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Outage Analysis of Hybrid VLC/RF Networks with an Energy Harvesting Relay and Random Receiver Orientation

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Abstract—In this paper, we explore an indoor downlink cooperative hybrid visible light communication (VLC)/radio frequency (RF) scenario using a relay node to reduce system outage probability. In particular, information can be transmitted to the end user either directly through the VLC link or via the relay node. To re-transmit the decoded information to the end user through the RF link the relay utilizes harvested energy from the source light emitting diode (LED) at the ceiling. We derive the analytical expression for the outage probability of the relay-aided hybrid VLC/RF system, considering the randomness of location and receiver orientation for both the relay and the end user. Furthermore, we investigate the effects of the direct current (DC) bias, data rate threshold, and different distributions for the location and orientation of the end user and relay on the outage probability of the system.

Index Terms—Energy harvesting, hybrid VLC-RF, outage probability, random receiver, relay.

I. INTRODUCTION

The growing demand for radio frequency (RF) spectrum, driven by the rise of wireless services and new technologies, has been a major challenge over the past decade. In response, visible light communications (VLC) is emerging as a helpful complement to traditional radio frequency wireless systems. VLC not only helps conserve spectrum resources by shifting users away from RF bands but also simultaneously provides illumination to the surrounding environment [1]. Recent studies highlight the importance of hybrid VLC/RF systems, which combine both technologies strategically. These systems are designed to support fast data transmission through VLC links while ensuring continuous coverage with RF links. The appeal of a hybrid VLC/RF system is particularly evident in indoor applications like the Internet of Things (IoT) and wireless sensor networks (see e.g., [2]). In these setups, where power is limited, a practical solution involves harvesting energy from the surroundings. This approach not only addresses the power constraints but also aligns with the broader goal of creating sustainable and effective wireless communication systems.

Receiver orientation and mobility present substantial obstacles to maintaining line-of-sight (LOS) connections in VLC networks. Their direct impact on the presence of LOS connections and signal quality becomes particularly pronounced, especially when both factors exhibit random variations. Therefore, it is essential to investigate the impact of receiver orientation and mobility on the performance of VLC networks.

In this paper, considering a relay-aided hybrid VLC/RF system as in Fig. 1, we investigate the impact of random receiver orientation and location, both for the end user and relay, on the statistical aspects of the system outage probability. In particular, we derive the outage probability of the VLC link based on the distribution of receiver orientation and location. Using the lower and upper bounds on the harvested energy at the relay, we then derive the outage probability of the RF link. We assume that a communication link goes into outage when the information rate drops below a set threshold, and this outage in our system happens when both the direct link and the relay link are simultaneously disrupted.

The remainder of this paper is organized as follows. Section II presents the literature review. In Section III, we describe our system model. The statistical distribution of the outage probability for the VLC and RF link are derived in Section IV along with providing the lower and upper bounds on the harvested energy. Section V presents the numerical results, and the paper concludes in Section VI.

Notations: \( N(\mu, \sigma^2) \) represents the real-valued Gaussian distribution, characterized by its mean \( \mu \) and variance \( \sigma^2 \). Furthermore, \( \mathcal{U}(a, b) \) denotes the continuous uniform distribution spanning the interval \( [a, b] \), and \( \mathcal{R}(b) \) represents the Rayleigh distribution with a scale parameter of \( b \). The trigonometric functions \( \cos^{-1}(\cdot) \) and \( \sin^{-1}(\cdot) \) correspond to the inverse operations of \( \cos(\cdot) \) and \( \sin(\cdot) \), respectively.

II. LITERATURE REVIEW

Recent research has explored the integration of energy harvesting into two hops hybrid VLC/RF systems. In this configuration, a relay harvests energy from a VLC link (first
hop) to re-transmit data over the RF link (second hop). For instance, stochastic geometry was employed by Pan et al. in [3] to calculate the outage probability of secrecy and statistical properties of the received signal-to-noise ratio in the presence of an eavesdropper for a hybrid VLC/RF system. In [4], Zhang et al. studied probability of outage and symbol error rate, considering random relay and destination locations with decode-and-forward (DF) and amplify-and-forward (AF) schemes. In [5], Pan et al. investigated IoT hybrid RF/VLC systems, providing analytical expressions for outage probability using stochastic geometry in a 3-D room. Peng et al. in [6], [7] considered a mobile relay in the system, analyzing end-to-end outage probability and its minimization under light emitting diodes (LEDs) source power constraints. In [8], Zhang et al. selected a relay among IoT devices, calculated the end-to-end probability of outage for various transmission schemes based on channel state information. Xiao et al. in [9] calculated outage probabilities for a cooperative hybrid VLC/RF relaying network, deriving a sub-optimal DC bias.

While the impact of receiver orientation on the VLC channel gain has been explored in the realm of VLC (see e.g., [10], [11]), to the best of the authors’ knowledge, there have been only sporadic efforts to study it in the context of hybrid VLC/RF networks [12], [13]. This requires assessing the efficiency of energy harvesting and its effects on the RF link, consequently influencing the system performance. In [12], an optimization problem is formulated and solved through exhaustive search to optimize both the DC bias and time allocation of the energy harvesting link simultaneously. The goal is to maximize the end-to-end information rate in the presence of random relay orientation. While they analyze the impacts of different design parameters on attainable data rates with and without joint optimization and investigate the related trade-offs in [13], they neglect the effect of end-user receiver orientation and random location.

III. SYSTEM MODEL

Fig. 1 illustrates the considered hybrid VLC/RF system. We assume an end user with random orientation can receive information from both the LED access point (AP) and the relay. The relative vertical distance between the LED AP and the end user, as well as the relay, is set to $h_{\Delta}$. The horizontal distance from the LED AP to the end user is $d_u$, and the distance to the relay is $d_r$. The distance between the end user and relay is assumed to be $d_{ur}$. Further, we consider a relay that is equipped with a single photo-detector (PD), energy-harvesting circuitry, and a transmit antenna for RF communications. At the relay side, the alternating current (AC) and DC components of the received signal are separated and used for information decoding and energy harvesting purposes, respectively.

The optical DC channel gain of the VLC link is given by [10]

$$H_{\text{VLC}} = \frac{\gamma + 1}{2\pi} \frac{A_p h_{\Delta}^2}{(h_{\Delta}^2 + d^2)^{-\frac{\gamma}{2}}} \cos(\theta) \Phi_{1/2}(\cos(\phi)),$$

where $d$ is the horizontal distance between the AP and the receiver which can be either the relay (i.e., $d_r$) or the end user (i.e., $d_u$). In (1), $\theta$ represents the incidence angle, $\phi$ is the irradiance angle, $\gamma$ is the Lambertian order and is defined as $\gamma = \frac{1}{\log_2(\cos(\Phi_{1/2}(\cos(\phi))))}$, where $\Phi_{1/2}$ is the half-power beamwidth of the LED. Additionally, $A_p$ denotes the receiver detection area, $g$ represents the gain of the optical concentrator calculated as $g = n^2_{ref} / \sin^2(\theta)$ with $n_{ref}$ being the refractive index, and $\Theta$ is the field of view (FOV) angle of the receiver. The function $\Pi(x)$ takes the value of 1 when $|x|$ is less than or equal to 1 and is 0 otherwise. The corresponding information rate can be expressed as [12]

$$R_{\text{VLC}} = B_{\text{VLC}} \log_2 \left( 1 + \frac{e (\eta P_{\text{LED}} A H_{\text{VLC}})^2}{\sigma^2_{\text{VLC}}} \right),$$

where $\eta$ is the photo-detector responsivity in A/W and $A$ denotes the peak amplitude of the input electrical signal. To satisfy the peak-intensity constraint of the optical channel, we assume that $A \leq \min(I_b - I_{\min}, I_{\max} - I_b)$, where $I_{\max}$ and $I_{\min}$ respectively represent the maximum and minimum input currents of the DC offset, and $I_b$ denotes the DC bias.

The harvested energy at the relay node is computed as [12]

$$E_h = 0.75\eta H_{\text{VLC}} P_{\text{LED}} I_b V_t \ln \left( 1 + \frac{\eta H_{\text{VLC}} P_{\text{LED}} I_b}{I_0} \right),$$

where $V_t$ and $I_0$ are the thermal voltage and the dark saturation current, respectively. In this context, as we are concentrating on a single time slot, the unit of the harvested energy can be expressed as joules per second, which is equivalent to watts. As it is observed in (3), the logarithmic term makes the closed-form analysis of outage probability intractable when dealing with a random VLC channel gain. As an alternative, by considering a deterministic VLC channel gain, the lower and upper bounds for harvested energy can be respectively written as

$$E_{h,\zeta} = 0.75\eta H_{\text{VLC},\zeta} P_{\text{LED}} I_b V_t \times \ln \left( 1 + \frac{\eta H_{\text{VLC},\zeta} P_{\text{LED}} I_b}{I_0} \right), \quad \zeta \in \{\min, \max\},$$

where $H_{\text{VLC,\min}}$ and $H_{\text{VLC,\max}}$ respectively denote the minimum and maximum VLC channel gain with the system parameter under consideration which results in minimum and maximum harvested energy.

Subsequently, the relay proceeds to re-transmit the information to the end-user through the RF link, utilizing the harvested energy. Mathematically speaking, the RF information rate can be written as

$$R_{\text{RF}} = B_{\text{RF}} \log_2 \left( 1 + \frac{E_h |h_{\text{RF}}|^2}{G_{\text{RF}} N_0} \right),$$

where $h_{\text{RF}}$ denotes the Rayleigh channel coefficients and $G_{\text{RF}}$ represents the RF link path loss model. Let $\lambda$ represents the used RF carrier wavelength. We can express $G_{\text{RF}}$ as

$$G_{\text{RF}} = \left( \frac{4\pi d_0}{\lambda} \right)^2 \left( \frac{d_u}{d_0} \right)^\beta,$$

where $d_0 = 1$ m is the reference distance, and $\beta$ denotes the path loss exponent, typically falling within the range of [1.6, 1.8] [12].
IV. Outage Probability Analysis

The outage of a communication link happens when the information rate falls below a predetermined threshold value. In our system, the outage happens when both the direct link and the relay link are in outage. Mathematically speaking, the overall outage of the system can be expressed as

$$P^O = P^O_{\text{direct}} + P^O_{\text{relay}} ,$$

where $P^O_{\text{direct}}$ represents the outage probability of the direct link (VLC to the end user link), and $P^O_{\text{relay}}$ represents the relay link outage probability. The relay link outage probability is further decomposed as

$$P^O_{\text{relay}} = P^O_{\text{relay-VLC}} + (1 - P^O_{\text{relay-VLC}}) P^O_{\text{relay-RF}},$$

where $P^O_{\text{relay-VLC}}$ is the outage probability of the VLC link to the relay, and $P^O_{\text{relay-RF}}$ is the outage probability of the RF link during re-transmission from the relay to the end user.

A. Outage Probability of VLC Link

The outage probability for the VLC link can be written as

$$P^O_{\text{direct}} = P(R_{\text{VLC}} < R_{\text{th}}),$$

where $P^O_{\text{direct}}$ is the outage probability of the VLC link to the end user. We can rewrite (1) as $H_{\text{VLC}} = h_dh_\theta h_c$, where $h_d = (h_\Delta^2 + d^2)^{-(\gamma+2)/2}$, $h_\theta = \cos(\theta)\Pi(|\theta|, \Phi)$, and $h_c$ is the remaining deterministic part. Now, the general form of $F_{H_{\text{VLC}}} (\cdot)$ is given by [10]

$$F_{H_{\text{VLC}}} (x) = 1 - F_\theta (\Theta) + \int_{R_y} f_{h_{\Delta}^2} (y) F_{h_{\Delta}^2} \left( \frac{y^2}{h_c^2} \right) dy,$$  \hspace{1cm} (10)

where $F_\theta (\cdot)$ is the CDF of the random incidence angle $\theta$, $f_{h_{\Delta}^2} (\cdot)$ denotes the probability density function (PDF) of $h_{\Delta}^2$, $F_{h_{\Delta}^2} (\cdot)$ denotes the CDF of $h_{\Delta}^2 = \cos(\theta)\Pi(|\theta|, \Phi)$, and $R_y$ represents the set of $y$ values for which the integrated function has a nonzero value. Further, the PDF of $h_{\Delta}^2$ can be written as [10]

$$f_{h_{\Delta}^2} (y) = c_d y^{\frac{\gamma+2}{2}} \frac{1}{\Gamma\left(\frac{\gamma+2}{2}\right)} \left[ y^{-1/2} - h_{\Delta}^2 \right]^{-1/2} \times \frac{c_d}{f_d} \left[ y^{-1/2} - h_{\Delta}^2 \right]^{1/2},$$ \hspace{1cm} (11)

for $0 \leq y \leq t^{-2(\gamma+2)}$, and 0 otherwise with $f_d(\cdot)$ being the PDF of the random distance $d$, respectively. Recalling [10], the CDF of $h_{\Delta}^2$ can be expressed as

$$F_{h_{\Delta}^2} (x) = \Delta_\Theta \left( \frac{1}{2} \cos^{-1} (2x - 1), \Theta \right) + 1 - F_\Theta (\Theta),$$ \hspace{1cm} (12)

for $0 \leq x \leq 1$, and equal to 0 for $x < 0$, and 1 for $x > 1$. In (12), $\Delta_\Theta (a, b)$ is an auxiliary function, given by

$$\Delta_\Theta (a, b) = \Pr \{ a < \theta \leq b \} \begin{cases} \quad F_\Theta (b) - F_\Theta (a) \text{ for } a \leq b \\ 0 \text{ for } a > b \end{cases}.$$ \hspace{1cm} (13)

It is worth mentioning that the general expression for VLC link outage, applicable to both direct (LED AP to end user) and relay (LED AP to relay) links, remains the same (i.e., (9)). The specific value for each link can be computed by employing the relevant distribution and parameters under consideration.

B. Outage Probability of RF Link

Now, the outage probability for the RF link can be written as

$$P^O_{\text{relay-RF}} = P(R_{\text{RF}} < R_{\text{th}}),$$

where $P^O_{\text{relay-RF}}$ is the outage probability of the RF link to the relay. We can rewrite (12) as $h_{\Delta}^2 d_{ur}^\beta$, denoted as $F_{Z^2}(\cdot)$, can be calculated by taking advantage of the independence between $h_{\Delta}$ and $d_{ur}$, and by applying the property on the distribution of the product of independent random variables as follows [14]

$$F_Z(x) = \int_{R_m} f_{|h_{\Delta}|^2 d_{ur}^\beta} \left( \frac{x}{(x/m)} \right) \, dm,$$ \hspace{1cm} (15)

where $R_m$ represents the set of $m$ values for which the integrated function has a nonzero value. The PDF of $|h_{\Delta}|^2$ can be written as

$$f_{|h_{\Delta}|^2}(m) = \frac{1}{2\sigma^2} \exp \left( -\frac{m}{2\sigma^2} \right),$$ \hspace{1cm} (16)

for $m \geq 0$ and 0 otherwise. In (16), $2\sigma^2$ is the power of $|h_{\Delta}|^2$ and is assumed to be 1.

To simplify (15), we assume that the end user and relay are two random points in a circle of radius $R$. The PDF of the corresponding distance (i.e., $d_{ur}$) can be expressed as [15]

$$f_{d_{ur}} (t) = \frac{4t}{\pi R^2} \cos^{-1} \left( \frac{t}{2R} \right) - \frac{t}{2R} \sqrt{1 - \left( \frac{t}{2R} \right)^2},$$ \hspace{1cm} (17)

for $t \in [0, 2R]$. The CDF of $d_{ur}$ then can be calculated as

$$F_{d_{ur}} (x) = \int_0^x f_{d_{ur}} (t) \, dt = \frac{4}{\pi R^2} \left[ \int_0^x \cos^{-1} \left( \frac{t}{2R} \right) \, dt \right]_{f_1(x)}^{f_2(x)} $$ \hspace{1cm} (18)

$$= \frac{2}{\pi R^2} \int_0^x \frac{t^2}{4R^2} \left[ \sqrt{1 - \frac{t^2}{4R^2}} \right] \, dt.$$
Utilizing 2.8332 of [16], we can solve $f_1(x)$ as

$$f_1(x) = \frac{4}{\pi R^2} \left[ -\frac{2x^2 - 4R^2}{4} \sin^{-1}\left( \frac{x}{4R^2} \right) - \frac{x}{4} \sqrt{4R^2 - x^2} \right].$$

Recalling 2.2623 of [16], $f_2(x)$ can be solved as

$$f_2(x) = \frac{2}{\pi R^3} \left[ -xR^2 \sqrt{\left( 1 - \frac{x^2}{4R^2} \right)^3} + \frac{R^2 x}{2} \sqrt{\frac{1 - x^2}{4R^2}} \right].$$

(20)

To simplify (20), we exploit 2.261 of [16] and (20) can be derived as

$$f_2(x) = \frac{2}{\pi R^3} \left[ -x \sqrt{\left( 1 - \frac{x^2}{4R^2} \right)^3} + \frac{x}{2} \sqrt{1 - \frac{x^2}{4R^2}} \right].$$

(21)

To calculate $F_{d_{\text{ur}}}^\beta$, we have

$$F_{d_{\text{ur}}}^\beta(x) = P(d_{\text{ur}} \geq x) = P(d \geq x^\beta) = 1 - F_{d_{\text{ur}}}(x^\beta).$$

(22)

Recalling (14), it is clear that the harvested energy has an inverse relationship with the outage probability of the RF link. However, if the increase in harvested energy comes at the cost of increasing the DC bias (equivalent to a decrease in peak amplitude), it will reduce the VLC data rate and, consequently, increase the outage probability of the VLC link.

V. NUMERICAL RESULTS

In this section, we illustrate the performance of the hybrid VLC/RF scheme depicted in Fig. 1 through computer simulations. For the convenience of the reader, unless otherwise stated, the channel and system parameters are summarized in Table I. In the upcoming figures, the plot for the direct link (referred to as $P_{O}^{\text{direct}}$) is generated using (9). The simulation employs the precise value of harvested energy, as shown in (3). Additionally, the lower and upper bound plots are created using the respective upper and lower bounds on harvested energy specified in (4).

Fig. 2 presents the effect of DC bias on the outage probability of the considered system. As a benchmark, we incorporate the plot for the direct VLC link (i.e., $P_{O}^{\text{direct}}$); indicating that the end user can receive information solely from the VLC link. Here, we assume the end user and relay location follow a uniform distribution; i.e., $d \sim U[0, 7]$ m. Further, we assume the receiver orientation of both the end user and relay follow a uniform distribution; i.e., $\theta \sim U[10^\circ, 40^\circ]$. As observed, increasing the DC bias results in an increase in the outage probability. For the direct link, this is because increasing the DC bias leads to a decrease in peak amplitude (i.e., $A$), resulting in a reduction in the VLC information rate. However, increasing the DC bias leads to an increase in the harvested energy (see (3)), thereby increasing the RF transmit power. Along the lines of this argument, the difference between the simulation results and the lower bound (i.e., $P_{O}^{\text{direct}}$) becomes practically negligible as the DC bias increases. In addition, we also assume the fixed orientation case, where $\theta = 0^\circ$. In line with expectations, the outage probability for the fixed orientation condition is smaller than the random case due to the fact that $\cos(\cdot)$ is a decreasing function for $[0, \pi]$ (see (1)).

Fig. 3 illustrates the outage probability of the system under consideration for different data rate thresholds. Similar to the previous scenario, we assume the end user and relay locations follow a uniform distribution ($d \sim U[0, 7]$ m), and receiver orientations for both entities follow a uniform distribution ($\theta \sim U[10^\circ, 40^\circ]$). We further consider two distinct values of DC bias; i.e., $I_b \in \{0.55, 0.95\}$ A. Results show that considering high value of DC bias causes outage for thresholds greater than 20 Mbps.

In Fig. 4, we assume $I_b = 0.85$ A and the location for both the end user and relay follows the Rayleigh distribution with

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Numerical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>LED power ($P_{\text{LED}}$)</td>
<td>1.5 W/A</td>
</tr>
<tr>
<td>Noise figure ($N_F$)</td>
<td>9 dB</td>
</tr>
<tr>
<td>RF signal bandwidth ($B_{RF}$)</td>
<td>10 MHz</td>
</tr>
<tr>
<td>VLC signal bandwidth ($B_{\text{VLC}}$)</td>
<td>10 MHz</td>
</tr>
<tr>
<td>Thermal noise ($P_{\text{Th}}$)</td>
<td>-174 dBm/Hz</td>
</tr>
<tr>
<td>RF frequency ($f_c$)</td>
<td>2.4 GHz</td>
</tr>
<tr>
<td>Minimum DC bias ($I_{\text{min}}$)</td>
<td>100 mA</td>
</tr>
<tr>
<td>Maximum DC bias ($I_{\text{max}}$)</td>
<td>1 A</td>
</tr>
<tr>
<td>Photo-detector responsivity ($\eta$)</td>
<td>0.4 A/W</td>
</tr>
<tr>
<td>Thermal voltage ($V_t$)</td>
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</tr>
<tr>
<td>Dark saturation current ($I_0$)</td>
<td>$10^{-10}$ A</td>
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<tr>
<td>Half FoV ($\Phi$)</td>
<td>60°</td>
</tr>
<tr>
<td>Induced current ($I_i$)</td>
<td>$5840 \times 10^{-6}$ A</td>
</tr>
<tr>
<td>PD detection area ($A_p$)</td>
<td>$10^{-4}$ m²</td>
</tr>
<tr>
<td>AP relative height ($h_\Delta$)</td>
<td>2 m</td>
</tr>
<tr>
<td>Data rate threshold ($R_{th}$)</td>
<td>$10^7$ bps</td>
</tr>
</tbody>
</table>

### Table I

System and channel parameters that are used to generate the numerical results.

![Fig. 2. Impact of DC bias on outage probability in the considered system. We assume the end user and relay locations follow a uniform distribution ($d \sim U[0, 7]$ m), and receiver orientations for both entities follow a uniform distribution ($\theta \sim U[10^\circ, 40^\circ]$).](image-url)
scale parameter of $b$. For the receiver orientation, we assume normal distribution for both the end user and relay. Specifically, we consider $\theta \sim \mathcal{N}(10^\circ, 5^\circ)$ and $\theta \sim \mathcal{N}(30^\circ, 20^\circ)$ as the receiver orientation distribution for both the end user and relay. The considered distribution of the location and orientation are the same for the end user and relay. Increasing the Rayleigh scale parameter results in an increase in the outage probability. This is due to the fact that increasing $b$ causes the distribution to become more spread out: a higher value of $d$ becomes more probable, and the distribution’s tail extends further which results in a reduction in information data rate. Further, as expected, increasing the mean and variance value for the receiver orientation decreases the quality of VLC link and the plot shows a higher value for the outage probability.

VI. CONCLUSIONS

In this paper, we proposed an indoor downlink cooperative hybrid VLC/RF scenario, employing a relay node to mitigate the outage probability in the system. We assumed the end user can receive information through the VLC link or via the relay node, which utilizes harvested energy to re-transmit decoded information through the RF link. The reliability of VLC channels is intricately tied to the availability and alignment of LOS links as well as the amount of energy that can be harvested at the relay. Our study investigates the impact of random receiver orientation and location on the system’s outage probability, deriving an analytical expression that accounts for the randomness of both relay and end-user locations and receiver orientations while capturing the effect of energy harvesting. The numerical results show that the impact of DC bias on outage probability is apparent, as an increase is linked to a decrease in VLC information rate caused by a reduction in peak amplitude ($A$). A high DC bias results in a constant outage beyond a critical data rate threshold.

REFERENCES


