Karst water resources in a changing world: Review of solute transport modelling approaches

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March 29, 2024

Abstract

Karst water resources are valuable freshwater sources for around 10 % of the world population. Nonetheless, anthropogenic factors and global changes have been seriously deteriorating the karst water quality and dependent ecosystems. Solute transport models are powerful tools to monitor, control, and manage the water quality and dependent ecosystem functioning. By representing and predicting the spatiotemporal behavior of solute migration in karst systems, the transport models enhance our understanding about the karst transport processes, thus enabling us to explore contamination risks and potential outcomes. This paper reviews the current state of knowledge on the modelling of solute transport processes in karst aquifers, thereby unveiling the fundamental challenges underlying a successful karst transport modelling. We discuss to what extent and in what ways we can handle these challenges and derive the key challenges and directions for reliable modelling of transport processes in karst systems in the present context of global changes.

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Key Points:

● Anthropogenic factors and global changes pose a serious threat on karst water quality
● We review solute transport process and modelling approaches in karst water resources
● Fundamental challenges afront a successful transport modelling practice are provided
● We address knowledge gaps and research directions in karst transport modelling
Abstract

Karst water resources are valuable freshwater sources for around 10% of the world population. Nonetheless, anthropogenic factors and global changes have been seriously deteriorating the karst water quality and dependent ecosystems. Solute transport models are powerful tools to monitor, control, and manage the water quality and dependent ecosystem functioning. By representing and predicting the spatiotemporal behavior of solute migration in karst systems, the transport models enhance our understanding about the karst transport processes, thus enabling us to explore contamination risks and potential outcomes. This paper reviews the current state of knowledge on the modelling of solute transport processes in karst aquifers, thereby unveiling the fundamental challenges underlying a successful karst transport modelling. We discuss to what extent and in what ways we can handle these challenges and derive the key challenges and directions for reliable modelling of transport processes in karst systems in the present context of global changes.

Plain Language Summary

Karst aquifers are valuable freshwater sources. Nonetheless, anthropogenic factors and global changes have been seriously deteriorating the karst water quality and dependent ecosystems. Therefore, understanding, characterizing, and predicting the impacts of such changes are essential for effectively managing and sustainably protecting the karst water quality and dependent ecosystem functioning. Solute transport models are valuable decision-support tools to explore the contamination risks and the potential outcomes of deteriorated water quality. Furthermore, model predictions can be used for introducing the efficient adaptation strategies and regulatory decisions for the contamination-related problems in karst systems. In this review, we provide the current state of knowledge on modelling of solute transport in karst aquifers. By synthesizing the existing body of knowledge, we unveil the fundamental modelling challenges for successful karst transport modelling practice.
1 Introduction

A safe and clean groundwater supply is necessary for our society and water-dependent ecosystems (Hartmann et al., 2021). Karst groundwater is a valuable freshwater source that provides the potable water for nearly 10% of the total world’s population (Stevanović, 2019). However, contamination issues have been seriously threatening karst water quality (Henry & Suk, 2018; Reberski et al., 2022; Savio et al., 2018) and karst groundwater-dependent ecosystems (Goldscheider, 2019; Siegel et al., 2023). For this reason, a reliable risk assessment for the contamination-related problems in karst water resources is essential to effectively manage and sustainably protect the karst water quality and dependent ecosystems.

Deterioration of karst water quality and ecosystem functioning can be linked to two main factors including anthropogenic factors and global changes. While the anthropogenic factors represent the direct human intervention that modifies the physical, chemical, and biogeochemical characteristics of karst systems, such as water management alterations (i.e., groundwater abstractions, reservoir, and dam buildings), the global changes are indirectly influencing the water quality. Urbanization, climate change, soil degradation, land use, and land cover changes are such serious factors that predominantly impact the water quality and ecosystem services (i.e., springs, caves, groundwater-fed wetlands) (e.g., Gutiérrez et al., 2014; Hartmann et al., 2014; Katz, 2019; Vilhar et al., 2022).

Karst aquifers are particularly vulnerable to contamination risks due to the substantial heterogeneity manifested by the preferential flow paths (i.e., fractures, conduits, cave streams) at differing spatial scales (e.g., Bakalowicz, 2005; Ford & Williams, 2007; Goldscheider & Drew, 2014; Padilla & Vesper, 2018; Stevanović, 2018; Quinn et al., 2006, among many others). Besides these (multiple) discrete flow paths that cause the rapid spreading of the contaminants throughout the karst systems (Field, 2018; Hartmann et al., 2014, 2021; White, 2018), the close interaction with the surface waters (i.e., rivers, lakes) and thin soil characteristics limit the attenuation of the contaminants released into the karst aquifers (Goldscheider, 2005; Sinreich, 2014; White, 2019).

Solute transport models with differing complexity have been used for solving complex environmental issues by various scientific communities (e.g., fractured rock, geothermal energy, mining, and soil sciences). However, as the multiscale karstic heterogeneity — referring to the spatial variations of the aquifer’s physical and chemical characteristics at varying scales — shapes the karst system’s physical, chemical, and biogeochemical characteristics, the successful application of the solute transport model in karst systems is a challenging task, particularly due to the (significant) lack of knowledge on the karst system’s physical and (biogeo)chemical characteristics that govern karst transport dynamics. While this impedes our proper understanding about contaminant mixing and transport problems in karst systems, finding a tradeoff between the proper representation of the karstic heterogeneity in the karst models and the model prediction’s reliability also remains a difficult task. Besides that, predicting the impacts of anthropogenic factors and global changes on the karst water quality is still a scientific and operational challenge.

This paper reviews the current state of knowledge on the modelling of transport processes in karst water resources by synthesizing the existing body of knowledge in the literature. We initially introduce the principle of the solute transport processes in karst aquifers. We then present the state-of-the-art for karst transport modelling approaches by providing the governing
transport equations that describe the process of interest with differing complexity. We discuss the methodological concerns about a solute transport modelling practice in karst water resources, and hence report the fundamental challenges afront the successful modelling practice. To move forward, we give a broader perspective on the transport modelling in karst water resources, thereby providing emerging directions with potential drawbacks for the karst community.

2 Principles of solute transport processes in karst water resources

Heterogeneity in karst water resources manifests itself at multiple spatial scales i.e., the regional scale, aquifer scale, local scale, and single fracture scale (Figure 1). This multiscale karstic heterogeneity governs both flow and solute transport processes (i.e., advection, diffusion, mechanical and turbulent dispersion, and chemical reactions) (Figure 1). As the flow velocity field in karst systems is often extremely variable (in the order of meters per day in the matrix and of meters per second in the conduits), advection is often the most influenced process by the flow variations in the karst systems. By comparison, the diffusive processes result from the random motion of the solute molecules, and they are not significantly affected by the heterogeneity in the system’s hydraulic properties. The (biogeo)chemical reactions (i.e., sorption, chemical transformation) are mostly affected by the heterogeneity in the hydrogeochemical conditions both in fractured/fissured carbonate rock (hereinafter referred to as karst matrix) and preferential flow paths within dissolution enlarged fractures and conduits of the karst systems (hereinafter referred to as karst conduits).

As compared to porous or fractured groundwater systems, the most peculiar characteristic of karst aquifers is the complex interaction between the fractured/fissured carbonate matrix and karst conduit, which leads to the duality in the flow and transport processes (Massei et al., 2003; Joodi et al., 2010; Kavousi et al., 2020). This duality is attributed to the marked difference in the water flow velocities — typically described by the laminar flow in the matrix and turbulent flow in the karst conduits (Kiraly, 1998; Hauns et al., 2001). The duality in the karst flow characteristics is therefore expressed by the definition of different characteristic spatial and temporal scales in karst transport processes over the matrix and karst conduits.

In the following, we describe the main transport processes occurring separately in matrix and conduit components, as well as karst-specific transport dynamics based on the system duality. We then briefly define the concepts of spreading, dilution, and mixing, as there is not a general agreement in the use of this terminology in the karst literature. Furthermore, we provide information on tracers as diagnostic tools to describe and/or quantify the karst transport processes, thereby only touching upon the concept of groundwater age and its primary role in identifying solute mixing and transport characteristics in karst systems.

2.1 Transport in the matrix

There is vast literature describing transport processes in the karst matrix. Recently, several review papers covered the topic by emphasizing the different characteristic aspects. Dentz et al. (2023) review concepts and approaches for the quantification of passive, non-reactive solute mixing in steady uniform porous media flows across scales. Rolle & Le Borgne (2019) focus both on the description of reactive mixing, on displacement scenarios and continuously released plumes approaching the transport problem from the pore scale to the aquifer scale. Valocchi et al. (2018) describe mixing-limited reactions and upscaling processes from the pore to the continuum scale. Berkowitz et al. (2016) emphasize the complex interplay
between reactive mixing and spreading, as well as the differences among the models to describe it. In our review, despite the undoubtful relevance of the pore-scale transport processes, we will describe transport mechanisms starting from the continuum scale (generally known as the Darcy-scale). Therefore, we refer the interested reader to the aforementioned review papers for the upscaling from the pore scale to the continuum scale.

Figure 1. Conceptual representations of solute transport processes in karst water resources at differing spatial scales. (a) regional scale, (b) aquifer scale, (c) borehole scale, and (d) single-
fracture scale. Here, the solute attenuation process is described by the chemical transformation process influenced by physical, chemical, and (biogeo)chemical processes. In the figure, anthropogenic contamination source is described only while geogenic contamination is not depicted. The figure also indicates the diffuse (areal) sources whereas the point source is depicted by the intentional problem in the site.

2.1.1 Darcy scale: Pore diffusion and mechanical dispersion

Aqueous diffusion results from the random motion of the molecules of a solute in water at microscopic scale and controls the occurrence of complete mixing at the pore scale (Rolle et al., 2012). When aqueous diffusion occurs in a porous material, it is often called pore diffusion (Bear, 1972). Aqueous diffusion coefficients are compound specific and range from $10^{-7}$ to $10^{-9}$ m$^2$/s. In the matrix, since the solute is not free to move in all directions due to the presence of the solid material, pore diffusion is slower than aqueous diffusion. At the Darcy scale, by comparison, a pore diffusion coefficient is described as the product between aqueous diffusion and a coefficient whose value ranges between 0 and 1, and it is parametrized by the tortuous nature of the pore space (Ghanbarian et al., 2013). Therefore, pore diffusion coefficients are typically between one half and one tenth of the aqueous diffusion coefficients. When the grain Péclet number — the ratio between the product of the groundwater flow velocity and the grain size over the aqueous diffusion coefficient — is smaller than 1, pore diffusion is the dominant transport process in the karst matrix.

Mechanical dispersion also contributes to the displacement of the solute of interest, and hence it reflects the effect of random velocity variations at the pore scale during advective transport (Bear, 1972). Generally, the advective velocity is laminar and the Reynolds number characteristic of the system is small (typically below one). If aqueous diffusion eliminates concentration gradients occurring in pore channels, mechanical dispersion at the Darcy scale is parameterized as the product of the velocity and a constant property of the porous material defined as dispersivity (Scheidegger, 1961). While dispersivity values differ in the longitudinal and transverse directions, their values are proportional to the grain diameter. If concentration gradients at the pore scale still exist, then mechanical dispersion is compound specific and depends non-linearly on the grain Péclet number (Bear, 1972; Chiogna et al., 2010).

The sum of pore diffusion and mechanical dispersion is called hydrodynamic dispersion.

2.1.2 Field scale: Effective and macro dispersion

Correctly characterizing pore diffusion and mechanical dispersion is fundamental to reproduce transport processes at the Darcy scale. Since the heterogeneity in the matrix hydraulic conductivity is hardly measurable, we are not able to assign spatially distributed values for the pore diffusion and mechanical dispersion to the karst systems. For this reason, to move from the Darcy scale to the field scale, the concepts of effective dispersion and macro dispersion are introduced as an additional upscaling mechanism. Here, the macro dispersion is a concept for explaining “the scale effect observed in experimental data of dispersion coefficients measured in the heterogeneous porous media at different spatial scales” by Dentz et al. (2023). However, we can distinguish between the definitions of ensemble dispersion coefficients and effective dispersion coefficients. While the ensemble dispersion defines the solute distribution due to the advective spreading, effective dispersion characterizes the width of the solute plume originating from a point-like injection (Dentz et al., 2023). As a typical characteristic of both parameters,
they are all scale-dependent, which means that the larger the area of interest, the larger the
coefficients. Furthermore, their values are often orders of magnitude larger than the
hydrodynamic dispersion coefficients.

2.2 Transport in the conduits

Transport in karst conduits shows similarities with the transport in rivers and/or pipes.
But depending on the saturation of the karst system, karst conduits can behave as a free surface
flow or a (saturated) pipe flow under pressurized conditions (Liedl et al., 2003; Loop & White,
2001; Reimann et al., 2014). While solute transport in conduits is mainly controlled by advection
and aqueous diffusion, typical flow velocities encountered in karst conduits are orders of
magnitude larger than the ones presented in the matrix. This leads typically to turbulent flow
conditions in the conduits, thus making aqueous diffusion a dominant process along the conduit,
although it is generally neglected (Hauns et al., 2001). At the field scale, however, the advective
flow field is often not known due to the multiscale karstic heterogeneity, as well as due to the
lack of knowledge about the distribution and shape of conduits.

2.2.1 Aqueous diffusion

Aqueous diffusion in conduits is the same process as in the matrix. However, we can
highlight three main differences here. First, in karst conduits we do not have to reduce the
aqueous diffusion coefficient by the presence of the porous medium and its tortuosity. Second, as
we often move from laminar to turbulent flow conditions, the magnitude of the aqueous diffusion
coefficients is much smaller than the turbulent dispersion coefficient. Third, incomplete mixing
is generally not controlled by aqueous diffusion along the conduit system, but it is mainly
controlled by the conduit network topology and the turbulence occurring in the conduits.

2.2.2 Turbulent dispersion

The distinguishing characteristics of turbulent flow are the rapid changes in flow velocity
over time and space, and the randomness of the velocity and pressure fields (Fischer et al., 1979).
Therefore, the rapid irregular and chaotic fluctuations in flow velocity and pressure lead to the
formation of swirling vortices, eddies (random complex patterns), and turbulent structures that
interact with each other. Because of these random motions, turbulent flows occur at several
characteristic scales. At large Reynolds number (typically larger than 500 in karst conduits), the
strong nonlinearity of the equation of fluid motion results in the spreading of the kinetic energy
over a range of eddy sizes, and thus causes the interaction between the large and small scales of
motion (Fischer et al., 1979). These varying flow velocity fields result in the spreading and
dispersion each of which is quantified by the turbulent dispersion coefficients. Therefore, being
process scale-dependent, the definition of the dispersion coefficient needs to be done considering
the specific spatial scale (Kinzelbach, 1986).

The turbulent dispersion coefficient is commonly assumed to be proportional to the flow
velocity, where the proportionality constant is called dispersivity. Taylor (1953, 1954) developed
a more process-based approach by correlating dispersion to the differences in flow velocity
within the flow cross-section and the consequent mass transfer processes. Under turbulent flow
conditions, however, the dispersion coefficient for a pipe is proportional to the pipe radius and
friction velocity (Taylor, 1954). In the field scale where the karst conduits exist, the dispersivity
is often required to adjust by the karst transport modelling during the model parametrization (see Sections 3 and 4).

2.3 Sorption, retardation, and chemical reactions in karst systems

Sorptions that are neither affected by sorption nor undergo chemical reactions are defined as conservative. In our review, for the sake of simplicity, we only review the single-phase flow and transport processes in karst aquifers which are much relevant to assessing natural attenuation processes (e.g., Geyer et al., 2007; Einsiedl et al., 2009; Flynn & Sinreich, 2010; Schiperski et al., 2016; Valiente et al., 2020).

Sorption refers to the process that transfers the solute concentration from the dissolved aqueous phase into an immobile phase attached to the solid geological matrix. It is particularly relevant for solute transport in the karst matrix and at the interface between the matrix and karst conduit because it requires direct contact between the solute and the porous material. Sorption can also occur on suspended particles (Lan et al., 2018; Massie et al., 2002; Schwarz et al., 2011), thus leading to the so-called particle-facilitated transport. In general, sorption has two main effects on the solute concentration. First, the solute concentration in the water is reduced due to the mass transfer towards the immobile sorbed phase. Second, the advective velocity of the solute of interest is reduced, and hence the mass exchange between the dissolved and the sorbed phase leads to an effective retardation in the solute.

Retardation is described as the delay of the solute transport relative to the groundwater flow. In karst systems, retardation is often assumed to be negligible due to the rapid transport in the karst conduits (Kresic & Stevanovic, 2009) whereas sorption reactions (adsorption or absorption) and ion-exchanges (combined sorption-desorption processes) can occur at the conduit walls/surfaces and within the surrounding carbonate matrix (Field & Pinsky, 2000; Geyer et al., 2007; Morales et al., 2010). However, both processes are generally relevant to local-scale transport problems (Hauns et al., 2001; Jeannin & Maréchal, 1997). To account for sorption and retardation processes in karst systems, sorption isotherms (i.e., linear, Langmuir, Freundlich isotherms) that ignore the complex sorption kinetics are often preferred (e.g., Frank et al., 2021; Hillebrand et al., 2015; Luhmann et al., 2015; Morales et al., 2010; Schiperski et al., 2016; Tinet et al., 2019, among others).

Chemical reactions can lead to the production and/or degradation of the solute of interest. A prerequisite for a reaction to occur is that all reaction partners are present in the same location at the same time while diffusive and dispersive processes are mainly responsible for this. As the diffusive and dispersive processes are slow in the karst matrix, chemical reactions are limited by transport mechanisms. On the contrary, reactive transport in the karst conduit systems is often limited by the reaction rate constant of the chemical reaction due to the high turbulent dispersion coefficient. Besides, the timescales of (biogeo)chemical reactions (i.e., degradation, nitrification, dissolution, mineralization, weathering) are relatively slow in the matrix (i.e., seasonal, decadal, or multidecadal) as compared to advective transport in karst conduit (i.e., days, months). For these reasons, the (biogeo)chemical reactions are generally overlooked in karst systems. But when they are considered, a (biogeo)chemical interaction term is often coupled with the advective-dispersive transport processes (Dvory et al., 2018a, 2018b; Pastore et al., 2021).

Overall, due to the multiscale variabilities in (hydro)geochemical conditions modified both by the karst matrix and by the composition of mobile water, the proper description of the
complex reactions is hardly possible in karst systems. For this reason, sorption and (biogeo)chemical processes are crudely represented in the karst literature while there is a general lack of knowledge of the hydrogeochemical compositions of karst systems.

2.4 Dynamics of solute transport processes in karst systems

The spatiotemporal dynamics of solute transport in karst aquifers are controlled by the relative contributions of the rapid (i.e., conduits, fractures) and slow (i.e., matrix, storage zones due to the vortices and eddies in conduit systems) flow components (Ewers et al., 2012; Field, 2002, 2018; Geyer et al., 2007; Hauns et al., 2001) (Figure 2). Solute transport in karst systems is often characterized by non-ideal transport phenomena — also referred to as non-Fickian or anomalous transport — controlled both by the preferential flow paths (mostly by karst conduits and conduit networks) and by the rate-limited mass transfer between the slow and fast flow components. Overall, the key transport mechanisms in karst systems are defined as (i) physical transport along the main flow direction controlled both by advection and dispersion processes, (ii) matrix diffusion and mechanical dispersion along the conduit surfaces in and out of the (fractured) carbonate matrix, and (iii) mass exchange between mobile water in karst conduit and matrix components via such as sorption, ion exchange, radioactive decay, and (biogeo)chemical reactions (Figure 2) (e.g., Field & Liej, 2014; Field & Pinsky, 2000; Sinreich, 2011; Sinreich et al., 2007).

Figure 2. Schematic representations of the anomalous transport in karst systems demonstrated with the mass exchange mechanism between the (saturated) karst conduit and surrounding fractured/fissured carbonate matrix. Here, the variability of mass transfer rate is controlled by the karst system’s hydraulic properties (i.e., the matrix porosity and conduit diameter). Due to the
greater hydraulic conductivity of the karst conduits, advection and dispersion processes dominate
the physical transport along the conduit whilst the (fractured/fissured) carbonate matrix acts as a
sink and/or source, mainly depending on the hydraulic head and concentration gradient at the
domain of interest.

In karst aquifers, conduits represent the region where contaminants can be rapidly
transferred (e.g., Ghasemizadeh et al., 2012; Goeppert et al., 2020; Pronk et al., 2009; Vesper et
al., 2001, among many others). Along the conduit, the nature of the transport is strongly
dependent on the flow regimes that range from laminar to turbulent flows (Field, 2002; Green et
al., 2006; White, 2002). The conduit flow velocity is particularly important because it shapes the
movement of the solute plume. Here, the (mean) flow velocity defines the rate of mass transfer
between the karst conduit and matrix (Bajracharya & Barry, 1997; Lehmann et al., 2022) while
the magnitude and direction of the mass exchange widely vary depending on the karst system’s
hydrological and antecedent conditions (i.e., soil and/or epikarst wetness). For instance, the high
velocity reduces the contact time of the contaminant with the (fractured/fissured) carbonate
matrix and thus limits the mass exchange between the conduit and matrix (Figure 2). In contrast,
when the flow is laminar — typically characterized by lower flow and recharge conditions —,
the mass exchange is generally favorable (Faulkner et al., 2009; Martin et al., 1999, 2012). While
the advective transport in the karst conduit are mainly influenced by the conduit’s structural
properties (i.e., conduit size, morphology, sinuosity) and conduit networks (Aliouache et al.,
2019; Field & Pinksy, 2000; Ronayne, 2013), the local and/or regional recharge characteristics
adjust the flow magnitude and direction of the mass transfer between the conduit and matrix
(Figure 2) (Doummar et al., 2018; Göppert & Goldscheider, 2008; Ji et al., 2022).

The karst matrix is characterized as the region where solutes can be stored for a long
period and are slowly released into the conduit network (e.g., Berglund et al., 2020; Ji et al.,
2022; Nativ et al., 1999; Padilla & Vesper, 2018; White & White, 2005; Worthington, 2011,
among many others). As the mass transfer from the karst conduit to matrix is often controlled by
the (matrix) pore diffusion and mechanical dispersion, these processes can be particularly
important during the storage of solute of interest within the matrix, especially in the low
permeable karst media, such as chalk (Zuber et al., 2011; Polak et al., 2002; Worthington, 2015).
Therefore, the rate of the solute transfer in and out of the matrix is regulated by several other
factors including matrix permeability, conduit flow velocity, irregularities of conduit surface,
solute retardation factor, as well as the (solute) matrix pore diffusion and mechanical dispersion
coefficients (Field, 2002, Joodi et al., 2010; Kresic & Stevanovic, 2009; Mosthaf et al., 2018;
Luo et al., 2024) (Figure 2).

2.5 Spreading, Dilution, and Reactive Mixing

A large amount of literature deals with the proper distinction among three different
concepts to quantify transport processes: spreading, dilution, and mixing (e.g., Dentz et al., 2023;
Kitanidis, 1994).

Spreading refers to a change of the solute plume shape due to the spatially varying
velocity, and thus to an increase in the average distance of the particles. This process is
quantified by computing the second spatial moments of the solute present in the system. If we
neglect aqueous diffusion, mechanical and turbulent dispersion, then spreading, if only caused by
advective transport, is a reversible process and does not lead to a change in the concentration of
the solute, but only to a different spatial distribution of the solute concentration.
Dilution, on the contrary, quantifies the increase in volume of the fluid occupied by the solute. Due to dilution, the solute plume tends to occupy a larger volume leading to a decrease in the solute concentration (Kitanidis, 1994). This process is irreversible and is controlled both by diffusive and dispersive processes. Dilution is typically used to characterize conservative solute transport. However, we can often find the term mixing to refer to dilution in the literature. Since the term 'mixing' is widely established for the description of conservative transport, we will use this term as a synonym for dilution in our review.

Reactive mixing is the process that brings two different chemical species together as the result of molecular diffusion, mechanical dispersion, and turbulent dispersion (Neupauer et al., 2014), which allows a chemical reaction to occur. Here, it is important to notice that dilution (mixing) is not necessarily affected by incomplete mixing at the pore scale whereas reactive mixing is significantly controlled by incomplete mixing (Berkowitz et al., 2016), particularly in the karst matrix. Therefore, the presence of incomplete mixing very often leads to the need for case study-specific reaction rate constants to match the experimental observations.

2.6 Tracers as diagnostic tool in describing transport processes in karst systems

Describing flow and transport processes in karst systems is difficult mainly due to the unknown configuration and geometry of the conduit network (Borghi et al., 2012; Fandel et al., 2022). Tracers are frequently used to define the karst system’s flow and transport dynamics (Geyer et al., 2007; Goldscheider et al., 2008; Martín-Rodríguez et al., 2023; Mudarra et al., 2014). Various substances (i.e., solutes, particles, solids, and gasses) and physical quantities (i.e., temperature or heat) can be used for quantifying flow and transport parameters (i.e., porosity, dispersivity, flow velocity) at the spatiotemporal scales of interest (Benischke, 2021; Field, 2020; Massei et al., 2006). Additionally, tracers are frequently utilized as additional information sources to confirm and/or invalidate for karst simulation models (e.g., Hartmann et al., 2014, 2017; Mudarra et al., 2019; Rusjan et al., 2019; Zhang et al., 2021; Çalli et al., 2023a).

Environmental tracers and artificial tracers are two main groups commonly used for the description of the karst transport processes. While the environmental tracers such as stable isotopes, radionuclides, dissolved gases (i.e., chlorofluorocarbons and Sulphur hexafluoride), hydro-chemicals, and sediments already exist in the karst groundwater, artificial tracers (i.e., fluorescent dyes, salts, and bacteria) are introduced into the karst system under pre-defined experimental conditions where the substance is supposed to be nonexistent under natural conditions (Benischke, 2021; Goldscheider et al., 2008).

Among environmental tracers, water isotopes such as Oxygen-18 and Deuterium are frequently used for defining hydrological system’s water age and (water) age distributions (i.e., travel time distributions (TTDs) and residence time distributions (RTDs) (Jasechko et al., 2017; Musgrove et al., 2023). As a terminology, water ‘age’ refers to the time elapsed since the meteoric water entered the system (Bethke & Johnson, 2008; Long & Putnam, 2009; Suckow, 2014). In groundwater studies, water age is often denoted as residence time which indicates the time between the water entrance and leave (Aquilina et al., 2003; Maloszewski et al., 1992; Maloszewski & Zuber, 1993; Plummer et al., 1998a, 1998b) whereas travel time is defined as the time required for a volume of water moving across the specified flow paths. The knowledge of water age and/or its distributions — distributions of the different water ages — is particularly useful for revealing the vulnerability of karst aquifers to the contamination risks (Malík et al., 2016; Musgrove et al., 2023; Çalli et al., 2023a) as it provides valuable insight into the karst
system’s underlying mixing, storage, and transport characteristics (i.e., piston flow, complete mixing) (Figure 3). For instance, the karst aquifers characterized by (relatively) younger water age (i.e., shorter travel time) are often at the risk of contamination problem due to the system’s rapid response whilst the karst systems represented with older groundwater ages (i.e., longer residence time) is often an indication of the sufficient time for the contaminant attenuation processes (Figure 3).

**Figure 3.** Sketch for idealized groundwater age distribution in two different karst systems. a) Conduit-dominated karst system resembling advection-dominated transport dynamics — characterized by relatively younger water age (i.e., shorter travel time), b) Matrix-dominated transport dynamics, which reflects diffusive-dispersive dominant transport characteristics — characterized by older groundwater age (i.e., longer residence time).

Artificial tracer test is powerful tool to estimate the karst system’s hydraulic properties (i.e., porosity, hydraulic conductivity, conduit volumes) and transport parameters (i.e., dispersion, mass exchange rate, sorption) (Aquilanti et al., 2003; Mudarra et al., 2019; Lauber et al., 2014; Peely et al., 2021; Perrin & Luetscher, 2008). In these tests, the investigation scale is very often a key factor in characterizing and quantifying the karst system’s flow and transport characteristics. For this reason, the observations/data obtained by the tracer tests are rather site-specific, and hence the estimated parameters are frequently dependent on the initial and boundary conditions of the experimental works (i.e., climatic, hydrological conditions) (Duran et al., 2016; Sivelle & Labat, 2019; Stevanović & Stevanović, 2021) (see discussion in Section 4.1).

The solute breakthrough curve (BTC) is the primary production of artificial tracer tests (Figure 4), which reflects solute transport behaviour at the scales of interest. BTC is often used as a karst system’s (characteristic) response obtained at the given spatial and/or temporal scales (e.g., Field, 2002; Labat & Mangin, 2015). However, due to the multiscale karstic heterogeneity, BTC is often described by the various anomalous characteristics such as power-law tailing, high skewness (positive or negative), and multi-modality in BTC (Figure 4) while each of which can resemble the karst system’s flow and transport dynamics. In general, the tailing effects in BTC are associated with the dispersion and (matrix) diffusion processes, as well as with the presence of conduit pools where the eddies and vortices form the (transient) storage zones to retain the solute of interest (Bodin et al., 2022; Hauns et al., 2001; Li & Loper, 2011; Morales et al., 2010). The multi-peaked BTC is considered as a resemble of anastomoses (or channeled) of the conduits and/or conduit networks (i.e., auxiliary conduits) along which advective transport is often the dominant mechanism (Figure 4) (e.g., Cen et al., 2021; Goldscheider et al., 2008; Deleu et al., 2023; Dewaide et al., 2018).
Figure 4. A conceptual description of the (typical) anomalous behaviours of the (saturated) karst systems. a) Contaminant plume migration along the different karst compartments under changing time. b) Solute Breakthrough Curve (BTC) characterized by the dominant transport characteristics at the domain of interest within the selected time-periods given in Panel-a.

3 The current status of solute transport modelling in karst water resources

3.1 A generic framework for a successful transport modelling in karst systems

To present the current status of solute transport modelling in karst water resources, we first define a workflow for a successful transport modelling application. A successful transport modelling practice starts with the definition of the karst transport problem for which some critical information about the properties of the source of nutrient or contaminant (i.e., source type such as point or areal sources, source locations) and the karst system’s physical and chemical boundaries are mainly gathered. Similar to the development of a flow model (Beven, 2002, 2011; Wagener et al., 2001, 2021), a successful implementation of a transport model in karst systems is a strategy that contain three main successive stages including (i) system conceptualization, (ii) development of mathematical model and model parametrization, and (iii) model evaluation (Figure 5). Here, each stage feeds one another with the multiple feedback loops to retrieve more robust and reliable results for the successful karst transport model. The overall success in this modelling practice, therefore, reflects the level of accumulated knowledge and observational inputs for the proper descriptions of the complex physical, chemical, and (biogeo)chemical processes in karst systems. A successful transport model application usually requires an a priori definition of a flow model that solves the water mass balance equations based on the changes of the variable of interest (i.e., hydraulic head, drawdown, or water pressure) within the flow domain where the dominant hydrodynamic conditions (i.e., regional and/or local flow directions and magnitudes) govern the karst transport processes, which has been extensively discussed in previous reviews (e.g. Hartmann et al., 2014).
3.1.1 Model conceptualization for karst transport processes

Perceptual understanding of transport processes is a key stage of successful transport modeling practice. A perceptual model is a qualitative description of the karst system’s physical, chemical, and (bio)geochmical transport characteristics, each of which forms the definition of the dominant physicochemical processes at the domain of interest (e.g., Hartmann et al., 2012a, 2013a, 2018). At this stage, the understanding of a dominant process is critical as it mainly conditions the concept of the karst transport mechanism (i.e., advection-dominated transport, diffusion-dominated transport) for the conceptual model development. Since the perceptual model is just a mental model amongst the multiple other possibilities, it is more likely to be updated by the accumulated scientific knowledge and collected data (Beven & Chappell, 2021; Wagener et al., 2021) before evolving into a conceptual model (Enemark et al., 2019; Gupta et al., 2012).

The development of a conceptual model for representing transport processes in karst systems is a critical stage of the successful transport modelling practice (Figure 5). This model is structured based on generic scientific knowledge and a set of decisions on how the karst system functions and/or it might function over the changing time and space (Chang et al., 2017, 2019; Hartmann et al., 2014). Model conceptualization helps to determine and/or select a suitable mathematical formulization (hereinafter referred to mathematical model) for karst transport processes, thereby refining the governing equations within the given range of spatiotemporal scales. While these equations represent the dominant (physicochemical) processes, the
mathematical formulation can be adjusted by which type of contaminant is of interest (i.e., reactive, conservative transport). In most cases, the formulation for the conservative solute transport is expanded by coupling reactive and/or chemical processes (i.e., such as by the organic chemicals that are sorbed onto the carbonate matrix and/or fracture filling) where these processes can play a role for karst transport dynamics. In such a process-coupling, system conceptualization can be extended by the development of a karst “hydrochemical conceptual model”. Here, a hydrochemical conceptual model refers to the spatial variations of different hydrochemical facies — or water type — in which similar geochemical characteristics are grouped (e.g., Güler & Thyne, 2004; Kresic & Stevanovic, 2009; Thyne et al., 2008). In this manner, building hydrochemical conceptual model is a complementary stage for a successful transport modeling practice because it provides valuable information on modelling chemical and/or reactive transport processes, particularly during the representation of the (different) origins of contaminant sources and their mixing characteristics at the domain of interest (i.e., borehole scale, aquifer scale).

3.1.2 Model parameterization in karst transport models

Mathematical models translate the system conceptualization into a set of governing equations that are formulated based on the karst system’s physical and (biogeo)chemical properties under (specified) boundary and initial conditions (Figure 5). The mathematical models (or numerical codes) solve solute mass balance to define the spatiotemporal dimension of the process of interest. In general, the structure of the mathematical model (or model structure) is formed and/or chosen by the modeler himself/herself depending on the adequate representation of the system and process of interest. But the selection of an appropriate model structure often depends on data availability and information collected. For this reason, different models with differing complexity can be successful for solving (different) karst transport problems at various spatial and/or temporal scales. To solve the solute mass balance, the mathematical models attribute specific values to the coefficients of the governing equations. These coefficients are called as model parameters (i.e., hydraulic conductivity, dispersivity, sorption constant). While the parameters are the main integrators of available data and/or information content, they are often formulated based on the karst system’s physical and chemical properties (see further discussion in Section 4.3).

Model parametrization is the stage for the (realistic) estimation of the parameter values to capture the spatial and/or temporal dimensions of the karst transport dynamics. The parameter values can be obtained by three common ways: (i) direct measurements (field and/or lab-scale measurements), (ii) model calibration (i.e., inverse solution, history matching), and (iii) the combination of thereof. Since the direct field measurements and/or laboratory scale experiments often resemble the spatial and temporal variabilities of the process of interest at the domain of interest, they are favorable for the improved model parameterization due to the well-defined parameter values. Nonetheless, the characterization methods for the data acquisition and/or interpretation techniques in karst aquifers is rather appropriate for the continuum (Darcy) scale applications where the effective parameters can be readily used for the entire model domain (e.g., Faulkner et al., 2009; Huntoon, 1995; Kresic & Stevanovic, 2009; Reimann et al., 2011a; Zhang et al., 2021). For this reason, when the direct measurements of transport parameters (i.e., dispersivity, mass transfer coefficient) are not practical and/or their values might not be representative in resembling the process of interest, model parameters are frequently estimated...
Model calibration refers to model tuning during which the parameter values are frequently adjusted to reduce the misfit the observations. In a successful model calibration, the measured variables (i.e., solute concentrations for the transport models, discharge and/or groundwater heads for the flow models) are required to reasonably meet the simulated counterparts (Dietrich & Uniyal, 2020; Neven & Sorab, 2021; Oehlmann et al., 2015; Sullivan et al., 2019). Such a model parameterization requires a calibration metric (i.e., objective function, performance criteria or likelihood) to indicate how well and to what degree the transport model represents the system’s reality (Hartmann et al., 2018; Gharari et al., 2021; Çalli et al., 2023). For the calibration of karst simulations models, a single variable (i.e., spring discharge, groundwater level) is often used to quantify the misfit between the simulated and observed variables. But when it is possible, introducing multiple sources of information (i.e., isotope concentrations, water age in formation) and/or soft data (i.e., knowledge and experience on the modelled area) into the model calibration phase is rather preferable, particularly for the improvement of model robustness and parameter reliability (e.g., Banusch et al., 2022; Hartmann et al., 2013a, 2013b; Çalli et al., 2022; Çalli et al. 2023b).

The number of parameters required for modelling of transport processes is typically greater than the flow models (Konikow, 1996, 2010; Reilly, 1987). For this reason, multiple parameters (i.e., degrees of freedom or parameter space) are often necessary to represent the system’s physical and chemical behaviors in karst solute transport models. Besides, the number of parameters can be increased when the multispecies and/or multiphase (air and water phases) transport is considered in the modelling framework (e.g., Faulkner et al., 2009; Dewaide et al., 2016; Loop & White, 2001; Zhang et al., 2017). The model calibration for solute transport models is often framed by how the transport model is coupled with the flow model. Here, this coupling can be done at two different levels, either fully or partly/loosely. Mainly depending on the model coupling, two approaches are frequently applied for the parametrization of karst transport models: (i) sequential calibration and (ii) simultaneous calibration (Schilling et al., 2019). In sequential calibration, the (calibrated) parameters of the karst flow model is served for the parametrization of the transport model (e.g., Husic et al., 2021; Sullivan et al., 2019; Çalli et al., 2023a) whereas the simultaneous calibration is performed for a fully coupled model (flow model coupled by transport model) and hence whole parameter set is considered against the available observations (e.g., both karst spring discharge and δ18O composition of spring discharge) (e.g., Hartmann et al., 2017, 2012a, Kavousi et al., 2022; Zhang et al., 2020a, 2020b).

3.1.3 Evaluation of karst transport model results

The successful application of a karst transport model requires a robust model evaluation to ensure the reliability of the model predictions (Hartmann et al., 2013b; Çalli et al., 2023a) (Figure 5). In general, the tools used for the model evaluation are (i) model performance assessment (similarity of the observed variable), (ii) model plausibility (alignment with the conceptual model), and (iii) uncertainty estimation (Dietrich & Uniyal, 2020; Hartmann, 2018; Jeannin et al., 2021; Jukić & Denić-Jukić, 2009). The model predictions might be greatly varied during the modelling practice depending on the modeler’s target, system conceptualization, model selection, and applied calibration scheme (i.e., selection of objective function, calibration-validation periods, multi-objective, and/or multivariable calibration) (e.g., Bittner et al., 2020; Mai, 2023; Mazzilli et al., 2012; Zhang et al., 2020b; Çalli et al., 2023a). However, as the karst
transport model often relies on the accuracy of a karst flow model, the transport model’s predictions are generally less reliable as compared to flow models (e.g., Konikow, 1996, 2011; Scanlon et al., 2003). Therefore, the predictions of transport models are always uncertain to some degree, which thus requires diagnosing and quantifying sources of uncertainty in the model results.

Nonetheless, uncertainty propagates and amplifies throughout the modelling practice (Figure 5). For that reason, the exploration of the principal sources of uncertainty is strictly limited (Beven, 2005; Hartmann, 2018). Similarly, the translation of each uncertainty source into a quantitative form — refers to the attribution of a numerical value for each uncertainty source — might change from one modeler to another and is mostly dependent on the progressive decisions and choices over the modelling practice. Therefore, different sources of uncertainty in model results (i.e., data uncertainty, model structure, and parameter uncertainties) could be reduced to a certain degree, only if a systematic model assessment in the model results is carried out. This assessment can identify what we need for the reduction of model uncertainty, and how much we shall put effort to reduce the different sources of uncertainty. Although this is not an easy task, some techniques such as Generalized Likelihood Uncertainty Estimation (GLUE) (Beven & Binley, 1992), Bayesian model averaging (BMA) (Hoeting et al., 1999; Neuman, 2003), and multi-model ensemble method (MME) can be useful for the systematic and robust uncertainty assessment.

3.2 Description and classification of solute transport models in karst water resources

3.2.1 Generic classification of karst simulation models

To deal with a wide variety of flow and transport problems in karst water resources, two fundamental approaches have been using for representing the physical heterogeneities of karst systems: (i) spatially lumped karst simulation models and (ii) spatially distributed karst simulation models (Hartmann et al., 2014) (Figure 6) while various forms of models in between (called semi-distributed or hybrid models) are available for the simulations of the karst systems (Kovács & Sauter, 2014; Hartmann et al., 2013a, Gill et al., 2013, 2021). Since the distributed models represent the physical processes more explicitly based on the physical laws by discretizing the karst systems into several grid cells, data requirement is often too high to adequately represent the karst flow and transport processes at varying spatial scales (Hartmann et al., 2014). By comparison, the spatially lumped karst simulation models do not explicitly consider the karst system’s spatial variabilities (i.e., geometric boundaries, flow pathways) (Hartmann et al., 2014), thereby implicitly representing the physical processes across the domain of interest.
**Figure 6.** Generic classification of karst simulation models based on the model parametrization considering process complexity and data requirement. (a) Conceptualization of the karst aquifer physical boundaries depicted by the gray-shaded area with a blue-indicated karst conduit network (the blank-circles also describe the swallets/sinkholes along the conduit network), (b) Spatially lumped karst simulation models depicted based on the solute concentration distribution over different karst compartments, (c) Spatially distributed karst simulation models described considering the spatial distribution of the solute concentration. The classification is adapted from Hartmann et al., 2014 (see the further details in Hartmann et al., 2014). Herein, an artificial tracer test is described for demonstrating only the spatial distributions of contamination plume across two main karst simulation approaches.

For karst systems, spatially distributed karst simulation models are classified as (i) Equivalent Porous Medium Approach (EPM), (ii) Double Continuum Approach (DC), and (iii) Combined Discrete-Continuum approach (CDC) (Ghasemizadeh et al., 2012; Hartmann et al., 2014). EPM is a continuum-based modelling approach that considers the Representative Elementary Volume (REV) and groundwater flow equations (Hartmann et al., 2014; Scanlon et al., 2003) (Figure 6c). REV is defined as the smallest volume over which the averaged flow and transport parameters can be represented by the spatial scale where we can still apply the Darcy law based on the laminar flow conditions. REV represents a threshold volume in which the hydrological properties are assumed to be invariant after then (Medici & West, 2021).

Nonetheless, due to the presence of the karst conduit and/or conduit networks, the flow in karst conduits is often characterized by turbulent flow characteristics, and hence representing the whole karst system with a single REV is not necessarily possible (Geyer et al., 2013; Kiraly, 2002). For that, EPM is often (more) suitable for simulating larger scale model applications where the karst flow and transport processes can be described by a single REV (Anderson et al.,...
...overlapping continua (i.e., karst conduit and matrix) where the physical and/or chemical interaction between these two continua is addressed by an exchange rate (i.e., flow and/or mass exchange) (Cherubini, 2008; Kiraly, 1998; Maréchal et al., 2008, Robineau et al., 2018; Teutsch & Sauter, 1998). Here, the rate of exchange is proportional to the differences of hydraulic head and/or concentration gradient between the karst conduit and matrix continua (Kavousi et al., 2020; Oehlmann et al., 2015; Reimann, 2011a, 2011b, 2014).

By comparison, CDC treats the karst conduits as the main flow and transport regions while an additional parameter set (i.e., mass exchange parameter) is defined for the matrix component. According to the CDC approach, karst conduits can be either treated as high permeable regions or characterized as drain (or pipe) (Figure 6c). This approach considers the discrete conduit and/or conduit networks, it is often more reasonable choice for the well-defined sites and/or small-scale karst modelling applications due to extensive data need and high computational effort for representing karst conduits and its topology (i.e., conduit networks, fracture intersection) (e.g., Kovács, 2003, Ravbar et al., 2011; Xanke et al., 2016). Here, the turbulent flow characteristics in karst conduit and/or discrete conduits is often simulated by Conduit Flow Process (CFP) (developed by Shoemaker et al., 2008 and later extended by Reimann et al., 2013, 2018, 2023). CFP has been successfully applied for well-characterized karst systems to simulate conduit flow and transport characteristics (Chang et al., 2019; Spieesl et al., 2017; Shirafkan et al., 2023; Sullivan et al., 2019; Xu et al., 2015, among many others). Therefore, CFP can consider both laminar and turbulent flow regimes by coupling solute and/or heat transport processes (Kavousi et al., 2020; Reimann et al., 2018).

Spatially lumped karst simulation models (i.e., bucket-type models, cascading reservoirs) conceptualize the entire karst systems based on the karst system’s compartments (i.e., matrix, conduit, epikarst) in which average physical properties of the system are represented (e.g., Charlier et al., 2012; Geyer et al., 2008; Jukić & Denić-Jukić, 2009; Mazzilli et al., 2019; Orban et al., 2010; Tritz et al., 2011, among many others) (Figure 6b). By quantifying the vertical and/or horizontal water exchange between these compartments, the lumped models are often coupled with the ‘mixing’ concept to describe the transport dynamics in karst systems (see following subsection).

### 3.2.2 Classification of solute transport models in karst systems

As there is not a generic classification for the solute transport models in karst water resources, we suggest the following classification for modelling approaches of solute transport processes in karst systems. This proposed classification is categorized based on the complexity of karst system conceptualization, (model) data requirement, and (model) numerical formulation. Therefore, the solute transport models are divided into two main classes: Lumped transport models (LTMs) and Advanced transport models (ATMs) (Figure 7).
Figure 7. The conceptual descriptions of the solute transport models often used in karst systems. (a) Lumped transport models (LTMs) including complete instantaneous mixing model (CIMM) and lumped parameter model (LPM), (b) Advanced transport models (ATM) such as Multi-rate Mass-transfer Model (MRMT).

Overall, LTMs consider the karst flow field as a single non-distributed black box by assuming the karst flow is constant at the domain of interest (Bayari, 2002; Ozyurt & Bayari, 2005, 2008). Since LTMs do not require specific knowledge about the various transport processes (i.e., advection, dispersion, mass exchange) within the flow domain, they are often preferable for karst modelling applications particularly at aquifer scale and/or regional scale (Małoszewski & Zuber, 1996; Toth & Katz, 2006). ATMs, by comparison, explicitly consider the impact of the karstic heterogeneities both on the flow velocity field and on the anomalous transport (i.e., late-time and/or early arrivals in BTC). Therefore, the underlying assumptions of ATMs are physically better than LTMs whereas the mathematical solutions are more complex and rather prone to the numerical errors.

Lumped transport models (LTMs)

Lumped transport models (LTMs) comprise the compartmental mixing models — often referred as complete and instantaneous mixing models (CIMMs) — and lumped parameter models (LPMs) (Figure 7).

CIMM is a compartmental model approach, in which a solute of interest is mixed in a model compartment (usually the bucket of a lumped model) based on an assumption of instantaneous and complete mixing. The mixing can be applied to each of the model’s compartments (i.e., epikarst, matrix, and conduits as depicted in Figure 6b) or to the entire simulated dynamic water storage. Also, partial mixing within the same storage has been applied in some studies (e.g., McMillan et al. 2012). In CIMMs, while the structure of the lumped karst flow model specifies the flow paths between the compartments, solute mass balance between those compartments is solved by the solute mixing ratios. Although simple, this approach brings
the advantage of requiring no (complete mixing, Hartmann et al. 2012b, 2013a, 2016a) or few (partial mixing, Zhang et al., 2020a) transport modeling parameters. However, CIMMs ignore advection and time-variant mixing transport characteristics of the karst systems. Therefore, their application for karst transport modeling is often limited.

LPMs were a common modeling approach in karst aquifers mainly due to its simplicity (significant simplifications with fewer parameters) and suitability for the data-rare sites (e.g., Bayari, 2002; Małoszewski et al., 1992, 2002; Zuber, 1986; Zuber & Małoszewski, 2001, among many others). LPM was first introduced by Małoszewski & Zuber (1982) to solve the transport problems in karst systems.

LPM is a groundwater age-based modeling approach that solves age distributions (i.e., TTDs, RTDs) of the solute of interest at the karst system outlet (i.e., karst spring, observation well). In doing so, LPM converts the input solute concentrations (i.e., δ¹⁸O concentrations of atmospheric precipitation) to the output solute concentrations (i.e., δ¹⁸O concentration of karst spring) using a convolution integral in Eq. 1.

\[
C_{\text{out}}(t) = \int_{0}^{\infty} C_{\text{in}}(t - \tau) g(\tau) \exp(-\eta \tau) d\tau
\]

where \( C_{\text{out}} \) [ML⁻³] and \( C_{\text{in}} \) [ML⁻³] are the input and output tracer concentrations, respectively. \( \tau \) [T] is the travel time (or residence time is often used in groundwater studies). \( g(\tau) \) is the impulse response function (or transfer function) that is characterized by the system’s TTDs. Here, \( t \) refers to the calendar (or current) time and \( \eta \) is a decay constant accounting for the non-conservative transport mechanisms. In Eq.1, when a conservative tracer is considered for the model application, the decay term is then set to zero (Małoszewski & Zuber, 1982; Zuber & Małoszewski, 2001). But the critical parameter for karst transport modelling studies in Eq.1 is the mean transit time of the solute of interest, referred to as mean transit time (MTT) for the time-invariant hydrological conditions (Eq. 2);

\[
M TT = \int_{0}^{\infty} t g(\tau) d\tau
\]

In LPMs, the response function plays an important role in quantifying the physical spreading of solutes of interest at the transport domain. It describes how different (groundwater) ages in the convolution integral contribute to the age distributions at the karst system’s outlet (i.e., springs, observation well). Consequently, a proper definition and/or selection of a suitable response function for the LTMs is a critical step in representing the underlying flow and transport processes (Małoszewski & Zuber, 1993; Zuber, 1986). This functional form can either be defined by a priori understanding or by comparing the simulations of different response functions to historical records of the solute concentrations at a particular sampling location (i.e., springs, observation wells). For instance, the simplistic LPMs resemble the individual flow and transport paths with no mixing in karst systems (i.e., for the conduit-dominated karst aquifers) or well-mixed (or perfect mixing) karst systems (i.e., for the matrix-dominated karst aquifers). While the former would be represented by the Piston Flow Model (PFM) response function, the latter one would be represented by the Exponential Model (EM) (e.g., Małoszewski & Zuber, 1982, 1993; Zuber, 1986). There are also different combinations of the response functions that are used to represent the transport dynamics of karst systems such as the combined exponential piston models (EPM) and dispersion models (DM) which resembles the solute mixing between perfect mixing and no mixing (Maloszewski et al., 2002; Małoszewski & Zuber, 1982, 2002; Zuber & Małoszewski, 2001).
Furthermore, as the response function in Eq. 1 defines groundwater age distributions in the karst system (i.e., TTDs or RTDs), it also enables us to link between LPMs and groundwater age models — known as Direct Age Models (DAMs) — (e.g., Cornaton & Perrochet, 2006a, 2006b; Ginn, 1999, 2009; Goode, 1996; Sanford, 2011; Suckow, 2014). These models predict the water age distributions by solving ADE in which the ‘solute mass’ concentration is replaced by a groundwater ‘age term’ (see more details in Ginn, 1999, 2009; Goode, 1996).

Single-fissure dispersion model (SFDM) and Multi-dispersion model (MDM) — also as the multi-flow dispersion model are two common types used for estimating karst flow and transport parameters (Figure 8) by including more process understanding compared to simple LPMs. These models differ from LTMs in that the additional parameters (i.e., dispersion parameter) are required for solving dispersive-diffusive processes in karst matrix. In general, as both SDFM and MDM analytically solve the karst transport problems, they are more suitable for the well-defined aquifer conditions where the initial and boundary conditions are known (e.g., Göppert & Goldscheider, 2008; Frank et al., 2021; Kübeck et al., 2013; Vincenzi et al., 2009). They are derived from the earliest analytical solutions of one-dimensional ADE by Lenda & Zuber (1970) and Kreft & Zuber (1978). The general form of the advection-dispersion model (ADM) is as followed by:

\[
C_f(x, t) = \frac{M}{Q_t_0} \sqrt{\frac{4\pi Pe^{-1}}{4. Pe^{-1}}} \left[ \frac{(1 - \frac{L}{t_0})^2}{4 Pe^{-1}} \right]
\]  

where \(C_f(x, t)\) [ML\(^{-3}\)] is solute concentration, \(Q\) [M\(^3\)T\(^{-1}\)] is discharge, and \(t_0\) [T] is the time of mean flow (or refers also to the mean residence time) [T]. \(D_L\) [L\(^2\)T\(^{-1}\)] is the longitudinal dispersion, and \(\alpha_L\) [L] is the longitudinal dispersivity.

In Eq. 3, \(Pe\) is the (dimensionless) Péclet number which is the characteristic flow length multiplied by the mean flow velocity (\(\theta\)) and divided by the diffusivity (Labat & Mangin, 2015) and it represents the ratio of the advective transport rate to the diffusive transport rate. Along this characteristic length, \(Pe\) is assumed to be constant and homogeneous. However, this rarely holds the reality in karst aquifers. For a (karst) aquifer scale, when \(Pe\) is below 0.4 [-], diffusion controls the solute transport while \(Pe\) > 6.0 [-] indicates the advection-dominated transport characteristics. 0.4 < \(Pe\) < 6.0 demonstrates the (transition) transport characteristics varying between the diffusion-dominated and advection-dominated transport (Field & Nash, 1997). In Eq. 3a, \(1/Pe\) is called as a dispersion parameter for describing the (matrix) diffusion effect on the solute transport behaviour (e.g., Małoszewski & Zuber, 1982, 1992; Zuber, 1986).
Figure 8. The conceptual descriptions of a) Single-fissure dispersion model (SFDM), b) Multi-dispersion model (MDM). Here, \( c \) refers to a single conduit while \( c_1 \) and \( c_2 \) are denoted as Conduit-1 and Conduit-2.

SFDM or Parallel fissure dispersion model (PFDM) by Mależewski & Zuber (1985) is based on the theory of the parallel-plate modelling approach introduced by Sudicky & Frind (1982). This approach considers the solute transport along the parallel fissure plates and into the matrix coupled by the matrix diffusion (Małoszewski & Zuber, 1985). For karst aquifers, SFDM is first applied into the fissured chalk aquifers by Mależewski & Zuber (1990) and modified later by Witthüser et al. (2003) to simulate transversal dispersion in chalk aquifers. To solve the karst transport problem, SFDM requires three parameters including dispersion parameter, diffusion parameter, and mean transit time (Mależewski & Zuber, 1985; Mależewski et al., 1994; Witthüser et al., 2003).

MDM is an extension of ADM, and it considers the multiple discrete flow and transport pathways. While MDM solves ADM in each flow path based on the specified parameters (i.e., flow rate, transit time, and dispersivity) (Małoszewski et al., 1992, 1994), it is often used for the (a large-scale) multi-peaked BTC analysis in karst systems (e.g., Goldscheider et al., 2008; Richter et al., 2022).

Advanced transport models (ATMs)

The anomalous behaviour in karst transport is frequently characterized by the rapid spread of the front of the contaminant plume due to existence of preferential paths, as well as by the slow diffusion owing to the mass exchange between the karst conduit and surrounding carbonate matrix (e.g., Berkowitz et al., 2006; Goeppert et al., 2020; Ewers et al., 2012; Wang et al., 2022). Diffusion and retardation processes often result in the late-time heavy tailing in BTC, and thus refers to time-nonlocality in karst transport behaviour. By comparison, highly skewed and/or multi-peaked BTC characteristics is defined by the space-nonlocality in solute transport behaviour (see also Figure 4) (e.g., Haggerty & Gorelick, 1995; Haggerty et al., 2000; Zhang et al., 2020; Zhang et al., 2009). In this context, ATMs intend to capture the spatiotemporal dynamics of the anomalous karst transport.

In this review, ATMs is divided into three main modelling concepts: (i) Continuous-time Random Walk (CTRW) that discretizes the solute concentration into the particles to describe the spatiotemporal evolution of each particle, (ii) Fractional Advection-Dispersion Equation (FADE) wherein the solute movement obeys the non-Gaussian distribution, and (iii) Multi-rate mass...
transfer (MRMT) models that divide flow and transport domain into (at least) matrix and conduits. Broadly speaking, MRMT and FADE represent the Eulerian-type (Continuum) modelling approaches while CTRW is the Lagrangian-based transport modelling approaches. Here, the Eulerian-type models consider the volume-based flow and transport by discretizing the problem domain into the different volumes while the Lagrangian-based approaches, such as Particle Tracking (PT), Random Walk Particle Tracking (RWPT) discretize the solute concentration into the particles, thereby considering the solute particles as a discrete phase without an explicit need for a flow field definition. CTRWs and FADEs are the robust transport modeling approaches, especially for capturing the time-nonlocalities in solute transport behaviors in heterogeneous groundwater systems (e.g., Benson et al., 2000; Meerschaert et al., 1999; Metzler & Klafter, 2000; Schumer et al., 2009) although the applications of both modelling approaches in solving karst transport problems are rather new. In the following, we will provide fundamental knowledge about these advanced karst transport models and their primer applications in modelling of karst solute transport processes.

CTRW considers the solute (particle) transport as a random walk in spatial and temporal scales (Berkowitz et al., 2006; Dentz & Berkowitz, 2003). It is a generalization of the Random walk methods in that the spatial displacement and temporal resolution (or time-step) are counted as random variables (Dentz et al., 2004). In fact, the formalism of CTRW originates from Einstein’s Brownian motion approach that considers the random motions of the solute particles. While CTRW has been extensively applied for the fractured rocks and aquifers (e.g., Berkowitz et al., 2001, 2006; Hansen & Berkowitz, 2020), the first application of CTRW in karst systems was done by Goeppert et al. (2020) for the simulation of the flow and transport processes in an alpine karstic aquifer. The CTRW approach is particularly useful for simulating the late-time and heavy-tailed characteristics of the solute transport behaviour (e.g., Berkowitz et al., 2006; Berkowitz & Scher, 1995; Dentz & Berkowitz, 2003). For this reason, it better resembles the dispersion and diffusion processes as compared to the particle tracking techniques (Tsang & Tsang, 2001). In general, the applications of the Particle-tracking based methods in karst aquifers are preferable for the delineation of capture zone characteristics in the well-head protection studies (e.g., Klaas et al., 2017; Rayne et al., 2001; Qiao et al., 2015).

FADE uses an additional “fractional” term in the classical ADE given in Eq. 4 for non-sorbing (immiscible, single dissolved) solute transport without a source term,

\[ R \frac{\partial c}{\partial t} = -\varphi \frac{\partial c}{\partial x} + D \frac{\partial^2 c}{\partial x^2} \]  

(4)

in which \( \varphi \) [LT\(^{-1}\)] is the flow velocity and \( x \) [L] is the transport distance in the main flow direction. \( C \) [ML\(^{-3}\)] is the solute concentration in time (t) [T]. \( D \) [L\(^2\)T\(^{-1}\)] is the (longitudinal) dispersion coefficient in the main flow direction. Here, \( R \) is denoted as a retardation factor and estimated by the following formula,

\[ R = 1 + \rho_s K_d \]  

(5)

where \( K_d \) [L\(^3\)M\(^{-1}\)] is denoted as the distribution (or partition) coefficient that is described as a ratio of the sorbed concentration into (geological) material and the dissolved concentration in groundwater at (thermodynamically) equilibrium conditions. In karst aquifers, \( K_d \) is often used to describe the sorption degree of any species in karst groundwater based on the distribution of solute between solid matrix and groundwater (e.g., Pastore et al., 2021). Therefore, as for the
estimation of retardation given in Eq.5, when $K_d$ is equivalent to 0, it equals 1, which means that the retardation is negligible during the karst transport mechanism.

The “fractional” term in FADE refers to the fractional derivatives of dispersion that are incorporated by time and/or space for solving time and/or space non-localities in BTC (see for latest discussions on FADEs by Sun et al. (2020)). For one-dimensional solute transport form of FADE is followed by Meerschaert et al. (1999) and Kelly & Meerschaert (2019):

$$\frac{\partial c}{\partial t} = -\theta \frac{\partial c}{\partial x} + D\rho \frac{\partial^{\alpha}c}{\partial x^{\alpha}} + D(1-\rho) \frac{\partial^{\alpha}c}{\partial(-x)^{\alpha}}$$

where $C$ [ML$^{-3}$] is the solute concentration in time (t) [T] and the transport distance (x) [L] in the direction of main flow. $\theta$ [LT$^{-1}$] is the mean flow velocity, and $D$ [L$^{2}$T$^{-1}$] is a dispersion coefficient in the main flow direction. Here, $0 \leq \rho \leq 1$ describes the skewness of the transport processes while $\alpha$ [-] is the order of fractional derivatives.

Depending on which nonlocality problem (time or space nonlocalities) is being solved, FADE is either called the time-in-fractional advection-dispersion equation or the space-in-fractional advection-dispersion equation. While the time-nonlocality is typically related with the diffusion and retardation processes resulting in late-time heavy tailing in breakthrough curve (BTC), the space-nonlocality leads to highly skewed and/or multi-modality on the BTC (e.g., Feehley et al., 2000; Haggerty & Gorelick, 1995; Haggerty et al., 2000; Zhang et al., 2020, among many others). For solving karst transport problems, Yin et al. (2022) proposed the efficient fractional-in-time transient storage model for simulating the multi-peaked BTC in karst aquifers which considers the karst solute transport in two domains by solving the tailing effects in BTC due to the mass exchange between these regions while Zang et al. (2024) introduced an improved version of the FADE to solve the late-time behaviors of BTC characterized by the multiple-conduit flow within an intermediate-scale karst aquifer, named as the impulsive tempered FADE approach.

MRMT models are the generalization of the mobile-immobile model (MIM) concept — known also as the classical dual domain approach — (Carrera et al., 1998; Coats & Smith, 1964; Gerke & Van Genuchten, 1993; Haggerty & Gorelick, 1995). MIM simulates the flow and solute transport processes based on two distinct flow and transport regions: mobile zone (e.g., conduit, fracture, fissure) and immobile zone (e.g., matrix, voids, vugs, dead-end passages along conduits and/or cave streams). Here, MRMT differs from the MIM approach in that it considers the (one) mobile domain with the multiple immobile regions (Figure 9). For this reason, MRMT considers the multi-rate mass transfer exchange terms for each interaction zone to solve the solute transport characteristics that are frequently characterized by heavy-tailing and multi-modality in BTC (Figure 9).
The schematic description of the Multi-Rate Mass-Transfer (MRMT) model. Here, \( c \) and \( m \) are denoted as karst conduit and matrix components, respectively.

In karst aquifers, adaptations of the MIM approach have been extensively used to simulate transport processes. In the MIM approach, the dominant transport is described by the advective-dispersive processes along the mobile zone while the mass transfer between the mobile and immobile regions is only controlled by the (matrix) diffusion without any fluid exchange (Becker & Bellin, 2013; Maloszewski & Zuber, 1990; Toride et al., 1995). Therefore, solute transport is set by the classical ADE with an additional term that describes the mass transfer gradient. For one dimensional solute transport with MIM approach is then written as follows:

\[
\begin{align*}
\beta \frac{\partial c_m}{\partial t} + (1 - \beta) \frac{\partial c_{im}}{\partial t} &= D_m \frac{\partial^2 c_m}{\partial x^2} - \vartheta_m \frac{\partial c_m}{\partial x} \\
\frac{\partial c_{im}}{\partial t} &= \omega (c_m - c_{im})
\end{align*}
\]  

(7a)

(7b)

where \( c_m \) [ML\(^{-3}\)] and \( c_{im} \) [ML\(^{-3}\)] refer to the solute concentrations in mobile and immobile regions, respectively. \( D_m \) [L\(^2\)T\(^{-1}\)] refers to the (longitudinal) dispersion in the mobile region while \( \vartheta_m \) [LT\(^{-1}\)] is the mean flow velocity in the mobile region. Higher values of \( \beta \) indicate that the substantial amount of water influences the solute transport (\( 0 \leq \beta \leq 1 \)). When \( \beta \) is equal to 0, however, the MIM model turns out to be the classical ADE (see Eq. 3). \( \omega \) [T\(^{-1}\)] is the first-order mass transfer coefficient that characterizes the mass exchange rate between the mobile and immobile regions. For instance, increasing values for \( \omega \) [T\(^{-1}\)] implies that the intensive (mass) transfer takes place between the mobile and immobile regions. In case multiple-rate mass transfer exchanges is considered, the term \( \frac{\partial c_{im}}{\partial t} \) can be defined based on the memory function that defines the (particle) travel time distributions over the immobile region.

To simulate karst transport, the two-region non-equilibrium (2RNE) approach is frequently used for simulating transport dynamics in karst aquifers (e.g., Assunção et al., 2023; Barberá et al., 2018; Birk et al., 2005; Ender et al., 2018; Geyer et al., 2007; Goeppert et al., 2020; Göppert & Goldscheider, 2008, among many others). Here, 2RNE only considers the flow along the karst conduit in which flow velocity is assumed to be constant whilst no water
exchanges are considered in between the matrix and conduit components (Field & Li, 2011; Li & Liu, 2014). For the simulation of reactive processes in karst aquifers, 2RNE was extended by the inclusion of the retardation factor by Geyer et al. (2007). Furthermore, similar approaches such as the physical non-equilibrium transport (PNE), the chemical non-equilibrium transport (CNE), and the physical and chemical non-equilibrium transport (PCNE) models by Field & Leij (2014) are also available. For more detailed representation of transport dynamics, the dual-domain based models (frequently dual-porosity models over the dual-permeability models) has also been extended by a triple porosity concept where the conduit flow is formulated based on the turbulent flow properties (Faulkner et al., 2009; Reimann et al., 2011a, 2014).

Other adaptations of the MRMT approach for karstic transport comprise the Dual-advection dispersion (DADE) model and the Transient storage model (TSM). Developed by Field & Leij (2012), DADE considers the solute transport characteristics between physically interacting two conduits (named for i and j in Eq. 8) whilst the mass exchange is approximated by a first-order mass exchange between the main and auxiliary conduits (Bodin, 2020; Dewaide et al., 2016, 2018; Zhao et al., 2017, 2021) formulated by:

$$\theta_i \frac{\partial C_i}{\partial x} = \theta_i D_i \frac{\partial C_i}{\partial x} - \theta_i \theta_j \frac{\partial C_i}{\partial x} + \omega (C_j - C_i) \quad (8)$$

where $\theta_i$ [L²T⁻¹] is the volumetric water content in Conduit-i while $C_i$ is the solute concentration in Conduit-j. $D_i$ [L²T⁻¹] is the (longitudinal) dispersion coefficient while $\theta_i$ [LT⁻¹] is the mean flow velocity in the Conduit-i. $\omega$ [T⁻¹] is an effective first-order rate coefficient for the mass exchange between the Conduit-i and Conduit-j. Here, $\theta_i = \frac{A_i}{A}$ and $A_i = A - A_j$ while $A_i$ represents the water-filled area for the Conduit-i (see more explanations by Field & Leij (2012)).

As for TSM, it is built upon the dual-domain model concept in which the preferential flow paths are considered the main flow and transport regions. Originally developed for solving transport problems in streams, TSM has been adapted to simulate karst transport problems (e.g., Morales et al., 2010; Hensley & Cohen, 2012). TSM was then reformulated by Dewaide et al. (2016) for the karst aquifers such that the karst conduits are counted as the main flow and transport regions whereas the (transient) storage zones are represented either by the pools in which eddies and vortices frequently occur (Figure 10) or by cave sediments and carbonate matrix (e.g., Deleu et al., 2023; Dewaide et al., 2018; Tinet et al., 2019; Zhao et al., 2017, 2019, 2021). TSM is formulated as followed by,

$$\frac{\partial C}{\partial t} = - \frac{Q}{A} \frac{\partial C}{\partial x} + \frac{1}{A} \frac{\partial}{\partial x} \left( A D_L \frac{\partial C}{\partial x} \right) + \alpha (C_s - C) \quad (9a)$$

$$\frac{\partial C_s}{\partial x} = \alpha \frac{A}{A_s} (C_s - C_s) \quad (9b)$$

where $Q$ [L³T⁻¹] is the volumetric flow rate and $D_L$ [L²T⁻¹] is the (longitudinal) dispersion coefficient. $C$ and $C_s$ [ML⁻³] represent the solute concentrations in the main flow channel and in the storage zone, respectively. While $A$ [L²] and $A_s$ [L²] are the cross-sectional areas of the main flow channel and storage zones, the term $\frac{Q}{A}$ [LT⁻¹] refers to the mean flow velocity along the main flow channel (i.e., karst conduit, cave stream). $\alpha$ [T⁻¹] is the (first order) mass exchange term (described also as a storage zone exchange coefficient, see Figure 10).
Figure 10. a) A perceptual schematic of the dual transport domain along the (reach) of karst conduit and an overlapping (fractured/fissured) carbonate matrix. b) The transformation of the conceptual model in Panel-a into the mathematical concept of the Transient Storage model (TSM). Here, the lateral inflow and outflow rates are not considered along the reach of karst conduit.

Overall, the MIM-based modelling approaches have strong limitations by representing the physical transport region in karst systems, particularly by representing the eddy mixing and inertial flow (Anwar & Sukop, 2009). There are newly proposed advanced transport models to solve the anomalous transport such as late-time, early arrivals, and/or multi-peaked BTC in karst systems while some of them were originally developed for the heterogeneous porous media and fractured aquifers. The fundamental concept of these advanced modelling approaches is (again) the consideration of two-region transport. But they require more sophisticated parameters in describing the mass exchange between the karst conduit and matrix. For instance, the dual heterogeneous domain model (DHDM) developed by Yin et al. (2022) solves the large-scale anomalous transport in karst systems that are characterized by the multiple peaks and strong tailing in BTC (Ji et al., 2022; Yin et al., 2022). DHDM considers the combination of MRMT and DADE (Leij et al., 2012) based on the memory function and fractional derivatives (see further details in Yin et al., 2022 and Ji et al., 2022). That way, two main parameter sets in DHDM describe the individual flow velocity (advection) and (hydrodynamic) dispersion for each mobile region. Similarly, the multi-region advection dispersion (MRAD) and Dual Region Advection Dispersion (DRAD) models proposed by Majdalani et al. (2018) consider the karst flow and transport process for (at least) two mobile regions in parallel and the mass exchange resulted from the concentration gradient (Majdalani et al., 2018).

4 Lessons learned: Challenges in the successful transport modelling practice

Considerable efforts have been devoted to representing transport processes in karst flow models over the last couple of decades (see Section 3). However, great challenges remain to successfully model solute transport dynamics in karst systems and to provide robust transport predictions. The challenges spread over all parts of the karst transport modeling practice (subsection 3.1, Figure 5) and will be elaborated in the following.
4.1 System conceptualization considering data collection and system understanding

System conceptualization for modelling of karst transport processes requires a proper understanding of the system’s physical, chemical, and bio(geo)chemical characteristics. This requires extensive data sets (Huntoon, 1995; Jeannin et al., 2013; Parise et al., 2015; Perrin et al., 2003a). However, given that the availability of such a massive data is very limited, a proper conceptualization of the spatiotemporal variabilities of karst transport processes (i.e., advection, dispersion, sorption and/or chemical transformations) is limited, too (e.g., Covington et al., 2011; Field & Pinsky, 2000; Hartmann et al., 2014; Quinlan et al., 1996).

Knowledge accumulation by the compiled data feeds the perceptual understanding of the karst system's characteristics, and thus allows the modelers to map the system’s conceptual definition out. For this reason, the required level of details for the karst transport problem often shapes our data collection strategy, and hence designs the monitoring networks (e.g., Hunkeler & Mudry, 2014; Nativ et al., 1999; Panno et al., 2019; Stevanović & Stevanović, 2021; Vucinic et al., 2022, among many others). In practice, however, there are certain difficulties during the karst system conceptualization including the (i) right definition of spatial scale for the transport problem, (ii) right definition of the temporal scale of karst transport dynamics with their temporal variabilities, (iii) realistic definitions of parameter values, and (iv) process dependency to the complex karst flow. Each of which will be pointed out in the following.

The right definition of the scale of karst transport problem is vital for the system conceptualization. However, due to the karstic heterogeneity at multiple scales, the investigated/modelled domain might substantially differ from the actual transport domain in which physical and (biogeo)chemical processes predominantly occur. This is often the case in karst systems in that both physical and chemical parameters controlling flow and transport processes rely on the scale being investigated (e.g., Göppert & Goldscheider, 2008; Medici & West, 2021). The spatial extent for the transport problem is generally defined by the field experts and/or modelers themselves. As an implicit rule, the scale of interest should not be less or greater than the scale of method (i.e., artificial tracer tests, pumping tests) applied for the characterization of that particular process (Bakalowicz, 2005; Geyer, 2008; Ghasemizadeh et al., 2016; Maréchal et al., 2008). Nonetheless, the methods for data acquisition and/or interpretation techniques are often (more) appropriate for the continuum (Darcy) scale applications where the effective parameters can often be used for the entire domain (Faulkner et al., 2009; Huntoon, 1995; Reimann et al., 2011; Zhang et al., 2021). For this reason, more sophisticated equipment and research techniques might be required for the descriptions of the karst transport dynamics. For instance, the characterization of nitrate or (dissolved) organic carbon migrations in karst systems are rather challenging due to the (contaminant) input variability (e.g., Husic et al., 2019; Orban et al., 2010; Opsahl et al., 2017; Yue et al., 2019, 2020) and less-known source history (e.g., Liu et al., 2021; Lu et al., 2023; Ren et al., 2022; Sullivan et al., 2019). Therefore, it might also be difficult to successfully capture the spatiotemporal dynamics of the contaminant migration depending on the contaminant of interest with the regular measurement devices and/or monitoring systems (e.g., Finsterle et al., 2012; Zhu et al., 2007).

The right identification of the transport timescale with its temporal resolution is crucial for the proper representation of the karst transport dynamics (e.g., Bailly-Comte et al., 2011; Covington et al., 2012; Zhang et al., 2019). However, given that karst conduit and matrix might have order-of-magnitude differences in their hydraulic and physicochemical properties (i.e., porosity, hydraulic conductivity, reactivity constants), a wide range of timescales can be
identified for the karst transport (e.g., Robinson & Hasenmueller, 2017; Sivelle et al., 2019). But in the context of experimental characterization, the (karst) transport timescale is related to the process-timescale along the karst conduits (i.e., hours, days), for which shorter-timescale observations with higher sampling resolution (i.e., in minute, hourly, daily) are usually enough for capturing the karst transport dynamics (Figure 11). Without a long-term monitoring and (further) data collection strategy, however, the chemical and (biogeo)chemical processes occurring at (rather) larger timescales (i.e., annual, seasonal, decadal) might be underestimated and/or overlooked in such a data collection (e.g., Mathias et al., 2005; Tran et al., 2020; Wynn et al., 2013; Zhang et al., 2020b). Therefore, proper understanding of the (actual) mass transfers between the karst conduit and matrix is strictly limited (Brettmann et al., 1993; Cherubini et al., 2013; Zhao et al., 2017).

Figure 11. Schematic representations of the model conceptualization for solute transport dynamics in karst system considering the spatial extent and temporal scale of the karst transport. The schematic also demonstrates what affects the temporal variability of the transport processes under different hydrological conditions. a) Contaminant plume migration under the high-water flow conditions, b) Contaminant plume migration under the low-water flow conditions. Here, $t_p$ depicts the time to the peak solute concentrations with respect to the hydrological variability and sampling frequency. $t_{p1}$ (high resolution) and $t_{p2}$ (low resolution) describe the time of the peak concentrations depending on sampling frequency.

The temporal variability of the data is another complement of the successful conceptualization because the karst transport dynamics are often controlled by the hydrological conditions being investigated (Figure 11). For instance, BTC resembles one of the solute
transport behaviours under certain flow conditions at certain time periods (Figure 11). However, actual transport can be broadly varied by the different timescales ranging from an event to seasonal and/or inter-annual timescales (e.g., Doummar et al., 2018; Luo et al., 2024; Remondi et al., 2018, 2019; Sivelle & Labat, 2019; Zhang et al., 2020a). As a matter of fact, the same shape for BTC might not actually be reproduced again if those particular conditions (both hydrological and physicochemical) at that particular timescale are not meet (e.g., Aydin et al., 2014; Chu et al., 2021; Kogovsek & Petric, 2014; Morales et al., 2007, 2018; Ravbar et al., 2012, among others). Therefore, the system conceptualization is restricted to the certain timescales and might not properly represent the overall karst transport dynamics especially when the hydrological or (biogeo)chemical conditions change or vary temporarily.

Another limitation in the system conceptualization is the identification of the realistic values for the system’s parameters since both hydraulic and transport parameters can vary by orders of magnitude at differing spatial scales — from a few centimeters to hundreds of meters or more — (Berkowitz, 2002; Giese et al., 2018; Medici et al., 2019; Medici & West, 2021). Hydraulic conductivity and dispersivity are such parameters very sensitive to the scale variations, thus causing substantial variations in the spreading of the contamination plume in the karst aquifer. Likewise, chemical parameters (i.e., sorption parameters) obtained under the laboratory conditions might largely differ from those measured at the field scale (Faulkner et al., 2009; Witthüse et al., 2003). For instance, the partition coefficient \( \left( K_p \right) \) and mass transfer coefficient (i.e., first-order mass transfer coefficient) could not be generalized (e.g., Stockmann et al., 2017; Swami et al., 2018) because their values are extremely varied (often by order of magnitudes) at the field-scale measurements depending both on the matrix hydraulic properties (e.g., surface area, matrix porosity, permeability) and on the fracture/conduit characteristics (Howroyd & Novakowski, 2021; Lehmann et al., 2022; Maloszewski & Zuber, 1993; Zhou et al., 2007). Similarly, the values for reactive transport parameters could also be misleading particularly when they are scaled up or extrapolated, because of the oversimplification of the multifaceted processes that are varied both by chemical heterogeneities in the matrix and local/global groundwater chemistry (e.g., Howroyd & Novakowski, 2021; Jiang et al., 2023; Katz et al., 1998; Kaufmann et al., 2014; Mohammadi et al., 2021; Maqueda et al., 2023, among many others). For this reason, a realistic definition of the reactive transport parameters is strictly limited by the well-defined karstic sites or site-specific applications (i.e., single-well push-pull test) (e.g., Hillebrand et al., 2012a, 2012b, 2015; Priebe et al., 2022; Schiperski et al., 2016; Tran et al., 2020, 2021).

Field-scale experiments, particularly artificial tracer tests, are frequently performed under highly vulnerable aquifer conditions in which sinkholes and/or swallets are frequently utilized as tracer injection points (e.g., Jones, 2019; Li et al., 2008; Morales et al., 2007; Sinreich, 2011). For such experimental designs, as the preferential flow paths predominantly shape the spatiotemporal behaviour of the contamination plume, karst transport dynamics are often characterized by the advection-dominated transport between the injection and sampling points (i.e., between sinkhole and observation well, Figure 11). As a result, the role of matrix diffusion and mass exchange processes between the karst conduit and surrounding matrix might be underestimated. Furthermore, due to the randomly varying flow velocities in karst conduits, physical parameters such as hydraulic conductivity, velocity, dispersivity, solute travel time (Kavousi et al., 2020; Rehrl & Birk, 2010; Zhao et al., 2017) and chemical parameters including sorption and mass exchange parameters are rarely understood (Barberá et al., 2018; Field & Pinsky, 2000; Haggerty et al., 2004). Consequently, the observed data might not provide a
complete picture about the karst system’s (overall) hydrochemical functioning and transport characteristics due to the limited transport distance and (certain) flow conditions under specific experimental conditions.

All these difficulties explain why a robust conceptualization for the karst transport dynamics is inherently limited, and why there are only few real-world applications of the karst transport models that solves the classical ADE (see Section 3.4). In this matter, the limitations in system conceptualization for karst transport point out the need for in-depth discussions on how to better align the investigated (transport) domain and the model domain in karst systems, as well as for finding better ways to collect physical and (biogeo)chemical data across the scales.

4.2 Selection and parameterization of the karst transport model

4.2.1 Model selection based on the (right) choice of dominant process

Model selection with a detailed understanding of its assumptions and limitations is one of the key steps for the successful transport modelling in karst systems. However, such a choice from a model plethora is not an easy task (Horton et al., 2022; Masciopinto et al., 2021) as each model has its own features (i.e., numerical formulations, parameters) that may be appropriate for a specific application and/or specified conditions (Figure 12). Certainly, the modeler’s perceptual understanding on the karst flow and transport processes can also be judicious in selecting a candidate model structure (e.g., Cousquer & Jourde, 2022; Hartmann et al., 2013b, 2014). Of the numerous criteria, however, the objective of the study, data availability, and model structural adequacy generally define our most-suitable choice in simulating karst transport dynamics (see Section 3).

In general, an (accurate) representation of the flow and transport processes in the model structure can be seen as a crux of the robust model selection (Figure 12). But, as we have often poor understanding about the complicated physical, chemical, and (biogeo)chemical processes, a special care is required for the representation of the karst transport dynamics in the (selected) model structure with the adequate numerical formulation and parameters. From a modelling point of view, however, the representations of multiple processes at differing spatiotemporal scales under a broad range of karstic environments — in which different transport problems might be encountered — with the similar model structures is neither possible nor enough to realistically simulate the karst transport dynamics (e.g., Chang et al., 2017, 2021; Cen et al., 2021; Hublart et al., 2015; Makropoulos et al., 2008; Massmann et al., 2005, among many others). For this reason, the overall success in the robust model selection is attributed to the ‘right’ choice of the key process that predominantly controls the karst system’s hydrological and/or physicochemical behaviors by the available data (Hartmann et al., 2013a, 2018).
Figure 12. Model selection based on the dominant processes across the different karstic environments. (a) (Generic) perceptual descriptions of the broad range of karstic environments under differing climatological, hydrological, and hydrogeochemical conditions, and (potential) karst transport dynamics. (b) The selected (conceptual) model structures depending on the right choice of the dominant process associated with corresponding karstic environments. Here, only lumped simulation models with its (possible) structures are depicted to emphasize that a wide range of the (parsimonious) model structure can also be possible depending on the proper definition of the dominant processes and the transport problem of interest. Here, the acronyms indicate the different components of karst systems such as E (Epikarst), C (Conduit), and M (Matrix), as well as the model state variables including P (Precipitation or Rainfall), ET (Evaporation/Evapotranspiration), Sb (Sublimation), and SWE (Snow Water Equivalent).

In karst systems, since the (accurate) process description is restricted by data availability, the principle of parsimony — the minimum level of the model complexity — (Schoups et al., 2008; Wagener et al., 2001, 2003) has been recommended to ensure the adequate model complexity in terms of process representation with the data we have (e.g., Hartmann et al., 2014, 2018; Labat & Mangin, 2015; Kavousi et al., 2023). In general, based on the data/knowledge on the spatial variabilities of the dominant processes, the model structure can be implemented as the lumped (parsimonious), distributed (more process-based), and semi-distributed way. Amongst them, the spatially lumped karst simulation models (i.e., CIMMs) are often preferable to simulate the karst transport dynamics in data-scarce karst regions (Barrett, 1996; Barrett & Charbeneau, 1997; Hartmann et al., 2014, 2016a; Scanlon et al., 2003) while spatially distributed process-based models (i.e., ATMs) require a large type of observations to resemble varying physical and chemical processes with a high dimensional of parameter sets (e.g., Assari & Mohammadi, 2017;
Chang et al., 2019; Kavousi et al., 2020; Xu et al., 2015). For instance, mass transfer between the karst conduit and matrix need to be considered with an additional parameter sets that resemble the mass exchange in such models i.e., 2RNE, TSM, DHDM (Assunção et al., 2023; Ender et al., 2018; Schiperski et al., 2022; Yin et al., 2022) (see subsection 3.2.2). As consequence, the successful applications of the advanced transport models are strictly limited by the well-defined sites, small-scale field and/or lab-scale investigations where the rather site-specific parameters (i.e., mass transfer rate, sorption constants) are obtained (e.g., Lehmann et al., 2022; Shu et al., 2020; Schiperski et al., 2022; Wang et al., 2022). Because of this limitation, the semi-distributed karst simulations models, like Varkarst (Hartmann et al., 2013a) and TCD-Dublin (Gill et al., 2013), are frequently served as the optimal model structures to ensure the balance between the process representation and data availability (Borghi et al., 2016; Hartmann et al., 2014). The complexity of such models is often enough to represent karst transport dynamics as compared to the parsimonious lumped models, as well as better choice with the lower/adapted data need than that of distributed counterparts (e.g., Hartmann, 2016b; Teixeira Parente et al., 2019; Zoghbi & Basha, 2020, among many others). But despite the semi-distributed, physically oriented flow path simulation of these models, a big gap remains in their presentation of transport dynamics which has been relying on simple CIMM approaches.

Overall, finding the most adequate transport model structure and degree of complexity is a particular challenge for karst transport modelling. But the optimal structure should be as simple as possible and as complex enough to better represent the karst system’s underlying key physical and chemical phenomena in accordance with the available data. In principle, the complexity of selected model structure can only be increased when data amount (i.e., multiple observations) and its quality ensure the proper representations of the multiple processes in the model structure (Milanović & Vasić, 2015; Hartmann et al., 2013a, 2014a, 2018), thereby increasing the model realism in terms of process representation. Nonetheless, seeking a trade-off between the (robust) representation of karst transport processes and their proper transcriptions in the model structures based on the principle of parsimony has not yet been fully explored yet. Therefore, which type of model we could use, and how complex it would be are those questions still open to discussion.

4.2.2 Model parametrization: Parameter non-uniqueness and transferability

A successful transport model parametrization must ensure robust parameter estimation with reliable model predictions (e.g., Chang et al., 2017; Hartmann et al., 2017; Sivelle et al., 2022; Çağlı et al., 2023a). Yet, a common tenet in karst modelling studies is to obtain physically reasonable parameters in comparison with the actual site characteristics. For that, a successful model parametrization is required. However, due to the limited system knowledge with the insufficient field estimates, a robust parameterization in karst transport models is a challenging task particularly in two ways. First, how to better translate the karst transport dynamics into the model structure with sufficient parameters is a serious matter because additional parameters are involved in the karst transport model (i.e., dispersivity, mass transfer coefficient) as compared to the flow models. Second, since most parameter values cannot be assessed from the direct measurements, inverse modelling (i.e., model calibration) is necessary to describe the key physical and (biogeo)chemical processes (Figure 13). The former task could be handled by sticking to the principle of parsimony where the minimum number of parameters meet with observational datasets (Hartmann et al., 2018; Kavousi et al., 2020, 2023), the later one is often rather difficult because it will possibly go along with overparameterization. This limits the physical interpretation of the parameter values, thus reducing the model reliability with a certain
degree of uncertainty (Figure 13) (Hartmann, 2016b; Teixeira Parente et al., 2019; Zhang et al., 2020b; Zogbi & Basha, 2020; Çallı et al., 2023a).

**Figure 13.** Description of the model calibration results for karst transport model based on the concept of model equifinality, both in model solutions and in parameters. (a) Calibration of a karst transport model ended up with the equifinal simulations within a certain range of uncertainty (depicted as an ensemble of the behavioral model simulations with the shaded area), (b) The (hypothetical) description of the equifinality concept in the transport model parameters resulting from the equifinal solutions in Panel a (In the figure, the acronyms are \( v_1 \) (flow velocity), \( K_1 \) (hydraulic conductivity), and \( d_1 \) (cross-sectional area). Here, multiple feasible parameter-sets provide equally-well hydrological (i.e., spring discharge, Q) and physicochemical (solute concentration, \( C_{out} \)) responses with respect to the (selected) calibration metric. In equifinal model results, finding the “best” or “most acceptable” solution is not necessarily possible, and the results might be misleading.

The direct field measurements and/or lab-scale experiments are favorable for the improved parameterization of the karst transport models in that the spatial and temporal variabilities of the transport processes could be defined by the well-defined parameter values. In principle, since the (more) physically based models (i.e., ATMs) consider the parameter heterogeneities along the domain of interest, they are often built upon the experimental data that describes the physical/chemical significance of the processes of interest with the effective values (e.g., Dentz, 2011; Li et al., 2008; Orban et al., 2010; Pérotin et al., 2021; Zhang et al., 2021). Nevertheless, it might not be still not possible to define the specified values for some of parameters, especially for the regional scale transport modelling such as conduit/fracture geometry (i.e., orientation, spacing, aperture) and conduit/fracture networks (i.e., distributions,
intersections, tortuosity factor). Consequently, such parameters are frequently unknown and
required to be calibrated (e.g., Barberá et al., 2018; Dreybrodt, 2012; Faulkner et al., 2009;
Florea & Wicks, 2001; Geyer et al., 2013; Robineau et al., 2018, among many others). For
instance, to parametrize Transient Storage Model (TSM) where transport distance is divided into
several smaller reaches needs several effective parameter-sets to represent the transport
processes along the karst conduit. Here, the length of each reach is defined by the (extensive)
field works whereas dispersion and cross-sectional areas for the main flow channel and transient
storage are still obtained by the model calibration (e.g., Cholet et al., 2017; Deleu et al., 2023;
Dewaide et al., 2016; Tinet et al., 2019; Zhao et al., 2021). Likewise, the mass transfer
coefficient is often considered as a calibration parameter, especially for the dual-
domain transport models (i.e., MIM, 2RNE, TSM). Considering all, the model calibration is an inevitable
part of the successful model applications, both for the lumped and advanced karst transport
models.

As an inverse solution of the karst system’s transport dynamics, a successful calibration
seeks to infer a (unique) parameter set which resembles the process of interest at the
spatiotemporal domain of interest. Yet, as there is not a unique solution for an inverse problem
(Hartmann, 2018; Sivelle et al., 2020; Valota et al., 2002), it is often difficult to link the karst
transport parameters with the actual transport dynamics. For that reason, karst transport models
are more prone to suffering from the issue of non-uniqueness as compared to the flow models,
particularly in describing and (physically) interpreting transport parameters (i.e., flow velocity,
dispersivity) (e.g., Adinehvand et al., 2017; Chang et al., 2017; Johnston et al., 2009; Field & Li,
2011; Massmann et al., 2005, among many others). This often results in overparameterization
and equifinality as exemplified in Figure 13. While the overparameterization is often a serious
problem for the advanced/distributed models where the multiple parameters are represented in
the model structure despite the less information available (Fischer et al., 2024; Kavousi et al.,
2020, 2023), it might be rather less of a problem for the lumped transport models (i.e., CIMMs,
LPMs) where the direct observations are not possible due to the aggregation of the karst flow and
transport processes along the domain of interest (e.g., Hartmann et al., 2014; Sivelle et al., 2022).

Model overparameterization or equifinality manifests itself larger uncertainties in the
model predictions and parameters (Figure 13). For this reason, finding the “best” or “most
acceptable” solution is not necessarily possible. Therefore, model predictions (or equifinal
solutions) can be strongly misleading (e.g., Beven, 2006; Husic et al., 2020; Zhang et al. 2019,
2020b; Wang et al., 2020). In theory, parameter identifiability tests can quantify the equifinality
and help to reduce it with additional information or model structure simplifications. However,
these improvements may be limited because the physical and (biogeo)chemical parameters that
control the migration of the contamination plume are highly sensitive to the karst system’s
physical and (biogeo)chemical heterogeneities (Bonanno et al., 2022; Jia et al., 2018; Zhang et
al., 2020b; Zoghbi & Basha, 2020). For instance, the interpretation of mass exchange rate,
spatially varying sorption properties, reactive kinetic surface areas for the dual-domain transport
models (e.g., 2RNE, MIM, TSM) are often problematic and less reliable for predicting the karst
transport dynamics due to the lack of exhaustive information. Therefore, the non-identifiability
problem in such transport parameters might be inherent.

A (successful) application of the karst transport model is often unique in solving
(particular) flow and transport conditions at the domain of interest. For this reason, the
transferability of the (model-specific) calibrated parameters for the (rather) larger and/or smaller
transport domains, for various karstic environments, and for different models under different concepts is strictly limited, too. As a result, using the literature values for some of the transport parameters might not be entirely possible since the karst transport parameters and their values are mostly unique in place and time.

Overall, it is particularly important to emphasize that the substantial limitations behind the successful model parametrization of karst transport models reside especially in the governing solutions (i.e., ADE) that are particularly developed for the porous aquifer-specific transport problems (e.g., Bear et al., 2012; Faulkner et al., 2009; Field & Pinsky, 2000; Zoghbi & Basha, 2020). For that reason, the targeted solutions cannot be easily transferred for the highly heterogeneous karst aquifers in which the additional mathematical complexities (e.g., inclusion of mass transfer coefficients, sorption parameters in classical ADE) are not easily incorporated and overparameterization is likely to arise.

4.3 Uncertainty of the model simulations

Characterization and reduction of the uncertainty is one of the priorities for the evaluation of transport model applications as it ensures the robustness and reliability of the model predictions (e.g., Chang et al. 2021; Hartmann et al., 2013b, 2018; Çalli et al., 2023a). For this reason, treating uncertainty with its (potential) sources requires a special care throughout the karst transport modelling practice, especially for identifying the ways for the model improvements.

Taxonomically, the larger source of uncertainty in the model predictions is attributed to the epistemistic uncertainty due to the lack of system knowledge and/or ignorance of some key processes in the formal model structure, rather than the stochastic uncertainty arising from the natural variability of the hydro(geo)logical systems (e.g., Beven et al., 2011; Beven, 2023; Reichert et al., 2021). This uncertainty type is particularly relevant for the karst flow and transport models because they are mostly built upon insufficient and site-specific information, while often shaped by the expert elicitations on the complex physical and chemical properties of the karst systems (Hartmann et al., 2014; Padilla & Vesper, 2018; Zhang et al., 2017). In general, uncertainty in karst transport modeling is raised from the different stages of the karst transport modelling practice but the main sources of uncertainty in the model predictions can be attributed to following sources: (i) data uncertainty, (ii) model structural uncertainty (both conceptual and numerical), (iii) parameter uncertainty, (iv) uncertainty associated with the unknown future behavior of the karst system.

Despite clear evidence that the uncertainties in groups (i)-(iii) are substantial (see previous subsections), only few karst transport studies with LTMs have quantified the uncertainty of their transport simulations (Chang et al. 2021; Mudarra et al., 2019; Çalli et al., 2023a). Amongst them, Chang et al. (2021) was the only one trying to reveal structural uncertainty by comparing different model structures. For ATMs, different modelling approaches (i.e., ADE, MIM, PCNE) were also tested for the simulations of the shape characteristics of BTCs (e.g., multi-modality and heavy tailing) obtained from different karst conduits by Field & Leij (2014). This model comparison research provided a better understanding about the suitable model structure and parameterization of the karst transport process (Massmann et al., 2005; Zhao et al., 2019). Therefore, the disagreements in the model predictions with differing complexity for the same applications might provide a complete understanding about the suitable model structure and improved parameterization (Assunção et al., 2023; Dong et al., 2023; Massmann et al., 2005;
Zhao et al., 2019) as well as indicate the scale dependency of the model parameters and process interactions (Goeppert et al., 2020; Kaufmann, 2003; Liu et al., 2021).

Concerning the uncertainty associated with the unknown future behavior of the karst system, realistic representations of a karst the system’s current and historical transport conditions may not guarantee that future conditions (e.g., unknown future state variables) will successfully met by the chosen model structure (Beven & Chappell, 2021; Oreskes et al., 1994; Wagener, 2022). Regarding this, one of the critical tasks for the karst modelers is to assess the model results by seeking (more) reasonable answers for to what extent the model simulates the process behaviour under (globally) changing hydrological and physiochemical conditions. Improving our physical understanding of karst systems and implementing it into new and better model structure is an ongoing challenge for the successful modelling practice.

5 Ways ahead of modelling karst transport dynamics and future perspectives

Studies on modelling of karst flow and transport processes have considerably increased over the last decades. Several factors can play a role in such a progress including the variety of collected data (i.e., multiple observations), its accuracy, and increments in computational power and capacity (both accuracy and efficiency). But the driving force is the need for solving transport problems in karst water resources across the world. Nonetheless, as a community, we are still at the stage of model improvements for the successful real-world applications of the karst transport models by confronting the main challenges (i.e., system conceptualization, model parametrization, uncertainty quantification) to ensure model robustness and reliability (see Section 4). Along with these (long-lasting) challenges, however, new challenges and ways are ahead of us, especially considering the deterioration of karst water quality in a changing world due to climate change impact on karst water quality and ecosystem services. For that, we highlight four key directions that could be addressed for better representing and predicting the solute transport behaviour in karst systems, as well as for exploring karst water quality risks and potential outcomes.

5.1 Collect and provide more data for improved karst transport understanding

To reasonably predict karst transport dynamics, the solute transport models often require spatially and temporally continuous and consistent datasets. However, the collection of chemical and (biogeo)chemical data (i.e., nitrate, arsenic, biological and microbial contaminants, microplastics) is still sparse, and mostly inconsistent with the temporal coverage of the physical data (i.e., spring discharge, groundwater head, water pressure). As a result, our proper understanding about the karst system-specific contaminant mixing and transport characteristics, as well as the knowledge on reactive transport processes and (biogeo)chemical transformations are substantially limited. Nonetheless, taking long-term precautions to protect karst water quality from anthropogenic factors and global changes (i.e., climate, land cover and land use changes, soil degradation) often rely on better understanding of chemical and biogeochemical processes. For this reason, a particular attention is required for improved understanding of the (multiple) physicochemical interactions between the preferential flow paths (i.e., fractures, conduits) and overlapping (fractured) carbonate matrix, thereby developing more integrated karst modelling approaches that simultaneously consider flow, transport, and (bio)geochemical reactions in karst systems.
Karst scientists and water authorities are collecting more data than ever before. However, the collected data are rarely published and shared with the community, especially those used for flow and transport modelling studies (i.e., karst spring δ¹⁸O isotope ratios). Some working groups under specified goals, for instance SISAL (Speleothem Isotope Synthesis and Analysis), provide a readily available global database for the speleothem isotopes collected from individual speleothems and grouped them by the cave locations across the world (Atsawawaranunt et al., 2018; Comas-Bru et al., 2020; Kaushal et al., 2023). There is, therefore, a need for the common database in which we can gather the original data on karst transport dynamics from different experimental and modelling works (i.e., tracer tests, hydrochemical studies, hydraulic tests, cave isotopic and hydrochemical experiments) similar to the few and valuable attempts to collect and provide karst data by Jourde et al. (2018), Petrič et al. (2020), Olarinoye et al. (2020).

5.2 Fill the gap between oversimplified LTMs and data-demanding ATMs

The critical discussion on how to find a tradeoff between the model complexity and the choice of a right process controlling the system’s physicochemical behavior (with a reasonable number of parameters) with respect to the data availability is a requirement for a completion of the successful transport modelling practice in karst systems. Due to suitability in data-rare sites, mathematically simpler LTMs have outpaced their sophisticated ATM fellows that are represented by more variables with multiple parameters. But although the LTMs are often served as valuable heuristic tools for modelling karst transport processes, they are still the most simplistic approach in representing rather complicated karst transport dynamics. By comparison, due to the (substantial) data need, ATMs are often inapplicable for resembling the karst transport dynamics, especially for the larger scale field applications. Therefore, we see a particular need for representing karst transport processes with more-process oriented and well-constrained models that can be used for alternative modeling approaches to better resemble the spatiotemporal dynamics of contaminant migration in karst systems.

In this direction, this review reveals that there is an emerging modelling concept that is coming outside of the karst. StorAge Selection (SAS) Function Approach (hereinafter referred to the SAS model) can be an alternative approach to simulate the karst transport dynamics (Figure 14). The SAS theory was built by Botter (2011), Harman (2015), and Van der Valde et al. (2012) for the catchment scale flow and transport modelling applications while the successful applications were also extended to simulate the karst transport processes by Zhang et al. (2021) and Çallı et al. (2023a). The SAS model defines the relationship between the distribution of water ages in the reservoir storage (i.e., soil, groundwater) and the ages exiting as system’s outflux (either discharge or evapotranspiration, or both) (Figure 14). Compared to LTMs, the SAS model considers the time-variant characteristics of TTDs by revealing the time-dependent mixing, storage, and discharge characteristics (e.g., Benettin & Bertuzzo, 2018; Botter et al., 2011; Botter, 2012; Harman, 2015; Rinaldo et al., 2015; Schwemmle & Weiler, 2024, among many others). Therefore, the SAS model is structured by rather less parameters as compared to ATMs, and it thus allows better identification of the model parameters while reducing overparameterization and equifinality issues.
Figure 14. Schematic representations of a) StorAge Selection functions, b) Precipitation and discharge age distributions in the SAS model conceptualization, c) Selected functional form for the description of age distributions in the SAS model (here, younger water age preference is described, often known as the power-law distribution), d) Hypothetical demonstration of karst groundwater age distributions at a given time period, $t_c$ (Here, the figures (a), (b), and (c) were
adapted for the karst systems based on the SAS model descriptions in Benettin et al. (2022), Harman et al. (2015) and Harman (2019). In the depicted figures, $S_T$ refers to the age ranked storage that describes the amount of water in storage whose age is younger than (or equal to) certain age $T$ at current time $t$. $\omega(S_T, t)$ is the probability density function of the travel time expressing each water age ($T$) removed from the storage (for further the details see Benettin & Bertuzzo (2018)).

5.3 Develop adequate approaches for sediment transport in karst systems

Karst transport model is frequently applied for the predictions of solute transport dynamics in karst systems. However, some agents such as vapor, heat, (suspended) sediments (i.e., particles and colloids) can also play a role in defining solute transport behaviours. Suspended sediments are such agents that are often induced by the turbulent flow along the karst conduits and cave streams (Mueller et al., 2023; Jukić et al., 2022; Richter et al., 2022). In this respect, understanding the (suspended) sediment transport is particularly important for karst transport processes in two ways. First, since (suspended) sediments can adsorb a wide range of contaminant types (i.e., organic substance, hydrocarbons, heavy, and trace metals, biological contaminants), they often act as storage and/or transformation regions for the contaminants (Herman et al., 2012; Mahler et al., 1999, 2000; Mueller et al., 2023). Second, they can attenuate the contaminant of interest as a zone of remobilization (Bettel et al., 2022; Nerantzaki et al., 2015). Furthermore, while the origin of the sediments provides the evidence of transport conditions during the contaminant events taking place (Bosch & White, 2018; Çelik et al., 2022), microbial transport in karst aquifers is frequently approximated by the suspended sediment transport (e.g., Goldscheider et al., 2010; Göppert & Goldscheider, 2008). But the transportation and transformation of the contaminants with the suspended materials are not directly measurable and observable. For these reasons, their characterizations require high-resolution monitoring and suitable technologies. Therefore, future research is recommended to address contaminant fate and transport processes due to the (suspended) sediment and/or particles at high temporal resolution.

5.4 Estimate the impact of climate and land use changes on karst water quality

Karst water resources are highly regulated, stressed, and managed by the human dimension, especially by the operational water management (i.e., dam and reservoir capacity) and increased water consumption (i.e., pumping rate, water needs). For this reason, a greater uncertainty is available for the karst system’s (natural) hydrological and hydrochemical functioning. In this matter, the impacts of the human-induced factors (i.e., pumping rate, sectoral water needs) on karst flow and transport dynamics need to be considered. But up until now, the human dimension on karst transport dynamics is overlooked and/or underestimated in the real-world applications of karst transport models while the focus is often more on naturalized aquifer conditions. Therefore, coupling the knowledge of human-induced factors with the (naturalized) system dynamics at differing scales under different scenarios (i.e., the impact of groundwater abstraction rates on karst water quality) is one of the key research areas to provide more realistic predictions, as well as to support decision-makers for sustainable water management in karst systems.

Contaminant mixing and transport problems in karst water resources are frequently denoted as (saturated) groundwater zone specific issues, and the role of epikarst in karst transport dynamics is significantly simplified or completely ignored from the (formal) model structures.
Nonetheless, epikarst is a karst critical zone whereby contaminant mixing, storage, and transport characteristics are largely controlled (e.g., Dal Soglio et al., 2020; Flynn & Sinreich, 2010; Li et al., 2017; Perrin et al., 2003b; Zhang et al., 2017, among many others). Furthermore, as the epikarst forms the reactive and biogeochemical processes (i.e., sorption, microbial activities, and oxidation), it significantly modifies the (initial) composition of the contaminants released from the ground surface before reaching the saturated groundwater table (Druhan et al., 2021; Dubois et al., 2014; Oster et al., 2021; Zeqiraj, 2022) while predominantly controlling the karst system’s (biogeo)chemical functioning (Anna-Neva et al., 2021; Laincz & Hays, 2014; Panno et al., 2001). For these reasons, better understanding of epikarst transport dynamics is a fundamental need to protect and manage karst water quality and dependent ecosystem functioning. Nevertheless, due to the unbearable complexity and substantial heterogeneity of epikarst, scientific attempts to develop epikarst-dedicated transport models are quite rare.

Degradation of karst water quality is not only a local scale issue in which we can only apply a site-specific solute transport model to predict the spatiotemporal behaviour of contaminant migration, but it is also a global phenomenon that we shall further consider global changes and their potential impacts on the karst water quality and dependent ecosystems. However, to the best of our knowledge, the effects of the global changes on the karst water quality in the context of the transport modelling are not adequately addressed at local/global scales. For that, the impact of local/global changes on the karst water quality brings us fundamental responsibility to make robust predictions under a changing world, despite the substantial uncertainty. Therefore, future works call attention to what extent (and how) karst transport dynamics can (could be) linked to the altered boundary conditions due to anthropogenic factors and global changes such as climate change, land-use and/or land-type changes, soil degradation, water management alterations, and groundwater depletion.

6 Conclusions

The threat of contamination in karst water resources has come to light over the last couple of decades. Nonetheless, the multiscale physical and (biogeo)chemical heterogeneities still pose an enormous challenge for properly characterizing, understanding, and modelling flow and transport dynamics in karst water resources. In this manner, we still have substantial limitations in providing more reasonable and reliable transport model predictions, thus handling the modelling issues such as parameter scaling, model non-uniqueness, and uncertainty. But beyond that, both anthropogenic factors and global changes (i.e., climate change, land use-land cover changes, soil degradation, urbanisation, agricultural practices, and water management alterations) have been seriously deteriorating the quality of karst water resources and dependent ecosystem functioning. As the karst system’s functioning both hydrodynamically and biogeochemically is controlled by such global changes in a way that they can shape the karst flow and transport characteristics, understanding karst transport dynamics is critical to successfully predict the impacts of such factors on the water quality and dependent ecosystems. With this review, we hope to provide the necessary overview of available methods, challenges and directions to support and improve the present and future management of water quality in different karst regions across the globe.
Acknowledgments

Kübra Özdemir Çalli, Daniel Bittner, and Andreas Hartmann are supported by the German Research Foundation (DFG, grant number: HA 8113/6–1, project “Robust Conceptualisation of Karst Transport (ROCKAT”). Gabriel Chiogna and Beatrice Richteri are supported by the German Research Foundation (DFG, grant number: CH 981/6–1, project “Robust Conceptualisation of Karst Transport (ROCKAT”). Süleyman Selim Çalli was financially supported by the Scientific and Technological Research Council of Türkiye (TÜBİTAK, grant number: 1059B142000592).

Open Research

No original/new data were collected and/or developed for this review. The paper is a synthesis of the published works.

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