Band-Stop Frequency Selective Surface (FSS) with Elliptic Response Designed by the Extracted-Pole Technique

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ABSTRACT This paper describes and validates an advanced synthesis design process of Frequency Selective Surfaces (FSS) with elliptic band-stop response. A systematic procedure based on the Generalized Chebyshev function and the extracted-pole technique enables to control the position of the transmission zeros and the attenuation level, to obtain and equiripple rejection response. A systematic process is followed to obtain the lumped LC values of the resonators circuits extracted as poles and the impedance inverters. Then, equivalent dipoles and transmission lines are obtained to carry out the electromagnetic design at normal incidence for a linearly polarized field. The impact of the high order modes of the periodic structure on the electrical response of the FSS, which can be relevant due to the stringent selected specifications, have been also analyzed. A four order band-stop filter with 3 (GHz) bandwidth centered at 30 (GHz) and attenuation 50 (dB) has been designed considering three different implementations: two filters using vacuum as transmission line with different connection lengths and a third one using dielectric substrate to enable its manufacturing. In order to verify the design procedure by experimental results, the third filter with printed dipoles in the dielectric substrate was manufactured and measured, validating the developed process.

INDEX TERMS Band-stop, Extracted-pole, Frequency selective surface (FSS), Spatial filter, Transmission zero, Unit-cell.
the most common way to increase the bandwidth and, at the same time, improve the attenuation response [9].

There are different approaches in the technical literature to achieve band-stop FFS, but most of them use direct brute force optimization of structures following heuristic arguments, or by trial and error processes [7], [10], [11], [12], [13], [14]. On the other hand, an equivalent circuit model has been proposed for larger fractional bandwidth response [15]. 3-D different structures with multiples transmission zeros or multiband response are shown in [16], [17], [18].

However, in this scenario a systematic synthesis procedure to design band-stop FSS with total control of the bandwidth and the attenuation level is missing. In this work, the powerful extracted-pole technique is applied, for the first time to the author’s knowledge, to design FSS with band-stop behavior. Previously implemented in waveguide technology [19], its relevant advantages are transferred now to open structures with periodicity conditions. In this sense, the total control of the position of the transmission zeros results in a equiripple attenuation bandwidth specified by the designer, who controls a priori the response by means of an analytical process well-established in circuit theory [20].

The paper is organized as follows. Section II presents the theoretical synthesis of the band-stop filter using the Generalized Chebyshev Function and the extracted-pole technique. Using this procedure, three different implementations of a FSS band-stop filter at Ka-band are proposed, and one of them is manufactured and measured for experimental validation purpose. In Section III, the dipoles implementing the LC resonators previously obtained, are calculated using the unit-cell with the suitable period to avoid the disturbance of the higher modes. Then, in Section IV, the full-wave design and optimization of the structure with dielectric substrate is carried out. Section V shows the experimental results, including an evaluation of the technology and measurements. In addition, the results of the developed design method are confronted with comparable designs in the technical literature. Finally, the contributions of the proposed work are summarized in Section VI.

II. THEORETICAL SYNTHESIS PROCEDURE

Band-stop filter response is necessary when a specific band of frequencies must be attenuated. Although this task can also be carried out by band-pass filters, when interfering frequencies are particularly strong, special actions have to be considered to suppress them. In these cases, the use of band-stop filters with high attenuation of unwanted frequencies is imperative.

Classical responses like Butterworth or Chebyshev can be implemented using the well developed insertion loss synthesis procedure. The circuit values for the maximally flat attenuation or equiripple response are obtained from tables or recursively computed [5]. After the reactive transformation from the normalized low-pass prototype to the band-stop is carried out, the frequency response is obtained.

However, the above classical responses do not allow the specific control of the attenuation over the desired bandwidth. In order to achieve this goal, a technique based on the extracted pole technique is presented in this work, introducing controlled transmission zeros to obtain an equiripple attenuation level in the band-stop [20]. This synthesis technique, that has been successfully accomplished in waveguide technology [19], is now extended to the spatial configuration of Frequency Selective Surfaces. Basically, the concept relies on the interchange of responses between the transmission and reflection coefficients $S_{21}(s)$ and $S_{11}(s)$, respectively. Therefore, the return loss response becomes the insertion loss and conversely. In this way, the synthesis process begins by finding the order of the filter $N$, the same value than the number of transmission zeros that fulfills the specification concerning the attenuation level. The position of the transmission zeros in the imaginary axis of the complex plane is determined by the Generalized Chebyshev Function [20].

| TABLE 1. Band-Stop Filter Specification |
| Order N | 4 |
| Center Frequency $f_o$ | 30 GHz |
| Bandwidth BW | 3 GHz |
| Attenuation | 50 dB |
| Return Loss | 18 dB |

The synthesis process extracts the transmission zeros at the initial position of the reflection zeros, obtaining the value of the residues and the phase-shifters. In addition, the sequence of the extraction provides different residue values, although the same response. Since the residue value specifies the impedance level of every resonator after the frequency transformation, it is worth analyzing different sequences of extraction in order to find the most suitable which provides the appropriate values for the physical implementation of the unit-cell (dipole, cross, etc.) of the FSS.

After extracting the poles with their respective residues and the phase lengths, the synthesis process ends with the extraction of a parallel inverter. Table 1 shows the specification of the band-stop filter, with equiripple attenuation level of 50(dB) in a 10% relative bandwidth centered at 30(GHz). These demanding specifications are a litmus test for an FSS.

Fig. 1 shows the topology of the normalized prototype and the circuit elements after the systematic extraction that verifies the specifications collected in Table 1. Fig. 2 shows the response of the normalized band-stop filter (S parameters).

Once the normalized band-stop prototype is synthesized, the next step is to accomplish the transformation to the center frequency $f_o$ and bandwidth $BW$ specified in Table 1. In this transformation, the first and last phase-shifters needed to extract the first and fourth poles can be eliminated since the magnitude response remains unchanged. In addition, every inductor in the normalized prototype is converted to
series LC resonant circuit with values given by equations (1) and (2). In this process, the four resonance frequencies are modified by the residue of every pole [21] to accomplish the desired transmission zero.

\[ L_k = \frac{1}{\omega_o b_k} (\alpha - \frac{\omega_k}{2}) \quad \alpha = \frac{\omega_o}{BW} \quad \omega_o = 2\pi f_o \quad (1) \]

\[ C_k = \frac{1}{\omega_o b_k} (\alpha + \frac{\omega_k}{2}) \quad \alpha = \frac{\omega_o}{BW} \quad \omega_o = 2\pi f_o \quad (2) \]

Fig. 3 shows the topology of the extracted pole \( k \), composed by the inductor \( (1/b_k) \) and the invariant reactance \( (-j\omega_k/b_k) \) of the normalized filter and the series LC resonator after the transformation composed by the inductor \( (L_k) \) and the capacitor \( (C_k) \) obtained by means of equations (1) and (2). Fig. 4 shows the topology of the band-stop filter after frequency transformation. It is composed by the cascade connection of series LC resonators in parallel arrangement and impedance inverters. As can be observed, the input-output impedance level is scaled to that of the vacuum level \( \eta = 120\pi(\Omega) \). The circuit element values and the frequencies of the four transmission zeros are shown in the figure caption. The response of the band-stop filter is shown after frequency transformation. It is composed by the cascade connection of series LC resonators in parallel arrangement and impedance inverters. As can be observed, the input-output impedance level is scaled to that of the vacuum level \( \eta = 120\pi(\Omega) \). The circuit element values and the frequencies of the four transmission zeros are shown in the figure caption. The response of the band-stop filter is shown in Fig. 5. The solid lines correspond to the case where the impedance inverters are ideal, i.e. frequency invariant. The dashed lines correspond to the case where the impedance inverters are implemented as transmission lines with \( \lambda/4 \) electrical length. As it is apparent, the main different lies in the return loss level, but the attenuation response maintains unchanged.

Once the theoretical synthesis is accomplished, the next task is to implement the four resonators, dipoles in this design, that are able to achieve in the FSS, the same response than the four lumped LC resonators.

**FIGURE 1.** Normalized band-stop filter topology and circuit values. Order \( N = 4 \). Transmission zeros: \( \omega_1 = -0.9239, \omega_2 = -0.3827, \omega_3 = 0.3827, \omega_4 = 0.9239 \). Residues: \( b_1 = b_4 = 1.8428, b_2 = b_3 = 4.4489 \). Phase-shifters: \( \theta_1 = 62.3560^\circ, \theta_2 = -90^\circ, \theta_3 = -62.3560^\circ, \theta_4 = -27.6440^\circ, \theta_5 = 90^\circ, \theta_6 = -62.3560^\circ \). Inverter \( K = 1 \).

**III. UNIT CELL OF THE FSS AND EQUIVALENT CIRCUITS**

With the intention of achieving the band-stop response by means of the FFS, distributed resonators equivalent to those lumped LC resonators shown in Fig.4 must be obtained. In the case of the inverters, their equivalent elements are transmission lines of electrical length \( n\lambda/4 \), being \( n \) a odd number. Fig. 6 shows the transmission response of the four LC lumped resonators and the values of the elements are shown in the caption of Fig. 4.

These partial responses are crucial to find the physical dimensions of the four dipoles and the period of the unit-cell. In order to obtain these dimensions from the equivalent circuit, a systematic procedure detailed in [22], [23] is accomplished. In this filter, the design process requires that the two fundamental modes of the periodic structure (homogeneous plane waves propagating at a certain angle of incidence) are not coupled to each other. At normal incidence, the two fundamental modes TEM, which are plane waves with vertical V and horizontal H polarization, are decoupled when the metallic scatters exhibit double symmetry with respect to the period. Thus, each mode can
be independently represented by a 2-port network. That is the reason for choosing symmetric and centered dipoles as scatters. Note that although the design is performed for vertical polarization, the same process can be used to obtain the same electrical filtering response for the $H$ polarization.

The LC lumped resonators are implemented by means of dipoles as shown in the inset of Fig.7. Every dipole is characterized by two parameters, i.e., length and width to fit simultaneously the resonant frequency and the impedance level (zero thickness is considered). In addition, the period of the unit-cell is chosen to control the cut-off frequencies of the higher modes.

As a validation proof, Fig.7 shows the excellent comparison between the circuit response of one of the four LC resonators and the full-wave response of its equivalent dipole inside the unit-cell. Resonator LC; $L_k = 4.58$ (nH), $C_k = 6.38$ (fF). Dipole dimensions: Length = 4.64 (mm), Width = 1.54 (mm), Period = 7.5 (mm). Fig. 8 shows the full wave response of the band-stop FSS with $\lambda/4$ length connection between dipoles (S parameters) including all dimensions as well as the layout of the unit-cell. As can be observed, the bandwidth is drastically reduced since two transmission zeros have disappeared. The reason lies on the disturbance due to the higher modes. In the simulation, three modes have been taken into account in the input-output ports. The excitation mode, the TEM mode that represents the wave-plane, and the first two higher modes, TE and TM modes with the same cut-off frequency of 40 (GHz) since the period or its equivalent, the dimension of the square waveguide is 7.5 (mm). In this context, the $s_{1(2)1(1)}$ and $s_{1(3)1(1)}$ responses represent the reflection coefficients in the input port of the two higher modes. Their values are lower than $-30$ (dB) in the band of interest but since we are dealing with a band-stop FSS structure their level is irrelevant. On the contrary, $s_{2(2)1(1)}$ and $s_{2(3)1(1)}$ responses represent the transmission coefficients in the output port of the two higher modes. As can be observed, their
value are higher than −50 (dB) in the upper part of the band of interest where the two transmission zeros have disappeared. Thus, the higher modes disturbance modifies the expected rejection in the band-stop.

The dimension of the square waveguide (the period of the FSS) is a design parameter to obtain the dimensions of the dipoles but, at the same time, determines the cut-off frequencies of the higher modes. Thus, there are two ways to minimize the higher order interaction.

The first one is reduce the period of the unit-cell in order to increase the cut-off frequencies of the higher modes, reducing the disturbance. Following this idea a redesign of the filter is carried out, where the period decreases from 7.5 (mm) to 6.75 (mm). The new full wave response and dimensions are shown in Fig. 9. As can be observed, the level of the transmission coefficients \( s_{2(2)1(1)} \) and \( s_{2(3)1(1)} \) are lower than −54 (dB) in the band on interest.

The second one is to extend the connection between dipoles from \( \lambda/4 \) to 3\( \lambda/4 \). Fig. 10 shows the full wave response of the band-stop FSS with distances between layers close to 3\( \lambda/4 \) length connection between dipoles (S parameters) and the layout of the unit-cell (dimensions are the same than in the previous cases). In this design, the level of the transmission coefficients \( s_{2(2)1(1)} \) and \( s_{2(3)1(1)} \) are lower than −60 (dB) in the band on interest. As a consequence, with this attenuation level, the four transmission zeros are in the response since the higher modes effect is negligible.

Accordingly, a systematic procedure to design band-stop FSS has been exposed in detail. The simulation of the unit-cell by means of the equivalent waveguide allows to control the higher mode interaction. In the next section, a band-stop filter implemented with dielectric substrate as support for the four layers of dipoles will be designed taking as starting point the results obtained in this section.


IV. FULL-WAVE DESIGN AND OPTIMIZATION

Once the design of the two band-stop filters with vacuum as transmission line connecting the layers is carried out, it is necessary to introduce the dielectric substrate with the printed dipoles for manufacturing. Therefore, a redesign of the dipole dimensions and the period of the unit-cell is imperative to take into account the thickness and permittivity of the dielectric substrate. Fig. 11 shows the layout of the band-stop filter composed of four layers with printed dipoles on a dielectric substrate and three layers of Rohacell [24] (a foam with permittivity close to unity), to ensure a compact structure.

A systematic full-wave procedure is followed to adjust the response corresponding to the first design with distance between dipoles $\lambda/4$ (Fig. 9), to the new physical structure using a dielectric substrate with parameters: thickness 0.762 (mm) and permittivity 3.2. Right now, the circuit model is no longer used since the higher modes are not enough attenuated to consider a mono-mode behaviour. However, the initial circuit design is an excellent starting point for the optimization.

In the optimization process, the vacuum permittivity is increased gradually from 1 to 3.2, and the thickness of the vacuum transmission line is decreased. In addition, the period of the unit-cell must be considerably reduced to minimize the higher modes interaction, eliminating their harmful effect, as it was detailed in the previous section.

By means of an optimization scheme with initial dimensions in the caption of Fig. 9 the new values of the lengths and widths of the dipoles are found. The Rohacell thickness is fixed to 1(mm) because is an standard dimension available from the supplier. Fig. 12 shows the full wave simulation and the final dimensions of the dipoles in the optimized structure ready for manufacturing and measurement. In order to take into account the accuracy of the etching process [25] of the dipoles in the four layers, a sensitivity analysis considering a normal distribution with $\pm50\mu$m of variation in the lengths and widths has been carried out. Fig.13 shows the range of variation of the rejection level by means of the $s_{21}$ parameter.

On the other hand, Fig. 14 shows the simulation of the rejection response, i.e. the $s_{21}$ parameter, considering oblique incidence for four different angles and TE excitation: $(\theta = 10^\circ, \phi = 0^\circ)$ $(\theta = 20^\circ, \phi = 0^\circ)$ $(\theta = 30^\circ, \phi = 0^\circ)$ $(\theta = 45^\circ, \phi = 0^\circ)$. As it can be observed, there is a progressive degradation in the rejection level decreasing as the $\theta$ angle increases. In addition, the bandwidth shifts to lower frequencies. Fig 15 shows the same simulation but considering oblique incidence for four different angles and TM excitation: $(\theta = 10^\circ, \phi = 90^\circ)$ $(\theta = 20^\circ, \phi = 90^\circ)$ $(\theta = 30^\circ, \phi = 90^\circ)$ $(\theta = 45^\circ, \phi = 90^\circ)$. In this case, the bandwidth shifts to higher frequencies. However, it must be clear that the design is accomplished for normal incidence.

In addition, note that a design considering oblique incidence requires of additional effort to control a priori the specifications by means of a synthesis procedure. For instance, in the best case, i.e. symmetrical metallizations and angles of incidence in the main planes ($(\phi = 0^\circ)$ or $\phi = 90^\circ$), a two-port circuit to describe electrically the TE wave (or the TM) would require transmission lines whose characteristic impedance are the TE (or TM) wave impedance (different from the terminal impedances, $(\eta_0 = 120\pi$). Therefore, the synthesis process become more elaborate and specific for each angle of incidence. In the worst case (non-symmetrical metallizations or angles outside the main planes), the equivalent circuits must be octopoles [23] and the design of the FSS would require to consider synthesis methods of 4-port networks (octopoles).

V. EXPERIMENTAL RESULTS

In order to verify experimentally the design procedure exposed in the previous sections, the band-stop filter is painstakingly measured. Fig.16 shows the measurement bench composed of the VNA and the horn antennas with lenses to obtain a locally plane wave impinging normally over the FSS, which allows its accurate characterization compared to an impinging spherical wave. The manufactured FSS is shown in the inset.

The first step is to characterize every layer separately for the purpose of controlling the suitable configuration of the measuring setup. Fig. 17 shows the comparison between the full-wave simulation and the measurement of the transmission coefficient $s_{21}$ for the four layers. As can be observed, the agreement is very good and every notch is located at its resonant frequency, although with slight difference in the level. The second step is to characterize the behavior of two layers, with the interaction between two notches. Fig. 18.

FIGURE 11. Layout of the band-stop filter composed of four layers of printed dipoles on a dielectric substrate and three layers of Rohacell.
FIGURE 12. Full wave response of the FSS unit-cell topology and circuit values for the band-stop response with dielectric substrate $\varepsilon_r = 3.2$, thickness = $0.762 \text{ (mm)}$.

Dipole 1: Length = $3.53 \text{ (mm)}$, Width = $0.13 \text{ (mm)}$
Dipole 2: Length = $3.43 \text{ (mm)}$, Width = $1.07 \text{ (mm)}$
Dipole 3: Length = $3.24 \text{ (mm)}$, Width = $1.44 \text{ (mm)}$
Dipole 4: Length = $2.83 \text{ (mm)}$, Width = $0.27 \text{ (mm)}$

Period: $4.45 \text{ (mm)}$
All lengths between substrates $1 \text{ (mm)}$

FIGURE 14. Simulated response of the rejection, i.e., the $s_{21}$ parameter of the FSS unit-cell under oblique incidence for four different angles and TE excitation; ($\theta = 10^\circ$, $\phi = 0^\circ$) ($\theta = 20^\circ$, $\phi = 0^\circ$) ($\theta = 30^\circ$, $\phi = 0^\circ$) ($\theta = 45^\circ$, $\phi = 0^\circ$).

FIGURE 15. Simulated response of the rejection, i.e., the $s_{21}$ parameter of the FSS unit-cell under oblique incidence for four different angles and TM excitation; ($\theta = 10^\circ$, $\phi = 90^\circ$) ($\theta = 20^\circ$, $\phi = 90^\circ$) ($\theta = 30^\circ$, $\phi = 90^\circ$) ($\theta = 45^\circ$, $\phi = 90^\circ$).

Sensitivity analysis considering a normal distribution with ±50µ of variation in the lengths and widths of the four dipoles: full wave response of $s_{21}$ parameter of the FSS unit-cell with nominal values (red color response) in the caption of Fig. [12].

FIGURE 13.

that the 2-port TRL calibration process used results in a value of 42 (dB) for the isolation in the measurement bench. As can be observed, this level is specifically the value measured for the attenuation in the band-stop, and the slope correspond a fourth order response. Thus, there is a good agreement between the electromagnetic simulation and the theoretical response, validating the developed designing procedure.

A comparison of different 2D-FSS implemented in PCB technology with band-stop response at normal incidence is shown in Table 2 (in the case of Reference [8] only theoretical results without measurements). It compares, among other parameters, the design process with an emphasising on the a priori capability to control the desired specifications,
TABLE 2. Comparison of 2D Band-Stop Frequency Selective Surfaces (FSS) Responses at Normal Incidence

<table>
<thead>
<tr>
<th>Reference</th>
<th>Theoretical synthesis</th>
<th>Transmission Zeros Control</th>
<th>Attenuation Level Control</th>
<th>Attenuation (dB) Sim. / Meas.</th>
<th>Center Frequency (GHz)</th>
<th>Bandwidth (%)</th>
<th>Filter order</th>
</tr>
</thead>
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<tr>
<td>[6]-2010</td>
<td>NO Heuristic</td>
<td>Heuristic</td>
<td></td>
<td>10 / 8</td>
<td>10.25</td>
<td>44</td>
<td>2</td>
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<tr>
<td>[18]-2016</td>
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<td>Heuristic</td>
<td></td>
<td>10 / 9</td>
<td>9.7</td>
<td>78</td>
<td>3</td>
</tr>
<tr>
<td>[15]-2017</td>
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<td>Heuristic</td>
<td></td>
<td>30 / 29</td>
<td>13.75</td>
<td>55</td>
<td>2</td>
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<tr>
<td>[10]-2018</td>
<td>NO Heuristic</td>
<td>Heuristic</td>
<td></td>
<td>40 / 35</td>
<td>13</td>
<td>17</td>
<td>2</td>
</tr>
<tr>
<td>[8]-2020</td>
<td>NO Heuristic</td>
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<td></td>
<td>10 / -</td>
<td>12</td>
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FIGURE 16. Measurement bench composed of the VNA and the radiation horns to obtain a plane wave impinging over the FSS. In the inset the FSS is shown.

FIGURE 17. Comparison between the full-wave simulation and the measurement of the transmission coefficient $s_{21}$ for the four layers.

FIGURE 18. Comparison between the full-wave simulation and the measurement of the transmission coefficient $s_{21}$ for the three groups of two layers.

VI. CONCLUSIONS

An advanced design process for the synthesis of FSS with equiripple band-stop response is exposed in detail. The Generalized Chebyshev function is the starting point to apply the extracted-pole technique to control the position of the transmission zeros to achieve the specified attenuation with the equiripple response. Once the lumped LC resonators and impedance inverters values are calculated, the equivalent dipoles and transmission lines are implemented in the electromagnetic design by means of a suitable correspondence between them and the unit-cell. The equivalent circuit was used as a predesign tool to achieve an excellent initial point...
for performing a full-wave optimization that includes thin dielectrics.

A four order band-stop filter centered at 30 (GHz) and bandwidth 3 (GHz) to achieve an attenuation of 50 (dB) is designed using three different structures to operate at normal incidence and linear vertical polarization. The two first with vacuum transmission line and different connection lengths and period to avoid the higher modes disturbance. The third one uses a dielectric substrate for the printed dipoles to enable its manufacturing. A comprehensive experimental process to characterize the four layers as well as the final response of the FFS is carried out.

The comparison between the electromagnetic simulation and the measurement validates the design process of the band-stop filter. The exposed procedure is controlled throughout, from the synthesis of the lumped circuit to its implementation by dipoles, allowing the achievement of the required specifications.

REFERENCES