Solution of Integral Equations by Physics-Informed Neural Networks For Electromagnetic Scattering

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Abstract—This work develops an approach to solve the integral equations for dynamic electromagnetic scattering problems based on the physics-informed neural networks (PINNs), which were originally proposed to solve partial differential equations (PDEs). Since different from the applications in most of the existing PINN studies, the function to be evaluated here is complex-valued, network structures to handle the phase information contained in the complex-valued numbers are investigated. An adaptive activation function is employed to improve the performance of the PINN solution. Numerical simulations on two-dimensional (2-D) electromagnetic scattering problems have been conducted to validate the proposed method.

Index Terms—Electromagnetic scattering, integral equation, physics-informed neural network.

I. INTRODUCTION

W ITH the rapid development of artificial intelligence (AI), there is a growing interest in utilizing the AI technology to tackle electromagnetic problems. Several well-established algorithms in machine learning and deep learning, such as support vector machines (SVMs) [1] and deep neural networks (DNNs) [2]–[10], have been successfully employed to solve electromagnetic problems with promising results. A comprehensive review of traditional machine learning-based computational electromagnetic (CEM) methods can be found in [11] and [12].

Unlike other machine learning algorithms [1]–[12], PINNs can eliminate the need for a large amount of training data because it integrates the laws of physics, in the form of PDEs or their variations, as constraints into neural networks. As a result, PINNs are particularly effective for solving problems with a limited or even zero amount of training samples. Thanks to their ability and great potential as a new scientific computing tool, PINNs [13], including their variations and improvements [14]–[19], have gained great attentions as a novel machine learning framework. As far as CEM problems are concerned, a PINN was developed to solve time domain electromagnetic problems by encoding initial conditions, boundary conditions, and Maxwell’s equations as constraints during the network training process [20]. Based on the U-Net framework, the so-called MaxwellNet for solving the scattered light field in free space was proposed [21] by employing the frequency-domain vector Helmholtz equation with respect to the electric field as a physics-driven loss function. To design electromagnetic devices and systems, a PDNN was proposed for solving 2D magnetostatic fields [22]. In the low-frequency electromagnetism region, PINNs were utilized to address specific problems [23], where transfer learning was utilized to generalize the PINN estimator as much as possible. In [24], a hypernetwork was proposed as a parametrized real-time field solver that allows the fast solution of inverse problems. In fact, many researches were conducted on PINNs from different aspects in solving electromagnetic problems [25]–[30]. As an alternative to the PDE form, the integral equation (IE) form of the Maxwell’s equations is another popular technical line to solve electromagnetic problems. However, in contrast to the PINN solution of former, much few reports can be found for the latter.

To improve the performance of a neural network, the choice of a suitable activation function is vital. Numerous activation functions have been developed and studied, such as Rectified Linear Units (ReLU) [31] and its variations, exponential linear unit (ELUs) [32] and gaussian error linear units (GELUs) [33], Swish [34], [35], Mish [36], and etc. In general, it is hard to find a good activation function working well for all kinds of networks and all types of problems. Most recently, the concept of the adaptive activation function was proposed to combine a set of activation functions together. Studies in [37]–[39] validated their effectiveness in improving the accuracy of PINNs.

This work develops an approach to solve the IE of the Maxwell’s equation for dynamic electromagnetic scattering problems based on main idea of the PINN, which was originally aimed to solve partial differential equations (PDEs). Since the dynamic electromagnetic fields in frequency domain are complex-valued, three network structures are proposed and studied where the complex-valued fields are tackled distinctly. In addition, the performance of the adaptive activation functions is studied in terms of the proposed PINN solution.

II. VOLUME INTEGRAL EQUATION AND METHOD OF MOMENTS

The configuration of a 2D scattering problem is depicted in Fig. 1 where \( \Omega \) denotes the computational domain. The background is the free space with the permittivity \( \varepsilon_0 \) and permeability \( \mu_0 \). In \( \Omega \), there are \( N \) dielectric scatterers with \( \varepsilon_{r,1}(\mathbf{r}), \varepsilon_{r,2}(\mathbf{r}), \ldots, \varepsilon_{r,N}(\mathbf{r}) \), respectively, the relative permittivity of each scatterer. Suppose the scatterers are illuminated by a transverse magnetic (TM) wave and \( z \)-axis denotes the invariant direction, the fields can then be uniquely represented by their \( z \)-components. The associated electric field

Fig. 1. The geometry configuration of a typical electromagnetic scattering problem with the two-dimensional (2-D) transverse magnetic (TM) incident plane wave. Here Tx and Rx represent the transmitting and receiving antennas, respectively.
The integral equation can be written as,

$$E_{x}^{\text{tot}}(r) = E_{x}^{\text{inc}}(r) + k_0^2 \int_{\Omega} G(r, r') (\varepsilon(r') - 1) E_{x}^{\text{tot}}(r') dr', r' \in \Omega \quad (1)$$

where $\omega$ is the angular frequency, $k_0 = \omega \sqrt{\varepsilon_d \mu_d}$ is the wavenumber, $r$ and $r'$ denote the field and source points, $E_{x}^{\text{inc}}(r)$ is incident electric field and $G(r, r')$ is the Green’s function. Often, we define the contrast $\chi(r')$ and the induced equivalent current $J_z(r')$ within the scatterers as,

$$\chi(r') = \varepsilon(r') - 1, \quad J_z(r') = \chi(r') E_{x}^{\text{tot}}(r'). \quad (2)$$

With Eqs. (2) and (3), the scattered field $E_{x}^{\text{ sca}}(r)$ can be obtained by

$$E_{x}^{\text{ sca}}(r) = k_0^2 \int_{\Omega} G(r, r') J_z(r') dr'. \quad (4)$$

To find $J_z(r')$, the method of moments (MoM) can be employed to solve Eq. (1). In the MoM solution, Eq. (1) is discretized with the pulse basis function and the delta test function after each scatter in $\Omega$ is divided into sufficiently small subdomains. The associated linear system can be written as,

$$E_{x}^{\text{tot}} = E_{x}^{\text{inc}} + \overrightarrow{G_D} \cdot \overrightarrow{\chi} \cdot E_{x}^{\text{tot}}. \quad (5)$$

Suppose the number of subdomains is $M$, $E_{x}^{\text{tot}} = [E_{x}^{\text{tot}}, E_{y}^{\text{tot}}, \ldots, E_{M}^{\text{tot}}]^T$ and $E_{x}^{\text{inc}} = [E_{x}^{\text{inc}}, E_{y}^{\text{inc}}, \ldots, E_{M}^{\text{inc}}]^T$ contain the total and incident electric field samples of all subdomains; $\overrightarrow{G_D}$ is the impedance matrix, $\overrightarrow{\chi}$ is the diagonal matrix storing the contrast of each subdomain. Both $\overrightarrow{G_D}$ and $\overrightarrow{\chi}$ are of size $M \times M$. Once Eq. (5) is solved, $\overrightarrow{J}$ can be obtained by

$$\overrightarrow{J} = \overrightarrow{\chi} \cdot E_{x}^{\text{tot}}. \quad (6)$$

Thus, the scattered field $E_{x}^{\text{ sca}}$ can be computed according to Eq. (4).

III. PINN SOLUTION OF IEs

A. Incorporation of MoM into PINNs

The essence of PINNs to solve a PDE is to embed the given PDE or its variations into the loss function of the neural network, which can be defined as the weighted summation of the $L_2$ norm of residuals for the PDE equation and boundary conditions. To feed into a PINN the underlying physics of the electromagnetic scattering in the form of Eq. (1), this work defines a residual based on Eq. (5) as,

$$\overrightarrow{\delta} = E_{x}^{\text{inc}} - (\overrightarrow{I} - \overrightarrow{G_D} \cdot \overrightarrow{\chi}) \cdot E_{x}^{\text{tot PINN}}, \quad (7)$$

where $E_{x}^{\text{tot PINN}}$ represents the predicted total field by the PINN. The loss function of the PINN is then defined as the mean-square error (MSE) of $\overrightarrow{\delta}$,

$$L = \frac{1}{M} \| \overrightarrow{\delta} \|_F^2 = \frac{1}{M} \left\| E_{x}^{\text{inc}} - (\overrightarrow{I} - \overrightarrow{G_D} \cdot \overrightarrow{\chi}) \cdot E_{x}^{\text{tot PINN}} \right\|_F^2, \quad (8)$$

where $\| \cdot \|_F$ is the Frobenius norm operator.

B. Network Structures To Tackle Complex-valued Numbers

The function to be approximated is complex-valued in our applications. As it is well-known, the phase information contained in complex numbers plays an important role in the underlying physics. It is therefore interesting to study the performance the PINN solution with different neural network structures where the phase information is handle distinctively. In this work, three network structures are investigated, as shown in Fig. 2. The three networks share a same backbone, i.e., the fully-connected network (FCN), to reach a fair comparative study. For the structure in Fig. 2(a), the complex-valued numbers (fields) are generated in the output layer. In contrast, the one in Fig. 2(b) employs two FCNs to generate the real- and imaginary-components of the fields separately. In Fig. 2(c), a complex-valued network structure in [40]–[42] is used, where all operations in the network are capable of handling complex-valued numbers properly.

C. Adaptive Activation Function

Encouraged by the studies in [37], [43], an adaptive activation function is employed in our PINN solution. The adaptive activation function $f(x)$ is a linear combination of $F$ candidate activation functions with learnable coefficients, which can be written as,

$$f(x) = \sum_{i=1}^{F} G(\beta_i) \sigma_i(x) \quad (9)$$

where $\sigma_i(\cdot)$ and $\beta_i$ denote the $i$th candidate activation function and the corresponding learnable coefficient, $G(\cdot)$ denotes a gate function,
in the form of a Softmax function here. In this work, $F = 2$ and the candidate activation functions are $[\sin, \cos]$. The learnable coefficients ($\beta_1$ and $\beta_2$) are initialized as zeros. Identical to [38], [39], the adaptive activation function is layer-wise locally, aiming to enhance the learnability of the activation function.

IV. NUMERICAL RESULTS

The performance of the proposed PINNs for the 2D IE is investigated on a server configured by a NVIDIA RTX 3070ti GPU. Four types of 2-D dielectric scatterers with different shapes are selected. They are the rectangular-shaped objects with dimensions of $1.6 \times 1.6$ m$^2$, a floral-shaped objects, the digit-shaped and letter-shaped objects that are generated from the Extended MNIST (EMNIST) database. Scaling all scatterers to a comparable size, we set the computational domain $\Omega$ to be $2 \times 2$ m$^2$ with the center of $(0, 0)$ m for all of them. The scatterers are discretized in such a way that the obtained subdomains are $\lambda_d/10$ in size, where $\lambda_d$ is the dielectric wavelength. The working frequency is 300 MHz. The scattering fields are computed by sampling 360 points uniformly on the circle with 5 m radius. The backbone FCN has four hidden layers, each having 50 neurons. The adaptive moment estimation with decoupled weight decay (AdamW) method is used to minimize the loss function described by Eq. (8). The learning rate is $1.0e^{-4}$ and the training process stops when the number of epochs reaches 50000.

In order to evaluate the accuracy of the proposed PINNs, the $L_2$ relative error (RE) is calculated as,

$$RE = \frac{\|E_{\text{PINN}}^{\text{tot}} - E_{\text{MoM}}^{\text{tot}}\|_F^2}{\|E_{\text{MoM}}^{\text{tot}}\|_F^2},$$

where $E_{\text{MoM}}^{\text{tot}}$ is obtained by the traditional MoM. In the following study, the equivalent currents obtained by the MoM, denoted by $J_{\text{MoM}}$, and the scattering fields from the MoM, denoted by $E_{\text{MoM}}^{\text{scat}}$, are also presented to reach a comprehensive comparison.

A. Network Structures

The proposed three different network structures are studied in terms of their performance. Table I lists the detailed information on the computational configurations and REs associated with the three different network structures for the digit-shaped scatterers. In the case with the relative permittivity of $(1.5, 0), M$ is 1024, whereas it is increased to 4096 in the case with the relative permittivity of $(3.0, 0.5)$. From the table, it can be seen that three networks with different structures can all provide acceptable solutions. The three networks exhibit comparable accuracy when the relative permittivity of the scatterer is $(1.5, 0)$. However, the network described by Fig. 2(b)