Extended Object Tracking by Rao-Blackwellized Particle Filtering for Orientation Estimation

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Abstract—A novel particle filter-based single extended object tracking system is proposed, which addresses the challenges arising from an unknown and changing orientation of the tracked object. To this end, we factorize the overall distribution of the extended object state, and employ Monte Carlo techniques for orientation estimation. A computation scheme for a single value from all particles scheme regarding the periodic nature of the orientation and the geometric representation of the overall state is proposed, and challenges regarding the computation of the measurement likelihood are discussed. Due to the bounded nature of the sampled (one-dimensional) state, convincing results can already be achieved with merely ten particles, ensuring the computational efficiency of the approach. Extensive evaluation is carried out, demonstrating a significant improvement in comparison with state-of-the-art methods for a variety of noise levels and measurement rates.

Index Terms—Extended object tracking, target tracking, particle filter, Bayesian filtering

I. INTRODUCTION

Tracking of moving objects is a challenging problem with applications in a variety of domains. With improving sensor resolution, the assumption that targets only emit a single measurement and can be tracked as points is becoming obsolete. Instead, accurate tracking of such objects requires parallel estimation of both, kinematic and shape parameters. This is referred to as Extended Object Tracking (EOT), and is relevant in a variety of fields, such as automotive [1], [2] or maritime [3], [4] applications. An overview of the field can be found in [5], [6]. Tracking multiple extended objects requires both a multi-object tracking system capable of handling the specific challenges of the measurement assignments, and additionally a filtering system that is capable of accurately estimating the kinematic and extent parameters of an individual object, once measurement assignment has been handled. This work is concerned with the latter, i.e., single extended object tracking. Different models for the target extent exist. It is possible to model the target as star-convex [7]–[9], giving many degrees of freedom for the object shape. One common simpler model is to approximate the target extent as elliptical [4], [10]–[15], which is also adopted here. Handling the estimation of the target orientation is central to accurate and robust filtering results, in particular for maneuvering targets. Assuming that the yaw angle and the heading angle coincide is a simple way of circumventing this problem. However, in practice, this assumption often does not hold.

Targets under influence of external forces, such as ships being moved by ocean currents and wind, present one common example for this.

The estimation problem in EOT is highly nonlinear. Closed-form solutions are only available for some special cases, such as in the Multiplicative Error Model Extended Kalman Filter* (MEM-EKF*) [13], where an Extended Kalman Filter (EKF) is employed. A different approach are Particle Filter (PF) methods, also referred to as Sequential Monte Carlo (SMC) methods. In these, the posterior is approximated by a set of weighted samples, i.e., the particles. A naive PF implementation suffers from the curse of dimensionality: Effective sampling of all involved dimensions leads to an exponential number of required particles. For real-time applications such as EOT, this is typically not feasible. One well-known approach to engage this issue is Rao-Blackwellization. In the Rao-Blackwellized Particle Filter (RBPF) [16]–[18], the state is partitioned into two parts, one linear and one nonlinear. The former is solved using analytic methods, whereas SMC techniques are utilized for the latter.

A. Contribution

The key observation underpinning this work is that elliptical EOT is much “simpler” in case the target orientation is known, i.e., in this case efficient closed-form estimators are available. Based on this observation, we apply the idea of Rao-Blackwellization to elliptical EOT. The target orientation is represented by particles, and for each particle (i.e., possible orientation) the mean and variance of the semi-axes lengths and kinematic properties are maintained. For this purpose, we derive closed-form expressions for the required marginal likelihood and show how the measurement update of the semi-axes lengths can be performed.

Experimental results show that by handling the estimation of the unknown and changing target orientation in this manner, the filtering task tackled by the enclosed algorithm is significantly alleviated. Existing related EOT approaches are significantly outperformed, while the computational overhead of the RBPF, which results from sampling a one-dimensional target orientation, is rather minor.

This article is an improved and extended version of [19], where the underlying idea of employing a PF for orientation estimation in EOT has been proposed (using a rather heuristic approach). In this article, however, we propose an improved formulation, which follows the well-known framework of the RBPF, and provide a more elaborate evaluation.

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B. Structure

The remainder of this article is structured as follows: First, Section II discussed related works. Afterwards, Section III illustrates the problem settings and defines the proposed algorithm in detail. Section IV presents the experimental analysis carried out to validate the approach as well as the observed results. Based on this, discussion of the results is carried out in Section V and the paper is then concluded in Section VI.

II. RELATED WORKS

Previously, different approaches to EOT have been proposed. One common algorithm is the Random Matrix (RM) method \[10, 11\]. Here, the shape of the target is modeled as an ellipse, represented by a symmetric positive definite shape matrix. Filtering is handled by means of an inverse Wishart distribution. A variety of extensions based on the original framework have been proposed, e.g., \[20\] uses multiple random matrices to track objects with non-elliptical extents. In \[21\], variational Bayes techniques were employed, allowing for explicit consideration of the target orientation. If objects are tracked as ellipses, it is also possible to explicitly parameterize the target shape as a combination of semi-axis lengths and orientation. A comparison of methods taking this approach was presented in \[22\]. Under the assumption that the yaw angle of the extent coincides with the velocity heading, the ellipse orientation does not need to be explicitly tracked. Based on this assumption, the Independent Axes Estimation (IAE) algorithm \[14\] was derived. The scattering matrix of measurements is used to derive a measurement for each of the semi-axis lengths, and the mean of measurements is used to update the target center kinematics filter. Explicit consideration of the orientation is performed in the MEM-EKF* \[13\]. The method processes measurements iteratively, rather than using a batch-based approach as the previous algorithms. It is based on a specifically derived EKF. An EKF is also used for tracking an elliptical target in \[23\]. Here, measurements are associated to points distributed on the object contour, and moment matching is used to ensure the target remains Gaussian under multiple association hypotheses. In contrast to this work, the employed measurement model is designed for Light Detection And Ranging (LIDAR). Using the same key assumption as the IAE regarding yaw angle and heading angle, in \[15\] a closed-form filter named Lambda:Omicron Multiplicative Error Model (L:OMEM) was proposed. This key assumption is in direct contrast to the setting this work is concerned with. Compared to the MEM-EKF*, minor accuracy gains were observed, but a major speed-up in computation time was achieved. By employing Eigenvalue Decomposition (EVD), it is possible to acquire explicit estimates for orientation and semi-axis lengths in a batch-based fashion. This is employed in the Principal Axes Kalman Filter (PAKF) \[4\]. The filter was adopted for maritime Radio Detection And Ranging (RADAR) measurements. For a variety of tracked objects, e.g., ships or vehicles, the semi-axes are unknown, but fix. This assumption is exploited in \[24\], where an iterative approach based on the Expectation-Maximization (EM) algorithm is proposed.

Particle filters have also received attention as a framework for tackling the EOT problem. In \[25\], an RBPF has been employed for tracking objects by making use of measurement generating points. The method is applied to spline-based tracking of a rectangular target, under varying perspectives and clutter. A mixed linear/non-linear state space model is assumed, and the RBPF is used for estimation of nonlinear parts. A different variant of a PF, namely a convolution particle filter, was examined for EOT in \[12\]. Non-maneuvering and maneuvering targets were considered. Since the entire state is filtered using SMC methods, the number of particles required is several orders of magnitude larger than the one of the RBPF-based approach proposed here. A box particle filter was presented in \[26\]. In contrast to the proposed method, the focus is on multiple extended object tracking. Experimental results are discussed for targets with a circular extension, i.e., a single semi-axis (the radius) needs to be estimated, compared to the elliptic model discussed here, where two semi-axis and the orientation must be tracked. A (standard) PF is employed to filter the target state in \[27\]. However, opposed to the focus of this article, the target extent is assumed to be known, so the estimation problem reduces to the kinematic properties of the tracked object. Based on the RM model, an RBPF is proposed as part of \[2\]. SMC is employed for the semi-axis lengths of the object, represented by the shape matrix. Similar to \[14\], \[15\], the orientation is assumed to coincide with the heading, meaning that the investigated setting differs substantially from the one in this article. Furthermore, a core contribution of \[2\] is the data-driven learning of a measurement model, whereas here we focus on model-based EOT.

III. PROPOSED METHOD

In the following, first, the problem setting with which this work is concerned will be illustrated. Afterwards, the RBPF employed for EOT will be defined. More details regarding the likelihood will be given, and finally the acquisition of an overall single estimate given a set of individual particles will be discussed.

Throughout the article, the following notation is used: Bold lower-case letters indicate vectors, whereas standard italic notation indicates scalars. Bold upper-case letters indicate matrices. A subscript, typically \(k\), is used to refer to the discrete time step index, with \(k\) referring to a value at time \(k\) with measurements up to time step \(l\) integrated. For indexing of particles or measurements within their respective set, a superscript is used.

A. Problem Setting

Throughout this article, we focus on the case of tracking a single elliptical extended object in two-dimensional space, i.e., the seven-dimensional state can be written as

\[
x_k = \begin{bmatrix} m_{1,k} & m_{2,k} & \dot{m}_{1,k} & \dot{m}_{2,k} & \theta_k & l_{1,k} & l_{2,k} \end{bmatrix}^T.
\]

Here, \(m_{1/2,k}\) and \(\dot{m}_{1/2,k}\) are the target position and velocity, respectively. The shape is parameterized explicitly by the two semi-axis lengths \(l_{1/2,k}\) and the ellipse orientation \(\theta_k\). Note that as we assume heading angle and shape yaw angle to be entirely decoupled, both \(\dot{m}_{1/2,k}\) and \(\theta_k\) must be part of the estimated state.
At a discrete time step $k$, a set of $M_k$ measurements $Z_k = \{z^i_k \mid i = 1 \ldots M_k \}$ is received. The expected number of measurements is $\lambda$, and $M_k \sim \text{Pois}(\lambda)$. Individual measurements are independent of each other, and are formed as

$$z^i_k = y^i_k + \nu^i_k,$$

where $y^i_k$ is the $i$-th measurement source at time $k$ on the target surface and $\nu^i_k$ is zero-mean additive noise with

$$\nu^i_k \sim \mathcal{N}(0, R).$$

For the measurement sources $y^i_k \in \mathbb{R}^2$, a uniform distribution across the target ellipse surface $S(x_k)$ is assumed, i.e.,

$$p(y_k | x_k) = \begin{cases} \frac{1}{\text{area}(S(x_k))}, & \text{if } y_k \in S(x_k) \\ 0, & \text{otherwise}. \end{cases}$$

The state transition of the orientation $\theta_k$ is modeled as

$$p(\theta_k \mid \theta_{k-1}) = \mathcal{N}(\theta_{k-1}, \sigma^2_\theta).$$

No initial value for any part of $x_k$ is known to the tracker. Instead, the initialization must be computed based on the first set of received measurements.

The scope of this work is limited to filtering, i.e., recursive estimation of the target quantities given only measurements up to and including the current time step.

**B. Rao-Blackwellized Particle Filtering**

In the following, details of the proposed RBPF will be given. For the general form of an RBPF, we closely follow [18], and refer there for a more detailed derivation.

In an RBPF, the state is split in two parts, a latent variable $u$ and a conditionally linear part $v$. Sequential importance resampling is used for $u$, whereas filtering of $v$ can be handled using the Kalman filter formulas. Here, we set $u = \theta$ and $v = x \setminus \theta$. Even conditioned on $\theta$, estimation of $v$ is not a linear problem for which a standard Kalman filter could be employed. Instead, we approximate it using an existing extended object tracker, which works given a fixed orientation. We select the IAE [13] as the method of choice. In this method, the yaw angle of the object was originally matched to its corresponding covariance given $z^i_k$ and the newly drawn latent variables $u^i_k$ using the IAE.

Incorporating newly received measurements $Z_k$ in the RBPF consists of five main steps:

1. Perform the predict step, computing $v^i_{k|k-1}$ and its corresponding covariance from $v^i_{k-1|k-1}$ and its covariance using the IAE.

2. Draw a new value for the latent variable, i.e., the orientation $\theta$, according to the importance distribution $q^i$:

$$\theta^i_k \sim q(\theta_k \mid \theta^i_{k-1}, Z_{1:k}),$$

3. Compute the unnormalized particle weights

$$w^{i,*}_k = w^{i}_{k-1} \cdot \frac{p(Z_k \mid \theta^i_k, Z_{0:k-1})p(\theta^i_k \mid \theta^i_{k-1})}{q(\theta_k \mid \theta^i_{k-1}, Z_{1:k})},$$

4. Compute $v^i_{k|k}$ and its corresponding covariance given $Z_k$ and the newly drawn latent variables $u^i_k$ using the IAE.

5. Resample the particles using $w^{i,*}_k$, resetting their weights to uniform values.

Resampling can be limited by certain conditions such as the effective number of particles computed from the variance of the particle weights [18], or be done in every time step, which was chosen for simplicity in this work.

**C. Likelihood Computation**

At the heart of the filter is the computation of the likelihood in (7), which is not straightforward due to the involved nonlinearities and approximations. Since we set

$$q(u^i_k \mid u^i_{k-1}, Z_{1:k}) = q(\theta_k \mid \theta_{k-1}) = p(\theta_k \mid \theta_{k-1}),$$

as in the bootstrap PF, computation of the particle weights (7) reduces to the computation of the likelihood, i.e.,

$$w^{i,*}_k = w^{i}_{k-1} \cdot p(Z_k \mid \theta^i_k, Z_{0:k-1}).$$

As resampling is performed in every time step, $w^{i,*}_k$ will always be uniform across all particles and can be dropped due to the normalization in (8), leading to

$$w^{i,*}_k = p(Z_k \mid \theta^i_k, Z_{0:k-1}).$$

Typically, this term would be computed using the marginal measurement likelihood of the Kalman filter, given the newly sampled latent variable, i.e., orientation. However, due to the approximations necessary to tackle the nonlinearities remaining in the EOT case, this is not as straightforward here. The IAE employs three individual Kalman filters, one for each of the semi-axes and one for the target center. Updating the filter for the target center is carried out based on the mean of all observed measurements. For the semi-axis lengths, it is assumed that the orientation of the elliptic shape aligns with the velocity heading of the tracked object. An EVD of the normalized scattering matrix of measurements followed by moment matching yields a Gaussian model for the individual semi-axis lengths, which is used for the update of the remaining two Kalman filters. The marginal measurement likelihood for the filter concerned with the target center is straightforward to compute using the average of the observed measurements. For the other two filters, the measurements can be constructed using the half-axis observation model derived in [14, Section IV]. In theory, these three could be viewed as independent, and the product of the individual marginal measurement likelihoods could be used to compute (11). However, in practice, we found this to yield inaccurate weighting of the particles, causing the particle filter to converge to an isotropic estimate of the elliptical target shape. Instead, we propose to
replace the two terms for the semi-axes filters with a single likelihood, namely by using the distribution of the normalized scattering matrix of $Z_k$, which is used in [14] to derive the half-axis observation model based on which the individual semi-axes filter measurements are computed. Following [14], the normalized spread matrix can be computed as

$$\bar{Z}_k = \frac{1}{M-1} \sum_{i=1}^M (z_k^i - \bar{z}_k)(z_k^i - \bar{z}_k)^T,$$

using the measurement mean

$$\bar{z}_k = \frac{1}{M} \sum_{i=1}^M z_k^i \quad .$$

$\bar{Z}_k$ is Wishart-distributed [10], [14] according to

$$Z_k \sim \mathcal{W}(M - 1, \frac{1}{M - 1}(X_k^{i|k-1} + R)) \quad ,$$

with $R$ being the measurement noise covariance matrix as defined in [3] and $X_k^{i|k-1}$ being the predicted shape matrix estimate of particle $i$ at time $k$, given measurements up to time $k-1$. The shape matrix can be computed according to

$$X_k^{i|k-1} = \frac{1}{4} \text{rot}(\theta_k^i) \begin{bmatrix} t_{1,k-1} & 0 \\ 0 & t_{2,k-1} \end{bmatrix}^2 \text{rot}(\theta_k^i)^T,$$

using $\text{rot}(\cdot)$ to create a standard rotation matrix of the argument. The scaling factor of $\frac{1}{4}$ is used to approximate a uniform distribution across the ellipse surface [11].

By maintaining the independence assumption between target center and shape, the overall likelihood can be computed as the product of the likelihood of the predicted target center $m_k^i$ with corresponding covariance matrix $P_k^{i,m}$ and the aforementioned likelihood of $Z_k$, yielding

$$w_k^{i,*} = \mathcal{N} \left( \bar{z}_k; m_k^i, P_k^{i,m}_{k|k-1} + \frac{1}{M} (X_k^{i|k-1} + R) \right) \cdot \mathcal{W} \left( \bar{Z}_k; M - 1, \frac{1}{M - 1} X_k^{i|k-1} + R \right) \quad .$$

An approximation that is made here is that the uncertainties of the semi-axis lengths are dropped, due to the representation of the shape by means of a Wishart distribution.

In addition to the likelihood defined in (16), we furthermore propose an alternative formulation. For increasing noise levels, i.e., as $X_k^{i|k-1} + R$ is being progressively dominated by the contribution of $R$, the particle weights tend to uniform values. This is because the quality of the estimate, represented by $X_k^{i|k-1}$, barely contributes to the overall likelihood evaluation anymore. A possible solution is to eliminate the measurement noise covariance from the computation of the likelihood entirely, giving rise to

$$w_k^{i,*} = \mathcal{N} \left( \bar{z}_k; m_k^i, P_k^{i,m}_{k|k-1} + \frac{1}{M} X_k^{i|k-1} \right) \cdot \mathcal{W} \left( \bar{Z}_k; M - 1, \frac{1}{M - 1} X_k^{i|k-1} \right) \quad .$$

In Section [IV] the empirical performance of the two likelihood formulations will be compared.

D. Estimate Acquisition

To acquire an overall estimate of the filter from the individual particle states, typically a (weighted) average is computed. However, this poses two issues for the state space defined by (11). The yaw angle $\theta$ is periodic in the half-open interval $[0, 2\pi)$, which needs to be accounted for. Furthermore, $[\theta \ l_1 \ l_2]^T$ has a specific geometric representation, namely that of an ellipse. Due to ambiguities with orientation and semi-axis lengths, the same ellipse can be represented numerically in different ways. These ambiguities must be regarded during the estimate acquisition process.

The challenge posed by the consolidation of particle estimates is closely related to the fusion of multiple elliptical extended object estimates, which has been discussed in detail in [28]. There, the Square Root Space (SRS) of ellipse representations was introduced. The SRS representation of an ellipse $[\theta_k^i \ l_{1,k}^i \ l_{2,k}^i]^T$ can be computed as

$$\sqrt{X_k^i} = \text{rot}(\theta_k^i) \begin{bmatrix} t_{1,k}^i \\ 0 \\ t_{2,k}^i \end{bmatrix} \text{rot}(\theta_k^i)^T \quad .$$

Following [28], the weighted average of ellipses can be computed using this representation as

$$\sqrt{\bar{X}_k} = \sum_{i=1}^P w_k^i \cdot \sqrt{X_k^i} \quad ,$$

where

$$\sum_{i=1}^P w_k^i = 1 \quad .$$

Given the overall estimate $\sqrt{\bar{X}}$ in SRS, the explicit semi-axes representation can be reconstructed using EVD of the full shape matrix

$$X_k = \sqrt{\bar{X}} \sqrt{\bar{X}} \quad .$$

To this end, the orientation can be computed from the eigenvectors, and the semi-axis lengths from the square root of the eigenvalues.

Consolidation of the remainder of the state, i.e., of $m_k^i = [m_{1,k}^i \ m_{2,k}^i]^T$ and $\dot{m}_k^i = [\dot{m}_{1,k}^i \ \dot{m}_{2,k}^i]^T$, can be carried out using a standard weighted average

$$\bar{m}_k = \sum_{i=1}^P w_k^i \cdot m_k^i \quad ,$$

and equivalently for $\dot{m}_k^i$.

This estimate acquisition scheme is equivalent to the one previously proposed in the conference version [19]. The full algorithm is summed up as pseudo-code in Algorithm [1] and source code for the implementation is available online [2].

IV. EXPERIMENTAL RESULTS

The following section will first describe the simulation used to study the previously derived filter in practice. Afterwards, an in-depth analysis of the performance of the filter will be carried out.

https://github.com/Fusion-Goettingen/
Algorithm 1: Predict/update cycle of the tracker.

1) For each particle $i \in \{1, \ldots, P\}$:
   a) Perform a predict step using the per-particle IAE instance following [14, Section III-a],
   b) Sample a new orientation $\theta^*_k$ using (5) and ensure it is in $[0, 2\pi)$ via
      $$\theta^*_k = \theta^*_k \mod 2\pi,$$
   c) Calculate the particle weight using (16) or (17),
   d) Perform a measurement update using the per-particle IAE instance and the newly drawn $\theta^*_k$ following [14, Section III-b].
2) Normalize the particle weights as in (8).
3) Resample the particles according to their updated weights $w_k$, leading to uniform $w_k$.
4) Create a single overall estimate using (19) and (22).

A. Simulation Setup

For the experiments, the object moved along a trajectory consisting of straight parts and four turns, visualized in Fig. 1a.
Throughout the trajectory, the object’s orientation was rotating by $\frac{1}{16} \pi$ in every time step, decoupled from its velocity. An example for this be seen in Fig. 1b where the third turn in the simulation is shown. The lengths of the semi-axes of the object were 5 m and 2 m, respectively.

In the experiments, two main parameters were varied: Firstly, the measurement noise covariance $R$, which was set to an identity matrix scaled by a factor between 0.1 and 4. Secondly, the expected number of measurements per time step, which was Poisson distributed with expected value $\lambda$ between 4 and 40. Each time step generated at least one measurement.

In order to evaluate the estimation accuracy of different tracking algorithms, a suitable metric must be chosen. For the specific problem of single elliptical EOT, the metric must simultaneously account for displacement of the object center and inaccuracies in the shape estimate. Furthermore, it is indispensable to take the ambiguous nature of the parameterization of the elliptic shape into account. Following [29], the Gaussian Wasserstein Distance (GWD) [30] has been chosen as a metric, as it considers all this. Errors will be reported in squared GWD, measured in square meters.

The following set of reference algorithms was chosen for the experiments: Even though its parameterization differs from the one explored in this article, the RM algorithm [11] was chosen as a baseline method, which does not explicitly account for the orientation. Both the MEM-EKF* [13] and the PAKF [4] explicitly parameterize the extent as semi-axis length and orientation, while tracking all three shape parameters. Hence, both were included in the evaluation. Finally, the IAE [14] has been included as a reference for an algorithm which assumes yaw angle and heading to coincide. It must be noted that this assumption does not hold for the conducted study, and therefore, the results of the IAE are merely to be taken as a reference for the importance of correctly tracking the orientation in such a setting. All filters were parameterized to the best of our knowledge to yield optimal results in the studied simulation environment. For details regarding the parameterization, we refer to our publicly available code, see Section III. The motion model used in all algorithms is a nearly constant velocity model, i.e., the four turns are mismatches which need to be accounted for by the tracker. To this end, the process noise of all methods was set to correspondingly high values. All methods were initialized in the same manner, using the measurements observed in the first time step. The kinematic state mean for all filters was set to correspondingly high values. All methods were initialized in the same manner, using the measurements observed in the first time step. The kinematic state mean for all filters was set to the mean of the measurements. Semi-axis lengths and orientation were determined using an EVD of the scattering matrix of measurements.

B. Performance Analysis

In the following, the tracking performance of the RBPF will be analyzed. In particular, the following subsection focuses
on choices regarding the likelihood and particle numbers and compares the filter to the one presented before in [19]. The next subsection [IV-C] will contain comparison to existing state-of-the-art extended object trackers. A comparison of the RBPF with a naive PF-based approach as well as an analysis of the EOT algorithm used for $v = x \setminus \theta$ were carried out in [19], and are not repeated here.

Fig. 2 shows a grid-based visualization of the difference in tracking accuracy when employing the two likelihood variants (16) and (17). For all evaluated measurement rates and noise levels, one cell of the grid indicates the difference in squared GWD, averaged over 50 Monte Carlo runs. The difference between the two is largest in scenarios with high noise, which is the scenario which lead to the design of (17) in the first place. This can be seen as confirmation of the initial hypothesis that large measurement noise values overshadow the influence of the shape matrix $X_i k$, preventing accurate particle weighting. However, in fact, even low-noise scenarios show improved performance using the likelihood (17). This indicates that, for the sake of computing the particle weights, eliminating the diluting effect that including the noise covariance has on the likelihood can be desirable. When interpreting Fig. 2 and the following grid-based visualizations, the fact that a difference between errors is visualized must be kept in mind. Naturally, for sparse and noisy data, the overall error will be larger for both evaluated approaches than for dense data with little noise.

The same experiment has been carried out to compare the tracking accuracy of the method proposed in this article with the accuracy of the previous version of the algorithm [19]. Results are presented in Fig. 3. Results show that for all settings, results can be improved. In settings with high signal-to-noise ratio, i.e., small measurement noise, the difference is smaller. In particularly challenging settings, characterized by high measurement noise and low numbers of received measurements, the discrepancy between the two methods is particularly noticeable. Note that for improved visibility, the scaling of Fig. 2 and Fig. 3 is not equivalent. The largest difference in the former is much smaller than the largest one in the latter.

To evaluate the impact that varying the number of particles has on the filtering quality, the tracker was run with different numbers of particles across 50 Monte Carlo runs for a measurement rate of $\lambda = 12$ and measurement noise of $R = I \cdot 2$. Results are shown in Table I comparing both the error in squared GWD as well as the average run time of the different settings. The estimation quality improves with increasing particle numbers. However, a convergence behavior is observed already around 25 particles. As expected, the run time of the algorithm grows linearly with the number of particles. A more detailed analysis of the computational cost of the algorithm in comparison to reference methods is presented in Section [IV-C]. Note that the exact numbers differ slightly between experiments due to change in background computations of the environment.

In summary, it was determined that using the likelihood given in (17) and 25 particles is suitable for a well-performing tracking system. All further experiments were hence conducted using these settings.

### C. Comparison to Reference Methods

In order to compare the performance of the proposed method with existing approaches, an “average” scenario was

**TABLE I: Evaluation of the impact of the number of particles on the filtering performance of the algorithm. Furthermore, the average run time across 50 Monte Carlo runs of the simulation with an expected value of eight measurements is shown.**

<table>
<thead>
<tr>
<th>Number of Particles</th>
<th>5</th>
<th>10</th>
<th>25</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Squared GWD / m²</td>
<td>3.22</td>
<td>2.38</td>
<td>2.23</td>
<td>2.22</td>
</tr>
<tr>
<td>Run time / ms</td>
<td>3.07</td>
<td>5.51</td>
<td>12.92</td>
<td>25.34</td>
</tr>
</tbody>
</table>
selected to evaluate how different trackers behave across the entire trajectory. For this, a measurement rate of $\lambda = 12$ and measurement noise of $R = I \cdot 2$ were chosen, the same as for the evaluation of the number of particles. Results of the experiment are shown in Fig. 4 averaged over 500 Monte Carlo runs. Average error values in squared GWD are given as part of the legend. As visible from the figure, the proposed method consistently outperforms all reference methods. The MEM-EKF* performs nearly as good, however, in turns, spikes in error can be observed. A similar behaviour, albeit with larger errors, is visible for the PAKF. Finally, both the IAE and the RM methods exhibit consistently large errors. For the IAE, this is expected, as its core underlying assumption that yaw angle and velocity heading coincide is violated in the evaluated scenario. This leads to isotropic shape estimates. By design, the RM algorithm also tends to isotropic shape estimates due to the turning object. This is due to the single scalar uncertainty associated with the shape estimate. Increasing the “forgetting factor” of the method to allow for faster change in orientation makes it susceptible to noise. Hence, for the evaluated scenario, the RM algorithm is noticeably outperformed by methods that explicitly estimate the target orientation. This setting used for this experiment is the same as used for the analysis of the impact of the number of particles on the RBPF, shown in Table I. Comparing the overall average of the MEM-EKF* ($3.09 \text{ m}^2$) with the error of the RBPF when the number of particles is set to five ($3.22 \text{ m}^2$) shows that indeed very few particles are required to match the estimation quality of the MEM-EKF* with the proposed methodology.

As a reference method for a more detailed comparison, the MEM-EKF* was chosen. This is for two reasons: In the preliminary comparison shown in Fig. 4 the MEM-EKF* performed best of all reference methods, and furthermore it uses the same explicit parameterization of semi-axes and yaw angle as the RBPF. Results of the comparison over thirty different settings are shown in Fig. 5 averaged over 50 Monte Carlo runs. In settings with high measurement rate, i.e., expected values of 20 or 40 measurements per scan, the difference between methods is minuscule, as both methods are able to accurately track the object. The difference between methods is largest for settings with very sparse measurements and medium noise levels. For very low noise, both methods require few measurements for accurate tracking. With increasing noise, but sparse measurements, the proposed method’s tracking performance noticeably exceeds that of the MEM-EKF*. For settings with very high measurement noise and lower measurement rates, the proposed filter still outperforms the reference method, albeit to a smaller degree. In summary, the proposed method compares favorably to the MEM-EKF* in settings with sparse measurements, and matches its performance in denser ones.

In order to compare the computational cost of the algorithms, the average run time across all steps of 500 Monte Carlo runs of the simulation has been evaluated. This evaluation was done using unoptimized python implementations of all methods on an Intel i5-12600K CPU. Results of this are given in Table II. Generally, the batch-based methods RM, IAE and PAKF are much faster than the iterative MEM-EKF* and the proposed method. The latter two are comparable in speed, yet scale with different factors: The run time of the MEM-EKF* mostly depends on the number of received measurements. The proposed method on the other hand scales...
with the number of employed particles. As it is based on the batch-based IAE, the number of measurements does not have notable impact on the run time. This is true for all methods except the MEM-EKF*. In fact, some methods actually have a marginally lower average computation time for the higher measurement count, likely due to randomness of background computations of the testing environment.

Qualitative results of a single turn of the trajectory are shown for the RBPF and the MEM-EKF* in Fig. 6, where excerpts from example Monte Carlo runs are presented. Two settings are visualized, one with lower measurement noise and higher measurement rate, and a second one with increased noise and sparse measurements. In the first setting shown in Fig. 6a both algorithms yield highly accurate tracks. In individual time steps, the MEM-EKF* estimate is slightly worse than that of the proposed method, yet overall the difference is negligible. In Fig. 6b, however, a clear distinction between the methods is visible. Before the turn, the object is tracked well by both methods. During the turn, the orientation can not be clearly estimated, and the MEM-EKF* orientation estimate diverges from the true value. Simultaneously, the RBPF accounts for this with a “wider” ellipse, which stems from the consolidation of particles of different orientations. As more measurements are received, the particle filter returns to the true object shape, with a noticeably lower error in the yaw angle compared to the reference method.

V. DISCUSSION

In general, the method has proven to yield highly accurate estimation performance. A variety of settings regarding measurement noise and measurement sparsity have been evaluated, and the method outperformed the state-of-the-art in almost all of them. Especially sparse measurements prove difficult for the reference methods. Since Kalman filter-based systems such as the MEM-EKF* maintain the state by means of a Gaussian random variable, the overall estimate is unimodal. When the target turns, and when little information is available due to sparse or noisy data, estimation of the orientation can prove difficult. Compared to this, the multi-modal approach taken by the RBPF, representing different orientations as individual particles, can allow adapting to such changes quickly, while still maintaining lower-likelihood hypothesis for a while. Of course, this comes at the cost of evaluating multiple hypothesis, which is at least linearly more expensive compared to updating a single tracker. Keeping the number of particles small is therefore integral to the applicability of the algorithm. Due to the Rao-Blackwellization approach, the number of sampled dimensions can however be kept to a single one, which is bounded in $[0, 2\pi]$. This means that very few particles are required for accurate estimation. In fact, as shown in Table I, merely five to ten particles already yield accurate results. In that case, the method is comparable in speed to the MEM-EKF*, even for sparse settings. Nevertheless, it is impossible to achieve the same speed as the batch-based methods.

Two different likelihoods were evaluated. It was found that incorporating the measurement noise into the likelihood used for particle weight computation leads to a diluting effect, which causes estimation accuracy to be reduced. As expected, this is particularly noticeable in noisy scenarios. Removing the measurement noise terms from the weight computation of the particle was chosen for the overall method.

It is important to note that the results do not imply that the filter always performs better than, e.g., the MEM-EKF*. The MEM-EKF* yields accurate tracking results, and there were simulation runs in which it outperformed the proposed approach. In sparse and noisy scenarios, accurate shape estimation is exceedingly difficult. Still, the average results over many Monte Carlo runs in a variety of settings show that, at least with respect to the GWD, the proposed method can indeed improve estimation quality over existing approaches.

VI. CONCLUSION

In this article, we propose an RBPF for EOT, focusing on the task of accurate orientation estimation under high
uncertainties and sparse measurements. Highly detailed comparisons across a variety of measurement settings were carried out to analyze the method and evaluate it with respect to existing reference tracking algorithms. Results show that in all evaluated settings, state-of-the-art methods could be outperformed. In particular if few measurements of the object to be tracked are available, the novel method yields noticeably improved estimates. Two different variants of the measurement likelihood used in the computation of the particle weights were investigated. It was found that incorporating the measurement noise covariance with the expected shape matrix of the track can have a diluting effect on the weights. The small search space of potential orientations means the filter requires very few particles to yield accurate estimation results, making it suitable for online filtering applications. Remarkably, merely five to ten particles were found to achieve acceptably high quality, and performance was found to barely improve beyond 25 particles.

In the future, integration of the proposed method into multi-target tracking architectures could be of prime interest. For Bayesian multi-object trackers, reliable uncertainties for the state estimate must be available. Future work may evaluate how this can be achieved with the method at hand. Another future research direction could be the application of the methodology to tracking of star-convex targets.

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