Equivalence of Electro-optic Modulator Electrical and Optical Sidebands via Neumann’s Addition Theorem for Bessel Functions

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May 09, 2024

Abstract

In photonic circuits and optical communications links, the electro-optic modulator (EOM) can be considered the heart of the entire system, allowing analog and digital data in the RF or microwave domain to be modulated onto an optical carrier for transmission over low-loss, high-bandwidth optical links. EOMs can be fabricated in a number of material systems and operated in various modes that optimize some measure of performance depending on the application. One common configuration is the push-pull configuration, where a phase modulator in each arm of a Mach-Zehnder interferometer is modulated with equal amplitude but opposite phase. The resulting optical fields of the EOM, and most importantly, the photodetected currents generated from those fields, are well-characterized in the literature. Here, we explicitly demonstrate how the complex optical fields, represented by a sinusoidal series with Bessel function amplitudes, are combined by the detector nonlinearity into RF and microwave currents, also represented by a sinusoidal series with Bessel function amplitudes, and extracted at the end of the optical link.
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1 Electric Field Representation of the Push-Pull EOM

The electric fields of the electro-optic modulator (EOM), particularly in the push-pull configuration, are well-characterized in the academic and technical literature. For a push-pull EOM driven by a DC-bias voltage $V_{DC}$ and an RF modulating voltage $V_{RF}$, the total applied voltage is given by

$$V_{in}(t) = V_{DC} + V_{RF} \sin(\Omega t),$$

where $\Omega$ represents the RF driving frequency, in contrast to $\omega$, which will be used to describe the optical carrier frequency. The electro-optic modulator with input optical field $E(t) = E_0 \cos(\omega t)$ has complex, complimentary optical outputs given by [1]

$$\begin{bmatrix} E_1(t) \\ E_2(t) \end{bmatrix} = \frac{\gamma}{2} e^{j\omega t} \begin{bmatrix} e^{j\phi(t)/2} - e^{-j\phi(t)/2} \\ j e^{j\phi(t)/2} + j e^{-j\phi(t)/2} \end{bmatrix}$$

The time-dependent phase $\phi(t)$ consists of a DC component due to the applied DC voltage and the DC half-wave voltage $V_g(0)$, given by $\phi_{DC} = \pi V_{DC}/V_g(0)$, and an RF phase proportional to the applied RF voltage and the RF half-wave voltage $V_g(\Omega)$, given by $\phi_{RF} = \pi V_{RF}/V_g(\Omega)$, such that $\phi(t) = \phi_{DC} + \phi_{RF}$. The constant $\gamma$ accounts for losses in the MZM and represents the optical field amplitude $E_0$ in terms of the incident optical power $P_0$.

These definitions can be applied to the upper electric field equations to solve for $E_1(t)$ in terms of its real and complex electric fields.

$$E_1(t) = \frac{\gamma}{2} e^{j\omega t} \begin{bmatrix} e^{j\phi(t)/2} - e^{-j\phi(t)/2} \\ e^{j\phi(t)/2} - e^{-j\phi(t)/2} \end{bmatrix}$$

$$= \frac{\gamma}{2} e^{j\omega t} \begin{bmatrix} e^{\frac{\phi_{DC}}{2}} e^{\frac{\phi_{RF}}{2} \sin(\Omega t)} - e^{-\frac{\phi_{DC}}{2}} e^{-\frac{\phi_{RF}}{2} \sin(\Omega t)} \\ e^{\frac{\phi_{DC}}{2}} e^{\frac{\phi_{RF}}{2} \sin(\Omega t)} - e^{-\frac{\phi_{DC}}{2}} e^{-\frac{\phi_{RF}}{2} \sin(\Omega t)} \end{bmatrix}$$

(3)
The complex exponential terms containing the RF driving frequency can be expanded using the appropriate Jacobi-Anger expansions.

\[
e^{j\frac{\Phi_{RF}}{2}\sin(\Omega t)} = \sum_{n=-\infty}^{+\infty} J_n \left( \frac{\Phi_{RF}}{2} \right) e^{jn\Omega t} \tag{4}
\]

\[
e^{-j\frac{\Phi_{RF}}{2}\sin(\Omega t)} = \sum_{n=-\infty}^{+\infty} (-1)^n J_n \left( \frac{\Phi_{RF}}{2} \right) e^{jn\Omega t} \tag{5}
\]

The coefficients of the expansion, \( J_n \), are Bessel functions of the first kind of order \( n \) with arguments that depend on the RF modulation index \( \pi V_{RF}/2 V_{DC}(\Omega) \). Inserting Eqs. 4 and 5 into Eq. 3, and expanding the complex exponential terms for the optical field and DC phase via the Euler identity, the output field can be written in terms of the real and imaginary components.

\[
E_1(t) = \frac{\gamma}{2} e^{j\omega t} \left[ e^{j\frac{\Phi_{DC}}{2}} \sum_{n=-\infty}^{+\infty} J_n \left( \frac{\Phi_{RF}}{2} \right) e^{jn\Omega t} - e^{-j\frac{\Phi_{DC}}{2}} \sum_{n=-\infty}^{+\infty} (-1)^n J_n \left( \frac{\Phi_{RF}}{2} \right) e^{jn\Omega t} \right]
\]

\[
= \frac{\gamma}{2} \left( e^{j\frac{\Phi_{DC}}{2}} \sum_{n=-\infty}^{+\infty} J_n \left( \frac{\Phi_{RF}}{2} \right) e^{j(\omega+n\Omega)t} - e^{-j\frac{\Phi_{DC}}{2}} \sum_{n=-\infty}^{+\infty} (-1)^n J_n \left( \frac{\Phi_{RF}}{2} \right) e^{j(\omega+n\Omega)t} \right)
\]

\[
= \frac{\gamma}{2} \left( \cos\left(\frac{\Phi_{DC}}{2}\right) + j \sin\left(\frac{\Phi_{DC}}{2}\right) \right) \sum_{n=-\infty}^{+\infty} J_n \left( \frac{\Phi_{RF}}{2} \right) \left[ \cos(\omega + n\Omega)t + j \sin(\omega + n\Omega)t \right]
\]

\[
- \frac{\gamma}{2} \left( \cos\left(\frac{\Phi_{DC}}{2}\right) - j \sin\left(\frac{\Phi_{DC}}{2}\right) \right) \sum_{n=-\infty}^{+\infty} (-1)^n J_n \left( \frac{\Phi_{RF}}{2} \right) \left[ \cos(\omega + n\Omega)t + j \sin(\omega + n\Omega)t \right] \tag{6}
\]

The real and imaginary optical fields are given respectively as

\[
Re(E_1) = \gamma \cos \left( \frac{\Phi_{DC}}{2} \right) \sum_{n=-\infty}^{+\infty} J_{2n+1} \left( \frac{\Phi_{RF}}{2} \right) \cos(\omega + (2n + 1)\Omega)t
\]

\[
- \gamma \sin \left( \frac{\Phi_{DC}}{2} \right) \sum_{n=-\infty}^{+\infty} J_{2n} \left( \frac{\Phi_{RF}}{2} \right) \sin(\omega + 2n\Omega)t \tag{7}
\]

\[
Im(E_1) = \gamma \cos \left( \frac{\Phi_{DC}}{2} \right) \sum_{n=-\infty}^{+\infty} J_{2n+1} \left( \frac{\Phi_{RF}}{2} \right) \sin(\omega + (2n + 1)\Omega)t
\]

\[
+ \gamma \sin \left( \frac{\Phi_{DC}}{2} \right) \sum_{n=-\infty}^{+\infty} J_{2n} \left( \frac{\Phi_{RF}}{2} \right) \cos(\omega + 2n\Omega)t \tag{8}
\]

The real and imaginary fields of the complementary output \( E_2 \) can be similarly derived, but are simply stated below as

\[
Re(E_2) = -\gamma \sin \left( \frac{\Phi_{DC}}{2} \right) \sum_{n=-\infty}^{+\infty} J_{2n+1} \left( \frac{\Phi_{RF}}{2} \right) \cos(\omega + (2n + 1)\Omega)t
\]

\[
- \gamma \cos \left( \frac{\Phi_{DC}}{2} \right) \sum_{n=-\infty}^{+\infty} J_{2n} \left( \frac{\Phi_{RF}}{2} \right) \sin(\omega + 2n\Omega)t \tag{9}
\]
Im(E2) = −γ sin \left( \frac{\phi_{DC}}{2} \right) \sum_{n=-\infty}^{+\infty} J_{2n+1} \left( \frac{\phi_{RF}}{2} \right) \sin[\omega + (2n + 1)\Omega] t \\
+ γ cos \left( \frac{\phi_{DC}}{2} \right) \sum_{n=-\infty}^{+\infty} J_{2n} \left( \frac{\phi_{RF}}{2} \right) \cos[\omega + 2n\Omega] t \quad (10)

These expressions for the EOM optical fields are well-documented, and their dependence on the DC bias voltage can be used to set the modulator operating point in peak, quadrature, or null (φ_{DC} = \pi, \pi/2, 0) depending on the required application. However, the optical fields of the modulator are normally not of direct interest in RF photonic links, but rather the photodetected current.

2 Photodetected Output of the Push-Pull EOM

The photodetected current at the output of a photodetector is given by \( I(t) = \mathcal{R}P_0 \), where \( \mathcal{R} \) is the detector responsivity in A/W and \( P_0 \) is the incident optical power proportional to \( E_{0}^2(t) = E_0(t)E_0^*(t) \). The resulting currents due to the complementary fields \( E_1(t) \) and \( E_2(t) \) are given in [1] by Eqs. 6.17 and 6.18 as

\[
I_{1,2}(t) = \frac{\mathcal{R}g_{o}l_{mzm}P_0}{2} [1 \mp \cos(\phi(t))]
\]

\[
I_{1,2}(t) = I_{dc,q} \mp \cos(\phi_{DC})I_{dc,q}J_0(\phi_{RF})
\]
\[
\mp 2 \cos(\phi_{DC})I_{dc,q} \sum_{m=1}^{\infty} J_{2m}(\phi_{RF}) \cos(2m\Omega t)
\]
\[
\pm 2 \sin(\phi_{DC})I_{dc,q} \sum_{n=0}^{\infty} J_{2n+1}(\phi_{RF}) \sin[(2m+1)\Omega t]
\]

The expression in Eq.12 is obtained in a rather straightforward manner due to the complex exponential nature of the fields by multiplying Eq.2 by its complex conjugate, substituting the expression for \( \phi(t) \), and applying the Jacobi-Anger expansion for the cosine term given by

\[
\cos(\phi_{RF} \sin(\Omega t)) = J_0(\phi_{RF}) + 2 \sum_{n=1}^{\infty} J_{2n}(\phi_{RF}) \cos(2n\Omega t)
\]

In the textbook treatment the upper sign represents the photocurrent due to \( E_1(t) \) and the lower sign represent the photocurrent due to \( E_2(t) \). The constant \( g_o \) represent gain/loss between the MZM and the photodiode, \( l_{mzm} \) is the loss in the MZM, and \( P_0 \) is the incident optical power into the MZM. These terms, along with the responsivity, are grouped together in Eq.12 to represent the DC photocurrent at quadrature bias.

An important observation is that the DC and RF phase terms in the photocurrents in Eq.12 are no longer divided by 2, and it may seem intuitive that this derives from the square law nature of the photodetector in converting the incident optical fields into photocurrents. Another consequence of squaring the fields is that the harmonics of the optical fields will mix in infinite combinations to produce the electrical response at a given harmonic of the applied RF field. For example, the RF current at the driving frequency \( \Omega \) is caused by the mixing of all optical harmonic pairs separated by \( \Omega \), and in general RF frequencies \( n\Omega \) are due to all optical harmonic pairs separated by \( n\Omega \).
3 Optical and RF Bessel Sideband Equivalence via the Neumann Addition Theorem

In this section we demonstrate how the Bessel function coefficients in the RF photocurrent expressions can be synthesized from the Bessel function coefficients in the optical field representation by applying the Neumann Addition Theorem of Bessel Functions. The Neumann Addition Theorem for Bessel functions of the first kind with integer order \( \nu \) is given in [2] as

\[
J_\nu(u \pm w) = \sum_{k=-\infty}^{+\infty} J_{\nu\pm k}(u)J_k(w)
\]  

(14)

Several special cases of the Neumann Addition Theorem are also useful.

\[
J_0^2(z) + 2 \sum_{k=1}^{\infty} J_k^2(z) = 1
\]  

(15)

\[
\sum_{k=0}^{2n} (-1)^k J_k(z)J_{2n-k}(z) + 2 \sum_{k=1}^{\infty} J_k(z)J_{2n+k}(z) = 0 \quad n \geq 1
\]  

(16)

\[
\sum_{k=0}^{n} J_k(z)J_{n-k}(z) + 2 \sum_{k=1}^{\infty} (-1)^k J_k(z)J_{n+k}(z) = J_n(2z)
\]  

(17)

The square of a complex field \( E = E_R + jE_I \) can be shown to be \( E^2 = E_R^2 + E_I^2 \). First, squaring the real field given by Eq.7 yields

\[
E_{1,R}^2 = \gamma^2 \left[ \sum_{n=-\infty}^{+\infty} \cos\left(\frac{\phi_{DC}}{2}\right)J_{2n+1}\left(\frac{\phi_{RF}}{2}\right)\cos[\omega + (2n+1)\Omega]t - \sin\left(\frac{\phi_{DC}}{2}\right)J_{2n}\left(\frac{\phi_{RF}}{2}\right)\sin[\omega + 2n\Omega]t \right]
\]

* \[
\sum_{p=-\infty}^{+\infty} \cos\left(\frac{\phi_{DC}}{2}\right)J_{2p+1}\left(\frac{\phi_{RF}}{2}\right)\cos[\omega + (2p+1)\Omega]t - \sin\left(\frac{\phi_{DC}}{2}\right)J_{2p}\left(\frac{\phi_{RF}}{2}\right)\sin[\omega + 2p\Omega]t
\]

(18)

Multiplying the two infinite sums term by term and grouping equivalent sinusoidal terms gives

\[
E_{1,R}^2 = \gamma^2 \sum_{n=-\infty}^{+\infty} \sum_{p=-\infty}^{+\infty} \cos\left(\frac{\phi_{DC}}{2}\right)J_{2n+1}\left(\frac{\phi_{RF}}{2}\right)J_{2p+1}\left(\frac{\phi_{RF}}{2}\right)\cos[2(n-p)\Omega]t +
\]

\[
\sin\left(\frac{\phi_{DC}}{2}\right)J_{2n}\left(\frac{\phi_{RF}}{2}\right)J_{2p}\left(\frac{\phi_{RF}}{2}\right)\cos[2(n-p)\Omega]t +
\]

\[
\cos\left(\frac{\phi_{DC}}{2}\right)\sin\left(\frac{\phi_{DC}}{2}\right)J_{2n+1}\left(\frac{\phi_{RF}}{2}\right)J_{2p}\left(\frac{\phi_{RF}}{2}\right)\sin[2(n-p)+1]\Omega]t +
\]

\[
J_{2p+1}\left(\frac{\phi_{RF}}{2}\right)J_{2n}\left(\frac{\phi_{RF}}{2}\right)\sin[2(p-n)+1]\Omega]t
\]

(19)

Applying the identities \( \cos^2\left(\frac{\phi_{DC}}{2}\right) = \frac{1}{2}[1 + \cos(\phi_{DC})] \), \( \sin^2\left(\frac{\phi_{DC}}{2}\right) = \frac{1}{2}[1 - \cos(\phi_{DC})] \), and \( \cos\left(\frac{\phi_{DC}}{2}\right)\sin\left(\frac{\phi_{DC}}{2}\right) = \frac{1}{2}\sin(\phi_{DC}) \) to Eq.19 eliminates the one-half dependence of the DC phase term. Collecting terms in DC phase gives
\[ E_{1,R}^2 = \frac{\gamma^2}{4} \sum_{n=-\infty}^{+\infty} \sum_{p=-\infty}^{+\infty} \left[ J_{2n+1} \left( \frac{\phi_{RF}}{2} \right) J_{2p+1} \left( \frac{\phi_{RF}}{2} \right) + J_{2n} \left( \frac{\phi_{RF}}{2} \right) J_{2p} \left( \frac{\phi_{RF}}{2} \right) \right] \cos[2(n-p)\Omega t] \]

\[ + \cos(\phi_{DC}) \left[ J_{2n+1} \left( \frac{\phi_{RF}}{2} \right) J_{2p+1} \left( \frac{\phi_{RF}}{2} \right) - J_{2n} \left( \frac{\phi_{RF}}{2} \right) J_{2p} \left( \frac{\phi_{RF}}{2} \right) \right] \cos[2(n-p)\Omega t] \]

\[ + \sin(\phi_{DC}) \left[ J_{2n+1} \left( \frac{\phi_{RF}}{2} \right) J_{2p} \left( \frac{\phi_{RF}}{2} \right) \sin [(2(n-p)+1)\Omega t] + J_{2p+1} \left( \frac{\phi_{RF}}{2} \right) J_{2n} \left( \frac{\phi_{RF}}{2} \right) \sin [(2(p-n)+1)\Omega t] \right] \]  

(20)

The two terms on the third line of Eq.20 appear to be the same, under the condition that the \( n \) and \( p \) variables are interchangeable and are summed over the same values. However, this is not the case and the sums must be evaluated separately. Applying the same analysis to the imaginary component of the optical field shows that the square of both the real and imaginary fields are equivalent, so the analysis will not be repeated. Therefore, the total squared field is

\[ E_1^2 = \frac{\gamma^2}{2} \sum_{n=-\infty}^{+\infty} \sum_{p=-\infty}^{+\infty} \left[ J_{2n+1} \left( \frac{\phi_{RF}}{2} \right) J_{2p+1} \left( \frac{\phi_{RF}}{2} \right) + J_{2n} \left( \frac{\phi_{RF}}{2} \right) J_{2p} \left( \frac{\phi_{RF}}{2} \right) \right] \cos[2(n-p)\Omega t] \]

\[ + \cos(\phi_{DC}) \left[ J_{2n+1} \left( \frac{\phi_{RF}}{2} \right) J_{2p+1} \left( \frac{\phi_{RF}}{2} \right) - J_{2n} \left( \frac{\phi_{RF}}{2} \right) J_{2p} \left( \frac{\phi_{RF}}{2} \right) \right] \cos[2(n-p)\Omega t] \]

\[ + \sin(\phi_{DC}) \left[ J_{2n+1} \left( \frac{\phi_{RF}}{2} \right) J_{2p} \left( \frac{\phi_{RF}}{2} \right) \sin [(2(n-p)+1)\Omega t] + J_{2p+1} \left( \frac{\phi_{RF}}{2} \right) J_{2n} \left( \frac{\phi_{RF}}{2} \right) \sin [(2(p-n)+1)\Omega t] \right] \]  

(21)

Factoring out the negative sign in the \( \cos(\phi_{DC}) \) term and making the change of variable \( n - p = l \) for the trigonometric terms of Eq.21 with argument \( n - p \), and \( p - n = -(l + 1) \) in the second half of the \( \sin(\phi_{DC}) \) term, the squared field is rewritten as

\[ E_1^2 = \frac{\gamma^2}{2} \sum_{l=-\infty}^{+\infty} \sum_{p=-\infty}^{+\infty} \left[ J_{2l+2p+1} \left( \frac{\phi_{RF}}{2} \right) J_{2p+1} \left( \frac{\phi_{RF}}{2} \right) + J_{2l+2p} \left( \frac{\phi_{RF}}{2} \right) J_{2p} \left( \frac{\phi_{RF}}{2} \right) \right] \cos[2l\Omega t] \]

\[ - \cos(\phi_{DC}) \left[ -J_{2l+2p+1} \left( \frac{\phi_{RF}}{2} \right) J_{2p+1} \left( \frac{\phi_{RF}}{2} \right) + J_{2l+2p} \left( \frac{\phi_{RF}}{2} \right) J_{2p} \left( \frac{\phi_{RF}}{2} \right) \right] \cos[2l\Omega t] \]

\[ + \sin(\phi_{DC}) \left[ J_{2l+2p+1} \left( \frac{\phi_{RF}}{2} \right) J_{2p} \left( \frac{\phi_{RF}}{2} \right) - J_{2l+2p+1} \left( \frac{\phi_{RF}}{2} \right) J_{2p+1} \left( \frac{\phi_{RF}}{2} \right) \right] \sin [(2l+1)\Omega t] \]  

(22)

Each line of Eq.22 represents a sum over all values of the variable \( p \), which can be re-written more compactly as

\[ E_1^2 = \frac{\gamma^2}{2} \sum_{l=-\infty}^{+\infty} \sum_{p=-\infty}^{+\infty} \left[ J_{2l+p} \left( \frac{\phi_{RF}}{2} \right) J_{p} \left( \frac{\phi_{RF}}{2} \right) \right] \cos[2l\Omega t] \]

\[ - \cos(\phi_{DC})(-1)^p J_{2l+p} \left( \frac{\phi_{RF}}{2} \right) J_{p} \left( \frac{\phi_{RF}}{2} \right) \cos[2l\Omega t] \]

\[ + \sin(\phi_{DC})(-1)^p J_{(2l+1)+p} \left( \frac{\phi_{RF}}{2} \right) J_{p} \left( \frac{\phi_{RF}}{2} \right) \sin [(2l+1)\Omega t] \]  

(23)

The Neumann Addition Theory can be applied to each line of Eq.23 by summing over the variable \( p \). For \( u = w = \frac{\phi_{DC}}{2} \), Eq.14 yields
The above result reflects that Bessel functions of the first kind \( J_\nu \) are zero at the origin for all orders \( \nu \) with the exception of \( J_0 \), which is unity at the origin. Another form of the Neumann Addition Theorem which is helpful here is

\[
\sum_{p=-\infty}^{+\infty} (-1)^p J_{n+p}(z) J_p(z) = J_n(2z)
\] (25)

Applying this identity to Eq.23 allows the following simplifications

\[
J_{2l}(\phi_{RF}) = \sum_{p=-\infty}^{+\infty} (-1)^p J_{2l+p} \left( \frac{\phi_{RF}}{2} \right) J_p \left( \frac{\phi_{RF}}{2} \right)
\] (26)

\[
J_{2l+1}(\phi_{RF}) = \sum_{p=-\infty}^{+\infty} (-1)^p J_{2l+1+p} \left( \frac{\phi_{RF}}{2} \right) J_p \left( \frac{\phi_{RF}}{2} \right)
\] (27)

Therefore, Eq.23 can be recast as

\[
E_1^2 = \frac{\gamma^2}{2} \left[ 1 - \cos(\phi_{DC}) \sum_{l=-\infty}^{+\infty} J_{2l} (\phi_{RF}) \cos(2l \Omega t) + \sin(\phi_{DC}) \sum_{l=-\infty}^{+\infty} J_{2l+1} (\phi_{RF}) \sin((2l + 1) \Omega t) \right]
\] (28)

The sum over \( l \) involving even-order Bessel functions takes advantage of their even symmetry \( J_{2l} = J_{-2l} \) and can be simplified to include only the zero and positive terms in the sum by doubling the positive terms and writing the zero term explicitly as \( J_0 (\phi_{RF}) \). The sum involving the odd-order Bessel functions \( J_{2l+1} = -J_{-2l-1} \) can be treated similarly since the odd symmetry of \( \sin((2l + 1) \Omega t) \) cancels the odd symmetry of the Bessel functions over negative indices. Applying these conditions gives a form equivalent to the photocurrent expansion derived in Eq.12.

\[
E_1^2(t) = \frac{\gamma^2}{2} \left[ 1 - \cos(\phi_{DC}) J_0(\phi_{RF}) - 2 \cos(\phi_{DC}) \sum_{l=1}^{+\infty} J_{2l}(\phi_{RF}) \cos(2l \Omega t) + 2 \sin(\phi_{DC}) \sum_{l=0}^{+\infty} J_{2l+1}(\phi_{RF}) \sin((2l + 1) \Omega t) \right]
\] (29)

The squared field expression given by Eq.29 has the same functional form as the upper photocurrent in Eq.12, differing only by a multiplicative constant equal to the responsivity. From this we can make a direct comparison between the Bessel function amplitudes in the optical domain to those in the RF domain, with even RF sideband amplitudes on the left side of Eq.26 given by the linear combination of optical sidebands on the right side of Eq.26, and the odd RF sideband amplitudes similarly determined according to Eq.27. The results are expected because RF harmonics of order \( n \Omega \) are due to all combinations of optical harmonics separated by \( n \Omega \). However, the exact relationships may not be intuitive.

References
