On the incompatibility of Maxwell’s addition and the Lorentz force in non-relativistic electrodynamics

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This article addresses whether the Lorentz force formula, as used in electrostatics and magnetostatics, is also applicable to non-relativistic electrodynamics. To answer this question, the article uses the Liénard-Wiechert potentials to calculate the general analytical solution of Maxwell’s equations for arbitrarily moving, non-relativistic point charges. It is then shown that the obtained solution only produces the correct force of a direct current on a moving test charge under illogical assumptions. This problem does not arise from the Maxwell equations but is probably related to the fact that the Lorentz force is already contained in Maxwell’s equations due to Maxwell’s addition. The Lorentz force should therefore not be added a second time in the form of an explicit supplementary formula. Instead, it is reasonable to integrate an old hypothesis by Carl Friedrich Gauss from 1835 into Maxwell’s electrodynamics. This integration turns Maxwell’s electrodynamics into Weber-Maxwell electrodynamics and resolves the contradictions.

I. Introduction

This article addresses a theoretical as well as historical question from the field of classical electrodynamics. Specifically, it discusses whether the adaptation of the Lorentz force formula would have been advisable when J. C. Maxwell introduced the displacement current to Ampère’s law in around 1870 (often referred to as Maxwell’s addition). Although this question is not directly related to current research topics in physics or electrical engineering, its answer has great indirect relevance.

Maxwell established his field equations for electric and magnetic fields over 30 years before Einstein presented the special theory of relativity. It is well known that Maxwell’s electrodynamics does not provide correct results for high relative velocities without the Lorentz transformation. It is much less known that Maxwell’s original electrodynamics without Lorentz transformation can also produce incorrect results even at very low relative velocities. In particular, the conservation of momentum is often violated, which is difficult to demonstrate only by means of Maxwell’s field equations themselves. However, the problem becomes much more obvious when the general solution for point charges is analyzed.

For this reason, the present article uses the Liénard-Wiechert potentials to calculate the fields that an arbitrarily moving point charge would generate on a resting test charge. To generalize the resulting solution to moving test charges, the Galilean principle of relativity is used. According to our modern conception, this procedure is correct for slow relative velocities. Furthermore, it reflects the approach that a scientist would have chosen at the end of the nineteenth century because the special theory of relativity did not yet exist. Remarkably, the resulting solution makes no sense if it is inserted into the Lorentz force formula.

Although the general solution for arbitrarily moving point charges is a fairly simple and user-friendly formula, it is not easy to calculate. The first attempts at such a solution were made by Liénard and Wiechert, who published their work a few years before Einstein at the beginning of the twentieth century. However, their work was only a first step. The first to calculate the full solution was likely Jefimenko at the end of the twentieth century [1]. For this reason, the complete non-relativistic solution for point charges was not yet accounted for when Einstein developed the special theory of relativity.

The problem pointed out in this article with non-relativistic electrodynamics do not seem to originate from the fields themselves, but is clearly related to the complementary Lorentz force formula. It is quite obvious how the problem should be resolved and we can speculate that the Lorentz force formula would probably not have survived into the twentieth century if Liénard and Wiechert had succeeded in calculating the complete solution and presented their work one or two decades earlier. This in turn would have had a major impact on all subsequent developments. For example, special relativity might be different today, since the proposed modification not only solves the problem with the conservation of momentum, but also makes the magnetic field superfluous and even eliminates the issues that led to the development of the Lorentz transformation.

II. Structure of the article

The article is structured as follows. First, we calculate the general solution to Maxwell’s equations for non-relativistic but otherwise arbitrarily moving point charges. The calculated formula is then reduced to the special case where the accelerations are small. This simplified formula is then used to calculate the force that a conductor with direct current exerts on a uniformly moving test charge (and vice versa). The results are then discussed.

All calculations in this article have been validated using the software Mathematica, and the corresponding script is available from the author on request.

III. General solution to Maxwell’s equations for point charges

In this section, we solve Maxwell’s equations for point charges. We start with the Liénard-Wiechert potentials, which
are the exact solution of the Maxwell equations for the electric field \( E \) and magnetic field \( B \) generated by a moving point charge with trajectory \( r_s(t) \) in the rest frame of a test charge located at time \( t \) and location \( r \).

The Liénard-Wiechert potentials are derived and well-described in textbooks (e.g. [2]). In summary, the Liénard-Wiechert potentials state that the fields \( E \) and \( B \) can be calculated using the formulas

\[
E = -\nabla \Phi - \frac{\partial}{\partial t} A, \tag{1}
\]

and

\[
B = \nabla \times A \tag{2}
\]

if the potentials

\[
\Phi = \frac{q_s c}{4\pi \varepsilon_0} \frac{(c^2 (t - \tau) - r_s(\tau) \cdot (r - r_s(\tau)))}{(c^2 (t - \tau) - r_s(\tau) \cdot (r - r_s(\tau)))^2}, \tag{3}
\]

and

\[
A = \frac{1}{c^2} \dot{r}_s(\tau) \Phi \tag{4}
\]

are known. The parameter \( \tau \) is a certain moment in the past defined by

\[
\tau = t - \frac{1}{c} ||r - r_s(\tau)||. \tag{5}
\]

The electromagnetic force \( F \) in Maxwell’s electrodynamics can be found by means of the Lorentz force

\[
F = q_d E + q_d u \times B \tag{6}
\]

where \( u \) is a velocity. This velocity has a somewhat vague definition in the literature [3], and its interpretation is therefore initially left open in this article.

To evaluate the right-hand side of equation (6), we must eliminate the derivatives in equations (1) and (2). For this reason, we calculate the derivatives of the potentials \( \Phi \) and \( A \). The difficulty in doing so is that \( \tau \) is an unknown function of \( r \) and \( t \). By calculating these derivatives, we obtain:

\[
\nabla \Phi = \frac{q_s c}{4\pi \varepsilon_0} \frac{h_2(\tau) \nabla \tau + q_s c \dot{r}_s(\tau)}{h_1(\tau)^2}, \tag{7}
\]

\[
\frac{\partial}{\partial t} A = \frac{q_s c}{4\pi \varepsilon_0} \frac{h_3(\tau) \frac{\partial}{\partial \tau} \dot{r}_s(\tau) + q_s c^2 \dot{r}_s(\tau)}{h_1(\tau)^2}, \tag{8}
\]

and

\[
\nabla \times A = \frac{q_s c h_3(\tau) \times \nabla \tau}{4\pi \varepsilon_0 c h_1(\tau)^2}. \tag{9}
\]

Here, \( h_1, h_2 \) and \( h_3 \) are auxiliary variables that were introduced so that the equations above could be presented more concisely. They are defined as

\[
h_1(\tau) := (r - r_s(\tau)) \cdot \dot{r}_s(\tau) - c^2 (t - \tau), \tag{10}
\]

\[
h_2(\tau) := c^2 - r_s(\tau) \cdot \dot{r}_s(\tau) + (r - r_s(\tau)) \cdot \dot{r}_s(\tau) \tag{11}
\]

and

\[
h_3(\tau) := h_1(\tau) \dot{r}_s(\tau) - h_2(\tau) r_s(\tau). \tag{12}
\]

In the next step, we substitute equations (7), (8), and (9) into equations (1) and (2) to obtain

\[
E = \frac{q_s h_3(\tau) \frac{\partial}{\partial \tau} \dot{r}_s(\tau) - q_s c^2 h_2(\tau) \nabla \tau}{4\pi \varepsilon_0 c h_1(\tau)^2} \tag{13}
\]

and

\[
B = \frac{q_s h_3(\tau) \times \nabla \tau}{4\pi \varepsilon_0 c h_1(\tau)^2}. \tag{14}
\]

Although the function \( \tau \) is usually an unknown function of \( r \) and \( t \), the derivatives \( \nabla \tau \) and \( \partial \tau / \partial t \) can be calculated in equations (13) and (14). To do so, we first apply the differential operators to both sides of equation (5). This yields

\[
\nabla \tau = \frac{- (r - r_s(\tau)) + (r - r_s(\tau)) \cdot \dot{r}_s(\tau)}{c ||r - r_s(\tau)||}, \tag{15}
\]

and

\[
\frac{\partial \tau}{\partial t} = \frac{1 + (r - r_s(\tau)) \cdot \dot{r}_s(\tau)}{c ||r - r_s(\tau)||} \frac{\partial \tau}{\partial \tau}. \tag{16}
\]

We now have two linear equations in terms of \( \nabla \tau \) and \( \partial \tau / \partial t \). These equations are easy to solve; using the equation \( ||r - r_s(\tau)|| = c (t - \tau) \) and the definition (10), we obtain the equations

\[
\nabla \tau = \frac{r - r_s(\tau)}{h_1(\tau)}, \tag{17}
\]

and

\[
\frac{\partial \tau}{\partial t} = \frac{- c^2 (t - \tau)}{h_1(\tau)}. \tag{18}
\]

These two equations can be substituted into equations (13) and (14), giving

\[
E = \frac{- q_s c (h_3(\tau) (t - \tau) + h_2(\tau) (r - r_s(\tau)))}{4\pi \varepsilon_0 c h_1(\tau)^3} \tag{19}
\]

and

\[
B = \frac{q_s h_3(\tau) \times (r - r_s(\tau))}{4\pi \varepsilon_0 c h_1(\tau)^3}. \tag{20}
\]

Equations (19) and (20) are the formal solutions of Maxwell’s equations for a resting test charge at location \( r \) at time \( t \). If we insert the calculated fields into the Lorentz force (6), we obtain the force that a resting charge \( q_d \) at location \( r \) would experience at time \( t \) due to the existence of the moving point charge \( q_s \).

We want to generalize this solution to moving test charges. For this generalization, we first set \( r = 0 \), because we are only interested in the field that a point-like test charge \( q_d \) can perceive in its own center-of-momentum frame. We therefore apply the substitution

\[
(r \rightarrow 0) \tag{21}
\]

in equations (19) and (20). We do the same in equations (10), (11) and (5).

Second, we assume that (i) the center-of-momentum frame of the test charge \( q_d \) is moving with trajectory \( r_d(t) \) and (ii) the differential speed between \( q_s \) and \( q_d \) is much smaller than the speed of light in a vacuum \( c \). Given these assumptions, non-relativistic mechanics and a Galilean transformation can be used. We exploit these techniques to generalize the solution to a moving test charge \( q_d \).

In the center-of-momentum frame of \( q_d \), only the source charge \( q_s \) seems to be moving with the trajectory \( r_s(t) \rightarrow r_d(t) \). We can therefore perform the replacements

\[
r_s(\tau) \rightarrow r_s(\tau) - r_d(\tau) := -r, \tag{22}
\]
\[ F_s(\tau) \rightarrow \dot{r}_s(\tau) - \ddot{r}_s(\tau) := -v, \] (23) and
\[ \dot{r}_s(\tau) \rightarrow \dot{r}_s(\tau) - \ddot{r}_s(\tau) := -a. \] (24)
Because of these replacements and equation \( c (t - \tau) = r \), we obtain
\[ E = \frac{q_s}{4\pi \varepsilon_0} \left( \frac{(r c + r v) \left(c^2 - v^2 - r \cdot a\right)}{(r c + r \cdot v)^3} \right) + \frac{r a}{(r c + r \cdot v)^2} \] (25) and
\[ B = \frac{q_s}{4\pi \varepsilon_0 c} \left( \frac{(r \times v) \left(c^2 - v^2 - r \cdot a\right)}{(r c + r \cdot v)^3} \right) + \frac{r \times a}{(r c + r \cdot v)^2}. \] (26)

In the present time, a Lorentz transformation would have been used to obtain these equations. The procedure chosen here reflects the approach that would have been taken at the end of the 19th or the beginning of the 20th century, before the Lorentz transformation had been formalized.

Incidentally, as can be seen directly, the equation
\[ B = \frac{r}{c} \times \frac{E}{c} \] (27)

applies. This equation shows that, in Maxwell’s electrodynamics, the force \( F \) that a point charge \( q_s \) with trajectory \( r_s(\tau) \) exerts on another point charge \( q_d \) with trajectory \( r_d(\tau) \) at time \( t \) can be expressed solely in terms of the electric field \( E \). Strictly speaking, the magnetic field is superfluous.

It should also be noted that the formulas (25) and (26) correspond to the formulas (4.4.34) and (4.5.2) by Jefimenko [1], although Jefimenko’s method for calculating the solutions seems to be different and more laborious. Because Jefimenko does not generalize his equations to arbitrarily moving test charges, the signs of \( v \) and \( a \) should be considered when the equations are being compared. Equation (27) can also be found as formula (3.2.13) in Jefimenko’s work. Why Jefimenko persists with the concept of the magnetic field in his work, despite demonstrating its superfluousness, is somewhat puzzling.

The final solution for the force experienced by a moving test charge can be obtained by inserting equation (27) into the Lorentz force (6):
\[ F = q_d \left( E + \frac{u}{c} \times \left( \frac{r}{c} \times E \right) \right). \] (28)
The force \( F \) is therefore a function of four parameters: \( r, v, a \) and \( u \), though \( u \), as noted earlier, is not consistently used in the literature. However, the first three parameters are well-defined as the retarded distance vector
\[ r := r_d(\tau) - r_s(\tau), \] (29) and the retarded relative acceleration
\[ a := \ddot{r}_d(\tau) - \ddot{r}_s(\tau). \] (31)
The time \( \tau < t \) is a moment in the past and defined by the equation
\[ \tau = t - \frac{r}{c}. \] (32)

It is important to note that \( r \) is the absolute value of \( r \) and a function of \( \tau \) due to definition (29). Because the parameter \( \tau \) therefore occurs on both sides of equation (32), the equation can usually only be solved numerically. However, this does not present a major practical problem, as equation (32) is one-dimensional and can always be solved quickly and unambiguously when the relative speed is below the speed of light \( c \). The proof that there is always a single unique solution is simple and can be performed using Banach’s fixed point theorem.

IV. SIMPLIFICATION OF THE SOLUTION FOR UNIFORMLY MOVING POINT CHARGES

If all the involved point charges are moving almost uniformly, equation (28) for the electromagnetic force can be simplified. First, we exploit the fact that \( a = \mathbf{0} \) given our assumption of uniform motion. This reduces equation (25) to
\[ E = \frac{q_s c}{4\pi \varepsilon_0} \left( \frac{(r c + r v) \left(c^2 - v^2\right)}{(r c + r \cdot v)^3} \right). \] (33)
This can be further transformed to
\[ E = \frac{q_s}{4\pi \varepsilon_0 \gamma(v)^2} \left( \frac{r + \frac{\gamma}{c} v}{r + \frac{r \cdot v}{c}} \right). \] (34) with \( \gamma(\cdot) \) being the Lorentz factor. This equation can now be substituted for \( E \) in the force (28), giving
\[ F = \frac{q_s q_d}{4\pi \varepsilon_0 \gamma(v)^2} \left( \frac{r + \frac{\gamma}{c} v}{r + \frac{r \cdot v}{c}} \right) \cdot \left( \frac{r}{c} \times E \right). \] (35)
Due to definition (29), \( r \) is a retarded quantity. However, the velocities \( u \) and \( v \) are not because we have assumed constant velocities. Retarded variables complicate calculations, so it is useful to express equation (35) in terms of the non-retarded distance vector
\[ s := r_d(t) - r_s(t). \] (36)
To remove \( r \), we use the fact that, for uniform velocities \( v \), the equation
\[ s = r + v (t - \tau) \] (37) applies. Because \( t - \tau = r/c \),
\[ r = s - \frac{r}{c} v. \] (38)
Equation (35) can therefore be simplified to
\[ F = \frac{q_s q_d}{4\pi \varepsilon_0 \gamma(v)^2} \left( \frac{s + \frac{\gamma}{c^2} u \times (s \times v)}{r + \frac{r \cdot v}{c}} \right). \] (39)
If we multiply both sides of equation (38) with $r/r$, we can see that
\[ r + r \cdot \frac{v}{c} = s \cdot \frac{r}{r}. \]  
(40)
Inserting equation (38) on the right-hand side of the equation above yields
\[ r + r \cdot \frac{v}{c} = \frac{s^2}{r} - \frac{s \cdot v}{c}. \]  
(41)
Based on equation (32), $r = c(t - \tau)$. Furthermore, due to equation (37), the relation $r = ||s - v(t - \tau)||$ holds. Together, these two equations give us
\[ r = ||s - \frac{v}{c}r||. \]  
(42)
This equation can be solved, giving
\[ r = \frac{c s^2}{s \cdot v + c \sqrt{s^2 - \frac{1}{c^2}||s \times v||^2}}. \]  
(43)
If we substitute this relation for $r$ on the right-hand side of equation (41), we get
\[ r + r \cdot \frac{v}{c} = \sqrt{s^2 - \frac{1}{c^2}||s \times v||^2}. \]  
(44)
This enables us to transform equation (39) into
\[ F(q_s, q_d, s, v, u) = \frac{e n q_d}{4\pi \varepsilon_0 \gamma(v)^2} \left( s + \frac{1}{c^2} u \times (s \times v) \right) \left( s^2 - \frac{1}{c^2}||s \times v||^2 \right)^{3/2}. \]  
(45)
This equation for the force between two uniformly moving point charges now depends only on the non-retarded distance vector $s$. Parameters that depend on the past time $\tau$ are no longer present. We can verify that equation (45) is identical to equation (33) by insert the relation (37) into equation (45).

V. Force exerted by a direct current on a test charge
Equation (45) can now be used, for example, to calculate the force that an infinitely long, straight, direct current exerts on a test charge that is moving uniformly at the speed $w$ in the rest frame of an observer. To remain general, we assume that both the negative and the positive charge carriers in the electrical conductor are moving at the average drift velocities $w_-$ and $w_+$. The definition of the velocity $u$ continues to remain open.

To keep the calculation simple, we assume that the charge carriers in the electrical conductor are always located on the $x$-axis. For the velocities $w_-$ and $w_+$, we can then define $w_- = w_+ e_x$ and $w_+ = w_+ e_x$. Furthermore, we assume that the test charge is at the location $r = r e_x$ and has the velocity $w = w_x e_x + w_z e_z$. The force $F_{wd}$ on the test charge $q_d$ can then be calculated by integrating over all the charge carriers in the conductor:
\[ F_{wd} = \int_{-\infty}^{\infty} F(n(-e), q_d, r - x e_x, w - w_-, u_-) \, dx + \int_{-\infty}^{\infty} F(n(+e), q_d, r - x e_x, w - w_+, u_+) \, dx. \]  
(46)
Here, $n$ is the number of charge carriers per meter of conductor, and $e$ represents the elementary charge. For the velocity $u$, we try $u_- := c_1 w - c_2 w_-$ and $u_+ := c_1 w - c_2 w_+$, with $c_1$ and $c_2$ being constants that we want to determine later. For $c_1 = 1$ and $c_2 = 1$, for example, $u$ would be identical to the corresponding relative velocity. For $c_1 = 1$ and $c_2 = 0$, on the other hand, we would have $u = w$, as is the case in electrostatics and magnetostatics.

The integrals in equation (46) can be solved to give
\[ F_{wd} = \frac{e n q_d (f_x e_x + f_z e_z)}{2 c^2 \varepsilon_0 \pi r \sqrt{1 - \frac{v^2}{c^2}}} \]  
(47)
with
\[ f_x := (1 + c_1) (w_+ - w_-) w_z \]  
(48)
and
\[ f_z := (w_+ - w_-) (c_2 (w_+ - w_-) - (c_1 + c_2) w_z). \]  
(49)
For $w_-, w_+, w_z \ll c$, we get the first-order approximation
\[ F_{wd} \approx \frac{e n q_d}{2 c^2 \varepsilon_0 \pi r} \left[ \begin{array}{c} (1 + c_1) (w_+ - w_-) w_z \\ 0 \\ -(c_1 + c_2) (w_+ - w_-) w_z \end{array} \right]. \]  
(50)
The force can also be calculated using classical magnetostatics [4]. First, the magnetic field $B$ is calculated and then explicitly substituted into the formula of the Lorentz force (6). This approach gives
\[ F_{wd} = -\frac{I q_d (w \times e_y)}{2 c^2 \varepsilon_0 \pi r} \]  
(51)
For the current $I$, the equation $I = e n (w_+ - w_-)$ applies. Substituting this definition into the equation above gives
\[ F_{wd} = \frac{e n q_d}{2 c^2 \varepsilon_0 \pi r} \left[ \begin{array}{c} (w_+ - w_-) w_z \\ 0 \\ -(w_+ - w_-) w_z \end{array} \right]. \]  
(52)
A comparison of equations (50) and (52) shows that, if we choose the parameters $c_1 = 1$ and $c_2 = 1$, the force obtained by electrodynamics would be twice as large as that obtained by magnetostatics. Setting $c_1 = 1$ and $c_2 = 0$ likewise does not produce a meaningful result. The only possible choice is $c_1 = 0$ and $c_2 = 1$.

This requirement for $c_1$ and $c_2$ means that the parameter $u$ in Equation (28) cannot be the relative velocity $v = r_d - r_s$ or the velocity $r_d$ of the test charge relative to the observer. Instead, $u$ must be the velocity $-r_s$, which is constant in time for uniform velocities. We can therefore conclude that the final formula for the force exerted by a direct current on a test charge that is moving uniformly has the form
\[ F = q_d \left( E - \frac{r_s}{c} \times \left( \frac{r_d}{r} \times E \right) \right), \]  
(53)
where $E$ depends only on relative quantities such as $r, v$ and $a$. Note that $r_s$ is not a relative quantity because it depends on the speed of an external observer. The electromagnetic force is therefore subjective with respect to an observer due to the
formula for the Lorentz force (6), whereas the electric field $E$ and thus the magnetic field $B$ are purely relative quantities which do not depend on the speed of any observer.

Equation (53) is not applicable in classical mechanics because it violates the conservation of momentum. Switching the source and receiver of the force does not result in a mere change in sign, as Newton’s third law would require of a valid force formula. This problem is well known in the scientific literature and is usually justified by arguing that the radiation field can also emit and absorb momentum (e.g., [5]).

However, we would still expect that the force exerted on the test charge by the long, straight conductor in the previous section is, in fact, inversely equal to the force exerted on the conductor by the test charge. To calculate this, we change the source and receiver of the force in equation (46). In doing so, we must substitute $-\mathbf{w}$ for $\mathbf{u}$, as required by equation (53). We obtain

$$F_{dw} = \int_{-\infty}^{\infty} F(q_{d}, n (-e), x e_\xi - r, w_\xi - w, - w) \, dx + \int_{-\infty}^{\infty} F(q_{d}, n (+e), x e_\xi - r, w_\xi - w, - w) \, dx. \quad (54)$$

For speeds well below the speed of light (i.e., $w_\xi, w_z \ll c$), the solution to equation (54) can be simplified to

$$F_{dw} = \frac{e n q_d}{2 c^2 \varepsilon_0 \pi r} \begin{pmatrix} -2 (w_\xi - w_z) w_\xi \\ 0 \\ (w_\xi - w_z) w_\xi \end{pmatrix}. \quad (55)$$

A comparison of equations (55) and (52) shows that $F_{wd} \neq - F_{dw}$. This non-equivalence means that, when using equation (6), the conservation of momentum is already violated even in simple experiments with direct currents and very slow test charges. This violation is, of course, unacceptable.

VI. Solution to the problem

The failure of these equations to adhere to the law of conservation of momentum raises questions as to (i) whether and how this problem can be eliminated and (ii) how it would have been solved at the end of the nineteenth century. One possible solution would be to replace the Lorentz force (6) with the equation

$$F = q_d \gamma(v) E. \quad (56)$$

The Maxwell equations themselves do not need to be changed. Given this alternative definition of the force, we find that equation (45) becomes

$$F(q_{d}, q_{d}, s, v) = \frac{q_s q_d}{4 \pi \varepsilon_0 \gamma(v)} \frac{s}{(s^2 - \frac{1}{c^2} ||s\times v||^2)^{3/2}}. \quad (57)$$

The unpleasant parameter $u$ is now gone. As a consequence, all conservation laws of classical mechanics are now satisfied [4]. Furthermore, we can verify that equations (46) and (54) produce the correct results when the revised force formula (57) is used.

If we expand the right-hand side of equation (57) with respect to $v$ into a Taylor series, we obtain the second-order approximation

$$F(q_{d}, q_{d}, s, v) \approx \frac{q_s q_d s}{4 \pi \varepsilon_0 s^3} \left(1 + \frac{v^2}{c^2} - \frac{3}{2} \left(\frac{s \cdot v}{r \cdot c}\right)^2\right). \quad (58)$$

This force formula corresponds to that of C. F. Gauss from 1835 [6, p. 617], which W. Weber used as the starting point for Weber electrodynamics. The specialist literature on Weber electrodynamics outlines how the electrodynamics of Gauss and Weber is capable of correctly reproducing the entire electrostatics and magnetostatics (e.g. [7]). Specifically, Weber electrodynamics contains the physical Lorentz force without having to add it explicitly as a supplementary formula.

VII. Summary

We have seen that the full set of Maxwell’s equations can be solved for arbitrarily moving point charges. However, the solutions for $E$ and $B$ do not concord with the Lorentz force formula (6) because the resulting force violates the conservation laws even in simple, non-relativistic situations. Furthermore, using the original Lorentz force equation (6) from electrostatics and magnetostatics invalidates the classical principle of relativity, even though this principle is clearly fulfilled for the fields $E$ and $B$.

This article demonstrates that the advantages of classical Weber electrodynamics can be inherited by replacing equation (6) with equation (56). The only caveat to this substitution is that the current and charge densities should be expressed with respect to the test charge, not with respect to any observer. This requirement means that a point charge that is at rest from the perspective of an uninvolved observer can still have a current density.

However, if subjective current and charge densities are used (i.e., densities from the perspective of an uninvolved observer), Maxwell’s equations provide the fields for a test charge that is resting together with the uninvolved observer. For a test charge that moves relative to the observer, the fields must be transformed in a separate additional step for which the Lorentz transformation was developed.

As shown in this article, the Lorentz transformation can be avoided by using the correct current and charge densities from the outset and rigorously excluding the uninvolved observer from the calculation. This approach simplifies the calculations considerably, especially in dynamic situations. However, we have seen that the calculated fields must not be substituted into the Lorentz force formula (6). Instead, the equation (56) should be used.

This small modification provides an electrodynamics that inherently complies with the usual conservation laws, is compatible with Einstein’s postulates, and still allows the user
to avoid applying the Lorentz transformation and relativistic mechanics to non-relativistic applications. Furthermore, this modification reintegrates the scientific heritage of some of the most important physicists of the nineteenth century into modern physics. The modern Maxwell equations themselves do not need to be changed.

References


