Distributed Joint Observation and Transmission Planning Method for Multiple Agile Satellites in Mega Constellation Networks

Ningxuan Guo\textsuperscript{1}, Yuqi Wang\textsuperscript{1}, Yufei Li\textsuperscript{1}, Liang Liu\textsuperscript{1}, Yupeng Gong\textsuperscript{1}, Ningyuan Wang\textsuperscript{1}, Anshou Li\textsuperscript{1}, Dan Liu\textsuperscript{1}, and Xiaoqing Zhong\textsuperscript{1}

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Abstract

With the improvement of satellites’ maneuverability, agile earth observation satellites (AEOSs) can pitch and roll themselves to observe targets with longer visible time window (VTW), which enables more targets to be observed while bringing greater uncertainties of mission planning and more conflicts of resources. Meanwhile, mega constellation networks (MCNs) provide powerful tools to transmit massive observation data. In MCNs, AEOSs can observe targets agilely and access communication satellites (CSs) by inter-satellite links (ISLs) to offload data. Based on this architecture, we propose a Distributed Joint Observation and Transmission Planning method for Multiple AEOSs (MA-DJOTP). This method uses a targets allocation strategy, a CS allocation model, and a targets reallocation strategy to transform the multi-AEOS problem into several single-AEOS subproblems. The Single-AEOS Joint Observation and Transmission Planning (SA-JOTP) model is formulated as a Mixed Integer Quadratic Constraint Programming (MIQCP) problem based on a mission-based time slot division method, which can help simplify the observation time determination and ISL handover modeling. The SA-JOTP model can realize both the benefit maximization and the transmission delay minimization based on practical constraints of mission transition time, laser ISLs’ characteristics, and limited onboard resources. We verify the effectiveness of the proposed MA-DJOTP algorithm in MCNs with 720 CSs and 3, 16, 36, 360 AEOSs. The results show that the proposed algorithm can obtain a solution very close to the global optimum of the centralized method and is applicable in MCNs with hundreds of satellites.
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Index Terms—Agile earth observation satellites, observation and transmission mission planning, mega constellation networks, inter-satellite links, distributed algorithm.

I. INTRODUCTION

Earth observation satellites (EOSs) can observe targets and transmit their data back to earth, which are vital in disaster emergency rescue, environmental monitoring, and meteorology [1]. The observation and transmission mission planning of EOSs has a great impact on the effectiveness of space observation systems.

As the improvement of EOSs’ maneuverability, agile earth observation satellites (AEOSs) are produced. Traditional non-agile EOSs can only observe targets when flying over them, while AEOSs can pitch and roll themselves to observe before or after flying over targets so that more missions can be executed [2]. When an AEOS can observe a target, the AEOS is in the visible time window (VTW) of the target. The exact imaging time is considered as the observation time window (OTW), as shown in Fig. 1. For non-agile EOSs, the OTW is the same as the VTW. While for AEOSs, the exact OTW needs to be determined in the VTW, which leads to greater uncertainties. Besides, the long VTWs of AEOSs result in more data to offload, more complexity of mission transition, and more conflicts of resources.

Meanwhile, many AEOSs can generate a large amount of data, which will put higher demands on data transmission. Communication satellites (CSs) can form a network (eg. Starlink [3], OneWeb [4]) to provide powerful tools to transmit massive observation data. In this paper, we consider a mega constellation network (MCN) shown in Fig. 1. CSs can help transmit data back to Earth through inter-satellite
links (ISLs) and satellite-earth links. AEOSs only need to observe targets agilely and access CSs concurrently to offload data. Consequently, timely transmission of massive data can be achieved.

Due to the rapid development of MCN and satellite platforms, it becomes more challenging for mission planning of AEOSs. On this issue, research efforts can be classified into three aspects.

The first aspect focuses on the observation planning of AEOSs, while not considering the offload of the image data. Many research [2], [5], [6] made efforts on the “time-dependent transition time”, which results from the fact that each pair of consecutive missions requires a transition time to maneuver the look angle of the AEOS from the previous target to the next target. Besides, Han et al. [7] studied the cloud coverage uncertainty in the multi-AEOS scheduling problem. He et al. [8] solved the AEOS scheduling problem by a genetic Markov decision process and the reinforcement learning method. Du et al. [9] proposed a task assignment strategy based on a probability prediction model to divide the multi-AEOS problem into several single-AEOS subproblems.

The second aspect focuses on the data transmission in MCN based on the time-varying topology. In this aspect, the snapshot graph is widely applied to divide the time domain evenly into several time slots and consider the network topology to be static during a time slot. Zhou et al. [10] proposed a benefits maximization method in terms of sum weighted transmitted data volume. Yan et al. [11], [12] optimized the ISLs’ topology to achieve the maximum throughput and the minimization of transmission delay. Guo et al. [13] proposed a distributed task-aware topology optimization method in a double-layered MCN to realize the maximization of data transmission benefits.

However, the above two aspects only studied one of the observation or transmission missions. The separation scheduling results in the low efficiency of AEOS’ mission performance. In practice, observation and transmission missions have strong coupling features since they share the same time domain and are both related to the onboard storage and energy resources. But few works studied the joint observation and transmission scheduling of AEOSs, which is the focus of this paper belonging to the third aspect.

In the third aspect, Zhang et al. [14] proposed a mixed integer programming model to formulate the imaging and data downloading scheduling problem. However, this paper only considered the data transmission to ground stations and did not include ISL in MCNs. To the best of the authors’ knowledge, He et al. [15] first studied the joint observation and transmission scheduling algorithm for AEOSs (i.e. JOTSAS) in satellite networks. The JOTSAS aims to realize more imaging data to be collected and offloaded. However, it is a centralized method that is hard to solve in MCNs with hundreds of satellites. Besides, JOTSAS has some problems such as the complex transition time modeling, the lack of connection between data transmission and ISLs, the lack of optimization in transmission delay, and the lack of consideration for limited onboard resources.

Motivated by [15] and the development of ISL in MCNs, we propose a Distributed Joint Observation and Transmission Planning method for Multi-AEOSs (MA-DJOTP) in MCNs. In this paper, we consider the ISLs as laser links and consider the same AEOS AS-01 as in [2], [5], [6], whose look angles are fixed during an observation mission. Comparison of JOTSAS in [15] and the proposed MA-DJOTP is listed in TABLE I. We propose a mission-based time slot division method to simplify the modeling of the complex transition time constraint. To make the algorithm applicable in MCNs, we also propose targets and CSs allocation strategies to transform the multi-AEOSs scheduling problem into single-AEOS subproblems and solve them in parallel.

Compared to the state of the art, the main contributions of this paper can be summarized as follows:

- A mission-based time slot division method that has different time slot durations for observation and transmission mission planning. This method can help simplify the OTW determination and the ISLs’ transition time modeling.
- A Single-AEOS Joint Observation and Transmission Planning (SA-JOTP) model, which is formulated as a Mixed Integer Quadratic Constraint Programming (MIQCP) problem. This model gives practical constraints considering the mission transition time, laser ISLs’ characteristics, and limited onboard resources. It realizes both the benefits maximization and the transmission delay minimization.
- A distributed algorithm (i.e. MA-DJOTP) for multiple AEOSs based on a targets allocation strategy with the ranking of AEOSs, a CS allocation model, a targets reallocation strategy, and the parallel solving of the SA-JOTP problems of each AEOS. This algorithm can obtain a solution very close to the global optimum of the centralized method and is applicable in MCNs with hundreds of satellites.

The rest of this paper is organized as follows. Section II presents the system model with the assumptions, some symbol definitions, and the mission-based time slot division method.

| TABLE I COMPARISON OF JOTSAS IN [15] AND THE PROPOSED MA-DJOTP |
|---------------------------------|-------------------|-------------------|
| **Architecture**                | JOTSAS [15]       | MA-DJOTP (proposed) |
| Time slot division              | Same for observation and transmission | Different for observation and transmission (mission-based) |
| Transmission mission transition | Transitions are required between the transmission of any different target data | Transitions are required between any different ISL linking objects |
| Modeling of transition time constraint | Complex | Simplified based on the mission-based time slot division method |
| Objective                        | Benefit maximization | Benefit maximization and transmission delay minimization |
| Limited energy and storage resources | Not considered | Considered |
| MCNs                             | Not applicable | Applicable |
Section III formulates the SA-JOTP model as an MIQCP problem with constraints and the multi-objective function. In Section IV, we propose an observation targets allocation method, a CS allocation method, and a targets reallocation method to form the distributed algorithm MA-DJOTP with the proposed SA-JOTP. Then, we verify the effectiveness of the proposed methods in MCNs with 720 CSs and 3, 16, 36, 360 AEOSs in Section V. At last, Section VI concludes this paper and discusses future work.

II. SYSTEM MODEL

In this section, we give the system model assumptions, symbol definitions, and the mission-based time slot division method in turn.

A. Assumptions

This study mainly focuses on the observation and transmission joint planning method for AEOSs. Therefore, some assumptions are made to simplify the problem:

1) This study only considers spot targets with one-pass observation. The observation duration for each target is considered a constant value.
2) The attitude transition time between observing different targets is considered a constant value.
3) The studied AEOS AS-01 does not adjust attitude during an observation mission.
4) This study considers the data transmission through ISLs from AEOSs to CSs. We will decide on the optimal ISLs for AEOSs to optimize the transmission. The routing among CSs and from CSs to ground stations is not considered.
5) The laser acquisition time between connecting to different CSs by ISLs is considered a constant value.
6) The energy and storage resource management is taken into account.

B. Definitions

In this multi-layer system with multiple targets, multiple AEOSs, and multiple CSs, we define some indices and sets to distinguish different nodes.

1) Index:
   - \(i\): index of AEOSs. The number of AEOS is \(N^a\).
   - \(k\): index of targets. The number of targets is \(N^o\).
   - \(j\): index of CSs. The number of CSs is \(N^c\).
   - \(t\): index of time slots.

2) Sets:
   - \(V^o_{i,K}\): The set of all visible targets of AEOS \(i\) during the planning horizon, where the number of elements is \(|V^o_{i,K}| = N^o_i\), satisfying \(\sum_{i=1}^{N^a} N^o_i = N^o\).
   - \(V^t_{i,k}\): The set of all AEOSs that can observe target \(k\) within the planning horizon, where the number of elements is \(|V^t_{i,k}| = N^a_i\), satisfying \(\sum_{k=1}^{N^c} N^a_k = N^a\).
   - \(V^c_{i,j}\): The set of all CSs that can connect to AEOS \(i\) by ISLs during the planning horizon, where the number of elements is \(|V^c_{i,j}| = N^c_i\), satisfying \(\sum_{i=1}^{N^a} N^c_i = N^c\).
   - \(V^a_{j,k}\): The set of all AEOSs that can connect to CS \(j\) by ISLs during the planning horizon, where the number of elements is \(|V^a_{j,k}| = N^a_j\), satisfying \(\sum_{j=1}^{N^c} N^a_j = N^a\).
   - \(T^o\): The set of observation time slots, where the number of elements is \(|T^o| = T^o\), each time slot duration is \(\tau^o\).
   - \(T^c\): The set of transmission time slots, where the number of elements is \(|T^c| = T^c\), each time slot duration is \(\tau^c\).

C. Mission-based Time Slot Division Method

In the snapshot graph, the network topology is considered unchanged during a time slot. Since we need to make the optimal decision with the unit of time slot, the duration of a time slot determines the number of variables and the optimization complexity. A long time slot can reduce complexity but may lose some optimality of the solution. While a too short time slot may result in a too huge problem to be solved.

To reduce the solving complexity while ensuring the solution performance, we propose a mission-based time slot division method, which is shown in Fig. 2. The main ideas are as follows:

1) The time slot duration for observation and transmission are different.
2) The observation time slot duration \(\tau^o\) is equal to the time needed for observing one target.
3) The transmission time slot duration \(\tau^c\) is equal to the time needed for laser acquisition when executing ISL handover.
4) The time slot duration \(\tau^o\) and \(\tau^c\) have an integer multiple relationship, so they can be unified with multiple or division operations.

For observation missions, the observation duration is generally shorter than the needed attitude transition duration [2], [7], so making \(\tau^o\) be equal to the observation duration is
practical. The advantage of diving time domain with $\tau^c$ is: for each mission, once an observation time slot is determined, the specific observing start and end time is determined. Thus, the observation-related decision variables can be defined as discrete integer variables in time domain. This will greatly reduce the difficulty of problem modeling and solving.

For transmission missions, dividing time domain with $\tau^c$ can reduce the nonlinear coupling relationship of time domain. Since only one time slot is needed when performing ISL handover, so when determining the link state of a certain time slot, only the link state of its previous time slot need to be considered. Thus, the ISL handover constraint can be modeled by the state difference between adjacent time slots. This will greatly simplify the modeling of the ISL handover constraint and reduce the solving complexity.

For joint observation and transmission, there are some interacting constraints between these two missions. As shown in the third case of Fig. 2, the AEOS cannot execute ISL handover and data transmission when adjusting attitude due to the stable requirements of the satellite for laser acquisition and ISL maintenance. Therefore, for modeling the interacting constraints, the time slots $\tau^o$ and $\tau^c$ should be able to convert to each other. The mapping between the time slots is as follows:

$$\gamma = \frac{\tau^c}{\tau^o}$$

$$t \in \tau^o \Rightarrow \lfloor \frac{t}{\gamma} \rfloor \in \tau^c$$

$$t \in \tau^c \Rightarrow [(t - 1)\gamma + 1, t \cdot \gamma] \in \tau^o$$

III. PROBLEM FORMULATION OF A SINGLE AEOS

In this section, we will propose the SA-JOTP model for a single AEOS $i$. The considered targets are in set $V^o_{i,k}$, and CSs are in set $V^c_{i,j}$. We will formulate the constraints of observation, transmission, the relation between observation and transmission, satellite transition state, energy, and storage in turn. At last, a linear weighted multi-objective optimization problem will be formulated for the single AEOS.

A. Observation Mission Constraints

1) Related Decision Variables:

- $\psi_i = \{\psi_i(k, t) | k \in V^o_{i,k}, t \in \tau^o\} \in \mathbb{R}^{N^o \times T^o}$: a 0-1 observation decision matrix, where its element $\psi_i(k, t)$ is ‘1’ when the AEOS $i$ decides to observe target $k$ at time slot $t$, and ‘0’ the opposite.

- $x_{i,k}$: a 0-1 scalar variable to represent whether target $k$ is scheduled to be observed by AEOS $i$.

- $p_{k,k'}$: a 0-1 scalar variable whose value is ‘1’ when target $k'$ is observed after $k$, and ‘0’ the opposite, where $k \neq k'$.

Constraints on the value range of decision variables are:

$$\psi_i(k, t), x_{i,k}, p_{k,k'} \in \{0, 1\} \quad \forall k \neq k' \in V^o_{i,k}, \forall t \in \tau^o$$

2) Target Visibility: Define $O_i \in \mathbb{R}^{N^o \times T^o}$ be the visible matrix of AEOS $i$, where its element $O_i(k, t)$ is ‘1’ when $k$ is in the visible range of $i$ at time slot $t$, and ‘0’ the opposite. We can calculate this parameter matrix $O_i$ according to the pitch and roll ability of AEOS. The matrix $O_i$ is an integer expression of the VTW. Assume $\{o_{i,k}, e_{i,k}\}$ is the VTW between AEOS $i$ and target $k$, then for $\forall t \in \{o_{i,k}, e_{i,k}\}, O_i(k, t) = 1$.

To guarantee AEOS $i$ can only observe targets when they are visible, the visibility constraint can be described as:

$$\psi_i \leq O_i$$

3) Number of Observed Targets: We assume each AEOS has one camera, so it can only observe one target at a time slot:

$$\sum_{k \in V^o_{i,k}} \psi_i(k, t) \leq 1 \quad \forall t \in \tau^o$$

4) Observation Duration: Since the time slot duration $\tau_o$ is equal to the observation mission duration, we only need to arrange a time slot for each target observation. It means the observation duration $l^o = 1$. That is, when $k$ is scheduled to be observed, $x_{i,k}$ and the observation duration both equal to 1, which can be expressed as:

$$\sum_{t \in \tau^o} \psi_i(k, t) = x_{i,k} \quad \forall k \in V^o_{i,k}$$

5) Observation Transition: Since an AEOS needs to adjust its attitude to point at a target, two targets cannot be observed continuously. Let $\Delta^o$ be the necessary attitude transition duration between adjacent observation missions, then $\Delta^o/\tau^o$ is the needed number of time slots. We use the Big-M model [16] with associated constant $M$ to formulate the observation transition constraint as follows:

$$\sum_{t \in \tau^o} \psi_i(k, t) \cdot t \leq \sum_{t \in \tau^o} \left[ \psi_i(k', t) \cdot (t + l^o + \frac{\Delta^o}{\tau^o}) \right] - M \cdot p_{k,k'} \quad \forall k \neq k' \in V^o_{i,k}$$

The above formulas realize the logical relation as:

if $\sum_{t \in \tau^o} \psi_i(k, t) \cdot t - \sum_{t \in \tau^o} \psi_i(k', t) \cdot (t + l^o + \frac{\Delta^o}{\tau^o}) < 0$, then $p_{k,k'} = 1$, $p_{k',k} = 0$

where $\sum_{t \in \tau^o} \psi_i(k, t) \cdot t$ obtains the observation time of $k$, and $\sum_{t \in \tau^o} \psi_i(k', t) \cdot (t + l^o + \frac{\Delta^o}{\tau^o})$ obtains the earliest observation time after observing $k'$. If $k$ does not satisfy the condition of observing after $k'$, then $k$ should be observed before $k'$ with $p_{k,k'} = 1$. Note that if $k$ is not observed, then $p_{k,k'} \equiv 1$ if $k'$ is observed.

B. Transmission Mission Constraints

1) Related Decision Variables:

- $\delta_i = \{\delta_i(j, t) | j \in V^c_{i,j}, t \in \tau^c\} \in \mathbb{R}^{N^c \times T^c}$: a 0-1 linking decision matrix, where its element $\delta_i(j, t)$ is ‘1’ when the AEOS $i$ connects to CS $j$ by an ISL at time slot $t$, and ‘0’ the opposite.

- $\beta_i = \{\beta_i(k, t) | k \in V^o_{i,k}, t \in \tau^c\} \in \mathbb{R}^{N^o \times T^c}$: a 0-1 transmission decision matrix, where its element $\beta_i(k, t)$ is ‘1’ when the AEOS $i$ starts to transmit observation data of target $k$ at time slot $t$, and ‘0’ the opposite.
• $st_{i,k} \in T^o$: an integer variable to represent target $k$’s data transmission start time expressed in $T^o$.

Constraints on the value range of decision variables are:

$$\delta_i(j, t), \beta_i(k, t) \in \{0, 1\} \quad \forall j \in V_{i,j}^o, k \in V_{i,K}^o, t \in T^c \quad (10)$$

2) ISL Connectivity: Define $S_i \in \mathbb{R}^{N_i \times T^c}$ be the connectable matrix between AEOS $i$ and all CSs at all time slots, where its element $S_i(j, t)$ is ‘1’ when CS $j$ is connectable to AEOS $i$ at time slot $t$, and ‘0’ the opposite. This parameter matrix $S_i$ can be calculated according to antenna angle and relative velocity of satellites [13]. AEOS $i$ can only establish ISLs with connectable CSs, which can be described as:

$$\delta_i \leq S_i \quad (11)$$

3) Number of ISLs: We assume each AEOS has one intersatellite communication antenna, so it can only connect to one CS at a time slot:

$$\sum_{j \in V_{i,j}^o} \delta_i(j, t) \leq 1 \quad \forall t \in T^c \quad (12)$$

4) Transmission Time: As $\beta_i(k, t)$ represents the transmission start time slot, then for each observed target $k$, we need to select one time slot for this target data to start transmitting. This constraint can be expressed as:

$$\sum_{t \in T^c} \beta_i(k, t) = x_{i,k} \quad \forall k \in V_{i,K}^o \quad (13)$$

Meanwhile, the selected transmission start time should not be too late so that all data of the observed target can be transmitted. Assume the data amount generated by observing each target is $q^o$, the data transmission rate of ISL is $v^c$, then the needed number of time slot for transmitting each target data is $t^c = \lceil q^o/(v^c \cdot T^c) \rceil$. Therefore, data transmission should not start at the last $t^c - 1$ time slot:

$$\sum_{k \in V_{i,K}^o} \beta_i(k, t) = 0 \quad \forall t \in [T^c - t^c + 2, T^c] \quad (14)$$

With constraints (13) and (14), we make sure that once a target is observed, all of its data should be offloaded within the planning horizon.

5) Relation of ISLs and Data Transmission: Observation data can be transmitted only when an ISL is available. If we decide to start transmitting data at $t - t^c + 1$, then the $t^c$ time slots in $[t - t^c + 1, t]$ should have available ISLs. That is:

$$\begin{align*}
\text{if} & \quad \sum_{k \in V_{i,K}^o} \beta_i(k, t - t^c + 1) = 1, \\
\text{then} & \quad \sum_{j \in V_{i,j}^o} \delta_i(j, t - t^c + 1) = 1, \cdot \cdot \cdot, \sum_{j \in V_{i,j}^o} \delta_i(j, t) = 1.
\end{align*}$$

In other words:

$$\begin{align*}
\text{if} & \quad \sum_{t - t^c + 1 \in V_{i,K}^o} \beta_i(k, t - t^c + 1) = 1, \text{ then } \sum_{j \in V_{i,j}^o} \delta_i(j, t) = 1.
\end{align*}$$

This constraint can be formulated as:

$$\sum_{j \in V_{i,j}^o} \delta_i(j, t) \geq \left\{ \sum_{k \in V_{i,K}^o} \beta_i(k, t') \right\} \quad \forall t \in [1, t^c - 1]$$

$$\sum_{t' = t - t^c + 1 \in V_{i,K}^o} \sum_{k \in V_{i,K}^o} \beta_i(k, t'), \quad \forall t \in [t^c, T^c] \quad (15)$$

Note that this constraint is based on the assumption that at most one target data can be transmitted by an ISL at a time slot, and the data transmission for a target is continuous.

6) ISL Handover: Since an AEOS needs to adjust the antenna to point at a CS for laser acquisition, a necessary duration for ISL handover is necessary. As introduced in Section II-C, the handover duration $\Delta^c$ is equal to the time slot duration $\tau^c$. So we need to guarantee that there is at least one time slot between linking with different CSs.

Define $d\delta_i(j, t) = \delta_i(j, t) - \delta_i(j, t - 1)$ be the link state difference between adjacent time slots $t$ and $t - 1$, where $\delta_i(j, 0)$ represents the initial link state between $i$ and $j$. Let $d\delta_i(t) = [d\delta_i(1, t), \cdot \cdot \cdot, d\delta_i(j, t), \cdot \cdot \cdot]^T \in \mathbb{R}^{N_i^o}$ be the link state difference vector with $N_i^o$ CSs. Different cases of the link state difference are listed in TABLE II.

<table>
<thead>
<tr>
<th>Cases (for a certain $t \in T^c$)</th>
<th>Value of $d\delta_i(j, t)$</th>
<th>Norm of $d\delta_i(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuously Link</td>
<td>${0}$</td>
<td>0</td>
</tr>
<tr>
<td>Disconnect Link</td>
<td>${-1, 0}$</td>
<td>1</td>
</tr>
<tr>
<td>Establish Link</td>
<td>${1, 0}$</td>
<td>1</td>
</tr>
<tr>
<td>Handover with no interval</td>
<td>${1, -1, 0}$</td>
<td>$\sqrt{2}$</td>
</tr>
</tbody>
</table>

According to TABLE II, to avoid link handover with no interval (i.e. $\delta_i(j, t - 1) = \delta_i(j', t) = 1$ with $j \neq j'$), the norm of the link state difference vector $d\delta_i(t)$ should be less than or equal to 1, which can be expressed as:

$$|d\delta_i(t)| \leq 1 \quad \forall t \in T^c \quad (16)$$

C. Relation Constraints of Observation and Transmission

For each target $k$, data transmission should not be earlier than observation, which can be formulated as:

$$st_{i,k} - \sum_{t \in T^o} |\psi_i(k, t) \cdot t| \geq 0 \quad \forall k \in V_{i,K}^o \quad (17)$$

where the data transmission start time slot $st_{i,k} \in T^o$ can be calculated by $\beta_i(k, t)$ as follows:

$$st_{i,k} = \left\{ \left( \sum_{t \in T^c} |\beta_i(k, t) \cdot t| - 1 \right) \gamma + 1 \right\} \cdot x_{i,k} \quad \forall k \in V_{i,K}^o \quad (18)$$

where $\sum_{t \in T^c} |\beta_i(k, t) \cdot t|$ can obtain the transmission start time slot expressed in $T^o$, and $\gamma$ is used to transform the time slot into $T^o$. Here, $x_{i,k}$ is multiplied to guarantee the value of $st_{i,k}$ be 0 when the target $k$ is not observed or transmitted.
D. Satellite Transition State Constraints

1) Related Decision Variables:

- \( \xi_{i,t} \): A 0-1 scalar variable to represent whether AEOS \( i \) is adjusting attitude at time slot \( t \), where \( t \in T^o \).
- \( \lambda_{i,t} \): A 0-1 scalar variable to represent whether AEOS \( i \) is performing laser acquisition (ISL handover) at time slot \( t \), where \( t \in T^c \).

Constraints on the value range of decision variables are:

\[
\xi_{i,t} \in \{0, 1\}, \quad \forall t \in T^o \quad \lambda_{i,t} \in \{0, 1\}, \quad \forall t \in T^c
\]  

(19)

2) Attitude Adjustment State: Before each observation, \( \Delta^o/\tau^o \) time slots are required for attitude adjustment. That is, for a certain time slot \( t \in T^o \):

if \( \sum_{k \in V^o_{i,K}} \psi_t(k, t) = 1 \),

then \( \xi_{i,t-\Delta^o/\tau^o} = 1, \xi_{i,t-\Delta^o/\tau^o+1} = 1, \cdots, \xi_{i,t-1} = 1 \).

In other words:

if one of the elements in set

\[
\left\{ \sum_{k \in V^o_{i,K}} \psi_t(k, t+1), \cdots, \sum_{k \in V^o_{i,K}} \psi_t(k, t+\Delta^o/\tau^o) \right\}
\]

is 1,

then \( \xi_{i,t} = 1 \).

This constraint can be formulated as:

\[
\xi_{i,t} = \begin{cases} 
\frac{1}{2} + \sum_{t=1}^{t+\Delta^o/\tau^o} \sum_{k \in V^o_{i,K}} \psi_t(k, t'), \quad \forall t \in [1, T^o - \Delta^o/\tau^o] \\
0, \quad \text{otherwise.}
\end{cases}
\]  

(20)

3) Laser Acquisition State: For each AEOS, Before establishing an ISL with a CS, \( \Delta^e/\tau^e = 1 \) time slot is required for laser acquisition. That is:

if \( \sum_{j \in V^e_{i,J}} d\delta_t(j, t) = 1 \), then \( \lambda_{i,t-1} = 1 \), else \( \lambda_{i,t-1} = 0 \)

Recall that \( d\delta_t(j, t) \) represents the link state difference between adjacent time slots \( t-1 \) and \( t \), so \( d\delta_t(j, t) = 1 \) indicates that an ISL is established with \( j \) at time slot \( t \). The constraint for the laser acquisition state \( \lambda_{i,t} \) is formulated as:

\[
\lambda_{i,t} = \begin{cases} 
\frac{1}{2} + \sum_{j \in V^e_{i,J}} d\delta_t(j, t+1), \left( \sum_{j \in V^e_{i,J}} d\delta_t(j, t+1) + 1 \right), \\
0, \quad \forall t \in [1, T^c - 1] \\
\end{cases}
\]  

(21)

According to TABLE II, the value of \( \sum_{j \in V^e_{i,J}} d\delta_t(j, t+1) \) can be -1 or 1, so we multiply the item \( \sum_{j \in V^e_{i,J}} d\delta_t(j, t+1) + 1 \) to guarantee that \( \lambda_{i,t} \) can only be 0 or 1.

4) State Exclusivity: Laser acquisition and ISL maintenance have certain requirements for the stability of the satellite platform. Therefore, when the satellite is adjusting attitude, laser acquisition and data transmission of ISL can not be performed. It means that the attitude adjustment state and the laser acquisition state or ISL connecting state are exclusive. These constraints can be formulated as:

\[
1 - \lambda_{i,t/\gamma} \geq \xi_{i,t} \quad \forall t \in T^o
\]

(22)

\[
1 - \sum_{j \in V^e_{i,J}} d\delta_t(j, t/\gamma) \geq \xi_{i,t} \quad \forall t \in T^o
\]  

(23)

where \( [t/\gamma] \) is the round up of \( t/\gamma \), which is performed to transform the time slot from \( T^o \) to \( T^c \).

5) ISL Continuity: To avoid unnecessary ISL handover, we hope the ISLs are as continuous as possible. That is, if the current time slot has established an ISL, then the next time slot should keep this ISL if the linking CS is still connectable and the AEOS does not adjust attitude. This constraint can be expressed as:

\[
d\delta_t(j, t+1) \geq d\delta_t(j, t) S_t(j, t+1) \left( 1 - \sum_{t=1}^{T^o-1} \xi_{i,t} \right) \quad \forall t \in T^c
\]

(24)

E. Energy Constraints

1) Related Decision Variables:

- \( E^C_{i,t} \): a scalar variable to represent the consumed energy amount of AEOS \( i \) during time slot \( t \), where \( t \in T^c \).
- \( E^H_{i,t} \): a scalar variable to represent the harvested energy amount by solar panels of AEOS \( i \) during time slot \( t \), where \( t \in T^c \).
- \( E^r_{i,t} \): a scalar variable to represent the remaining onboard energy of AEOS \( i \) at the end of time slot \( t \), where \( t \in T^c \).

2) Energy Consumption: The energy is consumed by five parts: observation with constant power \( P^o \), attitude adjustment with power \( P^a \), laser acquisition with power \( P^l \), data transmission with power \( P^t \), and some other conventional operations with power \( P^r \). Therefore, the energy consumption during each time slot \( t \in T^c \) is:

\[
E^C_{i,t} = \tau^c \cdot \sum_{t=(t-1)\gamma+1}^{t\gamma+1} \left[ P^o \sum_{k \in V^o_{i,K}} \psi_t(k, t') + P^a : \xi_{i,t'} \right] + \tau^c \cdot \left[ P^l \cdot \lambda_{i,t} + P^t \sum_{j \in V^e_{i,J}} d\delta_t(j, t) + P^r \right] \quad \forall t \in T^c
\]

(25)

where \( \gamma \) is used for transforming the time slot expressed in \( T^o \) into \( T^c \).

3) Energy Harvesting: For each AEOS, the harvested energy during each time slot should not exceed the maximum amount harvested during the sunlight duration. Let \( P^H \) be the output power of solar panels, and \( I_{i,t}^s \in \left[ 0, \tau^c \right] \) be the sunlight duration within time slot \( t \). Then, this constraint can be formulated as:

\[
0 \leq E^H_{i,t} \leq P^H \cdot I_{i,t}^s \quad \forall t \in T^c
\]

(26)
4) Energy Equilibrium and Capacity: The amount of energy variation per time slot is equal to the amount of harvested energy minus the consumed energy during this time slot:

\[ E_{i,t} - E_{i,t-1} = E_{i,t}^H - E_{i,t}^C \quad \forall t \in T^c \]  \hspace{1cm} (27)

where \( E_{i,0} \) is the initial onboard energy.

At each time slot, the onboard energy should not exceed the maximum battery capacity \( E_{i}^{max} \) and the maximum discharge depth \( \theta \), which can be expressed as:

\[ E_{i}^{max}(1-\theta) \leq E_{i,t} \leq E_{i}^{max} \quad \forall t \in T^c \]  \hspace{1cm} (28)

F. Storage Constraints

1) Related Decision Variables:

- \( b_{i,t} \): a scalar variable to represent the data amount stored on AEOS \( i \) at the end of time slot \( t \), where \( t \in T^c \).

2) Flow Equilibrium: The amount of data storage variation per time slot is equal to the amount of generated observation data minus the transmitted data during this time slot. Recall that the data amount of observing each target is \( q^o \), and with the observing duration \( l^o = 1 \), the generated data amount during a time slot in \( T^o \) is also \( q^o \). The average transmitted data amount during a time slot in \( T^c \) is \( q^c = q^o/l^c \). Let \( b_{i,0} \) be the initial data amount stored on board. The flow equilibrium constraint is formulated as (29), which is at the bottom of this page.

In both cases of this formula, the first term calculates the generated data amount during one time slot in \( T^c \), so the generated data during \([t - (t - 1)\gamma + 1, t \cdot \gamma]\) is summed for \( \psi_{i}(k,t) \) to transform the time slot set. The second item calculates the transmitted data amount during a time slot in \( T^c \). Since \( \beta_i(k,t) \) is the start transmission time, so any target data transmission starts from \([t - l^c + 1, t]\) (or \([1, t]\) in the second case) will result in data outflow at this time slot.

3) Storage Capacity: At each time slot, the stored data amount should not exceed the maximum memory capacity \( B_{i}^{max} \). The onboard storage amount constraint can be expressed as:

\[ 0 \leq b_{i,t} \leq B_{i}^{max} \quad \forall t \in T^c \]  \hspace{1cm} (30)

G. Optimization Problem Formulation

For each AEOS \( i \), the objective of the observation and transmission planning problem is to jointly maximize the benefits while minimizing the transmission delay. Let \( \omega_k \) be the benefit of observing and transmitting data of target \( k \). We use the linear weighting method [17] to formulate the multi-objective optimization problem SA-JOTP as follows:

\[
\begin{align*}
\max \quad & \alpha_1 \sum_{k \in V^o_{i,K}} \omega_k \cdot \psi_{i,k} - \alpha_2 \sum_{k \in V^o_{i,K}} \frac{\omega_k \cdot \delta_{i,k}}{T^o \cdot N^o_i} \\
\text{subject to:} \quad & (1), (4) \sim (30)
\end{align*}
\]

where \( \alpha_1 \) and \( \alpha_2 \) are the weights of the two objectives.

In (31), the first item realizes the maximization of benefits, where the numerator calculates the total benefits obtained by observing target \( k \) and transmitting its data, and the denominator is the maximum benefits value used for normalization. The second item realizes the minimization of transmission delay, where the numerator calculates the sum of the weighted transmission start time, and the denominator is also used for normalization.

This problem has integer decision variables \( \psi_{i,k}, x_{i,k}, p_{k,k'}, \delta_i, \beta_i, \delta_{i,k}, \epsilon_{i,t}, \lambda_{i,t} \) and continuous variables \( E_{i,t}^C, E_{i,t}^H, E_{i,t}^B, b_{i,t} \), \( b_{i,0} \). Constraints (18), (21), and (24) have multiplication operations between variables, so they are quadratic constraints. Other constraints are all linear. Therefore, (31) is an MIQCP problem.

Theorem 1. In problem (31), the value of \( \alpha_1 \) and \( \alpha_2 \) should meet \( \alpha_1/\alpha_2 \geq \varpi_k \), where \( \varpi_k \) is the average benefit of targets.

Proof. To achieve the objectives of (31), we need to guarantee that the objective function value of \( x_{i,k} = 1 \) is always greater than that of \( x_{i,k} = 0 \). That is:

\[ \alpha_1 \sum_{k \in V^o_{i,K}} \frac{\omega_k}{\omega_{k'}} - \alpha_2 \sum_{k \in V^o_{i,K}} \frac{\omega_{k'} \cdot \delta_{i,k}}{T^o \cdot N^o_i} > 0 \quad \forall k \in V^o_{i,K} \]  \hspace{1cm} (32)

By derivation, we have:

\[ \alpha_1 \frac{\omega_{k'}}{\omega_{k'}} - \alpha_2 \frac{\delta_{i,k}}{T^o \cdot N^o_i} > \frac{1}{T^o \cdot N^o_i} \quad \forall k \in V^o_{i,K} \]  \hspace{1cm} (33)

Since the maximum value of \( \delta_{i,k} \) can be approximated as \( T^o \), we can obtain:

\[ \alpha_1 \frac{\omega_{k'}}{\omega_{k'}} - \alpha_2 \frac{\delta_{i,k}}{T^o \cdot N^o_i} \Rightarrow \max \left\{ \frac{\delta_{i,k}}{T^o \cdot N^o_i} \right\} \approx \varpi_k \]  \hspace{1cm} (34)

IV. DISTRIBUTED SOLUTION ALGORITHM

In this Section, we propose an observation targets allocation method, a CS allocation strategy, and a targets reallocation method in turn. Then, we form the proposed methods and
the SA-JOTP model into a distributed algorithm for multiple AEOSs, named MA-DJOTP.

A. Observation Targets Allocation

1) Allocation Process: If each AEOS is completely autonomous in observing targets without interacting, then some targets might be observed many times and some targets may not be observed. Therefore, for mission planning of single AEOS, we firstly need to allocate all observation targets to their appropriate AEOSs. The allocation process aims to make each target have great possibility to be observed and allocate all targets as evenly as possible.

Define \( A = \{A(k, i) | \forall k, \forall i \} \in \mathbb{R}^{N^o \times N^a} \) be a 0-1 matrix to represent the target allocation result, where its element \( A(k, i) = 1 \) when target \( k \) is allocated to \( i \). Let \( L = \{L(k) | \forall k \} \in \mathbb{R}^{N^a} \) be the 0-1 vector to label whether each target has been allocated.

The allocation process is expressed in pseudo-code as a function \( A = \text{Tar_allocation}(\cdot) \) in Algorithm 1. The three crucial steps of the allocation process can be summarized as:

Step 1. First allocation: for each target \( k \), if it is visible to only one AEOS during the planning horizon (i.e. \( |V^a_{t,k}| = 1 \)), allocate \( k \) to this AEOS and label \( k \) as allocated with \( L(k) = 1 \). Meanwhile, if a target is not visible to any AEOSs, we also let \( L(k) = 1 \) to avoid repeated visits to this target in the program.

Algorithm 1: Allocate all observation targets to respective AEOSs: function \( A = \text{Tar_allocation}(\cdot) \).

\[
\begin{align*}
\text{Input:} & \quad \text{Parameters of all targets and all AEOSs.} \\
\text{Output:} & \quad \text{The target allocation results.} \\
& \quad \text{for } k = 1: N^o \text{ do} \quad \text{// First allocation} \\
& \quad \text{if } |V^a_{t,k}| = 1 \text{ then} \\
& \quad \quad A(k, V^a_{t,k}) \leftarrow 1; \quad \text{// allocate } k \text{ to } V^a_{t,k} \\
& \quad \quad L(k) \leftarrow 1; \quad \text{// label } k \text{ as allocated} \\
& \quad \text{else} \\
& \quad \quad \text{if } |V^a_{t,k}| = 0 \text{ then } L(k) \leftarrow 1; \\
& \quad \end{align*}
\]

\[
\begin{align*}
& \quad \text{end} \\
& \quad \text{end} \\
& \quad \text{for } i = 1: N^a \text{ do} \quad \text{// Second allocation} \\
& \quad \text{if } (1 \leq |V^a_{i,k}| \leq N^o / N^a) \text{ and } (L(V^a_{i,k}) = 0) \text{ then} \\
& \quad \quad A(V^a_{i,k}, i) \leftarrow 1; \quad \text{// allocate } V^a_{i,k} \text{ to } i \\
& \quad \quad L(V^a_{i,k}) \leftarrow 1; \\
& \quad \end{align*}
\]

\[
\begin{align*}
& \quad \text{end} \\
& \quad \text{end} \\
& \quad \text{for } k = 1: N^o \text{ do} \quad \text{// Formal allocation} \\
& \quad \text{if } L(k) = 0 \text{ then} \\
& \quad \quad R_k \leftarrow \text{Rank}(A, V^a_{i,k}); \quad \text{// rank AEOSs} \\
& \quad \quad A(k, R_k(k)) \leftarrow 1; \quad \text{// allocate } k \text{ to the top-ranked AEOS} \\
& \quad \end{align*}
\]

\[
\text{return } A
\]

Step 2. Second allocation: for each AEOS \( i \), if the number of visible targets is below average (i.e. \( |V^a_{i,k}| \leq N^o / N^a \)), then all visible targets are allocated to this AEOS. This step is to ensure the uniformity of allocation.

Step 3. Formal allocation: for each target \( k \) that has not yet been allocated, rank all AEOSs that can observe \( k \) and allocate \( k \) to the top-ranked AEOS.

2) Rank of AEOSs: In the target allocation process, the rank of AEOSs plays a vital role, as introduced in step 18 in Algorithm 1. For each target \( k \), all AEOSs that can observe \( k \) should be ranked. We propose a rank mechanism with four items. Each item has a rank value of 0~1, so the maximum total rank value is 4. The top-ranked AEOS has the smallest rank value.

Define \( R_k = \{R_k(i) | \forall i \in V^a_{i,k} \} \in \mathbb{R}^{N^a} \) be the rank value vector of all AEOSs that can observe target \( k \), where \( R_k(i) \) is the rank value of AEOS \( i \). The four rank items \( R_k^1(i), R_k^2(i), R_k^3(i), R_k^4(i) \) for \( \forall i \in V^a_{i,k} \) are described as follows.

- VTW: the earlier observation, the better ranking. For some emergency missions, we hope the targets can be observed as early as possible, so we prefer to allocate targets to those AEOSs that can observe them earlier. Recall that \( a_{i,k} \) is the start time of VTW between AEOS \( i \) and target \( k \). The rank value of this item can be calculated by:

\[
R_k^1(i) = \frac{a_{i,k}}{\sum_{i \in V^a_{i,k}} a_{i,k}}
\]

where the denominator is used to scale the rank value to 0~1 and make \( \sum_{i \in V^a_{i,k}} R_k^1(i) = 1 \).

Note that if \( a_{i,k} > T^o - \Delta^o + t^o + \tau^o + 1 \) for a certain AEOS, which means the observation and transmission cannot be finished in the planning horizon, then a big rank value \( M \) will be given to \( i \) to avoid allocating \( k \) to this AEOS.

- Observation distance: the closer the distance, the better the ranking. Let \( d_{i,k} \) be the closest distance between AEOS \( i \) and target \( k \) in the VTW. Generally, a smaller value of \( d_{i,k} \) means the target \( k \) is closer to the ground tracks of AEOS \( i \) so that a better image quality can be obtained. The rank value of this item can be calculated by:

\[
R_k^2(i) = \frac{d_{i,k}}{\sum_{i \in V^a_{i,k}} d_{i,k}}
\]

- Number of allocated targets: the fewer targets that have been allocated to this AEOS, the better ranking. We hope each AEOS to be allocated a similar number of targets so that more targets can be observed. This rank value can be calculated by:

\[
R_k^3(i) = \frac{\sum_{k \in V^a_{i,k}} A(k,i)}{\sum_{i \in V^a_{i,k}} \sum_{k \in V^a_{i,k}} A(k,i)}
\]

Note that in step 18 of Algorithm 1, the rank function is called in the for-loop with the input variable \( A \), so the allocation result of the subsequent targets is constantly adjusted according to the previous allocation result.

- Conflict degree to allocated targets: the fewer conflicting VTWs of allocated targets, the better ranking. For each target \( k' \) allocated to AEOS \( i \) (i.e. \( A(k', i) = 1 \)), if the VTW of \( k \) is overlapped with that of \( k' \), then the VTW of \( k \) and \( k' \)
are considered conflicting. Recall that \([a_{i,k}, e_{i,k}]\) is the VTW 
between AEOS \(i\) and target \(k\), then the VTW of \(k\) and \(k'\) are 
overlapped when \(a_{i,k'} \leq a_{i,k} \leq e_{i,k'}\) or \(a_{i,k'} \leq e_{i,k} \leq e_{i,k'}\). 
Let \(N_{i,k}^{\text{con}}\) be the number of conflicting VTWs between \(k\) and 
all targets \(k'\) that have been allocated to AEOS \(i\). A smaller 
value of \(N_{i,k}^{\text{con}}\) means this target \(k\) is more likely to be observed 
if allocated to AEOS \(i\). The rank value can be calculated by:

\[
R_k^i(i) = \frac{N_{i,k}^{\text{con}}}{\sum_{i \in V_{I,k}^a} N_{i,k}^{\text{con}}}
\]  

The rank function \(R_k = \text{Rank}(A, V_{I,k}^a)\) is described in 
Algorithm 2.

**B. Communication Satellites Allocation**

In MCNs, there are hundreds of CSs. It is difficult for 
each AEOS to choose an appropriate one among all CSs 
while planning observation missions. Therefore, to reduce 
the complexity of single-AEOS mission planning while avoiding 
the link conflict among AEOSs, we need to allocate appropriate 
CSs to AEOSs for data transmission during the planning 
horizon.

We propose a CS allocation method based on an Integer 
Linear Programming (ILP) model, which aims to achieve full 
link coverage of each AEOS during the planning horizon with 
a minimum number of CSs. This method is applicable in 
MCNs with an adequate number of CSs.

Define \(C = \{C(j,i)\} \forall j, i \in \mathbb{R}^{N_c \times N_a}\) be a 0-1 decision 
matrix to represent the CS allocation result, where its element 
\(C(j,i) = 1\) when CS \(j\) is allocated to AEOS \(i\). The ILP 
problem is named as function \(C = \text{Com_allocation()\) and is formulated as:

\[
\min \sum_{i=1}^{N_a} \sum_{j=1}^{N_c} C(j,i)
\]

subject to:

\[
\sum_{j=1}^{N_c} [C(j,i) \cdot S_i(j,t)] \geq 1 \quad \forall t \in T_c, \forall i \in [1,N_a] 
\]

\[
\sum_{i=1}^{N_a} C(j,i) \leq 1 \quad \forall j \in [1,N_c] 
\]

\[
C(j,i) \in \{0,1\} \quad \forall j \in [1,N_c], \forall i \in [1,N_a]
\]

where the objective function (39) is to minimize the total 
umber of allocated CSs; inequality (40) guarantees that each 
AEOS \(i\) has connectable CSs at each time slot \(t\); inequality 
(41) ensures that each CS is only allocated to one AEOS; (42) 
describes the decision variables are binary.

**C. Reallocation of Unscheduled Observation Targets**

After the mission planning of each AEOS by solving 
the SA-JOTP problem, some allocated targets may not be 
scheduled to be observed (i.e. \(A(k,i) = 1\), but \(x_{i,k} = 0\)). 
Therefore, we propose a reallocation method to enable more 
targets to be observed.

Define \(A' = \{A'(k,i)\} \forall k, i \in \mathbb{R}^{N_a \times N_a}\) be a 0-1 matrix 
to represent the target reallocation result. Let set \(U\) to mark 
the AEOSs that have changed the target allocation result.

**Algorithm 2: Rank of all AEOSs that can observe 
target \(k\): function \(R_k = \text{Rank}(A, V_{I,k}^a)\).**

**Input:** The current target allocation result \(A\); the 
parameters of AEOSs in set \(V_{I,k}^a\).

**Output:** The rank values \(R_k\) of AEOSs in set \(V_{I,k}^a\).

```plaintext
for \(\forall i \in V_{I,k}^a\) do // Calculate \(N_{i,k}^{\text{con}}\)
    \(N_{i,k}^{\text{con}} \leftarrow 0;\)
    for \(k' \in \{k' | A(k',i) = 1\}\) do
        if \((a_{i,k'} \leq a_{i,k} \leq e_{i,k'})\) or \((a_{i,k'} \leq e_{i,k} \leq e_{i,k'})\)
            then \(N_{i,k}^{\text{con}} \leftarrow N_{i,k}^{\text{con}} + 1;\)
    end
end
for \(\forall i \in V_{I,k}^a\) do // Calculate \(R_k(i)\)
    if \(a_{i,k} > T_o - \Delta^e + \Delta^o + \Delta^e\) then
        \(R_k(i) \leftarrow M;\)
    else
        \(R_k^1(i) \leftarrow \frac{a_{i,k} - a_{i,k}}{\sum_{i \in V_{I,k}^a} a_{i,k} - a_{i,k}};\)
        \(R_k^2(i) \leftarrow \frac{e_{i,k} - e_{i,k}}{\sum_{i \in V_{I,k}^a} e_{i,k} - e_{i,k}};\)
        \(R_k^3(i) \leftarrow 0;\)
        \(R_k^4(i) \leftarrow 0;\)
        if \(\sum_{i \in V_{I,k}^a} k \in V_{I,k}^a \) \(A(k,i) \neq 0\) then
            \(R_k^5(i) \leftarrow \frac{\sum_{i \in V_{I,k}^a} k \in V_{I,k}^a \ A(k,i)}{\sum_{i \in V_{I,k}^a} k \in V_{I,k}^a \ A(k,i)};\)
        if \(\sum_{i \in V_{I,k}^a} N_{i,k}^{\text{con}} \neq 0\) then
            \(R_k^6(i) \leftarrow \frac{N_{i,k}^{\text{con}}}{\sum_{i \in V_{I,k}^a} N_{i,k}^{\text{con}}};\)
        \(R_k^7(i) \leftarrow R_k^1(i) + R_k^2(i) + R_k^3(i) + R_k^4(i) + R_k^5(i);\)
    end
end
return \(R_k\).
```

**Algorithm 3: Reallocation of unscheduled observation 
targets: function \([A', U] = \text{Tar_reallocation()}\).**

**Input:** The original target allocation result \(A\); the 
mission planning result \(x_{i,k}\) of SA-JOTP.

**Output:** The target reallocation result \(A'\); the set \(U\) of 
AEOSs that need to replanning missions.

```plaintext
for \(k = 1 : N_o\) do
    if \(A(k,i) \neq x_{i,k} \neq 0\) and \(|V_{I,k}^a| \geq 2\) then
        \(A'(k,i) \leftarrow 0;\)
        \(R_k \leftarrow \text{Rank2}(A', V_{I,k}^a)\);
        \(A'(k, R_k(1)) \leftarrow 1;\) // reallocate
        \(U \leftarrow [U; R_k(1)];\) // mark the AEOS
    end
end
U \leftarrow \text{unique}(U); // remove duplicate elements
return \(A', U\).
```
The target reallocation function is described in Algorithm 3, which named \[ \text{Tar\_reallocation}() \]. In this algorithm, if target \( k \) is not scheduled and it has at least one other visible AEOS, then it can be reallocated. Here, function \( \text{Rank}() \) is the same as \( \text{Rank}(\cdot) \) except that \( \mathbf{R}_k(i) \leftarrow \mathbf{R}_k(i) + \mathbf{R}_k'(i) \) in step 15 of Algorithm 2. This is because the main goal of the reallocation is to make the observed targets as many as possible, so the number of allocated targets \( \mathbf{R}_k(i) \) and the conflict degree \( \mathbf{R}_k'(i) \) are influential items, while the observation time \( \mathbf{R}_k(i) \) and distance \( \mathbf{R}_k''(i) \) are not that important at this stage. In step 6 of Algorithm 3, we rank all AEOSs that can observe \( k \) except the original allocated AEOS \( i \). Then, we reallocate \( k \) to the top-ranked AEOS in step 7 and mark this AEOS in step 8.

D. Distributed Algorithm for Multiple AEOSs

With the above mission allocation methods and the SA-JOTP model, we propose the MA-DJOTP algorithm for multiple AEOSs. The algorithm is described in Algorithm 4.

Firstly, in steps 1~2, parameters are calculated based on the orbit information of satellites and the position of targets. Secondly in steps 3~4, targets and CSs are allocated to their respective appropriate AEOSs by calling Algorithm 1 and solving the ILP problem (39)~(42). Then, based on the allocation results, each AEOS can solve the SA-JOTP problem in parallel. Here in (4)~(31) of the SA-JOTP, set \( \mathbf{C}_{i,k} \) is replaced with \( \{ k \} | A(k,i) = 1 \} \) and \( \mathbf{C}_{i,k} \) is replaced with \( \{ j | C(j,i) = 1 \} \). With the mission planning result, we reallocate unscheduled targets to other AEOSs in step 8 by calling Algorithm 3. At last, for the AEOSs that have changed the target allocation results, they will solve the SA-JOTP problem again to realize more targets to be observed.

Assume the number of targets and CSs allocated to AEOS \( i \) are \( N_i^oA \) and \( N_i^cA \), respectively. The number of targets reallocated to \( i \) is \( N_i^oA' \). Then the complexity of the MA-DJOTP algorithm is discussed in Theorem 2.

**Theorem 2.** The time complexity of the MA-DJOTP algorithm is upper bounded by \( O(2^{N_i^oA}(T^o+T^c+1) + T^e(N_i^oA+1) + T^o+1) \).

**Proof.** The complexity of Algorithm 4 has four parts:

a) **Targets allocation in Algorithm 1.** The complexity of first allocation and second allocation are \( O(N^o) \) and \( O(N^o) \). The rank of AEOSs in Algorithm 2 has the worst complexity of \( O(N^o \cdot N^o - 1) + O(N^o) = O(N^o \cdot N^o A) \). Thus, the complexity of Algorithm 1 is upper bounded by \( O(N^o) + O(N^o \cdot N^o \cdot N^o A) \).

b) **The CS allocation problem (39)~(42).** This ILP problem can be solved by the branch-and-bound (B&B) algorithm which has exponential complexity related to the number of integer variables in the worst case [18]. Thus, the complexity is upper bounded by \( O(2^{N_i^cA-N^cA}) \).

c) **The SA-JOTP problem (31).** This MIQCP problem can be solved by the branch-and-cut algorithm which combines the B&B and cutting plane algorithms [19]. In the worst case, it has exponential complexity since it involves a B&B tree. The number of integer decision variables \( \psi_i \), \( x_{i,k} \), \( p_{k,k'} \), \( \delta_i \), \( \beta_i \), \( s_{i,k,l} \), \( \lambda_i,t \) is \( N_i^o T^o + N^o + N_i^c T^c + N_i^o T^o + N_i^o + T^o + T^c = N_i^o (T^o + T^c + 1) + T^e(N_i^o + 1) + T^o \). In Algorithm 4, steps 6 and 10 are solved in parallel. Thus, the complexity of steps 5~7 and 9~10 is upper bounded by \( O(2^{N_i^oA(T^o+T^c+1) + T^e(N_i^oA+1)+T^o}) \).

The complexity of Algorithm 3 is \( O(2^{N_i^oA}) \).

Consequently, the complexity of the MA-DJOTP algorithm is upper bounded by \( O(N^o \cdot N^o \cdot N^o A + O(2^{N_i^cA(T^o+T^c+1) + T^e(N_i^oA+1)+T^o}) + O(N^o \cdot N^o) \approx O(2^{N_i^oA(T^o+T^c+1) + T^e(N_i^oA+1)+T^o}) \).

From Theorem 2, we can notice that the time complexity of the MA-DJOTP is influenced by the number of integer decision variables in SA-JOTP. Therefore, the task allocation of Algorithm 1 and the CS allocation of problem (39)~(42) can reduce \( N_i^o \) and \( N_i^c \) to \( N_i^oA \) and \( N_i^cA \), which contribute a lot to reducing the time complexity.

V. SIMULATION AND ANALYSIS

In this section, we verify the effectiveness of the proposed MA-DJOTP algorithm. Firstly, the parameter settings are introduced. Then, result analysis in a small system and algorithm performance in large systems are given with figures and tables.

A. Parameter Settings

We simulate the MCN with 720 CSs and four AEOS systems with different sizes. The small system has 3 AEOSs and the large systems have 16, 36, and 360 AEOSs, respectively. The orbit parameters of satellites in the four cases are listed in TABLE III. The CSs are in Walker-Star configuration referred to the OneWeb [4]. The AEOSs are in solar synchronous orbits with Walker-Delta configuration, and their orbit parameters are
referred to the Shell 3 of Starlink [3]. We use the Satellite Tool Kit (STK) software to generate constellations and obtain their time-varying coordinates and sunlight information.

In the simulation, we consider AEOS AS-01 with the largest pitch degree of 45° and row degree of 45° [5]. We consider two scenarios of target distribution: ‘W’ represents worldwide distribution in 60°S-60°N and 180°W-180°E; ‘A’ represents Chinese area distribution in 3°N-53°N and 74°E-133°E. The number of targets ranges from 100 to 500. For example, ‘500_W’ indicates a scenario with 500 targets of worldwide distribution.

We assume that each observation mission takes 30 seconds [15], so the time slot duration $\tau_o$ is set to 30s. The attitude transition time $\Delta a$ is set to 60s [2], [7]. The ISL handover duration $\Delta$ is assumed to be 60 seconds [20], so the time slot duration $\tau_r$ is also 60s. The data amount $q^o$ of each image is 10 Gbit, and the data rate $v^c$ of ISL is 0.1 Gbit/s. The power of observation, attitude adjustment, laser acquisition, data transmission, and operation are set to $P^o = 0.6kW$, $P^a = 0.05kW$, $P^l = 0.1kW$, $P^d = 0.012kW$, $P^r = 0.1kW$, respectively [21]–[23]. The solar panels of each AEOS have an output power of $P_H = 3.2kW$, and the discharge depth of the battery is $\theta = 25\%$. The benefit $\omega_k$ of each target is randomly selected from the interval [1, 10]. According to Theorem 1, the weight values of objective function (31) are set to $\alpha_1 = 9$, $\alpha_2 = 1$. Other parameters, such as the target visible matrix $O_i$, the link connectable matrix $S_i$, and the sunlight duration $l^{s_{i,t}}$ are calculated based on the satellites’ and targets’ coordinates obtained from STK.

The simulation is executed on Dell 7080MT Optiplex with Intel(R) Core(TM) i7-10700 CPU@ 2.90GHz, 16 GB RAM with MATLAB R2022b on Windows 11. The Parallel Computing Toolbox in MATLAB is used for the parallel solving of the SA-JOTP problems in Algorithm 4. We solve the MIQCP and ILP problems in MA-DJOTP by solver Gurobi [24] and the modeling tool Yalmip [25].

### B. Result Analysis in the Small System

In this subsection, we analyze the simulation results in the small system with 3 AEOSs.

1) State Diagram in Time Domain: To verify the effectiveness of the observation and transmission planning result, we show a state diagram of a certain AEOS in a 500_W scenario in Fig. 3. The planning horizon is set as the orbital period of the AEOS, which is 96 minutes. The AEOS has four states: target observation, attitude transition, laser acquisition, and data transmission. It can be seen that the four operations are arranged appropriately in the time domain, which is consistent with the proposed optimization model (31):

- Enough attitude transition time before observing targets.
- Enough laser acquisition time before data transmission.
- Laser acquisition and data transmission are not performed during the attitude transition.
- ISLs’ continuity is maintained as much as possible.

2) Result Comparison with the Centralized Method: The centralized method is to plan the observation and transmission missions by taking all targets, AEOSs, and CSs in one optimization problem, which can obtain the global optimum. Compared to the SA-JOTP model (31), the centralized method has two other constraints:

$$\sum_{i=1}^{N^a} \delta_{i,j}(t) \leq 1 \quad \forall j \in [1, N^c], \forall t \in T^c$$  \hspace{1cm} (43)

$$\sum_{i=1}^{N^a} x_{i,k} \leq 1 \quad \forall k \in [1, N^o]$$  \hspace{1cm} (44)

where (43) guarantees each CS can only connect to one AEOS at each time slot, (44) constrains each target can be observed at most once. The objective of the centralized method is the sum of (31) for $i \in [1, N^a]$.

Fig. 4 shows the comparison of MA-DJOTP and the centralized method in benefits and solving time. From (a) and (c), we can see that the total benefit of MA-DJOTP is very close to the global optimum obtained by the centralized method. It indicates that MA-DJOTP can obtain excellent results with varying planning horizons and different numbers of targets. From (b) and (d), we can see that the solving time of MA-DJOTP increases slowly with the increase of the problem scale, while the solving time of the centralized method increases sharply. Therefore, the centralized method is not applicable in large constellation systems since the problem is too huge to be solved. Consequently, MA-DJOTP can obtain a solution very close to the global optimum in much less time than the centralized method.
3) Effects of Energy and Storage Resources: The onboard resources of AEOSs have effects on the obtained benefits. Fig. 5 shows the total benefits under varying energy capacity $E_{\text{max}}^i$ and memory capacity $B_{\text{max}}^i$. It can be seen that the benefits increase as the memory capacity increases below 128Gbit, and as the energy capacity increases below 1500kJ. This is because when the memory capacity is limited, the observation data that can be stored is limited, so increasing memory can obtain more benefits. Similarly, when the energy capacity is limited, the missions that can be performed by AEOSs are limited, so increasing energy can obtain more benefits. However, when the memory and energy are sufficient, the observed targets and transmitted data reach the upper limit due to the planning horizon and number of targets, so the benefit no longer increases.

C. Algorithm Performance in Different Sizes of Systems

In this subsection, we verify the algorithm’s effectiveness in different sizes of constellation systems.

1) Solving Time in Different Scenarios: We compare the solving time of MA-DJOTP in scenarios 500_W, 100_A, and 100_W with varying numbers of AEOSs in Fig. 6. Generally, the solving time has the relation: 500_W > 100_A > 100_W. It indicates that more targets and a smaller target distribution range can both make the problem more complex and need more time to be solved. An unusual finding is that in 500_W, the solving time does not increase linearly as the number of AEOSs increases. This is because when there are 16 AEOSs and 500 targets, each AEOS would be allocated lots of targets, so the solving of the SA-JOTP problem will be more complex to select high-benefit targets to be observed and transmit data in the limited planning horizon. With 36 AEOSs, each AEOS would not be allocated too many targets, so the mission planning of each AEOS will be more easier. In general, the solving time performance of MA-DJOTP is excellent under varying numbers of AEOSs and target distribution scenarios.

2) Indicators Analysis and Algorithms Comparison: We compare the simulation results of MA-DJOTP with the JOTSAS algorithm proposed in [15]. The differences between JOTSAS and MA-DJOTP are listed in TABLE I. In this subsection, we analyze four indicators as follows:

- $\text{Abenefit}$: Average benefits of each AEOS.
- $\text{Adelay}$: Average transmission delay (in minutes), where the transmission delay of each target $k$ is calculated by $(s_{t,k} - \sum_{t \in T_{\psi}} t)$.
- $\text{vNum}$: The total number of visible targets of all AEOSs.
within the planning horizon.

- oNum: The total number of observed targets.

The indicators of JOTSAS and MA-DJOTP under different scenarios with the planning horizon of 96 minutes are listed in TABLE IV. We only list the result of JOTSAS in 100_W with 3 AEOSs here, because the other cases cannot be solved by JOTSAS within 60 hours due to the centralized model and the complex modeling of transition time constraints.

From TABLE IV, several conclusions can be obtained as:

- JOTSAS can not obtain the optimal number of observed targets and benefits as MA-DJOTP, because some unnecessary transition time in the JOTSAS. Besides, targets' data need to wait for a long time for transmission since JOTSAS does not consider the second objective of (31).
- As for benefits: 1) in the same targets scenario, the more AEOSs, the less average benefit of each AEOS; 2) with the same number of AEOSs and targets, the smaller the range of targets distribution, the less benefits; 3) with the same number of AEOSs and targets distribution range, the more targets, the more benefits.
- As for transmission delay: 1) in the same target scenario, the more AEOSs, the shorter the transmission delay for each target; 2) with the same number of AEOSs, the denser the target distribution, the longer the transmission delay.
- As for observed targets: the denser the target distribution, the smaller the rate of oNum/vNum.

3) Effectiveness in Mega Constellation Networks: We verify the algorithm performance in MCN with 360 AEOSs and 720 CSs in Fig. 7. It can be seen that in 10 minutes, the 360 AEOSs can observe 463 targets and transmit their data, and the solving time is only 41.04 seconds. All targets can be observed and transmitted data in 30 minutes, and the solving time is at the minute level. The result indicates that MA-DJOTP has fast mission planning capability for the entire MCN, which can effectively deal with some emergencies.

VI. CONCLUSION

In this paper, we propose a distributed joint observation and transmission planning algorithm for multiple AEOSs (i.e., MA-DJOTP) in MCNs. This algorithm uses a targets allocation strategy, a CS allocation model, and a targets reallocation strategy to transform the multi-AEOS problem into several single-AEOS subproblems and solve them in parallel. For each AEOS, a joint observation and transmission planning model (i.e., SA-JOTP) is formulated as an MIQCP problem based on a mission-based time slot division method, which can help simplify the OTW determination and ISL handover time modeling. The SA-JOTP model can realize both the benefit maximization and transmission delay minimization based on practical constraints of mission transition time, laser ISLs' characteristics, and limited onboard resources. We verify the effectiveness of MA-DJOTP in MCNs with 720 CSs and 3, 16, 36, 360 AEOSs. The results show that the proposed algorithm can arrange AEOSs' observation and transmission missions effectively in the time domain. The onboard energy and storage capacity can both affect the optimal solution of benefits. The MA-DJOTP algorithm can obtain a solution close to the global optimum of the centralized method and has minute-level solving time in different target scenarios and hundreds of satellites. Compared to JOTSAS in the existing research, the proposed MA-DJOTP shows great application potential in MCNs.

In the future, we will study the multi-AEOS collaborative mission planning problem and consider some other observation modes for different target shapes and imaging requirements.

REFERENCES


