gives an unexpected and interesting result, that it to say that a wire screen with an interwire distance of slightly more than half a wavelength can be more effective than a finer grid. Unfortunately, the author has not been able to find in the literature any measures that confirm or denies such result.

In case the thick model is confirmed by measures, it would be interesting to extend it to the case of arbitrary incidence.

REFERENCES


