Stability Margin Improvement of Multi-Inverter, Multi-Vendor Power Station using Gradient-based Black-Box Models

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Abstract- Inverters in a power station (PS) may have different control algorithms and be made by different manufacturers. To ensure small-signal stability at different equilibrium points, systematic analyses and design are required. However, analytical small-signal models of each inverter are not available to the PS operator due to intellectual property (IP), which hinders the stability analysis and improvement. To overcome this problem, firstly, a gradient-based black-box modeling technique of the inverter is proposed. The gradient-based black-box model should be provided by the manufacturers. Based on it, an optimization method is proposed to improve the stability margin of the PS, which should be completed by the PS operator. Under the given work-flow, the IP rights would not be infringed.

I. INTRODUCTION

The increasing penetration of the renewable energy generation (REG) introduce inverter (or converter)-driven stability (IDS) issue over wide frequency range [1]. Small-signal-based analyses and design methods have been proven to be effective to address such stability issue. In a REG-based power station (PS), IDS may happen due to the interaction between a single inverter and the transmission line, or that among different inverters. The former interaction is formulated as the weak-grid problem [1]. Different control algorithms for weak-grid problem, applied on single inverter level, have been proposed [2].

Those algorithms can suppress the oscillations due to weak-grid condition. However, it is still not sufficient to restrain the oscillations due to the interaction among different inverters, which is a systematic problem. In a PS, the operator wants enough stability margin with different equilibrium points, i.e., different output-levels of the inverters. As to this aim, in addition to obtain the small-signal model (SSM) of the whole PS, an effective algorithm, instead of trial-and-error, is necessary for parameter tuning of each inverter. Because the number of the tunable parameters is huge.

There are two ways for small-signal modeling of a PS, in the time-domain or frequency domain [3]. In the time-domain, the SSM is a state-space model where the control of each inverter should be provided. But inverters in a PS may come from different manufacturers, which means their control strategies are not accessible to each other and the PS operator due to intellectual property (IP). Hence, it is hard to obtain a system-level state space model. In the frequency domain, the SSM is an impedance network [4]. One could obtain the impedance model of each single inverter at first, and then combine them as a network. Finally, Nyquist theorem can be used for stability margin analyses. Hence, if the black-box impedance model of each inverter is provided by the manufacturers, impedance network of the whole PS can be built by the PS operator.

Neural network (NN)-based black-box impedance modeling for grid-tied inverters with different control schemes have been developed [5]. For a grid-tied inverter, mathematical impedance model can only be built off-grid, while the NN-based model can be trained either off-grid or on-grid. Then stability margin of the whole PS can be measured at different existed, even potential, equilibrium points.

However, different from traditional analytical impedance model, the existing NN-based impedance model cannot be used for stability margin improvement. Firstly, the control parameters are not included in the inputs of the existing NN-based impedance model. Secondly, the stability of the inverter itself, which is always an assumption when Nyquist criterion is applied, cannot be verified through black-box impedance model. To solve the first problem, one can using an NN-based impedance model, with all the control parameters included as its inputs, to fit the analytical impedance model, which is a regression problem, and not the focus of this paper. To solve the second problem, an auxiliary black-box model based on Rouse-Hurwitz criterion and optimization theory is proposed.

On the other hand, it seems that efficient parameter tuning algorithm for margin improvement of a PS has not been proposed. To bridge the gap, the stability margin improvement is formulated as an optimization problem. The gradient-descent (GD) based algorithm is applied to solve it, where the back-propagation (BP) algorithm is used to obtain the gradients.

The contribution of this paper is the proposal of an optimization-based work-flow for stability margin improvement of a PS including modeling and tuning techniques. The auxiliary models and black-box impedance models of the inverters should be provided by the manufacturers. Based on these models, the GD-based algorithm can be used by the PS operator for tuning of each inverter. Hence, the stability margin can be improved without compromising on IP rights.

The rest of this paper is arranged as follows. Section II introduces how to formulate an optimization problem for stability margin improvement of the REG based PS. Section III introduces the proposed auxiliary black-box model and how to use the BP algorithm to enable optimization. Section IV gives examples to show the effectiveness of the proposed work-flow. Conclusions and discussions are given in the Section V.

II. PROBLEM FORMULATION FOR STABILITY MARGIN IMPROVEMENT OF THE REG BASED PS

A. System Description

The diagram of a REG-based PS with daisy-chain-based structure, which is a commonly used topology, is shown in Fig. 1 (a). It can be linearized and then represented as multi-
paralleled structure as shown in Fig. 1 (b), where $Z_k$ denotes the equivalent transmission line from the $k$th inverter to the PCC; $Z_{inv}$ and $Z_g$ denote the impedance of the $k$th inverter and the grid impedance, respectively; $\Delta i_k$ and $\Delta u_k$ denote the current of the $k$th branch and the grid current, respectively; $\Delta u_{PCC}$ denotes the PCC voltage; $\Delta u_{inv}$ and $\Delta u_g$ denote the voltage disturbance brought by the $k$th inverter and the power grid, respectively. The impedance of the $k$th branch can be defined as follows.

$$Z_k(s) = \frac{\Delta u_{PCC}(s) - \Delta u_{inv}(s)}{\Delta i_k} = Z^*_k + Z^*_g \tag{1}$$

It worth noting that, $Z^*_k$ can be $dq$-impedance, sequence-impedance or simplified version of them and not specified here.

Then the system can be described as follows.

$$\Delta u_{inv} - \Delta u_{PCC} = Z_k \Delta i_k$$

$$\Delta u_{PCC} - \Delta u_g = Z_g \left( \sum_{j=1}^{N} \Delta i_j \right) \tag{2}$$

where $N$ denotes the number of the branches; $1 \leq k \leq N$; $\Delta u_k$ and $\Delta u_g$ can be considered as inputs; $\Delta i_k$ and $\Delta u_{PCC}$ can be considered as outputs. Letting every $\Delta u_{inv}$ equal to zero, one has,

$$\Delta i_k = -\frac{\Delta u_g}{Z_k + Z^*_g + \sum_{j=1}^{N} Z_g \frac{Z_k}{Z^*_g}} = \frac{1}{Z^*_g + 1} \left( Z_k + Z^*_g \right) \sum_{j=1}^{N} \frac{1}{Z_g} \Delta u_g$$

$$\Delta u_{PCC} = Z^*_k \Delta i_k = \frac{1}{1 + Z^*_g \sum_{j=1}^{N} \frac{1}{Z_g}} \Delta u_g \tag{3}$$

An open-loop transfer-function can be defined as

$$G_{open} = Z_g \sum_{j=1}^{N} \frac{1}{Z^*_g} \tag{4}$$

It can be seen that, if every $1/Z^*_k$ is stable and $G_{open}$ satisfies the Nyquist criterion, the PS will be stable under the disturbance of $\Delta u_g$. Considering (2) is a linear system where superposition principle is guaranteed, and $\Delta u_{inv}$ and $\Delta u_g$ are interchangeable in the network, the stability of the whole PS can be ensured by the former conditions.

**B. Formulate the Stability-Margin Improvement as an Optimization Problem**

Providing that every $1/Z^*_k$ is stable, the whole PS will be stable, if and only if the Nyquist curve of $G_{open}$ does not enclose $(-1, j0)$ on the complex plain. Because $G_{open}$ is stable. To ensure sufficient stability margin, a forbidden region (FR), shown in Fig. 2 (a), can be defined as follows.

$$FR = \{ (R,a) | R < a, \ abs(X) < b \} \tag{5}$$

In which $a$ and $b$ are pre-selected constants. If any part of the Nyquist curve is not included in the FR, it will not enclose $(-1, j0)$, and have enough stability margin.

Providing the black-box impedance model of every inverter is available, the Nyquist curve of $G_{open}$ can be sampled on the frequency domain as $(R_s, \omega_s)$, $R_s = Re[\{G_{open}(j\omega_s)\}]$, $\omega_s = Im[\{G_{open}(j\omega_s)\}]$ where $(\omega_s)$ is the set of the sampled frequency points.

Three cost-functions can be defined as

$$J_s = \frac{1}{2} \sum_{s \in \{R_s, \omega_s\} \in FR} (\alpha - R_s)^2 \frac{1}{2} \sum_{s \in \{R_s, \omega_s\} \in FR} (e^s)^2 \tag{6}$$

Similarly, one can obtain $J_{XP}$, $J_{XN}$ and $\partial R_s/\partial p^*_j$. These three gradients are named as PS-Stability-margin-improvement-gradient (PS-SMIG). They convey the direction of how to change the $p^*_j$ to minimize the $J_s$, $J_{XP}$, $J_{XN}$, respectively, i.e., push the Nyquist curve outside the FB from left to right, from the bottom up and from the top down,
respectively. One can chose one or two of them to update the $p'_j$, e.g.,

$$p'_j = p'_j - \gamma_1 \frac{\partial}{\partial p'_j} J_x$$  \hspace{2cm} (9)

where $\gamma_1$ is the step-size.

### III. Building AUXILIARY MODEL AND USING BACK-PROPAGATION

#### A. Proposed Auxiliary Model

The optimization method mentioned above make sense only if every $1/Z_k$ is stable. This condition is equivalent to that the $j^{th}$ branch is stable when it is connected to an ideal voltage source. Generally, the transmission lines of each inverter can be described by a simple RLC circuit with different parameters. And such information can be provided by the PS operator. Hence, it is easy to obtain a group of differential equations describing the dynamics between the $j^{th}$ branch and a stiff grid. Then, these equations can be linearized.

An auxiliary modeling method is proposed based on those linearized equations, Rouse-Hurwitz criterion and optimization. One can obtain a $n^{th}$ order close-loop transfer function whose poles can represent the stability of the linearized system. Then, the eigen equation can be expressed as

$$\sum a_{i,j} e^{\lambda t} = 0$$  \hspace{2cm} (10)

where $a_i = f(P_j, E_j, L_j)$. $P_j$ and $E_j$ denote the set of the tunable parameters and equilibrium points of the $j^{th}$ inverter, respectively; $L_j$ denotes the set of the parameters of the transmission line of the $j^{th}$ inverter. Then, according to Rouse-Hurwitz criterion, one can obtain several factors $\{F_m = f_m(a_1, a_2, ..., a_n)\}$. For a stable system, all the $F_m$ should be positive.

Hence, one can define a set of positive number, $\{F_m^{\text{limit}}\}$, to describe a stability margin and a cost function as follows.

$$J_{\text{self}} = \sum_{m=1}^{n} \frac{1}{2} (T_m - F_m)^2$$  \hspace{2cm} (11)

where $T_m > F_m^{\text{limit}}$. When all $T_m > F_m^{\text{limit}}$, it means the stability margin is large enough and the cost function is equal to zero. Otherwise, one can enlarge the stability margin by minimize $J_{\text{self}}$ using GD-based method as follows.

$$p'_j = p'_j - \gamma_2 \frac{\partial}{\partial p'_j} J_{\text{self}}$$  \hspace{2cm} (12)

where $p'_j \in P_j$; $\gamma_2$ is the step-size. And the gradient, $\partial J_{\text{self}}/\partial p'_j$, is named as single-inverter-stability-margin-improvement-gradient (SI-SMIG).

#### B. Total Black-Box Model and Optimization Workflow

The diagram of the total black-box model of a grid-tied inverter is shown in Fig.3 (a) which should be provided by the inverter manufacturers. It contains a NN-based impedance model, and the proposed auxiliary model based on Rouse criterion. The input contains $P_j, E_j, L_j$, and the frequency, noted as $f$. The output contains the impedance values, and SI-SMIGs. The iterative optimization method of the PS stability-margin improvement is shown in Fig. 3 (b).

#### C. Utilizing the Back-Propagation

As shown in (7) and (8), to obtain the PS-SMIGs, the key is to obtain the partial derivatives of the impedance against the parameters. To achieve this, one only needs to apply the well-known BP algorithm.

An arbitrary neuron in the input layer of the NN-based impedance model can be expressed as

$$y = w^T [P_j, E_j, f] + b$$  \hspace{2cm} (13)

where $w$ denotes the weight vector and $b$ denotes the bias. The BP algorithm is used to derive the partial derivatives, e.g., $\partial Z_k / \partial w_k (w_k \in w)$, during training [7]. It can be seen from (13), the weight vector $w$ and the $P_j$ are interchangeable. Hence,

$$\frac{\partial Z_k^2}{\partial p'_j} = \frac{\partial Z_k^2}{\partial w_k} \frac{\partial w_k}{\partial p'_j}$$  \hspace{2cm} (14)

which means one can easily obtain the PS-SMIGs, combining with (7) and (8)

### IV. EXAMPLES

#### A. Stability Margin Improvement of a REG-based PS

To verify the proposed optimization-based work-flow, a PS design example including two grid-tied inverter is taken into account. These two inverters are stable when they are connected to the stiff-grid, with the initial parameters. Then, they are connected to a PCC. The inverters have same control algorithm (with PI-based current controller and the basic SRF-PLL [8]) shown in Fig. 4 (a). The nominal grid phase voltage is 690V. $Z_k^2 = sL_k, Z_g = sL_g, L_g = 4mH, L_{gg} = 2mH, L_{gg} = 5mH$. The filters of theses inverters are same $L$-filter with 3.3mH inductance.

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**Fig. 3 (a)** Total black-box model containing impedance model and auxiliary model. (b) Optimization work-flow.

**Fig. 4 (a)** Control strategy of the grid-tied inverters. (b) Control parameters.
A cost function is defined as the frequency range between 1 to 100 Hz by tuning the phase into the range between -90 to 90 degrees over the parameter set, the phase of the inverters is shown in Fig. 5 (b). It can be seen that, the system based platform shown in Fig. 5 (a). The phase A current of the improved a lot. The experiment was conducted on the RT-box parameter group 2 which shows the stability margin is margin. The blue trajectory denotes the Nyquist curve with parameter group 1 which indicates an extremely small stability (b). The red trajectory denotes the Nyquist curve with parameter group 2 which shows the stability margin is improved a lot. The experiment was conducted on the RT-box-based platform shown in Fig. 5 (a). The phase A current of the inverters is shown in Fig. 5 (b). It can be seen that, the system with parameter group 2 is stable at first. After switching to group 1, the system is unstable, due to the very limited margin. Hence, the effectiveness of the work-flow can be confirmed.

B. Impedance Shaping using Back-Propagation

It worth noting, during the optimization of the example above, the PS-SMIG is obtained through analytical impedance model. To verify the BP-based gradients calculation. An impedance shaping example is shown here. The NN-based impedance model is provided in [5], which is open source. The inputs of this model are active power, reactive power, PCC voltage and frequency, noted as [P, Q, V, f], while outputs are the real and imaginary part of the four dq-domain impedances, noted as \([Z_{dd}^R, Z_{dq}^R, Z_{qd}^R, Z_{qq}^R, Z_{dd}^I, Z_{dq}^I, Z_{qd}^I, Z_{qq}^I]\). With an initial parameter set, the phase of the dq-impedance, defined as follows, is shown in Fig. 6 with blue line.

\[
\theta_f = 180 \arctan \left( \frac{Z_{dq}^I(P,Q,V,f)}{Z_{dd}^I(P,Q,V,f)} \right) / \pi.
\]  

Then the BP based gradients calculation is applied to shape the phase into the range between -90 to 90 degrees over the frequency range between 1 to 100 Hz by tuning \(P\). To achieve that, a cost function is defined as

\[
J_{\text{phase}} = \sum_{f | f \geq 90} \left( \frac{-90 - \theta_f}{2} \right)^2 + \sum_{f | f \leq -90} \left( \frac{90 - \theta_f}{2} \right)^2.
\]

Shown in Fig. 6, the \(\epsilon_P\) denotes the distance between the \(\theta_f\) to the line of -90 degrees; the \(\epsilon_P\) is the distance between the \(\theta_f\) to the line of 90 degrees, respectively. Hence, by minimizing the \(J_{\text{phase}}\), the \(\theta_f\) can be restricted to the range between -90 to 90 degrees. The gradient of \(J_{\text{phase}}\) against \(P\) can be calculated as

\[
\frac{\partial J_{\text{phase}}}{\partial P} = - \sum_{f | f > 90} \epsilon_P \frac{\partial \theta_f}{\partial P} - \sum_{f | f < -90} \epsilon_P \frac{\partial \theta_f}{\partial P}.
\]

where

\[
\frac{\partial \theta_f}{\partial P} = \frac{\partial \theta_f}{\partial Z_{dd}^R} \frac{\partial Z_{dd}^R}{\partial P} + \frac{\partial \theta_f}{\partial Z_{dq}^R} \frac{\partial Z_{dq}^R}{\partial P} + \frac{\partial \theta_f}{\partial Z_{qd}^R} \frac{\partial Z_{qd}^R}{\partial P} + \frac{\partial \theta_f}{\partial Z_{qq}^R} \frac{\partial Z_{qq}^R}{\partial P}.
\]

\(\partial Z_{qq}^R/\partial P\) and \(\partial Z_{qq}^R/\partial P\) are calculated using BP algorithm. The result is shown in Fig. 6 using orange line.

V. CONCLUSIONS

Small-signal stability is the foundation for efficient operation of the REG-based PS whose inverters may come from different manufacturers. To ensure sufficient stability margin, an optimization workflow is proposed under the premise of IP rights protection. Based on the existing black-box impedance modeling technology, an auxiliary model, with SI-SMIG as input, is proposed to facilitate the use of Nyquist theorem. Based on the definition of FB and AL, cost functions are defined. BP-based algorithm is used to enable the use of GD-based optimization to minimize the cost functions improving the margin. The conclusions are confirmed with simulation and experiment.

REFERENCES


