Contamination impact mitigation in water distribution networks through consumer demand control

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Abstract

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Key Points:

• A novel control framework to mitigate the impact of a contamination threat by regulating consumer demand flow is proposed and tested.
• Consumer flows are regulated to create a pressure gradient from the contamination source to a flushing point, ensuring efficient flushing.
• The control problem is an optimization problem with pressure gradients as constraints and methods for identifying them are presented.

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Abstract
Amidst the time-consuming process of conducting laboratory tests to confirm contamination incidents, utilities must promptly implement measures to mitigate the impact of potential contamination without undue overreaction. This paper proposes a novel approach for mitigating the impact of contamination threats (an unconfirmed contamination event) in smart water networks. By leveraging smart valves, the consumer flows are regulated to create a decreasing pressure gradient from the contamination source to a flushing point, facilitating efficient contamination flushing with minimal consumer inconvenience. The decreasing pressure gradients are formulated as constraints within an online optimization problem. Additionally, two methods for identifying these pressure constraints are presented. To assess the efficacy of the proposed framework, testing is conducted using a realistic water network model.

1 Introduction
Water distribution networks have a distributed structure. Adding to this, ageing infrastructure, threats from cascading events, and insufficient disinfection make them susceptible to potential contamination, whether deliberate or accidental (Preis & Ostfeld, 2006). Contaminants in the water pose substantial threats to public health as highlighted in studies such as Islam et al. (2015), Kuhn et al. (2017), Bjelkmar et al. (2017) and Kirmeyer (2001), which document various cases of disease outbreaks caused by pathogens infiltrating these distribution networks.

When there is information or warning about contamination, a utility needs to promptly identify and initiate responsive actions to mitigate its impact. Such warnings could be based on consumer complaints, unusual readings from water quality sensors, coupled with contamination detection algorithms, or information about a security breach. A response to a contamination warning is not a one-time action but a continually evolving, interactive, and adaptable approach that adjusts based on the ongoing acquisition of new information about the contamination (Rasekh & Brumbelow, 2014). The United States Environmental Protection Agency (USEPA, 2006, 2004) presents a toolbox and guidelines on how to respond to a drinking water contamination event. A utility needs to first assess if a contamination event is a contamination threat or a contamination incident. An event in which contamination may have occurred but there is no conclusive evidence yet is said to be a contamination threat, and when there is confirmation, the event is classified as a contamination incident. In the case of a contamination incident, and based on the severity of the contamination, the utility would need to take remediation steps by closing down the network and flushing out the contaminant to bring the water network to safe and normal operation. Confirmation of a contamination incident typically involves a lab test, which requires several hours if not days. The initial period of a contamination event is critical, as the number of exposed consumers rises rapidly over time (Schiijven et al., 2016). Hence, during the period when the lab tests are being conducted, utilities need to take appropriate steps to mitigate the impact of potential contamination, which may include isolating the contaminated area through valve operations and alerting consumers via media or alert messages (Shafiee & Berghlund, 2017; Islam et al., 2015; Baranowski & LeBoeuf, 2008). However, it should also not be an overreaction if that might inconvenience the customers, cause panic, and harm the utility’s credibility.

Once a contamination event has been confirmed, most likely the utilities would need to isolate the contaminated area and flush the contaminated water through hydrants (Shafiee & Berghlund, 2017; Islam et al., 2015; Baranowski & LeBoeuf, 2008). Multiple studies have investigated optimal valve operation for flushing, utilizing various methodologies. Most of these studies assume a certain level of prior knowledge about contamination characteristics and the location of its source. For instance, Shafiee and Berghlund (2015) pre-
presented a decision tree obtained offline using a noisy genetic algorithm (NGA) to guide utilities in opening and closing valves and hydrants for different contamination events.

Numerous studies have framed the contaminant flushing problem as a single or multiple objective optimization problem. This problem involves defining the operation of actuators (valves, hydrants, pumps) in the network as decision variables. The objective function includes minimizing consumer health impacts, service disruption, and the number of operations. Various methods, such as genetic algorithms (Preis & Ostfeld, 2008; Guidorzi et al., 2009), deep reinforcement learning (Hu et al., 2022), evolutionary optimization (Alfonso et al., 2010), and swarm optimization (Moghaddam et al., 2020), have been employed to solve this optimization problem. These studies provide valuable insights and solutions for flushing contaminated water in the event of a contamination incident; however, there is limited literature on responsive actions in the case of a contamination threat. When facing a contamination threat where the contamination has not yet been confirmed, it may be unnecessary to completely close and flush the network. Instead, it is crucial to mitigate the impact of potentially contaminated water while awaiting confirmation of the contamination. This work presents a novel solution to address this specific problem.

1.1 Next Generation of Smart Water Networks

Information and Communication Technology (ICT) has undergone significant technological advancements in recent decades. These innovations have been integrated into electrical networks, evolving them into smart electrical grids. This transformation has allowed electric utilities to remotely monitor and better control electricity usage in real-time (Tuballa & Abundo, 2016). Similarly, the water sector is moving towards automation and digitalization by incorporating the Internet of Things (IoT) and communication technology, leading to the upgrade of conventional water networks into smart water networks. These networks utilize smart water technologies such as smart sensors and intelligent actuators, empowering water utilities to achieve real-time network monitoring and control. Studies by Gupta et al. (2020), Vrachimis et al. (2022) and Mutchek and Williams (2014) provide comprehensive insights into state-of-the-art smart water technologies and their utilization in better management of water networks. They also present examples of smart water networks being implemented in practice. These technologies include smart sensors for pressure measurement, smart water meters for flow measurement, and contaminant or biosensors for detecting contamination. Research and development efforts in smart water meters have notably increased in the last two decades (Zapata-Sierra et al., 2023). With these meters installed at consumer endpoints, both consumers and utilities can monitor water consumption in real-time (Mudumbe & Abumahfouz, 2015; Gabrielli et al., 2014; Cattani et al., 2017). Smart water networks also include intelligent actuators, such as smart valves and variable-speed pumps. As part of operational management strategies, variable-speed pumps can autonomously adjust their speed to meet the network's requirements, while smart valves can be controlled remotely. Wang et al. (2018) and Alshattanawi (2017) discuss the utilization of smart electric valves in water distribution networks for control and pressure management.

While current smart water networks typically incorporate only smart water meters at the consumer end, Sammaneh and Al-Jabi (2019) introduces the concept of smart valves situated at the consumer end, enabling the regulation of pressure for individual consumers and limiting consumer flow when necessary. Envisioning the next generation of smart water networks with further technological advancements, we foresee the integration of smart valves alongside smart water meters. These smart valves will provide a greater degree of freedom in controlling and operating the network, allowing for the regulation of individual flows within the network. With this vision in mind, we propose a novel solution in the form of a control framework for reducing the impact of contam-
ination in case of contamination threat within these advanced water distribution networks.

1.2 Contributions

Similar to most previous studies on contamination flushing, our method relies on initial knowledge of the contamination source location. The underlying idea is to create a decreasing pressure gradient from the contamination source to a flushing point. This pressure gradient is achieved by regulating consumer flows, an action we anticipate being plausible with the incorporation of smart valves in the next-generation smart water networks. This differs from methods where the network is closed, and shut-off valves are employed to flush out contamination. Since the nature and severity of contamination are unconfirmed, consumption is not stopped but rather regulated to limit the spread. This ensures a measured response, preventing an overreaction, maintaining water supply to critical services and minimizing inconvenience to consumers while mitigating the impact of potential contamination. This work can be seen as a first step leading to the development of a more comprehensive methodology. The control for the contamination impact mitigation problem is formulated as an optimization problem. Our approach entails solving the optimization problem online. A mathematical model of water distribution networks based on graph theory serves as the foundation for this optimization problem. This stands in contrast to prior studies (Shafiee & Berglund, 2015; Rasekh & Brunelow, 2014), where the flushing strategy was obtained offline. In those cases, coupling with hydraulic simulators like EPANET (Rossman et al., 2000) was often necessary, resulting in prolonged simulation times for convergence. Moreover, considering various contamination scenarios (injection location, concentration, time, and duration) imposed impractical computational burdens. In this work, we also present two different methods to identify pressure constraints on the junctions for creating the decreasing pressure gradient. These pressure constraints serve as inequality constraints in the optimization problem. Finally, the contamination impact mitigation control framework is tested on an EPANET model of a district metered area in a realistic water network.

To summarize, this work brings forth three key contributions: a) It proposes a control framework to mitigate the impact of a contamination threat, which has not yet been confirmed; b) The framework introduces a novel approach of regulating consumer demand flow to achieve the objective; c) The control problem is formulated as an online optimization problem, utilizing a mathematical model of the network.

The paper proceeds as follows: section 1.3 introduces the common notations used throughout this work. As a preliminary to this work, a graph theory-based mathematical model for a water distribution network is recalled in section 2 to provide foundational context. The design of the contamination impact mitigation control framework is described in section 3. In section 4 two distinct approaches for identifying the pressure constraints, part of the contamination impact mitigation control optimization problem, are presented. Following that, section 5 presents and discusses the results from the test on a benchmark water network. Finally, section 6 summarizes and concludes the study.

1.3 Notations

This section presents some of the common notations used throughout the paper. For a vector $x$, $x \in \mathbb{R}_{\geq 0}^{n}$ denotes $\forall i \ x_i \geq 0$, similarly $x \in \mathbb{R}_{\leq 0}^{n}$ denotes $\forall i \ x_i \leq 0$. The vector $\mathbb{1}$ is a column vector of ones in all positions and of appropriate dimension. The contamination impact mitigation optimal control developed in this work is based on a graph theory water network model. In this model, the variables corresponding to the spanning tree of the graph are denoted by the subscript $\mathcal{T}$ and the corresponding chord by the subscript $\mathcal{C}$. Any matrix or vector with a row corresponding to the reference node
removed is denoted with a bar, for example, \( \bar{H}, \bar{d} \). Furthermore, the model variables will be updated at discrete time instants and these time instants will be denoted by \( [k] \).

2 Graph Theory Model of a Water Distribution Network

This section recalls the graph theory based mathematical model of a water distribution network, introduced by Kallesøe et al. (2015). This model has been previously employed for leakage diagnosis in Rathore et al. (2021), Jensen and Kallesøe (2016) and Rathore, Kallesøe, and Wisniewski (2023), and for contamination mitigation control in Rathore, Misra, et al. (2023). Nonetheless, given its foundational role in formulating the optimization problem for the contamination impact mitigation control, it is presented here as a preliminary section in this paper. A detailed derivation of the model can be found in Kallesøe et al. (2015) or Rathore, Misra, et al. (2023).

A water distribution network with \( n \) junctions and \( m \) pipes can be represented as a directed graph, \( G \), with \( m \) edges and \( n \) nodes (Deo, 2017). Moreover, the \( n^{th} \) node is set as the reference node. The mathematical model for the water distribution network at \( k^{th} \) time instance can be given by (1) and (2).

\[
\lambda_c(q_c[k]) - \bar{H}_C^\top \bar{H}_C - \lambda_T(-\bar{H}_T^{-1}\bar{H}_C q_c[k] + \bar{H}_T^{-1}\bar{d}[k]) = 0. \tag{1}
\]

\[
p[k] = \bar{H}_T^{-\top} \lambda_T(-\bar{H}_T^{-1}\bar{H}_C q_c[k] + \bar{H}_T^{-1}\bar{d}[k]) - (\bar{z} - \mathbb{1}z_n) + \mathbb{1}p_n[k] \tag{2}
\]

The underlying network graph has been divided into arbitrary spanning tree \( T \) and its corresponding chords \( C \). Here, \( q_c[k] \) is the vector of flows through the chords at time \( k \). \( \bar{H}_C \) is the reduced incidence matrix corresponding to the chords and \( \bar{H}_T \) to the spanning tree of the graph. \( \lambda_T(\cdot) \) and \( \lambda_c(\cdot) \) are vector maps representing the flow-dependent pressure drops in the spanning tree and the chords edges respectively. Moreover, \( \lambda_i : \mathbb{R} \rightarrow \mathbb{R} \) is assumed to be of form, \( \lambda_i(q_i) = f_i |q_i| q_i \) with \( f_i > 0 \) (Swamee & Sharma, 2008). In this model, the supply flows and the consumer demands are modelled by assigning independent nodal flows to a subset of the nodes. \( \bar{d}[k] \in \mathbb{R}^{(n-1)} \) is the vector of independent nodal demands at the non-reference nodes. \( p[k] \in \mathbb{R}^{(n-1)} \) is the vector of pressures at the non-reference nodes and \( p_n[k] \in \mathbb{R} \) is the pressure at the reference node. Finally, \( \bar{z} \in \mathbb{R}^{(n-1)} \) and \( z_n \in \mathbb{R} \) is the vector of pressures due to elevation at the non-reference nodes and reference node respectively. Also, \( \mathbb{1} \) represents a vector of 1s.

The utilization of this model for the development of the contamination impact mitigation strategy is detailed in section 3.1.

3 Contamination Impact Mitigation Control Framework

In this section, the contamination impact mitigation control framework for drinking water networks is presented. As previously stated, a next-generation smart water network is considered, with automated valves at the consumer end assumed to control consumer flows. The automated valves serve as actuators within the network, providing a greater degree of freedom in controlling and manipulating pressure heads. A plausible scenario might involve automated valves being installed solely for a subset of consumers. Consequently, the framework would only have control over a subset of consumer flows or have control with a certain degree of freedom. However, given that this is a pilot study, we assume the capability to regulate all consumer demands for the sake of analysis. Moreover, a pressure-dependent demand control may be considered instead of a demand flow control in future work.
It is also assumed that a contaminant enters the water network from a single node at a time, referred to as the contamination source node. Contamination detection and localization have been addressed in the literature (Vrachimis et al., 2020; Seth et al., 2016). The emphasis of this work lies in a reactive strategy post-contamination threat detection. Hence, it is assumed that contamination intrusion has been reported or detected, and the potential source location is known. Typically, contamination localization methodologies do not pinpoint a single node as the source of contamination, but rather a set of nodes in the neighbourhood of each other, with some likelihood (Eliades & Polycarpou, 2012). In these scenarios, the identified neighbouring set of nodes can be treated as a singular node. However, for the sake of simplicity in this study, our starting point is a singular node identified as the source of contamination.

Upon detection of a contamination intrusion, the subsequent action involves preventing the spread and removal of contaminated water from the network. The concept involves guiding water flow from the contamination source node to a flushing node, where the water is flushed out. Typically, fire hydrants within the network are utilized as flushing nodes; however, any node capable of diverting water into the wastewater network or other disposal sites may serve as a flushing node.

This work proposes the utilization of the hydraulic pressure head gradient within the network to direct the flow toward the flushing node. The hydraulic pressure head at a node is defined as the sum of pressure due to water and pressure due to geodesic level, and is given as,

$$h_i[k] = p_i[k] + z_i,$$

where $h_i$ is the hydraulic pressure head, $p_i$ is the water pressure and $z_i$ is the pressure due to geodesic level at $i^{th}$ node.

To illustrate this concept, consider a small example network, as shown in Figure 1. In this network, node #1 serves as the flushing point, while contamination enters from node #4. Overlaying the network is a colour map representing the hydraulic pressure head across the network. Regions with the highest hydraulic pressure head are depicted in red, while those with the lowest pressure constraints are in violet-blue. The figure depicts a decreasing hydraulic pressure head gradient from the contamination source node to the flushing node. This decreasing pressure gradient facilitates the flow of contaminated water from the source to the flushing point, where it can be subsequently flushed out. As the pressure gradient is not limited to a specific path but in a general direction from the contamination source to the flushing point, multiple paths for the contaminated water to reach the flushing point exist. Moreover, the higher hydraulic pressure head surrounding these paths prevents the contaminated water from spreading throughout the network.

These hydraulic pressure head gradients can be established from the regulation of consumer demands and supply pressures. Determination of consumer demands and supply pressures involves solving an optimization problem, wherein the hydraulic pressure head gradients are formulated as constraints within this optimization problem.

In section 3.1 the optimization problem for the contamination impact mitigation control is defined based on the graph theory model developed in section 2. Further, in section 4, two different approaches are presented for defining the hydraulic pressure head gradient constraints.

### 3.1 The Control Optimization Problem

The contamination impact mitigation control intends to direct the flow from a known contamination source node towards a predefined flushing node by controlling supply pressures and consumer flows, thereby facilitating the removal of contaminants. To minimize the impact of the contamination, it is crucial to maximize the flow from the contami-
nation source node to the flushing node, which can occur when the hydraulic pressure head difference between these nodes is maximum. However, in doing so, the controller must also ensure minimal impact on consumers. This necessitates a multi-objective controller, formulated as an optimization problem with weighted objectives. The contamination impact mitigation control solves this optimization problem by minimizing a multi-objective cost function under specified constraints.

In this work, the impact on the consumer is construed as consumer water demands not being met. This can be defined as the difference between the nominal consumer demands or the water requested, $d^c_k$, and the controlled consumer demands or the water supplied, $d^c_k$. This forms the first objective of the optimization which is to be minimized and is given by,

$$\mathcal{J}_1[k] = (d^c_k - d^c_k)^\top (d^c_k - d^c_k).$$

As previously indicated, the effective mitigation of contamination relies on maximizing the hydraulic pressure head difference between the contamination source node and the flushing node. This is formulated as the second objective in the cost function of the optimization problem,

$$\mathcal{J}_2[k] = (h_{\text{source}}[k] - h_{\text{flush}}[k])^\top (h_{\text{source}}[k] - h_{\text{flush}}[k]),$$

where $h_{\text{source}}$ is the hydraulic pressure head at the contamination source node and $h_{\text{flush}}$ is the hydraulic pressure head at the flushing node.

Finally, to ensure the network functions and meets consumer satisfaction in terms of network pressure, a desired level of network pressure is to be maintained. This network pressure can be defined as the pressure at either all nodes throughout the network or a specific subset of nodes, which might adequately represent the pressure within a particular area. Opting for a subset of nodes could reduce the computational load of the optimization problem. The discretion to select this subset of nodes could lie with the operator or utility, employing a tool to balance both network requirements and computational load. Henceforth the nodes included in this selected subset will be referred to as targeted nodes. With that, the third objective is formulated as,

$$\mathcal{J}_3[k] = (p_t[k] - p^*_t[k])^\top (p_t[k] - p^*_t[k]),$$

Figure 1. Visual representation of the network-wide hydraulic pressure head established to facilitate the flow of contaminated water from the contamination source node to the flushing node, using a colour map.
where, $p_t$ is the network pressure at targeted nodes, $p^*_t$ is the desired pressure set-point.

The targeted node pressures can also be represented in terms of network node pressures as,

$$p_t[k] = F_t p[k], \quad (7)$$

where $F_t$ is an $n_t \times n$ binary matrix to extract the targeted node pressures, where $n_t$ is the number of target nodes. In the case of pressure control at all the nodes, the matrix $F_t$ would be an identity matrix. With that, equation (6) is given as,

$$J_3[k] = (F_t p[k] - p^*_t[k])^\top (F_t p[k] - p^*_t[k]). \quad (8)$$

The optimization problem is subjected to multiple constraints. The first set of constraints are the constraints on the hydraulic pressure head at the nodes to form a decreasing pressure gradient in the network from the contamination source node to the flushing node. These hydraulic pressure head constraints at the nodes are represented as,

$$h_a \leq h_b, \quad a \in A, \quad b \in B \quad (9)$$

where $A$ and $B$ are a set of nodes, having a one-to-one element correspondence. These sets are defined in two different ways using two different approaches in section 4.

Additionally, the capacity of the supply nodes, dictated by factors such as pump and pressure-reducing valve (PRV) capacities, imposes practical constraints. These constraints limit the supply nodes to deliver within specified minimum and maximum values for both pressure and flow. These constraints in the optimization problem are formulated as,

$$p_s^{\text{min}} \leq p_s[k] \leq p_s^{\text{max}}, \quad (10a)$$

and,

$$0 \leq d_s[k] \leq d_s^{\text{max}}, \quad (10b)$$

where $p_s$ is a vector of pressure at the supply node and $d_s$ is a vector of supply flow. The minimum and maximum pressure are denoted by $p_s^{\text{min}}$ and $p_s^{\text{max}}$ respectively, and a maximum flow is denoted by $d_s^{\text{max}}$.

As previously discussed, the assumption is that consumer demands can be regulated by varying closing degrees of automated valves on consumer connections. However, consumption at outlet points, such as taps, remains under the control of the consumer. While the automated valves can limit consumption, they cannot entirely dictate it. Consequently, controlled consumer demands can only be lower than the nominal consumer demands. Moreover, consumers cannot supply water to the network, meaning that the consumer demand can be reduced to a minimum of zero. This particular constraint is formulated as,

$$d_c^c[k] \leq d_c[k] \leq 0. \quad (11)$$

It is important to note that, according to the sign convention adopted in this study, consumer demands are regarded as negative independent flows out of the network. Hence, mathematically, the controlled consumer demand should be greater than the nominal consumer demand but still less than zero. However, the magnitude or absolute value of the nominal consumer demands will be greater than that of the controlled consumer demands.

There would also be some limitations on the maximum flow from the flushing point depending on the hydrant or sink capacity. In addition to that, we implement a minimum flushing flow limitation to ensure that contaminated water is flushed out consistently rather than merely maintaining a pressure gradient. These are also to be considered in the optimization problem and these constraints are formulated as,

$$-d_{\text{flush}}^{\text{max}} \leq d_{\text{flush}}[k] \leq -d_{\text{flush}}^{\text{min}}[k]. \quad (12)$$
where $d_{\text{flush}}^{\text{min}}$ and $d_{\text{flush}}^{\text{max}}$ is the magnitude of the minimum and maximum flushing flow respectively from the flushing point and $d_{\text{flush}}$ is the flushing flow. Similar, to consumer demand, according to the sign convention adopted in this study, the flows out of the network are regarded as negative independent flows. Hence, mathematically, the flushing flow should be greater than the negative of the maximum flushing flow but still less than the minimum flushing flow.

The third objective, outlined in (8), aims to operate the network at a desired pressure. Nevertheless, (13) is added as a constraint to ensure that a minimum network pressure for supply and firefighting is maintained under all conditions. This also prevents the creation of low-pressure points in the network, which increases the risk of contamination infiltration.

$$ p_{\text{min}} \leq p[k]. \quad (13) $$

Finally, with the multi-objective cost function and all the constraints, the contamination impact mitigation control optimization problem is given as,

$$ \min_{d_c[k], p[k]} \left( (d_c[k] - d_c^c[k])^\top Q(d_c[k] - d_c^c[k]) ight. $$
$$ \left. - (h_{\text{sou}}[k] - h_{\text{flush}}[k])^\top R(h_{\text{sou}}[k] - h_{\text{flush}}[k]) \right) $$

subject to

$$ \lambda_c(q_c[k]) - \hat{H}_c^\top H_T^{-1} \lambda_T (-\hat{H}_T^{-1} \hat{H}_c q_c[k] + \hat{H}_T^{-1} \bar{d}[k]) = 0. \quad (15a) $$

$$ \bar{p}[k] = \hat{H}_T^{-1} \lambda_T (-\hat{H}_T^{-1} \hat{H}_c q_c[k] + \hat{H}_T^{-1} \bar{d}[k]) - (\bar{z} - 1 z_n) + 1 p_n[k] \quad (15b) $$

$$ h_a \leq h_b, \quad a \in A, \quad b \in B \quad (16a) $$

$$ p_s^{\text{min}} \leq p_s[k] \leq p_s^{\text{max}}, \quad (16b) $$

$$ 0 \leq d_s[k] \leq d_s^{\text{max}}, \quad (16c) $$

$$ d_c^c[k] \leq d_c[k] \leq 0. \quad (16d) $$

$$ -d_{\text{flush}}^{\text{max}} \leq d_{\text{flush}}[k] \leq -d_{\text{flush}}^{\text{min}}, \quad (16e) $$

$$ p_{\text{min}} \leq p[k]. \quad (16f) $$

The cost function, (14), of the optimization problem, which is to be minimized, is a weighted sum of the three objective functions (4), (5) and (8). $Q$, $R$ and $S$ are the weight for objectives $J_1$, $J_2$ and $J_3$ respectively. $Q$ would be a $n_c \times n_c$ matrix, where $n_c$ is the number of consumer nodes; considering a singular contamination source node, $R$ would be a scalar; $S$ would be a $n_t \times n_t$ matrix, where as before $n_t$ is the number of target nodes. The weights are typically diagonal matrices assigned to each objective in proportion to its relative importance within the overall cost function. The order of magnitude of the objective function values must also be considered while assigning the weights. Particularly, $Q$ can be employed to assign higher weights to vital consumers, such as hospitals, thereby minimizing the impact on their demand. Note that, the second objective of the pressure difference between the contamination source node and the flushing node is to be maximized. However, the cost function is to be minimized in the optimization problem, therefore in (14), the objective, (5), has been added with a negative sign. The optimization variables or the variables which are controllable here are the consumer demands, $d_c[k]$ and the supply pressures, $p_s[k]$, with respect to which the cost function is minimized. Further, the optimization problem is subjected to hydraulic system model constraints given by (15). Finally, the constraints presented in (9), (10), (11), (12) and (13) are included in (16).
3.2 Selection of the Flushing Node

In a typical water network, multiple locations are usually available for conducting flushing operations. In these cases, the most efficient node, for a specific contamination scenario, under this control framework must be selected among all the possible flushing locations. In selecting the best flushing node, it is essential to consider a trade-off between several factors: the contaminated water consumed, the time it takes for the network to become contaminant-free, and the change in consumption needed. This decision-making process involves weighing these parameters in a minimization problem, where each factor is assigned specific weights to determine the most effective flushing node. This minimization problem is given by,

$$f_{n_{\text{best}}} = \arg \min_i (w_{\text{CWC}} C_{\text{WC}} + w_{\text{TTCF}} T_{\text{TTCF}} + w_{\text{CC}} C_{\text{CC}}),$$  \hspace{1cm} (17)$$

where, $f_{n_{\text{best}}}$ denotes the best flushing node. $C_{\text{WC}}$, $T_{\text{TTCF}}$ and $C_{\text{CC}}$ respectively indicate the percentage changes in contaminated water consumption, the percentage change in the time until the network is contaminant-free, and the percentage change in consumption given the mitigation solution that considers the $i^{th}$ node as the flushing node in the feasible solution subset. Correspondingly, the weights assigned to these parameters are denoted as $w_{\text{CWC}}$, $w_{\text{TTCF}}$, and $w_{\text{CC}}$, following the same order.

The weights specified in (17) can be determined by the utility, taking into account the network conditions and risk assessment of the contamination scenario. Subsequently, the argument yielding the minimum value is considered in the application of the control framework on the water network for that specific contamination scenario.

4 Pressure Constraints Identification

The following section presents two distinct approaches for identifying the constraints on hydraulic pressure head, as expressed in (16a) within the optimization problem. These pressure constraints are the key element of the proposed methodology as they establish a decreasing hydraulic pressure head gradient from the contamination source node to a flushing node, enabling the contaminated water to move towards the flushing node. Furthermore, the pressure gradient should not be confined to a specific path; instead, it should be in a general direction from the contamination source to the flushing point. This design would ensure the creation of multiple pathways for the contaminated water to reach the flushing point.

The two approaches, viz. breadth-first search and the shortest path approach, may yield different pressure constraints. However, the ultimate objective of a decreasing pressure gradient from the contamination source node to the flushing node remains consistent. In general terms, the breadth-first search approach explores multiple pathways between the contamination source node and the flushing node simultaneously. On the other hand, the shortest path approach begins with the shortest pathway between the contamination source node and the flushing node, subsequently exploring parallel pathways to the shortest path. A comparative analysis of these two approaches is conducted through tests on a water network model, and the results are presented in Section 5.

4.1 The Breadth-First Search Approach

In this approach, the hydraulic pressure head gradient in the network is determined using the breadth-first search algorithm (Cormen et al., 2022). This algorithm traverses the graph’s edges to uncover every reachable node from a root node, thereby constructing a breadth-first tree. Beginning at the root node, the algorithm identifies all nodes at the present depth-specifically, the neighbours of the root node in this instance, before progressing to nodes at the subsequent depth level. Subsequently, all nodes at that depth level are visited before advancing further to the subsequent depth level. This it-
Figure 2. Graphical representation of a small water network used as an example to illustrate the concept of hydraulic pressure head gradient identification using the breadth-first search approach.

The iterative process continues until all nodes reachable from the root node have been visited. It is essential to emphasize that each node is visited only once.

Now, for the determination of pressure gradient constraints concerning the contamination impact mitigation problem, we start from the flushing node, which serves as the root node for the breadth-first search. This results in a breadth-first tree graph representing the network, exhibiting nodes positioned at various depth levels relative to the flushing node. The root node’s level is denoted as the 1st level, while subsequent levels are denoted as 2, ⋅⋅⋅, n. These depth levels are considered as the hydraulic pressure head levels of the nodes, serving as the basis for defining hydraulic pressure head constraints on the nodes. The pressure constraints are defined such that the hydraulic pressure head at nodes within depth level $x$ needs to be higher than the nodes within depth level $(x-1)$. For a clearer grasp of this process, we will reference the network graph illustrated in Figure 2 as an example.

The graphical representation in figure 2 depicts a small water network, which can also be considered as a small section of a larger water network, comprising 16 nodes, $v_1, \cdots, v_{16}$, and 22 edges. In the context of the example, let us consider node $v_2$ as the flushing node and node $v_7$ as the contamination source node within the network, as illustrated in the figure with a red star and a brown diamond. Figure 3 presents the breadth-first tree graph, with node $v_2$ as the root node. The breadth-first search algorithm starts at the root node $v_2$, which resides at depth level 1. Nodes $v_1$, $v_3$ and $v_8$, neighbouring $v_2$, are situated at depth level 2. Subsequent nodes are categorized into deeper levels accordingly. Notably, at level 4, node $v_5$ neighbours $v_6$; however, as $v_6$ has been visited at level 3, it is not represented under $v_5$ in level 5.

To eliminate the contaminant from the network, achieving a flow from the contamination source node to the flushing node necessitates a decreasing hydraulic pressure head gradient along this path. A trivial method involves imposing constraints on hydraulic pressure heads at the nodes, up to the nodes one level below the contamination source node. As previously mentioned, the hydraulic pressure head at nodes on a level is to be...
higher than the hydraulic pressure head at nodes on the level above. In the example network (Figure 2), with $v_2$ as the flushing node, the starting level is level 1. Thus, the hydraulic pressure head at nodes $v_1$, $v_3$, and $v_8$ is to exceed that at node $v_2$. Similarly, the hydraulic pressure heads at nodes $v_4$, $v_6$, $v_7$ and $v_{10}$ is to be greater than those at nodes $v_1$, $v_3$ and $v_8$. Ultimately, the hydraulic pressure heads at nodes $v_5$, $v_{16}$, $v_{14}$, $v_9$, and $v_{11}$ need to be higher than that at nodes $v_4$, $v_6$, $v_7$, and $v_{10}$. These pressure constraints are visually presented in Figure 4, using a colour bar: nodes with the highest hydraulic pressure head are depicted in yellow, nodes with the lowest hydraulic pressure head in aqua, and nodes with no pressure constraints in violet-blue. Solving the optimization problem with these constraints ensures the desired flow direction from the contamination source node towards the flushing node.

However, directly applying these pressure constraints using the breadth-first tree graph might impose unnecessary constraints on nodes that have minimal or no role in guiding the flow from the contamination source to the flushing node. In the case of the example network illustrated in Figure 4, nodes $v_4$, $v_{11}$ and $v_{16}$ are evident examples of this phenomenon—they do not contribute to directing the flow, yet they bear pressure constraints. This occurs because the breadth-first search algorithm simultaneously explores all directions from the flushing node. Considering all nodes from various directions, up to a level after the level containing the contamination source node, when defining pressure constraints results in imposing unnecessary constraints on nodes in directions away from the contamination source node. To mitigate this issue, we implement two heuristic rules to refine the results obtained from the breadth-first tree graph.

The branches starting from nodes neighbouring to the flushing node, i.e. the nodes on level 2, are denoted as the primary branches. Now, only the primary branch which contains the contamination source node is considered while defining the pressure constraints, this forms Rule 1. With that, for the example network (Figure 2), only nodes...
Figure 4. Visually representation of the hydraulic pressure head constraints defined using the breath-first tree graph in figure 3 and presented using a colour bar.

Figure 5. The refined levels of pressure constraints at nodes defined using the breath-first tree graph, in Fig. 3, and the heuristic Rule 1.

$v_2$, $v_3$, $v_6$, $v_7$, $v_{14}$ and $v_9$ have pressure constraints. The pressure constraint levels refined using Rule 1 for the example network are presented in figure 5. With these pressure constraints, it is ensured that the direction of flow would be from the contamination source node to the flushing node.

However, without imposing constraints on the neighbouring nodes, except for the primary branch node, of the flushing node, contaminated water might potentially flow further than the flushing node. To prevent this scenario, all neighbouring nodes to the flushing node are also taken into account, imposing pressure constraints to ensure they maintain a higher hydraulic pressure head than the flushing node. Similarly, neighbouring nodes to nodes within the primary branch that currently lack pressure constraints are also included in this consideration. Nodes within the final level should inherently possess a greater hydraulic pressure head compared to the contamination source node due to already imposed pressure constraints. Consequently, there is no necessity to evaluate neighbouring nodes to nodes within the last level of the primary branch. This establishes Rule 2, wherein from the second-to-last level to the 1st level, all neighbouring nodes
Figure 6. Visually representation of the hydraulic pressure head constraints defined using the breath-first tree graph and the two heuristic rules and presented using a colour bar.

to each node encompassed within the refined pressure constraint levels, which are not already part of the refined pressure constraint levels, are to be taken into account. These nodes should adhere to pressure constraints ensuring a higher hydraulic pressure head than the node to which they are neighbouring. Once again, considering the example network (Figure 2) and the refined pressure constraint levels in figure 5, our starting point is $v_6$, whose neighbouring nodes include $v_3$, $v_5$, $v_7$, and $v_{14}$. However, among these nodes, only $v_5$ is not already part of the refined pressure constraint levels. Thus, a pressure constraint is imposed to $v_5$, such that its hydraulic pressure head is to be higher than that at $v_6$. Likewise, pressure constraints are enforced on $v_8$, $v_{10}$, and $v_1$, such that the hydraulic pressure head at $v_8$ and $v_{10}$ is to exceed that at $v_7$, and at $v_1$ and $v_8$ is to exceed that at $v_2$. These additional constraints aimed at preventing the spread of the contaminant are termed outer layer constraints, while the constraints derived from the refined breadth-first search tree graph results are termed inner layer constraints. Figure 6 visually represents the overall pressure constraints for the example network using a colour bar. As before, nodes with the highest hydraulic pressure head are depicted in yellow, those with the lowest hydraulic pressure head in aqua, and nodes with no pressure constraints in violet-blue.

The overall pressure constraints derived from the breadth-first search approach are given by (18). The inner layer constraints which are based on the refined pressure constraint levels are given by (18a), whereas the outer layer constraints are given by (18b).

\[ h_n \leq h_m, \quad \forall (n, m) \in \mathcal{L}_i \times \mathcal{L}_{i+1}, \quad \forall i \in \{1, \ldots, N-1\} \quad (18a) \]

\[ h_n \leq h_m, \quad \forall (n, m) \in \mathcal{L}_i \times \Gamma_n, \quad \Gamma_n := \mathcal{N}(n) \setminus \bigcup_{j=1}^{N} \mathcal{L}_j \quad (18b) \]

In (18) $h_n$ and $h_m$ denote the hydraulic pressure head at nodes $n$ and $m$ respectively. $\mathcal{L}_i$ denotes the set of nodes on $i^{th}$ level of the refined pressure constraint levels and $\mathcal{N}(n)$ denotes the set of nodes neighbouring the node $n$.

The pressure constraints, (18) in association with the breadth-first tree levels, are to be used in the optimization problem for (16a), in the contamination impact mitigation control framework, when using the breadth-first search approach.
Figure 7. Graphical representation of a small water network used as an example to illustrate the concept of hydraulic pressure head gradient identification using the shortest path approach. The green line marks the shortest path between the contamination source node, \( v_4 \), and the flushing node, \( v_1 \).

4.2 The Shortest Path Approach

In this approach, the pressure gradient constraints are determined using the shortest path algorithm. The starting point is to identify the shortest path, in terms of the shortest distance, between the contamination source node and the flushing node. The task of finding this shortest path is essentially a single-pair shortest-path problem, which can be effectively solved using Dijkstra’s algorithm (Cormen et al., 2022). This algorithm is specifically designed to find the shortest paths between two vertices in a directed graph with non-negative edge weights (Cormen et al., 2022). Within the water network graph, the lengths of pipes serve as the edge weights.

Furthermore, to establish a flow direction from the contamination source node to the flushing node, it is essential to have a decreasing hydraulic pressure head gradient along the path from the contamination source node to the flushing node. One approach to achieving this is by ensuring the highest pressure head at the contamination source node and subsequently creating a decreasing pressure head at the nodes along the shortest path leading to the flushing node. Again for better understanding, we will reference the network graph presented in Figure 7 as an example.

Figure 7 presents a graphical representation of a small water network, again it can be considered as a small section of a larger water network. This network section comprises 22 nodes \( v_1, \ldots, v_{22} \), and 26 edges, \( e_1, \ldots, e_{26} \), with equal lengths and diameter.

In the given scenario, let us designate \( v_1 \) as the flushing node, which is marked with a red star in Figure 7, and \( v_4 \) as the contamination source node, which is marked with a brown diamond, within the network. Consequently, the shortest path from \( v_4 \) to \( v_1 \) would be \( \{ v_4 \rightarrow e_1 \rightarrow v_3 \rightarrow e_2 \rightarrow v_2 \rightarrow e_3 \rightarrow v_1 \} \); the path is marked with a green line in the figure. Ensuring that the hydraulic pressure head at \( v_4 \) is higher than at \( v_3 \), at \( v_3 \) higher than at \( v_2 \), and at \( v_2 \) higher than at \( v_1 \) would facilitate a flow from the contamination source node towards the flushing node. Additionally, neighbouring nodes adja-
Figure 8. Graphical representation of a small water network, from Figure 7, with an inner layer formed along the shortest path and an outer layer formed around the inner layer.

Cent to the nodes on the shortest path should possess a higher hydraulic pressure head to prevent the spread of contaminated water in the network. Thus, it is necessary that the hydraulic pressure head at $v_9$ and $v_{10}$ is higher than at $v_4$, and the hydraulic pressure head at $v_7$ and $v_8$ is greater than that at $v_3$, continuing in the same manner for $v_2$ and $v_1$.

However, with this approach, there is only one path for the contaminated water to be flushed out and this might put too much constraint on the network as consumer demands would be regulated to achieve this. Also, the idea of this work is to have multiple paths for the contaminated water to reach the flushing node with a decreasing pressure gradient in a general direction from the contamination source node to the flushing node. To achieve this parallel paths to the shortest path are identified such that the contaminated water could take any of these paths to reach the flushing node. We form two layers around this shortest path, viz. the inner layer and the outer layer. The inner layer is formed of parallel paths or nodes on the parallel paths to the shortest path, through which the contaminated water could reach the flushing node. This inner layer is formed around the shortest path. The hydraulic pressure head at the nodes in the inner layer has a decreasing gradient from the contamination source node to the flushing node. The outer layer is formed of the nodes around the inner layer. The purpose of the outer layer is to prevent the spread of the contaminated water to the network. Therefore, the nodes in the outer layer should have a higher hydraulic pressure head than the nodes in the inner layer.

For the network in figure 7, the shortest path from the contamination source node to the flushing node is again depicted by a green line in figure 8. Additionally, figure 8 showcases an inner layer, distinguished by a shade of blue along the shortest path, and an outer layer, indicated by a shade of red surrounding the inner layer. The identification of nodes within these layers is established based on two heuristic rules. The inner layer comprises nodes adjacent to those on the shortest path, excluding the contamination source node and the flushing node. As mentioned, the hydraulic pressure head should exhibit a decreasing gradient from the contamination source node to the flushing node along the shortest path. Moreover, nodes in the inner layer must display a similar pres-
Figure 9. Levels of pressure constraints at nodes in the inner layer defined using the shortest path approach for the example network presented in Figure 7.

According to Rule 1 applied to the network illustrated in Figure 7, the inner layer consists of nodes $v_7$ and $v_8$, which are neighbours to $v_3$, and nodes $v_5$ and $v_6$, which are neighbours to $v_2$. It is worth noting that $v_7$ is also adjacent to $v_2$; however, since our starting point is $v_3$, and $v_7$ is already part of the inner layer from $v_3$, it is not reiterated, maintaining the pressure constraints identical to those of $v_3$. Consequently, the pressure constraints are as follows: the hydraulic pressure head at $v_4$ is to be higher than at $v_3$, $v_7$, and $v_8$; at $v_3$, $v_7$ and $v_8$ is to be higher than at $v_2$, $v_5$ and $v_6$; and at $v_2$, $v_5$ and $v_6$ is to be higher than at $v_1$. These pressure constraint levels for the inner layer nodes are visually represented in Figure 9 for clarity. Nodes placed at level $x$ should all possess a hydraulic pressure head greater than each node placed at level $(x - 1)$.

Much like the outer layer concept in the breadth-first search approach, here, an analogous outer layer is established surrounding the nodes of the inner layer, employing a similar heuristic rule to impede the spread of the contaminant. Rule 2 dictates the formation of this outer layer: All neighbouring nodes associated with each node contained within the inner layer including the shortest path, starting from the contamination node to the flushing node, which are not part of the inner layer, are to be taken into account. These nodes should adhere to pressure constraints ensuring a higher hydraulic pressure head than the node to which they are neighbouring. Considering the inner layer (Figure 9) within the example network (Figure 7), the starting node is $v_4$, which neighbors nodes $v_3$, $v_9$, $v_{10}$, and $v_{15}$. However, since $v_3$ is already part of the inner layer, $v_9$, $v_{10}$, and $v_{15}$ are considered as part of the outer layer, subject to pressure constraints such that the hydraulic pressure head at nodes $v_9$, $v_{10}$, and $v_{15}$ is to be higher than that at $v_3$. Similarly, the hydraulic pressure head at $v_{14}$ and $v_{15}$ is to be higher than that at $v_7$; at $v_{17}$ and $v_{18}$ is to be higher than at $v_8$; at $v_{11}$, $v_{13}$ and $v_{14}$ is to be higher than than at $v_5$; at $v_{16}$ is to be higher than that at $v_6$; and at $v_{11}$ and $v_{12}$ is to be higher than at $v_1$. Figure 10 visually represents the overall pressure constraints for the example network defined using the shortest path approach using a colour bar. As before, nodes with

<table>
<thead>
<tr>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5 2 6</td>
<td>7 3 8</td>
<td>4</td>
</tr>
</tbody>
</table>

Figure 9. Levels of pressure constraints at nodes in the inner layer defined using the shortest path approach for the example network presented in Figure 7.
the highest hydraulic pressure head are depicted in yellow, those with the lowest hydraulic pressure head in aqua, and nodes with no pressure constraints in violet-blue.

Similar to the breadth-first search approach, the overall pressure constraints, including the inner layer and the outer layer constraints at nodes, are given as,

\[ h_n \leq h_m, \quad \forall (n, m) \in \mathcal{L}_i \times \mathcal{L}_{i+1}, \quad \forall i | i \in \{1, \ldots, N - 1\}, \quad (19a) \]

\[ h_n \leq h_m, \quad \forall (n, m) \in \mathcal{L}_i \times \Gamma_n, \quad \Gamma_n := \mathcal{N}(n) \setminus \bigcup_{j=1}^{N} \mathcal{L}_j. \quad (19b) \]

Again in (19) \( h_n \) and \( h_m \) denote the hydraulic pressure head at nodes \( n \) and \( m \) respectively. \( \mathcal{L}_i \) denotes the set of nodes on \( i^{th} \) level of the inner layer constraint levels including the shortest path and \( \mathcal{N}(n) \) denotes the set of nodes neighbouring the node \( n \).

When solving the problem of contamination impact mitigation using the shortest path approach, the pressure constraints, (19), are used in (16a) of the optimization problem.

The contamination impact mitigation control framework, with both breadth-first search approach and shortest path approach, is implemented and tested on a simulated water network model. The implementation and the results are presented in section 5.

5 Results and Discussion

In this section, the performance of the contamination impact mitigation control framework is evaluated on a simulated district-metered area (DMA) of a realistic water network called CY-DMA. CY-DMA replicates a real DMA in Cyprus, previously utilized by Vrachimis et al. (2021) and Vrachimis et al. (2020) to evaluate the performance of algorithms developed in their respective works.

In this study, we employ a modified version of CY-DMA. The EPANET model of the network, referred to as CY-DMA 2, is illustrated in Figure 11. This network comprises 90 junctions and 121 pipes. The original CY-DMA (Vrachimis et al., 2020) consists of only one reservoir or supply point, denoted as \( R1 \) and marked in the figure with a red circle. The contamination source is located at junction 1, and the flushing point is located at junction 10.
a pink square. With a singular supply point in the network, the general flow direction—assuming no pipe closures—remains the same. This may pose feasibility issues for contamination impact mitigation control if the flushing node resides in the opposite direction from the contamination source node with respect to the network flow direction. Introducing multiple supply points assists the controller in altering the network’s flow direction, thereby yielding improved results for contamination flushing. Consequently, in CY-DMA 2 version, another reservoir, \( R_2 \), has been added at the bottom of the network. Realistic water demands were generated based on actual observations of consumption within the DMA and allocated to the network nodes. The network includes 16 distinct flushing locations, aligning with the fire hydrant placements in the real DMA. These flushing locations are marked with red stars in Figure 11, with the numbers adjacent representing the node numbers within the EPANET model.

5.1 CY-DMA 2 Test Results

To assess the framework’s performance, 74 distinct test cases were examined, wherein each of the 74 nodes within the network, excluding the flushing nodes, were considered as the contamination source node in individual cases. However, due to space constraints, only detailed results for the contamination source node being node #60 are presented here. Nevertheless, a comprehensive summary of the framework’s overall performance, derived from results across all test cases, is provided later in the section. In Figure 11, the contamination source node, #60, is indicated by a brown diamond.

The CY-DMA network is simulated in EPANET using the using the MATLAB-EPANET toolbox (Eliades et al., 2016). The simulation is carried out with a sampling time of 30 min and a simulation time of 48 hours. Commencing at the 0th hour in the simulation, contamination is injected from the source node as source type *Mass Booster*,

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**Figure 11.** The CY-DMA 2 network marked with the 16 flushing points and one of the contamination source nodes considered in this work.
in which a fixed mass flow of contamination is added to the network (Rossman et al., 2000). In this work, for all the test cases a fixed contamination mass flow of 40 [mass/minute] is injected from the source node for a duration of 2 hours. For each pairing of the contamination source node and the flushing node, the simulation is executed three times. Once without employing any contamination impact mitigation control strategy, secondly using contamination impact mitigation control with pressure constraints obtained via the breadth-first search approach, and lastly with contamination impact mitigation control using pressure constraints obtained via the shortest path approach.

Once more, considering this as a pilot study, we assume zero delays in contamination detection and localization, i.e. the contamination impact mitigation control is initiated immediately upon the onset of contamination injection. The first step within the contamination impact mitigation control framework involves identifying the pressure constraints. As detailed in section 4, two distinct approaches are employed to identify these constraints: the breadth-first search approach and the shortest path approach. In each test case, the pressure constraints are determined using these two methods for every pair of the contamination source node and the 16 flushing nodes individually.

Once more, considering this as a pilot study, we assume zero delays in contamination detection and localization, i.e. the contamination impact mitigation control is initiated immediately upon the onset of contamination injection. The first step within the contamination impact mitigation control framework involves identifying the pressure constraints. As detailed in section 4, two distinct approaches are employed to identify these constraints: the breadth-first search approach and the shortest path approach. In each test case, the pressure constraints are determined using these two methods for every pair of the contamination source node and the 16 flushing nodes individually.

The pressure constraints for the test case with node #60 as the contamination source node and node #35 as the flushing node are visually illustrated in Figure 12 using a colour bar: nodes with the highest hydraulic pressure head are represented in yellow, the nodes with the lowest hydraulic pressure head are represented in aqua, and the nodes without pressure constraints are represented in violet-blue. For this specific pairing of contamination source and flushing nodes, the pressure constraints are identified through two approaches: the subplot (a) in Figure 12 displays the constraints obtained using the breadth-first search approach, while the subplot (b) depicts those identified via the shortest path approach. Similarly, considering the same contamination source node but now with node #48 as the flushing node, the pressure constraints are visually depicted in Figure 13. Correspondingly, the subplot (a) in Figure 13 demonstrates the pressure constraints obtained from the breadth-first search approach, and the subplot (b) presents those obtained using the shortest path approach. Observing both sets of figures, Figure 12 and 13, it is evident that the defined pressure constraints enable the contaminated water to flow from the contaminated source node towards the flushing node without spreading throughout the network. Similarly, pressure constraints for other test cases were also obtained, but due to space limitations, they are not presented here.

The identified pressure constraints form the constraints of the optimization problem given by (14), (15), and (16). In the tests, the supply node $R2$ is arbitrarily designated as the reference node. It is essential to note that this choice has no impact on the simulation test, as mentioned earlier. In relation to the third objective in the optimization problem described in (14), node #51 is arbitrarily selected as the target node(s); specifically, $p_t = p_{51}$. Further parameters pertaining to the optimization problem are detailed in Table 1, which are based on the nominal operating conditions of the real network.

The optimization problem is minimized with respect to the decision variables, consumer flows, $d_c$, and the supply pressure, $p_s$. The optimization problem has been implemented in CasADi, which is an open source tool for nonlinear optimization (Andersson et al., 2019), in its MATLAB (MATLAB, 2022) interface. The optimal values of the decision variables are then applied to the EPANET simulation model to perform a hydraulic and quality analysis. All the cases are simulated with the same contamination characteristics. The optimization problem is solved and the network is subsequently simulated using the obtained solution at each time step.

The results from the three simulation tests for each pair of the contamination source node and flushing node are compared on various parameters, which are listed below:
Figure 12. Visually representation of the hydraulic pressure head constraints in CY-DMA network for the test case with node #60 as the contamination source node and node #35 as the flushing node defined using (a) the breadth-first search approach and (b) the shortest path approach, and presented using a colour bar.

Figure 13. Visually representation of the hydraulic pressure head constraints in CY-DMA network for the test case with node #60 as the contamination source node and node #48 as the flushing node defined using (a) the breadth-first search approach and (b) the shortest path approach, and presented using a colour bar.
Table 1. Parameters for the optimization problem.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p^*_{t}$</td>
<td>3.5 bar</td>
<td>$p^{min}$</td>
<td>2.5 bar</td>
</tr>
<tr>
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</tr>
<tr>
<td>$d^{max}$</td>
<td>80 m$^3$/h</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- **Contaminated water consumed** [m$^3$]: This represents the total water consumed with the contamination concentration higher than 0.01 mg/L, during the simulation period.

- **Time till contaminant free network** [hr]: This represents the time-frame from the initiation of contamination injection to the point where the contamination concentration at every node and link in the network falls below 0.01 mg/L. Note that the simulation sampling time is set at 30 min and therefore this parameter is also measured every 30 min.

- **Contaminated water flushed from the flushing node** [m$^3$]: This represents the total water flushed from the flushing node with the contamination concentration higher than 0.01 mg/L, during the simulation period.

- **Change in consumption** [%]: This represents the percentage change in the consumer flow resulting from contamination impact mitigation control.

To illustrate, a summary of the results for the case with node #60 as the contamination source node is presented in Table 2 and Table 3.

The first rows in Table 2 and Table 3 present the results of contamination impact under nominal conditions. If no control strategies are applied the contaminated water consumed is 49.75 m$^3$ and it would take 12.5 hours for the network to be contamination-free.

Further, Table 2 illustrates the outcomes of contamination impact mitigation control, employing pressure constraints determined through the breadth-first search approach. The values in parentheses beneath the headings ‘Contaminated water consumed’ and ‘Time till contamination-free network’ indicate the percentage change from the nominal values provided in the first row of the table. Moreover, a downward arrow denotes a reduction, while an upward arrow denotes an increase in the value. Similarly, Table 3 showcases the results of contamination impact mitigation control, employing pressure constraints determined through the shortest path approach.

From these results, it is evident that contamination impact mitigation control is able to effectively diminish the impact of contamination with minimal inconvenience to consumers, regardless of the approach employed. However, the results from both approaches yield varying degrees of effectiveness in flushing from different nodes. For instance, as shown in Table 3, when node #48 serves as the flushing node, a 96% reduction in contaminated water consumption is achieved, with only 7.63% of consumer demands remaining unmet. Conversely, when node #68 is the flushing node, only a 71% reduction in contaminated water consumption is attained, with 14.49% of consumer demands not being met. Furthermore, the two approaches produce different outcomes for the same flushing node. For node #35 as the flushing node, the control with the shortest path approach results in an 83% reduction in contaminated water consumption, with only 3.31% of consumer demands unmet. Conversely, the control with the breadth-first search approach
<table>
<thead>
<tr>
<th>Flushing Node</th>
<th>Contaminated water consumed $[m^3]$</th>
<th>Time till contaminant free network $[hr]$</th>
<th>Contaminated water flushed from the flushing node $[m^3]$</th>
<th>Change in consumption [%]</th>
</tr>
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<tbody>
<tr>
<td>1</td>
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<td>N/A</td>
</tr>
<tr>
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</tr>
<tr>
<td>15</td>
<td>2.15 (96% ↓)</td>
<td>Inf (Inf% ↑)</td>
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<tr>
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<td>N/A</td>
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</tr>
<tr>
<td>35</td>
<td>3.03 (94% ↓)</td>
<td>5 (60% ↓)</td>
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<tr>
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<tr>
<td>46</td>
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<tr>
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<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Table 2. Contamination impact results with contamination impact mitigation control in which pressure constraints are obtained via the breadth-first search approach. The values in parentheses indicate the percentage change from the nominal values provided in the first row of the table.
<table>
<thead>
<tr>
<th>Flushing Node</th>
<th>Contaminated water consumed ([m^3])</th>
<th>Time till contaminant free network ([hr])</th>
<th>Contaminated water flushed from the flushing node ([m^3])</th>
<th>Change in consumption [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>49.75</td>
<td>12.5</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>1</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>11</td>
<td>1.59 (97% ↓)</td>
<td>Inf (Inf% ↑)</td>
<td>0</td>
<td>89.19%</td>
</tr>
<tr>
<td>21</td>
<td>N/A</td>
<td>N/A</td>
<td>35</td>
<td>3.31%</td>
</tr>
<tr>
<td>24</td>
<td>8.28 (83% ↓)</td>
<td>9 (28% ↓)</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>35</td>
<td>N/A</td>
<td>9 (28% ↓)</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>43</td>
<td>2.26 (95% ↓)</td>
<td>4.5 (64% ↓)</td>
<td>30</td>
<td>3.43%</td>
</tr>
<tr>
<td>46</td>
<td>2.16 (96% ↓)</td>
<td>4.5 (64% ↓)</td>
<td>12</td>
<td>7.63%</td>
</tr>
<tr>
<td>48</td>
<td>N/A</td>
<td>4.5 (64% ↓)</td>
<td>12</td>
<td>7.63%</td>
</tr>
<tr>
<td>51</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>59</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>68</td>
<td>14.58 (71% ↓)</td>
<td>12 (4% ↓)</td>
<td>35.72</td>
<td>14.49%</td>
</tr>
<tr>
<td>71</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>75</td>
<td>7.92 (84% ↓)</td>
<td>10 (20% ↓)</td>
<td>40</td>
<td>16.02%</td>
</tr>
<tr>
<td>82</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>90</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

*Table 3.* Contamination impact results with contamination impact mitigation control in which pressure constraints are obtained via the shortest path approach. The values in parentheses indicate the percentage change from the nominal values provided in the first row of the table.
yields a 94% reduction in contaminated water consumption, with 3.44% of consumer demands not being met.

Among the 16 flushing nodes, results for 10 nodes in both Table 2 and Table 3 are indicated as ‘N/A’, signifying an infeasible problem. Discussion regarding these feasibility issues is provided in Section 5.2. An intriguing observation pertains to the flushing node #15, as detailed in Tables 2 and 3, where the time until the network becomes contaminant-free is given as ‘Inf’, indicating that the network remains contaminated throughout the simulation period. In such instances, contaminants persist in one or multiple pipelines due to zero flow within them. Additionally, another noteworthy finding relates to the flushing node #51, as shown in Table 2, where there is an 88% reduction in consumed contaminated water, yet the flushed contaminated water quantity is nil. These scenarios arise when the optimization process fails to establish a pressure gradient to direct contaminated water toward the flushing node, yet effectively containing its spread. Consequently, although a higher concentration of contaminated water is consumed, the quantity remains limited, resulting in a reduction in overall contaminated water consumption.

Among the feasible solution nodes, the most efficient node for flushing can be selected using (17). In this test, we prioritize the reduction in contaminated water consumption as the highest priority, followed by minimizing the consumer demands not being met, and lastly, minimizing the time it takes for the network to be contaminant-free. Accordingly, assigning the weights $w_{CWC} = 1$, $w_{TTCF} = 0.2$, and $w_{CC} = 0.7$ in (17), node #48 emerges as the most effective flushing node for node #60 being the contamination source node, under the breadth-first search approach. Here, a substantial 96% reduction in contaminated water consumption is achieved, with only 6.76% of consumer demands remaining unmet, and the network becoming contaminant-free 64% faster. Conversely, employing the shortest path approach designates node #46 as the optimal flushing node, resulting in a similar 95% reduction in contaminated water consumption, with 3.43% of consumer demands remaining unmet, and once again, the network becoming contaminant-free 64% faster.

As mentioned earlier, similar tests were carried out considering each of the other 73 nodes as the contamination source node individually. In certain cases, the contamination impact mitigation control could not provide a feasible solution with any of the flushing nodes. Again, this infeasibility issue is discussed in section 5.2. For the remaining cases (60 test cases), summary statistics results considering the most efficient node for flushing are presented using box plots in Figures 14, 15, and 16.

Figure 14 presents a box plot illustrating the distribution of the percentage change in contaminated water consumed under both breadth-first search and shortest path approaches. Under the breadth-first search approach, the percentage reduction in contaminated water consumed ranges from 60.99% to 100%, with outliers represented by red crosses. The median reduction is 94.07%, indicating that for half of the test cases, the reduction is more than 94.07%, while only for a quarter of the cases, the reduction is less than 81.83%. Similarly, under the shortest path approach, the percentage reduction in contaminated water consumed ranges from 80.65% to 100%. The 75th percentile is 97.94%, the median is 94.94%, and the 25th percentile is 89.59%. There are 4 outliers under the breadth-first search approach, with the worst performance showing only an 11.38% reduction in contaminated water consumed. Under the shortest path approach, there are 5 outliers, with the worst performance demonstrating a 32.40% reduction in contaminated water consumed. These results stem from test cases where the optimization problem fails to create the desired pressure gradient with the given freedom on consumer demand regulation.

In Figure 15, a box plot illustrating the distribution of the percentage change in the time until the network is contamination-free is presented. Under the breadth-first
Breadth-first search Shortest path
Pressure constraint identification approaches
10
20
30
40
50
60
70
80
90
100

Figure 14. Box plot illustrating the distribution of percentage change in contaminated water consumed with respect to the nominal conditions.

search approach, it ranges between -3.45% and 89.70%, while under the shortest path approach, it ranges between 4.35% and 91.04%. The negative sign indicates that the network becomes contaminant-free slower than the nominal case. In certain instances, the control framework can only establish a minor pressure gradient, resulting in a low flow of contaminated water. Therefore, we observe varying degrees of change in the time until the network is contaminant-free, with an outlier taking more than twice the time to achieve contamination-free status with the control framework compared to the nominal case.

Figure 16 depicts a box plot illustrating the distribution of the percentage change in water consumption. With the breadth-first search approach, this change ranges between 17.20% and 0.06%, while with the shortest path approach, it spans from 12.75% to 0.06%. In both scenarios, around three-fourths of the cases exhibit a reduction in consumer demands of less than approximately 7.35%, with half of the cases even falling below 3.5%. However, there are notable outliers where the change in consumption is substantially higher. Specifically, under the breadth-first search approach, the highest change in consumption reaches 61.78%, whereas under the shortest path approach, it peaks at 21.39%.

In the further section, the feasibility issues of the optimization problem are discussed.

5.2 Optimization Problem Feasibility

The contamination impact mitigation control relies on an optimization problem, making it susceptible to feasibility issues. While a comprehensive feasibility study has not been conducted in this work, we would like to highlight some observations from the tests regarding the feasibility of the optimization problem.

From Table 2 and Table 3, it is evident that the optimization problem yields infeasibility errors for multiple cases. Similar observations were noted for contamination source nodes located at a distance from the flushing node. In such instances, the opti-
Figure 15. Box plot illustrating the distribution of percentage change in the time until the network is contamination-free with respect to the nominal conditions.

Figure 16. Box plot illustrating the distribution of percentage change in the consumption of water with respect to the nominal conditions.
mization problem is unable to maintain the pressure constraints (the required pressure
gradient), given the limited controllability over the network.

We also observe that for the same pair of the contamination source node and the
flushing node, the shortest path approach typically yields a lower number of constraints
compared to the breadth-first search approach. This results in a higher computational
load for the breadth-first search approach. This disparity could be attributed to the breadth-
first search approach exploring multiple pathways from the flushing node to the contamina-
tion source node simultaneously, thereby leading to more constraints. Conversely, in
the shortest path approach, we initiate from the shortest path between the flushing node
and the contamination source node and construct layers around it, consequently limit-
ing the number of constraints. However, despite the shorter computational load asso-
ciated with the shortest path approach, there are certain cases where the breadth-first
search approach outperforms it in terms of contamination impact mitigation. The breadth-
first search approach might be better suited for denser networks with highly intercon-
nected pipelines and pathways. Further studies are required to compare the two approaches
in greater detail.

Moreover, the framework encounters an infeasibility error for contamination source
nodes where the direction of flow in the connected edges remains fixed, regardless of the
chosen flushing node. One such scenario is a leaf node, which refers to a node with a de-
gree of one, meaning it is connected to the network via only one edge. In the CY-DMA
network (Figure 11), nodes #4, #12, and #22 are examples of leaf nodes. In these in-
stances, water flow consistently moves from the network toward the leaf node, render-
ing a decreasing pressure gradient from the contamination source node to any flushing
node unattainable. Consequently, attempting to solve the optimization problem invari-
ably leads to an Infeasible Problem error. However, it is noteworthy that in these cases,
contamination does not spread within the network; instead, all contamination is contained
and consumed within the leaf node.

Further analytical feasibility analysis of the optimization problem is reserved for
future work.

6 Conclusion

The work introduces a control framework designed to mitigate the potential im-
 pact of contamination threats. By establishing a decreasing pressure gradient from the
contamination source node to a flushing node, the framework facilitates the movement
of contaminated water towards the flushing node. Two distinct methods, viz. the breadth-
first search approach and the shortest path approach, are proposed to define these de-
creasing pressure gradients as hydraulic pressure head constraints. Formulated as an op-
timization problem, the control framework incorporates these hydraulic pressure head
constraints as constraints of the optimization problem. The decision variables of the op-
timization problem include the supply pressure and consumer demands, which are reg-
ulated to create the desired pressure gradient. To evaluate the framework’s efficacy, ex-
periments were conducted using a model of a real-world network, CY-DMA. Results in-
dicate that in over 75% of cases, the framework successfully reduces the consumption
of contaminated water by a minimum of 81.84%, with only a maximum of 8.22% of con-
sumer demands left unmet.

Despite making strong assumptions regarding knowledge of contamination char-
acteristics and the response time, this pilot study illustrates the potential for achieving
significant reductions in contamination impact with minimal inconvenience to consumers.
This study lays the groundwork for further development of contamination mitigation strate-
gies and other applications centred around hydraulic pressure head regulation through
consumer flow control. Future research should focus on validating the framework using
diverse networks with different configurations. Additionally, conducting a detailed feasibility study of the optimization problem, devising optimal strategies for identifying pressure constraints and formulating optimizations to reduce computational load are areas of further investigation.

Data Availability Statement

The models, code and data generated during this study are available at https://zenodo.org/doi/10.5281/zenodo.11082064.

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Contamination impact mitigation in water distribution networks through consumer demand control

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Key Points:

• A novel control framework to mitigate the impact of a contamination threat by regulating consumer demand flow is proposed and tested.
• Consumer flows are regulated to create a pressure gradient from the contamination source to a flushing point, ensuring efficient flushing.
• The control problem is an optimization problem with pressure gradients as constraints and methods for identifying them are presented.
Abstract

Amidst the time-consuming process of conducting laboratory tests to confirm contamination incidents, utilities must promptly implement measures to mitigate the impact of potential contamination without undue overreaction. This paper proposes a novel approach for mitigating the impact of contamination threats (an unconfirmed contamination event) in smart water networks. By leveraging smart valves, the consumer flows are regulated to create a decreasing pressure gradient from the contamination source to a flushing point, facilitating efficient contamination flushing with minimal consumer inconvenience. The decreasing pressure gradients are formulated as constraints within an online optimization problem. Additionally, two methods for identifying these pressure constraints are presented. To assess the efficacy of the proposed framework, testing is conducted using a realistic water network model.

1 Introduction

Water distribution networks have a distributed structure. Adding to this, ageing infrastructure, threats from cascading events, and insufficient disinfection make them susceptible to potential contamination, whether deliberate or accidental (Preis & Ostfeld, 2006). Contaminants in the water pose substantial threats to public health as highlighted in studies such as Islam et al. (2015), Kuhn et al. (2017), Bjelkmar et al. (2017) and Kirmeyer (2001), which document various cases of disease outbreaks caused by pathogens infiltrating these distribution networks.

When there is information or warning about contamination, a utility needs to promptly identify and initiate responsive actions to mitigate its impact. Such warnings could be based on consumer complaints, unusual readings from water quality sensors, coupled with contamination detection algorithms, or information about a security breach. A response to a contamination warning is not a one-time action but a continually evolving, interactive, and adaptable approach that adjusts based on the ongoing acquisition of new information about the contamination (Rasekh & Brumbelow, 2014). The United States Environmental Protection Agency (USEPA, 2006, 2004) presents a toolbox and guidelines on how to respond to a drinking water contamination event. A utility needs to first assess if a contamination event is a contamination threat or a contamination incident. An event in which contamination may have occurred but there is no conclusive evidence yet is said to be a contamination threat, and when there is confirmation, the event is classified as a contamination incident. In the case of a contamination incident, and based on the severity of the contamination, the utility would need to take remediation steps by closing down the network and flushing out the contaminant to bring the water network to safe and normal operation. Confirmation of a contamination incident typically involves a lab test, which requires several hours if not days. The initial period of a contamination event is critical, as the number of exposed consumers rises rapidly over time (Schijven et al., 2016). Hence, during the period when the lab tests are being conducted, utilities need to take appropriate steps to mitigate the impact of potential contamination, which may include isolating the contaminated area through valve operations and alerting consumers via media or alert messages (Shafiee & Berglund, 2017; Islam et al., 2015; Baranowski & LeBoeuf, 2008). However, it should also not be an overreaction if that might inconvenience the customers, cause panic, and harm the utility’s credibility.

Once a contamination event has been confirmed, most likely the utilities would need to isolate the contaminated area and flush the contaminated water through hydrants (Shafiee & Berglund, 2017; Islam et al., 2015; Baranowski & LeBoeuf, 2008). Multiple studies have investigated optimal valve operation for flushing, utilizing various methodologies. Most of these studies assume a certain level of prior knowledge about contamination characteristics and the location of its source. For instance, Shafiee and Berglund (2015) pre-
Numerous studies have framed the contaminant flushing problem as a single or multiple objective optimization problem. This problem involves defining the operation of actuators (valves, hydrants, pumps) in the network as decision variables. The objective function includes minimizing consumer health impacts, service disruption, and the number of operations. Various methods, such as genetic algorithms (Preis & Ostfeld, 2008; Guidorzi et al., 2009), deep reinforcement learning (Hu et al., 2022), evolutionary optimization (Alfonso et al., 2010), and swarm optimization (Moghaddam et al., 2020), have been employed to solve this optimization problem. These studies provide valuable insights and solutions for flushing contaminated water in the event of a contamination incident; however, there is limited literature on responsive actions in the case of a contamination threat. When facing a contamination threat where the contamination has not yet been confirmed, it may be unnecessary to completely close and flush the network. Instead, it is crucial to mitigate the impact of potentially contaminated water while awaiting confirmation of the contamination. This work presents a novel solution to address this specific problem.

1.1 Next Generation of Smart Water Networks

Information and Communication Technology (ICT) has undergone significant technological advancements in recent decades. These innovations have been integrated into electrical networks, evolving them into smart electrical grids. This transformation has allowed electric utilities to remotely monitor and better control electricity usage in real-time (Tuballa & Abundo, 2016). Similarly, the water sector is moving towards automation and digitalization by incorporating the Internet of Things (IoT) and communication technology, leading to the upgrade of conventional water networks into smart water networks. These networks utilize smart water technologies such as smart sensors and intelligent actuators, empowering water utilities to achieve real-time network monitoring and control. Studies by Gupta et al. (2020), Vrachimis et al. (2022) and Mutchek and Williams (2014) provide comprehensive insights into state-of-the-art smart water technologies and their utilization in better management of water networks. They also present examples of smart water networks being implemented in practice. These technologies include smart sensors for pressure measurement, smart water meters for flow measurement, and contaminant or biosensors for detecting contamination. Research and development efforts in smart water meters have notably increased in the last two decades (Zapata-Sierra et al., 2023). With these meters installed at consumer endpoints, both consumers and utilities can monitor water consumption in real-time (Mudumbe & Abu-Mahfouz, 2015; Gabrielli et al., 2014; Cattani et al., 2017). Smart water networks also include intelligent actuators, such as smart valves and variable-speed pumps. As part of operational management strategies, variable-speed pumps can autonomously adjust their speed to meet the network’s requirements, while smart valves can be controlled remotely. Wang et al. (2018) and Alshattanawi (2017) discuss the utilization of smart electric valves in water distribution networks for control and pressure management.

While current smart water networks typically incorporate only smart water meters at the consumer end, Sammaneh and Al-Jabi (2019) introduces the concept of smart valves situated at the consumer end, enabling the regulation of pressure for individual consumers and limiting consumer flow when necessary. Envisioning the next generation of smart water networks with further technological advancements, we foresee the integration of smart valves alongside smart water meters. These smart valves will provide a greater degree of freedom in controlling and operating the network, allowing for the regulation of individual flows within the network. With this vision in mind, we propose a novel solution in the form of a control framework for reducing the impact of contam-
ination in case of contamination threat within these advanced water distribution networks.

1.2 Contributions

Similar to most previous studies on contamination flushing, our method relies on initial knowledge of the contamination source location. The underlying idea is to create a decreasing pressure gradient from the contamination source to a flushing point. This pressure gradient is achieved by regulating consumer flows, an action we anticipate being plausible with the incorporation of smart valves in the next-generation smart water networks. This differs from methods where the network is closed, and shut-off valves are employed to flush out contamination. Since the nature and severity of contamination are unconfirmed, consumption is not stopped but rather regulated to limit the spread. This ensures a measured response, preventing an overreaction, maintaining water supply to critical services and minimizing inconvenience to consumers while mitigating the impact of potential contamination. This work can be seen as a first step leading to the development of a more comprehensive methodology. The control for the contamination impact mitigation problem is formulated as an optimization problem. Our approach entails solving the optimization problem online. A mathematical model of water distribution networks based on graph theory serves as the foundation for this optimization problem. This stands in contrast to prior studies (Shafiee & Berglund, 2015; Rasekh & Brumbelow, 2014), where the flushing strategy was obtained offline. In those cases, coupling with hydraulic simulators like EPANET (Rossman et al., 2000) was often necessary, resulting in prolonged simulation times for convergence. Moreover, considering various contamination scenarios (injection location, concentration, time, and duration) imposed impractical computational burdens. In this work, we also present two different methods to identify pressure constraints on the junctions for creating the decreasing pressure gradient. These pressure constraints serve as inequality constraints in the optimization problem. Finally, the contamination impact mitigation control framework is tested on an EPANET model of a district metered area in a realistic water network.

To summarize, this work brings forth three key contributions: a) It proposes a control framework to mitigate the impact of a contamination threat, which has not yet been confirmed; b) The framework introduces a novel approach of regulating consumer demand flow to achieve the objective; c) The control problem is formulated as an online optimization problem, utilizing a mathematical model of the network.

The paper proceeds as follows: section 1.3 introduces the common notations used throughout this work. As a preliminary to this work, a graph theory-based mathematical model for a water distribution network is recalled in section 2 to provide foundational context. The design of the contamination impact mitigation control framework is described in section 3. In section 4 two distinct approaches for identifying the pressure constraints, part of the contamination impact mitigation control optimization problem, are presented. Following that, section 5 presents and discusses the results from the test on a benchmark water network. Finally, section 6 summarizes and concludes the study.

1.3 Notations

This section presents some of the common notations used throughout the paper. For a vector $x$, $x \in \mathbb{R}^n_{\geq 0}$ denotes $\forall i x_i \geq 0$, similarly $x \in \mathbb{R}^n_{\leq 0}$ denotes $\forall i x_i \leq 0$. The vector $\mathbb{1}$ is a column vector of ones in all positions and of appropriate dimension. The contamination impact mitigation optimal control developed in this work is based on a graph theory water network model. In this model, the variables corresponding to the spanning tree of the graph are denoted by the subscript $T$ and the corresponding chord by the subscript $C$. Any matrix or vector with a row corresponding to the reference node
removed is denoted with a bar, for example, $\bar{H}$, $\bar{d}$. Furthermore, the model variables will be updated at discrete time instants and these time instants will be denoted by $[k]$.

## 2 Graph Theory Model of a Water Distribution Network

This section recalls the graph theory based mathematical model of a water distribution network, introduced by Kallesøe et al. (2015). This model has been previously employed for leakage diagnosis in Rathore et al. (2021), Jensen and Kallesøe (2016) and Rathore, Kallesøe, and Wisniewski (2023), and for contamination mitigation control in Rathore, Misra, et al. (2023). Nonetheless, given its foundational role in formulating the optimization problem for the contamination impact mitigation control, it is presented here as a preliminary section in this paper. A detailed derivation of the model can be found in Kallesøe et al. (2015) or Rathore, Misra, et al. (2023).

A water distribution network with $n$ junctions and $m$ pipes can be represented as a directed graph, $G$, with $m$ edges and $n$ nodes (Deo, 2017). Moreover, the $n^{th}$ node is set as the reference node. The mathematical model for the water distribution network at $k^{th}$ time instance can be given by (1) and (2).

$$\lambda_C(q_C[k]) - \bar{H}_C^T \bar{H}_C^{-1} \lambda_T(-\bar{H}_T^{-1} \bar{H}_C q_C[k] + \bar{H}_T^{-1} \bar{d}[k]) = 0. \quad (1)$$

$$\bar{p}[k] = \bar{H}_T^{-1} \lambda_T(-\bar{H}_T^{-1} \bar{H}_C q_C[k] + \bar{H}_T^{-1} \bar{d}[k]) - (\bar{z} - \mathbb{1} \bar{z}_n) + \mathbb{1} \bar{p}_n[k] \quad (2)$$

The underlying network graph has been divided into arbitrary spanning tree $T$ and its corresponding chords $C$. Here, $q_C[k]$ is the vector of flows through the chords at time $k$. $\bar{H}_C$ is the reduced incidence matrix corresponding to the chords and $\bar{H}_T$ to the spanning tree of the graph. $\lambda_T(\cdot)$ and $\lambda_C(\cdot)$ are vector maps representing the flow-dependent pressure drops in the spanning tree and the chords edges respectively. Moreover, $\lambda_i : \mathbb{R} \rightarrow \mathbb{R}$ is assumed to be of form, $\lambda_i(q_i) = f_i |q_i|$, with $f_i > 0$ (Swamee & Sharma, 2008). In this model, the supply flows and the consumer demands are modelled by assigning independent nodal flows to a subset of the nodes. $\bar{d}[k] \in \mathbb{R}^{(n-1)}$ is the vector of independent nodal demands at the non-reference nodes. $\bar{p}[k] \in \mathbb{R}^{(n-1)}$ is the vector of pressures at the non-reference nodes and $\bar{p}_n[k] \in \mathbb{R}$ is the pressure at the reference node. Finally, $\bar{z} \in \mathbb{R}^{(n-1)}$ and $\bar{z}_n \in \mathbb{R}$ is the vector of pressures due to elevation at the non-reference nodes and reference node respectively. Also, $\mathbb{1}$ represents a vector of 1s.

The utilization of this model for the development of the contamination impact mitigation strategy is detailed in section 3.1.

## 3 Contamination Impact Mitigation Control Framework

In this section, the contamination impact mitigation control framework for drinking water networks is presented. As previously stated, a next-generation smart water network is considered, with automated valves at the consumer end assumed to control consumer flows. The automated valves serve as actuators within the network, providing a greater degree of freedom in controlling and manipulating pressure heads. A plausible scenario might involve automated valves being installed solely for a subset of consumers. Consequently, the framework would only have control over a subset of consumer flows or have control with a certain degree of freedom. However, given that this is a pilot study, we assume the capability to regulate all consumer demands for the sake of analysis. Moreover, a pressure-dependent demand control may be considered instead of a demand flow control in future work.
It is also assumed that a contaminant enters the water network from a single node at a time, referred to as the contamination source node. Contamination detection and localization have been addressed in the literature (Vrachimis et al., 2020; Seth et al., 2016). The emphasis of this work lies in a reactive strategy post-contamination threat detection. Hence, it is assumed that contamination intrusion has been reported or detected, and the potential source location is known. Typically, contamination localization methodologies do not pinpoint a single node as the source of contamination, but rather a set of nodes in the neighbourhood of each other, with some likelihood (Eliades & Polycarpou, 2012). In these scenarios, the identified neighbouring set of nodes can be treated as a singular node. However, for the sake of simplicity in this study, our starting point is a singular node identified as the source of contamination.

Upon detection of a contamination intrusion, the subsequent action involves preventing the spread and removal of contaminated water from the network. The concept involves guiding water flow from the contamination source node to a flushing node, where the water is flushed out. Typically, fire hydrants within the network are utilized as flushing nodes; however, any node capable of diverting water into the wastewater network or other disposal sites may serve as a flushing node.

This work proposes the utilization of the hydraulic pressure head gradient within the network to direct the flow toward the flushing node. The hydraulic pressure head at a node is defined as the sum of pressure due to water and pressure due to geodesic level, and is given as,

\[ h_i[k] = p_i[k] + z_i, \tag{3} \]

where \( h_i \) is the hydraulic pressure head, \( p_i \) is the water pressure and \( z_i \) is the pressure due to geodesic level at \( i^{th} \) node.

To illustrate this concept, consider a small example network, as shown in Figure 1. In this network, node #1 serves as the flushing point, while contamination enters from node #4. Overlaying the network is a colour map representing the hydraulic pressure head across the network. Regions with the highest hydraulic pressure head are depicted in red, while those with the lowest pressure constraints are in violet-blue. The figure depicts a decreasing hydraulic pressure head gradient from the contamination source node to the flushing node. This decreasing pressure gradient facilitates the flow of contaminated water from the source to the flushing point, where it can be subsequently flushed out. As the pressure gradient is not limited to a specific path but in a general direction from the contamination source to the flushing point, multiple paths for the contaminated water to reach the flushing point exist. Moreover, the higher hydraulic pressure head surrounding these paths prevents the contaminated water from spreading throughout the network.

These hydraulic pressure head gradients can be established from the regulation of consumer demands and supply pressures. Determination of consumer demands and supply pressures involves solving an optimization problem, wherein the hydraulic pressure head gradients are formulated as constraints within this optimization problem.

In section 3.1 the optimization problem for the contamination impact mitigation control is defined based on the graph theory model developed in section 2. Further, in section 4, two different approaches are presented for defining the hydraulic pressure head gradient constraints.

### 3.1 The Control Optimization Problem

The contamination impact mitigation control intends to direct the flow from a known contamination source node towards a predefined flushing node by controlling supply pressures and consumer flows, thereby facilitating the removal of contaminants. To minimize the impact of the contamination, it is crucial to maximize the flow from the contami-
nation source node to the flushing node, which can occur when the hydraulic pressure head difference between these nodes is maximum. However, in doing so, the controller must also ensure minimal impact on consumers. This necessitates a multi-objective controller, formulated as an optimization problem with weighted objectives. The contamination impact mitigation control solves this optimization problem by minimizing a multi-objective cost function under specified constraints.

In this work, the impact on the consumer is construed as consumer water demands not being met. This can be defined as the difference between the nominal consumer demands or the water requested, $d_{oc}[k]$ and the controlled consumer demands or the water supplied, $d_{c}[k]$. This forms the first objective of the optimization which is to be minimized and is given by,

$$J_1[k] = (d_{c}[k] - d_{oc}[k])^\top (d_{c}[k] - d_{oc}[k]).$$  
(4)

As previously indicated, the effective mitigation of contamination relies on maximizing the hydraulic pressure head difference between the contamination source node and the flushing node. This is formulated as the second objective in the cost function of the optimization problem,

$$J_2[k] = (h_{source}[k] - h_{flush}[k])^\top (h_{source}[k] - h_{flush}[k]),$$  
(5)

where $h_{source}$ is the hydraulic pressure head at the contamination source node and $h_{flush}$ is the hydraulic pressure head at the flushing node.

Finally, to ensure the network functions and meets consumer satisfaction in terms of network pressure, a desired level of network pressure is to be maintained. This network pressure can be defined as the pressure at either all nodes throughout the network or a specific subset of nodes, which might adequately represent the pressure within a particular area. Opting for a subset of nodes could reduce the computational load of the optimization problem. The discretion to select this subset of nodes could lie with the operator or utility, employing a tool to balance both network requirements and computational load. Henceforth the nodes included in this selected subset will be referred to as **targeted nodes**. With that, the third objective is formulated as,

$$J_3[k] = (p_t[k] - p_{oc}[k])^\top (p_t[k] - p_{oc}[k]),$$  
(6)
where, $p_t$ is the network pressure at targeted nodes, $p^*_t$ is the desired pressure set-point.

The targeted node pressures can also be represented in terms of network node pressures as,

$$p_t[k] = F_t p[k],$$  \hspace{1cm} (7)

where $F_t$ is a $n_t \times n$ binary matrix to extract the targeted node pressures, where $n_t$ is the number of target nodes. In the case of pressure control at all the nodes, the matrix $F_t$ would be an identity matrix. With that, equation (6) is given as,

$$J_3[k] = (F_t p[k] - p^*_t[k])^\top (F_t p[k] - p^*_t[k]).$$  \hspace{1cm} (8)

The optimization problem is subjected to multiple constraints. The first set of constraints are the constraints on the hydraulic pressure head at the nodes to form a decreasing pressure gradient in the network from the contamination source node to the flushing node. These hydraulic pressure head constraints at the nodes are represented as,

$$h_a \leq h_b, \hspace{0.5cm} a \in A, \hspace{0.5cm} b \in B$$  \hspace{1cm} (9)

where $A$ and $B$ are a set of nodes, having a one-to-one element correspondence. These sets are defined in two different ways using two different approaches in section 4.

Additionally, the capacity of the supply nodes, dictated by factors such as pump and pressure-reducing valve (PRV) capacities, imposes practical constraints. These constraints limit the supply nodes to deliver within specified minimum and maximum values for both pressure and flow. These constraints in the optimization problem are formulated as,

$$p_s^{\text{min}} \leq p_s[k] \leq p_s^{\text{max}},$$  \hspace{1cm} (10a)

and,

$$0 \leq d_s[k] \leq d_s^{\text{max}},$$  \hspace{1cm} (10b)

where $p_s$ is a vector of pressure at the supply node and $d_s$ is a vector of supply flow. The minimum and maximum pressure are denoted by $p_s^{\text{min}}$ and $p_s^{\text{max}}$ respectively, and a maximum flow is denoted by $d_s^{\text{max}}$.

As previously discussed, the assumption is that consumer demands can be regulated by varying closing degrees of automated valves on consumer connections. However, consumption at outlet points, such as taps, remains under the control of the consumer. While the automated valves can limit consumption, they cannot entirely dictate it. Consequently, controlled consumer demands can only be lower than the nominal consumer demands. Moreover, consumers cannot supply water to the network, meaning that the consumer demand can be reduced to a minimum of zero. This particular constraint is formulated as,

$$d_o^c[k] \leq d_c[k] \leq 0.$$  \hspace{1cm} (11)

It is important to note that, according to the sign convention adopted in this study, consumer demands are regarded as negative independent flows out of the network. Hence, mathematically, the controlled consumer demand should be greater than the nominal consumer demand but still less than zero. However, the magnitude or absolute value of the nominal consumer demands will be greater than that of the controlled consumer demands.

There would also be some limitations on the maximum flow from the flushing point depending on the hydrant or sink capacity. In addition to that, we implement a minimum flushing flow limitation to ensure that contaminated water is flushed out consistently rather than merely maintaining a pressure gradient. These are also to be considered in the optimization problem and these constraints are formulated as,

$$-d_{\text{flush}}^{\text{max}} \leq d_{\text{flush}}[k] \leq -d_{\text{flush}}^{\text{min}}[k],$$  \hspace{1cm} (12)
where $d_{\text{flush}}^\text{min}$ and $d_{\text{flush}}^\text{max}$ is the magnitude of the minimum and maximum flushing flow respectively from the flushing point and $d_{\text{flush}}$ is the flushing flow. Similar, to consumer demand, according to the sign convention adopted in this study, the flows out of the network are regarded as negative independent flows. Hence, mathematically, the flushing flow should be greater than the negative of the maximum flushing flow but still less than the minimum flushing flow.

The third objective, outlined in (8), aims to operate the network at a desired pressure. Nevertheless, (13) is added as a constraint to ensure that a minimum network pressure for supply and firefighting is maintained under all conditions. This also prevents the creation of low-pressure points in the network, which increases the risk of contamination infiltration.

$$p_{\text{min}} \leq p[k].$$

Finally, with the multi-objective cost function and all the constraints, the contamination impact mitigation control optimization problem is given as,

$$\min_{d_c[k], p_s[k]} \left( (d_c[k] - d_c^c[k])^\top Q(d_c[k] - d_c^c[k]) 
- (h_{\text{sou}}[k] - h_{\text{flush}}[k])^\top R(h_{\text{sou}}[k] - h_{\text{flush}}[k]) 
+ (F_i p[k] - p_i^s[k])^\top S(F_i p[k] - p_i^s[k]) \right)$$

subject to

$$\lambda_c(q_c[k]) - \dot{H}_c^\top \dot{H}_c \lambda_T (-\dot{H}_c^\top \dot{H}_c q_c[k] + \dot{H}_c^\top d[k]) = 0.$$  \hspace{1cm} (15a)

$$\dot{p}[k] = \dot{H}_c^\top \lambda_T (-\dot{H}_c^\top \dot{H}_c q_c[k] + \dot{H}_c^\top \lambda_T d[k]) - (\dot{z} - 1 z_n) + 1 p_n[k]$$  \hspace{1cm} (15b)

$$h_a \leq h_b, \quad a \in A, \quad b \in B$$  \hspace{1cm} (16a)

$$p_s^\text{min} \leq p_s[k] \leq p_s^\text{max},$$  \hspace{1cm} (16b)

$$0 \leq d_s[k] \leq d_s^\text{max},$$  \hspace{1cm} (16c)

$$d_c^c[k] \leq d_c[k] \leq 0.$$  \hspace{1cm} (16d)

$$-d_{\text{flush}}^\text{max} \leq d_{\text{flush}}[k] \leq -d_{\text{flush}}^\text{min}.$$  \hspace{1cm} (16e)

$$p_{\text{min}} \leq p[k].$$  \hspace{1cm} (16f)

The cost function, (14), of the optimization problem, which is to be minimized, is a weighted sum of the three objective functions (4), (5) and (8). $Q$, $R$ and $S$ are the weight for objectives $J_1$, $J_2$ and $J_3$ respectively. $Q$ would be a $n_c \times n_c$ matrix, where $n_c$ is the number of consumer nodes; considering a singular contamination source node, $R$ would be a scalar; $S$ would be a $n_t \times n_t$ matrix, where as before $n_t$ is the number of target nodes.

The weights are typically diagonal matrices assigned to each objective in proportion to its relative importance within the overall cost function. The order of magnitude of the objective function values must also be considered while assigning the weights. Particularly, $Q$ can be employed to assign higher weights to vital consumers, such as hospitals, thereby minimizing the impact on their demand. Note that, the second objective of the pressure difference between the contamination source node and the flushing node is to be maximized. However, the cost function is to be minimized in the optimization problem, therefore in (14), the objective, (5), has been added with a negative sign. The optimization variables or the variables which are controllable here are the consumer demands, $d_c[k]$ and the supply pressures, $p_s[k]$, with respect to which the cost function is minimized. Further, the optimization problem is subjected to hydraulic system model constraints given by (15). Finally, the constraints presented in (9), (10), (11), (12) and (13) are included in (16).
3.2 Selection of the Flushing Node

In a typical water network, multiple locations are usually available for conducting flushing operations. In these cases, the most efficient node, for a specific contamination scenario, under this control framework must be selected among all the possible flushing locations. In selecting the best flushing node, it is essential to consider a trade-off between several factors: the contaminated water consumed, the time it takes for the network to become contaminant-free, and the change in consumption needed. This decision-making process involves weighing these parameters in a minimization problem, where each factor is assigned specific weights to determine the most effective flushing node. This minimization problem is given by,

$$f_{n_{\text{best}}} = \arg \min_{i} (w_{CW}CW_{i} + w_{TTC}TTC_{i} + w_{CC}CC_{i}),$$

where, $f_{n_{\text{best}}}$ denotes the best flushing node. $CW_{i}$, $TTC_{i}$ and $CC_{i}$ respectively indicate the percentage changes in contaminated water consumption, the percentage change in the time until the network is contaminant-free, and the percentage change in consumption given the mitigation solution that considers the $i^{th}$ node as the flushing node in the feasible solution subset. Correspondingly, the weights assigned to these parameters are denoted as $w_{CW}$, $w_{TTC}$, and $w_{CC}$, following the same order.

The weights specified in (17) can be determined by the utility, taking into account the network conditions and risk assessment of the contamination scenario. Subsequently, the argument yielding the minimum value is considered in the application of the control framework on the water network for that specific contamination scenario.

4 Pressure Constraints Identification

The following section presents two distinct approaches for identifying the constraints on hydraulic pressure head, as expressed in (16a) within the optimization problem. These pressure constraints are the key element of the proposed methodology as they establish a decreasing hydraulic pressure head gradient from the contamination source node to a flushing node, enabling the contaminated water to move towards the flushing node. Furthermore, the pressure gradient should not be confined to a specific path; instead, it should be in a general direction from the contamination source to the flushing point. This design would ensure the creation of multiple pathways for the contaminated water to reach the flushing point.

The two approaches, viz. breadth-first search and the shortest path approach, may yield different pressure constraints. However, the ultimate objective of a decreasing pressure gradient from the contamination source node to the flushing node remains consistent. In general terms, the breadth-first search approach explores multiple pathways between the contamination source node and the flushing node simultaneously. On the other hand, the shortest path approach begins with the shortest pathway between the contamination source node and the flushing node, subsequently exploring parallel pathways to the shortest path. A comparative analysis of these two approaches is conducted through tests on a water network model, and the results are presented in Section 5.

4.1 The Breadth-First Search Approach

In this approach, the hydraulic pressure head gradient in the network is determined using the breadth-first search algorithm (Cormen et al., 2022). This algorithm traverses the graph’s edges to uncover every reachable node from a root node, thereby constructing a breadth-first tree. Beginning at the root node, the algorithm identifies all nodes at the present depth—specifically, the neighbours of the root node in this instance, before progressing to nodes at the subsequent depth level. Subsequently, all nodes at that depth level are visited before advancing further to the subsequent depth level. This it-
Figure 2. Graphical representation of a small water network used as an example to illustrate the concept of hydraulic pressure head gradient identification using the breadth-first search approach.

The graphical representation in figure 2 depicts a small water network, which can also be considered as a small section of a larger water network, comprising 16 nodes, $v_1, \ldots, v_{16}$, and 22 edges. In the context of the example, let us consider node $v_2$ as the flushing node and node $v_7$ as the contamination source node within the network, as illustrated in the figure with a red star and a brown diamond. Figure 3 presents the breadth-first tree graph, with node $v_2$ as the root node. The breadth-first search algorithm starts at the root node $v_2$, which resides at depth level 1. Nodes $v_1$, $v_3$, and $v_{11}$, neighbouring $v_2$, are situated at depth level 2. Subsequent nodes are categorized into deeper levels accordingly. Notably, at level 4, node $v_5$ neighbours $v_{15}$; however, as $v_6$ has been visited at level 3, it is not represented under $v_5$ in level 5.

To eliminate the contaminant from the network, achieving a flow from the contamination source node to the flushing node necessitates a decreasing hydraulic pressure head gradient along this path. A trivial method involves imposing constraints on hydraulic pressure heads at the nodes, up to the nodes one level below the contamination source node. As previously mentioned, the hydraulic pressure head at nodes on a level is to be
Figure 3. Breadth-first tree graph of the graph presented in figure 2 considering node $v_2$ as the root node.

higher than the hydraulic pressure head at nodes on the level above. In the example network (Figure 2), with $v_2$ as the flushing node, the starting level is level 1. Thus, the hydraulic pressure head at nodes $v_1$, $v_3$ and $v_8$ is to exceed that at node $v_2$. Similarly, the hydraulic pressure heads at nodes $v_4$, $v_6$, $v_7$ and $v_{10}$ is to be greater than those at nodes $v_1$, $v_3$ and $v_8$. Ultimately, the hydraulic pressure heads at nodes $v_5$, $v_{16}$, $v_{14}$, $v_9$, and $v_{11}$ need to be higher than that at nodes $v_4$, $v_6$, $v_7$, and $v_{10}$. These pressure constraints are visually presented in Figure 4, using a colour bar: nodes with the highest hydraulic pressure head are depicted in yellow, nodes with the lowest hydraulic pressure head in aqua, and nodes with no pressure constraints in violet-blue. Solving the optimization problem with these constraints ensures the desired flow direction from the contamination source node towards the flushing node.

However, directly applying these pressure constraints using the breadth-first tree graph might impose unnecessary constraints on nodes that have minimal or no role in guiding the flow from the contamination source to the flushing node. In the case of the example network illustrated in Figure 4, nodes $v_4$, $v_{11}$ and $v_{16}$ are evident examples of this phenomenon—they do not contribute to directing the flow, yet they bear pressure constraints. This occurs because the breadth-first search algorithm simultaneously explores all directions from the flushing node. Considering all nodes from various directions, up to a level after the level containing the contamination source node, when defining pressure constraints results in imposing unnecessary constraints on nodes in directions away from the contamination source node. To mitigate this issue, we implement two heuristic rules to refine the results obtained from the breadth-first tree graph.

The branches starting from nodes neighbouring to the flushing node, i.e. the nodes on level 2, are denoted as the primary branches. Now, only the primary branch which contains the contamination source node is considered while defining the pressure constraints, this forms Rule 1. With that, for the example network (Figure 2), only nodes
Figure 4. Visually representation of the hydraulic pressure head constraints defined using the breath-first tree graph in figure 3 and presented using a colour bar.

Figure 5. The refined levels of pressure constraints at nodes defined using the breath-first tree graph, in Fig. 3, and the heuristic Rule 1.

$v_2, v_3, v_6, v_7, v_{14}$ and $v_9$ have pressure constraints. The pressure constraint levels refined using Rule 1 for the example network are presented in figure 5. With these pressure constraints, it is ensured that the direction of flow would be from the contamination source node to the flushing node.

However, without imposing constraints on the neighbouring nodes, except for the primary branch node, of the flushing node, contaminated water might potentially flow further than the flushing node. To prevent this scenario, all neighbouring nodes to the flushing node are also taken into account, imposing pressure constraints to ensure they maintain a higher hydraulic pressure head than the flushing node. Similarly, neighbouring nodes to nodes within the primary branch that currently lack pressure constraints are also included in this consideration. Nodes within the final level should inherently possess a greater hydraulic pressure head compared to the contamination source node due to already imposed pressure constraints. Consequently, there is no necessity to evaluate neighbouring nodes to nodes within the last level of the primary branch. This establishes Rule 2, wherein from the second-to-last level to the 1st level, all neighbouring nodes...
to each node encompassed within the refined pressure constraint levels, which are not already part of the refined pressure constraint levels, are to be taken into account. These nodes should adhere to pressure constraints ensuring a higher hydraulic pressure head than the node to which they are neighbouring. Once again, considering the example network (Figure 2) and the refined pressure constraint levels in figure 5, our starting point is \( v_6 \), whose neighbouring nodes include \( v_3, v_5, v_7, \) and \( v_{14} \). However, among these nodes, only \( v_5 \) is not already part of the refined pressure constraint levels. Thus, a pressure constraint is imposed to \( v_5 \), such that its hydraulic pressure head is to be higher than that at \( v_6 \). Likewise, pressure constraints are enforced on \( v_8, v_{10}, \) and \( v_1 \), such that the hydraulic pressure head at \( v_8 \) and \( v_{10} \) is to exceed that at \( v_7 \), and at \( v_1 \) and \( v_8 \) is to exceed that at \( v_2 \). These additional constraints aimed at preventing the spread of the contaminant are termed outer layer constraints, while the constraints derived from the refined breadth-first search tree graph results are termed inner layer constraints. Figure 6 visually represents the overall pressure constraints for the example network using a colour bar. As before, nodes with the highest hydraulic pressure head are depicted in yellow, those with the lowest hydraulic pressure head in aqua, and nodes with no pressure constraints in violet-blue.

The overall pressure constraints derived from the breadth-first search approach are given by (18). The inner layer constraints which are based on the refined pressure constraint levels are given by (18a), whereas the outer layer constraints are given by (18b).

\[
\begin{align*}
    h_n & \leq h_m, \quad \forall (n, m) \in \mathcal{L}_i \times \mathcal{L}_{i+1}, \quad \forall i | i \in \{1, \cdots, N - 1\}. \quad (18a) \\
    h_n & \leq h_m, \quad \forall (n, m) \in \mathcal{L}_i \times \Gamma_n, \quad \Gamma_n := \mathcal{N}(n) \setminus \bigcup_{j=1}^{N} \mathcal{L}_j \quad (18b)
\end{align*}
\]

In (18) \( h_n \) and \( h_m \) denote the hydraulic pressure head at nodes \( n \) and \( m \) respectively, \( \mathcal{L}_i \) denotes the set of nodes on \( i^{th} \) level of the refined pressure constraint levels and \( \mathcal{N}(n) \) denotes the set of nodes neighbouring the node \( n \).

The pressure constraints, (18) in association with the breadth-first tree levels, are to be used in the optimization problem for (16a), in the contamination impact mitigation control framework, when using the breadth-first search approach.
4.2 The Shortest Path Approach

In this approach, the pressure gradient constraints are determined using the shortest path algorithm. The starting point is to identify the shortest path, in terms of the shortest distance, between the contamination source node and the flushing node. The task of finding this shortest path is essentially a single-pair shortest-path problem, which can be effectively solved using Dijkstra’s algorithm (Cormen et al., 2022). This algorithm is specifically designed to find the shortest paths between two vertices in a directed graph with non-negative edge weights (Cormen et al., 2022). Within the water network graph, the lengths of pipes serve as the edge weights.

Furthermore, to establish a flow direction from the contamination source node to the flushing node, it is essential to have a decreasing hydraulic pressure head gradient along the path from the contamination source node to the flushing node. One approach to achieving this is by ensuring the highest pressure head at the contamination source node and subsequently creating a decreasing pressure head at the nodes along the shortest path leading to the flushing node. Again for better understanding, we will reference the network graph presented in Figure 7 as an example.

Figure 7 presents a graphical representation of a small water network, again it can be considered as a small section of a larger water network. This network section comprises 22 nodes \( v_1, \ldots, v_{22} \), and 26 edges, \( e_1, \ldots, e_{26} \), with equal lengths and diameter.

In the given scenario, let us designate \( v_1 \) as the flushing node, which is marked with a red star in Figure 7, and \( v_4 \) as the contamination source node, which is marked with a brown diamond, within the network. Consequently, the shortest path from \( v_4 \) to \( v_1 \) would be \( \{ v_4 \to e_1 \to v_3 \to e_2 \to v_2 \to e_3 \to v_1 \} \); the path is marked with a green line in the figure. Ensuring that the hydraulic pressure head at \( v_4 \) is higher than at \( v_3 \), at \( v_3 \) higher than at \( v_2 \), and at \( v_2 \) higher than at \( v_1 \) would facilitate a flow from the contamination source node towards the flushing node. Additionally, neighbouring nodes adja-
Figure 8. Graphical representation of a small water network, from Figure 7, with an inner layer formed along the shortest path and an outer layer formed around the inner layer.

The hydraulic pressure head at the nodes on the shortest path should possess a higher hydraulic pressure head to prevent the spread of contaminated water in the network. Thus, it is necessary that the hydraulic pressure head at $v_9$ and $v_{10}$ is higher than at $v_4$, and the hydraulic pressure head at $v_7$ and $v_8$ is greater than that at $v_3$, continuing in the same manner for $v_2$ and $v_1$.

However, with this approach, there is only one path for the contaminated water to be flushed out and this might put too much constraint on the network as consumer demands would be regulated to achieve this. Also, the idea of this work is to have multiple paths for the contaminated water to reach the flushing node with a decreasing pressure gradient in a general direction from the contamination source node to the flushing node. To achieve this parallel paths to the shortest path are identified such that the contaminated water could take any of these paths to reach the flushing node. We form two layers around this shortest path, viz. the inner layer and the outer layer. The inner layer is formed of parallel paths or nodes on the parallel paths to the shortest path, through which the contaminated water could reach the flushing node. This inner layer is formed around the shortest path. The hydraulic pressure head at the nodes in the inner layer has a decreasing gradient from the contamination source node to the flushing node. The outer layer is formed of the nodes around the inner layer. The purpose of the outer layer is to prevent the spread of the contaminated water to the network. Therefore, the nodes in the outer layer should have a higher hydraulic pressure head than the nodes in the inner layer.

For the network in figure 7, the shortest path from the contamination source node to the flushing node is again depicted by a green line in figure 8. Additionally, figure 8 showcases an inner layer, distinguished by a shade of blue along the shortest path, and an outer layer, indicated by a shade of red surrounding the inner layer. The identification of nodes within these layers is established based on two heuristic rules. The inner layer comprises nodes adjacent to those on the shortest path, excluding the contamination source node and the flushing node. As mentioned, the hydraulic pressure head should exhibit a decreasing gradient from the contamination source node to the flushing node along the shortest path. Moreover, nodes in the inner layer must display a similar pres-
sure gradient to ensure flow direction from the contamination source node to the flushing node. This guideline constitutes Rule 1, wherein nodes neighbouring those on the shortest path, from the second node to the second-to-last node, are included in the inner layer. These nodes maintain the same pressure constraint as their neighbouring nodes on the shortest path, provided they are not already part of the inner layer or the shortest path.

According to Rule 1 applied to the network illustrated in Figure 7, the inner layer consists of nodes $v_7$ and $v_8$, which are neighbours to $v_3$, and nodes $v_5$ and $v_6$, which are neighbours to $v_2$. It is worth noting that $v_7$ is also adjacent to $v_2$; however, since our starting point is $v_3$, and $v_7$ is already part of the inner layer from $v_3$, it is not reiterated, maintaining the pressure constraints identical to those of $v_3$. Consequently, the pressure constraints are as follows: the hydraulic pressure head at $v_4$ is to be higher than at $v_3$, $v_7$, and $v_8$; at $v_3$, $v_7$ and $v_8$ is to be higher than at $v_2$, $v_5$ and $v_6$; and at $v_2$, $v_5$ and $v_6$ is to be higher than at $v_1$. These pressure constraint levels for the inner layer nodes are visually represented in Figure 9 for clarity. Nodes placed at level $x$ should all possess a hydraulic pressure head greater than each node placed at level $(x - 1)$.

Much like the outer layer concept in the breadth-first search approach, here, an analogous outer layer is established surrounding the nodes of the inner layer, employing a similar heuristic rule to impede the spread of the contaminant. Rule 2 dictates the formation of this outer layer: All neighbouring nodes associated with each node contained within the inner layer including the shortest path, starting from the contamination node to the flushing node, which are not part of the inner layer, are to be taken into account. These nodes should adhere to pressure constraints ensuring a higher hydraulic pressure head than the node to which they are neighbouring. Considering the inner layer (Figure 9) within the example network (Figure 7), the starting node is $v_4$, which neighbours nodes $v_3$, $v_9$, $v_{10}$, and $v_{15}$. However, since $v_3$ is already part of the inner layer, $v_9$, $v_{10}$, and $v_{15}$ are considered as part of the outer layer, subject to pressure constraints such that the hydraulic pressure head at nodes $v_9$, $v_{10}$, and $v_{15}$ is to be higher than that at $v_4$. Similarly, the hydraulic pressure head at $v_{14}$ and $v_{15}$ is to be higher than that at $v_7$; at $v_{17}$ and $v_{18}$ is to be higher than at $v_8$; at $v_{11}$, $v_{13}$ and $v_{14}$ is to be higher than than at $v_5$; at $v_{16}$ is to be higher than that at $v_6$; and at $v_{11}$ and $v_{12}$ is to be higher than that at $v_1$. Figure 10 visually represents the overall pressure constraints for the example network defined using the shortest path approach using a colour bar. As before, nodes with

Figure 9. Levels of pressure constraints at nodes in the inner layer defined using the shortest path approach for the example network presented in Figure 7.
the highest hydraulic pressure head are depicted in yellow, those with the lowest hydraulic pressure head in aqua, and nodes with no pressure constraints in violet-blue.

Similar to the breadth-first search approach, the overall pressure constraints, including the inner layer and the outer layer constraints at nodes, are given as,

\[
\begin{align*}
    h_n &\leq h_m, \quad \forall (n, m) \in L_i \times L_{i+1}, \quad \forall i \mid i \in \{1, \ldots, N - 1\}, \quad (19a) \\
    h_n &\leq h_m, \quad \forall (n, m) \in L_i \times \Gamma_n, \quad \Gamma_n := \mathcal{N}(n) \setminus \bigcup_{j=1}^{N} L_j. \quad (19b)
\end{align*}
\]

Again in (19) \( h_n \) and \( h_m \) denote the hydraulic pressure head at nodes \( n \) and \( m \) respectively. \( L_i \) denotes the set of nodes on \( i^{th} \) level of the inner layer constraint levels including the shortest path and \( \mathcal{N}(n) \) denotes the set of nodes neighbouring the node \( n \).

When solving the problem of contamination impact mitigation using the shortest path approach, the pressure constraints, (19), are used in (16a) of the optimization problem.

The contamination impact mitigation control framework, with both breadth-first search approach and shortest path approach, is implemented and tested on a simulated water network model. The implementation and the results are presented in section 5.

5 Results and Discussion

In this section, the performance of the contamination impact mitigation control framework is evaluated on a simulated district-metered area (DMA) of a realistic water network called CY-DMA. CY-DMA replicates a real DMA in Cyprus, previously utilized by Vrachimis et al. (2021) and Vrachimis et al. (2020) to evaluate the performance of algorithms developed in their respective works.

In this study, we employ a modified version of CY-DMA. The EPANET model of the network, referred to as CY-DMA 2, is illustrated in Figure 11. This network comprises 90 junctions and 121 pipes. The original CY-DMA (Vrachimis et al., 2020) consists of only one reservoir or supply point, denoted as \( R1 \) and marked in the figure with...
Figure 11. The CY-DMA 2 network marked with the 16 flushing points and one of the contamination source nodes considered in this work.

5.1 CY-DMA 2 Test Results

To assess the framework's performance, 74 distinct test cases were examined, wherein each of the 74 nodes within the network, excluding the flushing nodes, were considered as the contamination source node in individual cases. However, due to space constraints, only detailed results for the contamination source node being node #60 are presented here. Nevertheless, a comprehensive summary of the framework's overall performance, derived from results across all test cases, is provided later in the section. In Figure 11, the contamination source node, #60, is indicated by a brown diamond.

The CY-DMA network is simulated in EPANET using the MATLAB-EPANET toolbox (Eliades et al., 2016). The simulation is carried out with a sampling time of 30 min and a simulation time of 48 hours. Commencing at the 0th hour in the simulation, contamination is injected from the source node as source type Mass Booster,
in which a fixed mass flow of contamination is added to the network (Rossman et al., 2000). In this work, for all the test cases a fixed contamination mass flow of 40 [mass/minute] is injected from the source node for a duration of 2 hours. For each pairing of the contamination source node and the flushing node, the simulation is executed three times. Once without employing any contamination impact mitigation control strategy, secondly using contamination impact mitigation control with pressure constraints obtained via the breadth-first search approach, and lastly with contamination impact mitigation control using pressure constraints obtained via the shortest path approach.

Once more, considering this as a pilot study, we assume zero delays in contamination detection and localization, i.e. the contamination impact mitigation control is initiated immediately upon the onset of contamination injection. The first step within the contamination impact mitigation control framework involves identifying the pressure constraints. As detailed in section 4, two distinct approaches are employed to identify these constraints: the breadth-first search approach and the shortest path approach. In each test case, the pressure constraints are determined using these two methods for every pair of the contamination source node and the 16 flushing nodes individually.

The pressure constraints for the test case with node #60 as the contamination source node and node #35 as the flushing node are visually illustrated in Figure 12 using a colour bar: nodes with the highest hydraulic pressure head are represented in yellow, the nodes with the lowest hydraulic pressure head are represented in aqua, and the nodes without pressure constraints are represented in violet-blue. For this specific pairing of contamination source and flushing nodes, the pressure constraints are identified through two approaches: the subplot (a) in Figure 12 displays the constraints obtained using the breadth-first search approach, while the subplot (b) depicts those identified via the shortest path approach. Similarly, considering the same contamination source node but now with node #48 as the flushing node, the pressure constraints are visually depicted in Figure 13. Correspondingly, the subplot (a) in Figure 13 demonstrates the pressure constraints obtained from the breadth-first search approach, and the subplot (b) presents those obtained using the shortest path approach. Observing both sets of figures, Figure 12 and 13, it is evident that the defined pressure constraints enable the contaminated water to flow from the contaminated source node towards the flushing node without spreading throughout the network. Similarly, pressure constraints for other test cases were also obtained, but due to space limitations, they are not presented here.

The identified pressure constraints form the constraints of the optimization problem given by (14), (15), and (16). In the tests, the supply node $R_2$ is arbitrarily designated as the reference node. It is essential to note that this choice has no impact on the simulation test, as mentioned earlier. In relation to the third objective in the optimization problem described in (14), node #51 is arbitrarily selected as the target node(s); specifically, $p_t = p_{51}$. Further parameters pertaining to the optimization problem are detailed in Table 1, which are based on the nominal operating conditions of the real network.

The optimization problem is minimized with respect to the decision variables, consumer flows, $d_c$, and the supply pressure, $p_s$. The optimization problem has been implemented in CasADi, which is an open source tool for nonlinear optimization (Andersson et al., 2019), in its MATLAB (MATLAB, 2022) interface. The optimal values of the decision variables are then applied to the EPANET simulation model to perform a hydraulic and quality analysis. All the cases are simulated with the same contamination characteristics. The optimization problem is solved and the network is subsequently simulated using the obtained solution at each time step.

The results from the three simulation tests for each pair of the contamination source node and flushing node are compared on various parameters, which are listed below:
Figure 12. Visually representation of the hydraulic pressure head constraints in CY-DMA network for the test case with node #60 as the contamination source node and node #35 as the flushing node defined using (a) the breadth-first search approach and (b) the shortest path approach, and presented using a colour bar.

Figure 13. Visually representation of the hydraulic pressure head constraints in CY-DMA network for the test case with node #60 as the contamination source node and node #48 as the flushing node defined using (a) the breadth-first search approach and (b) the shortest path approach, and presented using a colour bar.
Table 1. Parameters for the optimization problem.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_t^*$</td>
<td>3.5 bar</td>
<td>$p_{min}^*$</td>
<td>2.5 bar</td>
</tr>
<tr>
<td>$p_{smin}$</td>
<td>3 bar</td>
<td>$p_{smax}$</td>
<td>6 bar</td>
</tr>
<tr>
<td>$d_{min}^{flush}$</td>
<td>4 m³/h</td>
<td>$d_{max}^{flush}$</td>
<td>10 m³/h</td>
</tr>
<tr>
<td>$d_{smax}$</td>
<td>80 m³/h</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- **Contaminated water consumed [m³]:** This represents the total water consumed with the contamination concentration higher than 0.01 mg/L, during the simulation period.

- **Time till contaminant free network [hr]:** This represents the time-frame from the initiation of contamination injection to the point where the contamination concentration at every node and link in the network falls below 0.01 mg/L. Note that the simulation sampling time is set at 30 min and therefore this parameter is also measured every 30 min.

- **Contaminated water flushed from the flushing node [m³]:** This represents the total water flushed from the flushing node with the contamination concentration higher than 0.01 mg/L, during the simulation period.

- **Change in consumption [%]:** This represents the percentage change in the consumer flow resulting from contamination impact mitigation control.

To illustrate, a summary of the results for the case with node #60 as the contamination source node is presented in Table 2 and Table 3.

The first rows in Table 2 and Table 3 present the results of contamination impact under nominal conditions. If no control strategies are applied the contaminated water consumed is 49.75 m³ and it would take 12.5 hours for the network to be contamination-free.

Further, Table 2 illustrates the outcomes of contamination impact mitigation control, employing pressure constraints determined through the breadth-first search approach. The values in parentheses beneath the headings ‘Contaminated water consumed’ and ‘Time till contamination-free network’ indicate the percentage change from the nominal values provided in the first row of the table. Moreover, a downward arrow denotes a reduction, while an upward arrow denotes an increase in the value. Similarly, Table 3 showcases the results of contamination impact mitigation control, employing pressure constraints determined through the shortest path approach.

From these results, it is evident that contamination impact mitigation control is able to effectively diminish the impact of contamination with minimal inconvenience to consumers, regardless of the approach employed. However, the results from both approaches yield varying degrees of effectiveness in flushing from different nodes. For instance, as shown in Table 3, when node #48 serves as the flushing node, a 96% reduction in contaminated water consumption is achieved, with only 7.63% of consumer demands remaining unmet. Conversely, when node #68 is the flushing node, only a 71% reduction in contaminated water consumption is attained, with 14.49% of consumer demands not being met. Furthermore, the two approaches produce different outcomes for the same flushing node. For node #35 as the flushing node, the control with the shortest path approach results in an 83% reduction in contaminated water consumption, with only 3.31% of consumer demands unmet. Conversely, the control with the breadth-first search approach
<table>
<thead>
<tr>
<th>Flushing Node</th>
<th>Contaminated water consumed [m$^3$]</th>
<th>Time till contaminant free network [hr]</th>
<th>Contaminated water flushed from the flushing node [m$^3$]</th>
<th>Change in consumption [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>49.75</td>
<td>12.5</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>11</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>15</td>
<td>2.15 (96% ↓)</td>
<td>Inf (Inf% ↑)</td>
<td>0</td>
<td>97.18%</td>
</tr>
<tr>
<td>21</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>24</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>35</td>
<td>3.03 (94% ↓)</td>
<td>5 (60% ↓)</td>
<td>35</td>
<td>3.44%</td>
</tr>
<tr>
<td>43</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>46</td>
<td>2.39 (95% ↓)</td>
<td>4.5 (64% ↓)</td>
<td>25</td>
<td>16.43%</td>
</tr>
<tr>
<td>48</td>
<td>2.16 (96% ↓)</td>
<td>4.5 (64% ↓)</td>
<td>12</td>
<td>6.76%</td>
</tr>
<tr>
<td>51</td>
<td>6.03 (88% ↓)</td>
<td>9.5 (24% ↓)</td>
<td>0</td>
<td>24.13%</td>
</tr>
<tr>
<td>59</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>68</td>
<td>7.73 (84% ↓)</td>
<td>28.5 (128% ↑)</td>
<td>45</td>
<td>9.92%</td>
</tr>
<tr>
<td>71</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>75</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>82</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>90</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

**Table 2.** Contamination impact results with contamination impact mitigation control in which pressure constraints are obtained via the breadth-first search approach. The values in parentheses indicate the percentage change from the nominal values provided in the first row of the table.
Table 3. Contamination impact results with contamination impact mitigation control in which pressure constraints are obtained via the shortest path approach. The values in parentheses indicate the percentage change from the nominal values provided in the first row of the table.
yields a 94% reduction in contaminated water consumption, with 3.44% of consumer demands not being met.

Among the 16 flushing nodes, results for 10 nodes in both Table 2 and Table 3 are indicated as ‘N/A’, signifying an infeasible problem. Discussion regarding these feasibility issues is provided in Section 5.2. An intriguing observation pertains to the flushing node #15, as detailed in Tables 2 and 3, where the time until the network becomes contaminant-free is given as ‘Inf’, indicating that the network remains contaminated throughout the simulation period. In such instances, contaminants persist in one or multiple pipelines due to zero flow within them. Additionally, another noteworthy finding relates to the flushing node #51, as shown in Table 2, where there is an 88% reduction in consumed contaminated water, yet the flushed contaminated water quantity is nil. These scenarios arise when the optimization process fails to establish a pressure gradient to direct contaminated water toward the flushing node, yet effectively containing its spread. Consequently, although a higher concentration of contaminated water is consumed, the quantity remains limited, resulting in a reduction in overall contaminated water consumption.

Among the feasible solution nodes, the most efficient node for flushing can be selected using (17). In this test, we prioritize the reduction in contaminated water consumption as the highest priority, followed by minimizing the consumer demands not being met, and lastly, minimizing the time it takes for the network to be contaminant-free. Accordingly, assigning the weights $w_{CWC} = 1$, $w_{TTCF} = 0.2$, and $w_{CC} = 0.7$ in (17), node #48 emerges as the most effective flushing node for node #60 being the contamination source node, under the breadth-first search approach. Here, a substantial 96% reduction in contaminated water consumption is achieved, with only 6.76% of consumer demands remaining unmet, and the network becoming contaminant-free 64% faster. Conversely, employing the shortest path approach designates node #46 as the optimal flushing node, resulting in a similar 95% reduction in contaminated water consumption, with 3.43% of consumer demands remaining unmet, and once again, the network becoming contaminant-free 64% faster.

As mentioned earlier, similar tests were carried out considering each of the other 73 nodes as the contamination source node individually. In certain cases, the contamination impact mitigation control could not provide a feasible solution with any of the flushing nodes. Again, this infeasibility issue is discussed in section 5.2. For the remaining cases (60 test cases), summary statistics results considering the most efficient node for flushing are presented using box plots in Figures 14, 15, and 16.

Figure 14 presents a box plot illustrating the distribution of the percentage change in contaminated water consumed under both breadth-first search and shortest path approaches. Under the breadth-first search approach, the percentage reduction in contaminated water consumed ranges from 60.99% to 100%, with outliers represented by red crosses. The median reduction is 94.07%, indicating that for half of the test cases, the reduction is more than 94.07%, while only for a quarter of the cases, the reduction is less than 81.83%. Similarly, under the shortest path approach, the percentage reduction in contaminated water consumed ranges from 80.65% to 100%. The 75th percentile is 97.94%, and the median is 94.94%, and the 25th percentile is 89.59%. There are 4 outliers under the breadth-first search approach, with the worst performance showing only an 11.38% reduction in contaminated water consumed. Under the shortest path approach, there are 5 outliers, with the worst performance demonstrating a 32.40% reduction in contaminated water consumed. These results stem from test cases where the optimization problem fails to create the desired pressure gradient with the given freedom on consumer demand regulation.

In Figure 15, a box plot illustrating the distribution of the percentage change in the time until the network is contamination-free is presented. Under the breadth-first
search approach, it ranges between -3.45% and 89.70%, while under the shortest path approach, it ranges between 4.35% and 91.04%. The negative sign indicates that the network becomes contaminant-free slower than the nominal case. In certain instances, the control framework can only establish a minor pressure gradient, resulting in a low flow of contaminated water. Therefore, we observe varying degrees of change in the time until the network is contaminant-free, with an outlier taking more than twice the time to achieve contamination-free status with the control framework compared to the nominal case.

Figure 16 depicts a box plot illustrating the distribution of the percentage change in water consumption. With the breadth-first search approach, this change ranges between 17.20% and 0.06%, while with the shortest path approach, it spans from 12.75% to 0.06%. In both scenarios, around three-fourths of the cases exhibit a reduction in consumer demands of less than approximately 7.35%, with half of the cases even falling below 3.5%. However, there are notable outliers where the change in consumption is substantially higher. Specifically, under the breadth-first search approach, the highest change in consumption reaches 61.78%, whereas under the shortest path approach, it peaks at 21.39%.

In the further section, the feasibility issues of the optimization problem are discussed.

5.2 Optimization Problem Feasibility

The contamination impact mitigation control relies on an optimization problem, making it susceptible to feasibility issues. While a comprehensive feasibility study has not been conducted in this work, we would like to highlight some observations from the tests regarding the feasibility of the optimization problem.

From Table 2 and Table 3, it is evident that the optimization problem yields infeasibility errors for multiple cases. Similar observations were noted for contamination source nodes located at a distance from the flushing node. In such instances, the opti-
Figure 15. Box plot illustrating the distribution of percentage change in the time until the network is contamination-free with respect to the nominal conditions.

Figure 16. Box plot illustrating the distribution of percentage change in the consumption of water with respect to the nominal conditions.
The optimization problem is unable to maintain the pressure constraints (the required pressure gradient), given the limited controllability over the network.

We also observe that for the same pair of the contamination source node and the flushing node, the shortest path approach typically yields a lower number of constraints compared to the breadth-first search approach. This results in a higher computational load for the breadth-first search approach. This disparity could be attributed to the breadth-first search approach exploring multiple pathways from the flushing node to the contamination source node simultaneously, thereby leading to more constraints. Conversely, in the shortest path approach, we initiate from the shortest path between the flushing node and the contamination source node and construct layers around it, consequently limiting the number of constraints. However, despite the shorter computational load associated with the shortest path approach, there are certain cases where the breadth-first search approach outperforms it in terms of contamination impact mitigation. The breadth-first search approach might be better suited for denser networks with highly interconnected pipelines and pathways. Further studies are required to compare the two approaches in greater detail.

Moreover, the framework encounters an infeasibility error for contamination source nodes where the direction of flow in the connected edges remains fixed, regardless of the chosen flushing node. One such scenario is a leaf node, which refers to a node with a degree of one, meaning it is connected to the network via only one edge. In the CY-DMA network (Figure 11), nodes #4, #12, and #22 are examples of leaf nodes. In these instances, water flow consistently moves from the network toward the leaf node, rendering a decreasing pressure gradient from the contamination source node to any flushing node unattainable. Consequently, attempting to solve the optimization problem invariably leads to an Infeasible Problem error. However, it is noteworthy that in these cases, contamination does not spread within the network; instead, all contamination is contained and consumed within the leaf node.

Further analytical feasibility analysis of the optimization problem is reserved for future work.

6 Conclusion

The work introduces a control framework designed to mitigate the potential impact of contamination threats. By establishing a decreasing pressure gradient from the contamination source node to a flushing node, the framework facilitates the movement of contaminated water towards the flushing node. Two distinct methods, viz. the breadth-first search approach and the shortest path approach, are proposed to define these decreasing pressure gradients as hydraulic pressure head constraints. Formulated as an optimization problem, the control framework incorporates these hydraulic pressure head constraints as constraints of the optimization problem. The decision variables of the optimization problem include the supply pressure and consumer demands, which are regulated to create the desired pressure gradient. To evaluate the framework’s efficacy, experiments were conducted using a model of a real-world network, CY-DMA. Results indicate that in over 75% of cases, the framework successfully reduces the consumption of contaminated water by a minimum of 81.84%, with only a maximum of 8.22% of consumer demands left unmet.

Despite making strong assumptions regarding knowledge of contamination characteristics and the response time, this pilot study illustrates the potential for achieving significant reductions in contamination impact with minimal inconvenience to consumers. This study lays the groundwork for further development of contamination mitigation strategies and other applications centred around hydraulic pressure head regulation through consumer flow control. Future research should focus on validating the framework using
diverse networks with different configurations. Additionally, conducting a detailed feasibility study of the optimization problem, devising optimal strategies for identifying pressure constraints and formulating optimizations to reduce computational load are areas of further investigation.

Data Availability Statement

The models, code and data generated during this study are available at https://zenodo.org/doi/10.5281/zenodo.11082064.

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