Endo-exo framework for a unifying classification of episodic landslide movements

Qinghua Lei\textsuperscript{1} and Didier Sornette\textsuperscript{2}

\textsuperscript{1}Uppsala University
\textsuperscript{2}SUSTech

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Abstract

We propose the “endo-exo” conceptual framework to account for the varied and complex episodic landslide movements observed during progressive maturation until collapse/stabilization. This framework captures the interplay between exogenous stressors such as rainfall and endogenous damage/healing processes. The underlying physical picture involves cascades of local triggered mass movements due to fracturing and sliding. We predict four distinct types of episodic landslide dynamics (exogenous/endogenous-subcritical/critical), characterized by power-law relaxations with different exponents, all related to a single parameter . These predictions are tested on the dataset of the Preonzo landslide, which exhibited multi-year episodic movements prior to a final collapse. All episodic activities can be accounted for within this classification with \( ?0.45\pm0.1 \), providing strong support for our parsimonious theory. We further show that the final catastrophic failure of this landslide is clearly preceded by an increased frequency of large velocities corresponding to a transition to a supercritical regime with amplifying positive feedbacks.

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Qinghua Lei¹, Didier Sornette²

¹Department of Earth Sciences, Uppsala University, Uppsala, Sweden
²Institute of Risk Analysis, Prediction and Management, Academy for Advanced Interdisciplinary Studies, Southern University of Science and Technology, Shenzhen, China

Correspondence: Qinghua Lei (qinghua.lei@geo.uu.se)

Key Points:
• A novel endo-exo framework is established to classify episodic landslide dynamics into four types and validated on a real landslide
• The theory is very parsimonious with a single adjusted parameter accounting for all four power-law regimes of episodic landslide movements
• The transition of the landslide from episodic to catastrophic movements is clearly preceded by an increased frequency of large velocities
Abstract

We propose the “endo-exo” conceptual framework to account for the varied and complex episodic landslide movements observed during progressive maturation until collapse/stabilization. This framework captures the interplay between exogenous stressors such as rainfall and endogenous damage/healing processes. The underlying physical picture involves cascades of local triggered mass movements due to fracturing and sliding. We predict four distinct types of episodic landslide dynamics (exogenous/endogenous-subcritical/critical), characterized by power-law relaxations with different exponents, all related to a single parameter $\theta$. These predictions are tested on the dataset of the Preonzo landslide, which exhibited multi-year episodic movements prior to a final collapse. All episodic activities can be accounted for within this classification with $\theta \approx 0.45 \pm 0.1$, providing strong support for our parsimonious theory. We further show that the final catastrophic failure of this landslide is clearly preceded by an increased frequency of large velocities corresponding to a transition to a supercritical regime with amplifying positive feedbacks.

Plain Language Summary

Landslides involve complex gravity-driven downslope movements developing over days to years before a possible major collapse, which are commonly boosted and/or driven by external events like precipitations and earthquakes. The reasons behind these episodic movements, characterized by alternating cycles of accelerating-decelerating creeps (marked by intermittent bursts of displacement followed by sustained periods of relaxation dynamics) and how they relate to the final instability remain poorly understood. Here, we propose a new “endo-exo” theory to classify episodic landslide movements into four fundamental types, based on the precursory/recovery properties of their associated intermittent velocity peaks. We provide a thorough demonstration of our theory based on the long-term monitoring dataset of a rainfall-induced landslide at Preonzo, Switzerland, which episodically moved over many years and eventually collapsed catastrophically in 2012. We observe all four types of episodic dynamics in the Preonzo landslide with their precursory/recovery properties consistent with our theoretical prediction. We further propose a new metric, obtainable from slope displacement monitoring data, as a precursor to catastrophic landslides. Our novel conceptual framework points at the existence of a deep quantitative relationship between episodic landslide movements, external triggering events (e.g., rainfall, snowmelt, and seismicity), and internal slip, damage, and healing processes within the landmass.
1. Introduction

Landslides, a widespread form of mass wasting, occur in various Earth surface environments and pose significant threats to life and property worldwide (Palmer, 2017). Due to rapid population growth and urbanization, human habitats are increasingly exposed to landslide hazards, with the situation becoming even more severe under climate change, where extreme rainfall, permafrost thaw, and glacier retreat have promoted fatal landslides (Garino & Guzzetti, 2016; Lacroix et al., 2022; Patton et al., 2021). Extensive field observations show that landslides commonly exhibit episodic movements characterized by intermittent acceleration-deceleration sequences that are boosted by external events like precipitations and earthquakes (Agliardi et al., 2020; Bontemps et al., 2020; Cappa et al., 2014; Crosta et al., 2014, 2017; Finnegan et al., 2022; Handwerger et al., 2013; Lacroix et al., 2014). Some landslides have episodically creeped over hundreds or thousands of years, while others could evolve into a major collapse after episodically deforming over days to years (Lacroix et al., 2020). The reasons behind these episodic movements (marked by intermittent bursts of displacement followed by sustained periods of relaxation dynamics) and how they relate to a possible final catastrophic failure remain poorly understood, inhibiting our capability to predict landslide behavior and mitigate the associated risks.

We identify the following fundamental questions: (a) Are episodic landslide movements of an exogenous or endogenous origin? (b) What are their underlying mechanisms? (c) How do they relate to catastrophic failures? In this Letter, we propose to answer these questions by establishing a novel “endo-exo” framework to quantitatively classify landslide episodic movements as well as decipher their exogenous/endogenous origins and triggering mechanisms. The rest of the Letter is organized as follows. Section 2 elaborates the mathematical foundation of this endo-exo framework. Section 3 presents a demonstration of how our theory applies, based on a real landslide dataset. Finally, a discussion on the above fundamental questions is given in Section 4.

2. Theory

We conceptualize a landslide as a complex system consisting of numerous geomaterial masses interacting via cohesive or frictional contacts. The displacement activity of the landslide results from a combination of external forces like precipitations and earthquakes, and of internal influences where each past moved mass may prompt other masses in its network of interactions to move as a result of the redistribution of mechanical stress, pore pressure, and possibly other physico-chemical fields. This impact of a mass on other masses is not instantaneous, due to the time-dependent nature of the relevant geomechanical processes like creep, damage, and friction (Scholz, 2019). This latency can be described by a memory kernel $\psi(t - \tau)$, giving the probability that the movement of a mass at time $\tau$ leads to the movement at a later time $t$ by another mass in direct interaction with the first moved mass. This memory kernel $\psi(t - \tau)$ can be seen as a fundamental macroscopic description of how long it takes for a mass to be triggered to move following the interaction with an already moved neighbouring mass. In other words, it is a “bare” propagator, describing the distribution of waiting times between “cause” and “action” for a mass to move, which may obey a power-law characterizing a long-memory process (Saichev & Sornette, 2010b; Sornette, 2006b; Sornette & Helmstetter, 2003):

$$\psi(t - \tau) \propto 1/(t - \tau)^{1+\theta}, \text{ with } 0 < \theta < 1 \text{ and for } t - \tau > c$$

(1)
where the exponent $\vartheta > 0$ controls the persistence of memory and $c$ is a small characteristic time scale regularizing the singularity at $t - \tau = 0$. For instance, one way to implement the regularization is to replace $1/(t - \tau)^{1+\vartheta}$ by $1/(t - \tau + c)^{1+\vartheta}$. Such a regularization is essential to make the integral of $\psi(t)$ finite and thus ensure a valid theory. Physically, this ensures the finiteness of the number of mass movements triggered by a preceding one. The assumption that $\psi(t)$ has a power-law tail is supported by many empirical observations such as Andrade’s law of material creep and Omori’s law of aftershock activity (see Helmstetter & Sornette, 2002; Nechad et al., 2005a, 2005b; Saichev & Sornette, 2005; Sornette & Sornette, 1999; and references therein).

Starting from an initial moved mass, i.e., the “mother” mass, which first displaces due to either external forces or internal fluctuations, it may trigger the movements of first-generation “daughter” masses nearby, which themselves trigger their own daughter masses to move, and so on. Such an epidemic process can be captured by a conditional self-excited point process (Hawkes & Oakes, 1974), which can be mapped exactly onto a branching process, such that the average of the displacement rate (i.e., velocity) of the mass system is governed by the following self-consistent equation (Sornette, 2006b; Sornette & Helmstetter, 2003):

$$ v(t) = V(t) + n \int_{-\infty}^{t} \psi(t - \tau) v(\tau) d\tau, \quad (2) $$

where $V(t)$ is the exogenous source that is not triggered by any epidemic effect in the system and $n \geq 0$ is the effective branching ratio defined as the average number of moving daughter masses triggered by a mother mass that moved in the past. Expression (2) is the equation for the first-order moment (or average) of the velocity, whose underlying dynamics is given by a self-excited point process. The branching ratio $n$ depends on the network topology of geomaterial masses and the spreading behavior of disturbances in the system, therefore reflecting the maturation of the landslide, with $n < 1$, $n \approx 1$, and $n > 1$ corresponding to the subcritical, critical, and supercritical regimes, respectively (Harris, 1963; Sornette, 2006a). Here, we mainly focus on the subcritical and critical regimes with $n \approx 1$ to ensure stationarity, whereas the transition into the supercritical regime $n > 1$ related to the emergence of a catastrophic failure (Helmstetter & Sornette, 2002; Sornette & Helmstetter, 2002) will be discussed in section 4.

Considering the exogenous source $V(t)$ given by a delta function $\delta(t)$ centered at the origin of time, we obtain the Green function of equation (2), also called a dressed or renormalized memory kernel $\Psi(t - \tau)$, which is the solution of (Helmstetter & Sornette, 2002; Sornette & Helmstetter, 2003)

$$ \Psi(t) = \delta(t) + n \int_{-\infty}^{t} \psi(t - \tau) \Psi(\tau) d\tau, \quad (3) $$

such that

$$ v(t) = \int_{-\infty}^{t} V(\tau) \Psi(t - \tau) d\tau, \quad (4) $$

which is the solution of equation (2). Equation (4) expresses the fact that the present velocity $v(t)$ results from all past exogenous sources $V(\tau)$ mediated to the present by the dressed memory kernel $\Psi(t - \tau)$ incorporating all the generations of cascades of influences (Sornette, 2006b). For the case where the bare propagator is given by equation (1), the recovery dynamics of the system after a strong external event $V(\tau) \propto \delta(\tau - t_c)$ is fully controlled by the dressed memory kernel (Sornette & Helmstetter, 2003), such that:
\[ v(t) = \Psi(t) \propto \begin{cases} 
1/(t - t_c)^{1-\vartheta}, & \text{for } c < t - t_c < t^*, \\
1/(t - t_c)^{1+\vartheta}, & \text{for } t - t_c > t^*, 
\end{cases} \]

where \( t_c \) is the critical time chosen as the time of the peak and \( t^* \) is the characteristic time given by (Helmstetter & Sornette, 2002):

\[ t^* = c \left( \frac{n\Gamma(1-\vartheta)}{|1-n|} \right)^{1/\vartheta} \propto |1-n|^{-1/\vartheta}. \]

Thus, it can be seen that, as \( n \to 1 \) (critical regime), \( t^* \to +\infty \), so that the short-term response prevails \( (t - t_c < t^*) \); as \( n \to 0 \) (pure noncritical regime), \( t^* \to c \) and the long-term response dominates \( (t - t_c > t^*) \); if \( 0 < n < 1 \) (subcritical regime), \( t^* \) has a finite value and the system may manifest a coexistence of both short- and long-term responses.

In the absence of strong external events, a peak in landslide velocity can also spontaneously occur due to the interplay of a continuous stochastic flow of small external perturbations and the amplifying impact of the epidemic cascade of endogenous interactions. The average velocity trajectory before and after the peak, conditioned on the existence of a peak, is given by \( \langle v(t) | v(t_c) \rangle \propto \text{Cov}(v(t), v(t_c)) \), so the precursory and recovery dynamics associated with the peak are governed by (Helmstetter et al., 2003; Sornette & Helmstetter, 2003):

\[ v(t) \propto \int_{-\infty}^{t-c} \Psi(t - t_c - \tau) \Psi(-\tau) d\tau \propto 1/|t - t_c|^{1-2\vartheta}, \text{ for } c < |t - t_c| < t^*, \]

or equivalently for \( n \to 1 \) (critical regime). If \( n < 1 \) (subcritical regime), the system response is essentially a noise process largely driven by random fluctuations (Crane & Sornette, 2008):

\[ v(t) \propto 1/|t - t_c|^0, \text{ for } |t - t_c| > t^*. \]

From the above derivations, we can see that landslide velocities around a peak at time \( t_c \) can be described by a generalized power-law as:

\[ v(t) \propto 1/|t - t_c|^p, \]

where the exponent \( p \) depends on the parameter \( \vartheta \) and the regime. This allows us to classify episodic landslide movements into four fundamental types based on a combination of the origin of disturbance (exogenous/endogenous) and the cascading behavior (subcritical/critical) (Crane & Sornette, 2008):

- **Type I: Exogenous-subcritical**, with \( p = 1 + \vartheta \) for \( t - t_c > t^* \). Here, the system is not “ripe” and the cascading propensity is limited (\( n < 1 \)), meaning that the exogenously induced displacement activity at time \( t_c \) does not cascade beyond the first few generations of triggered masses. The post-peak velocity relaxation is thus governed by the bare memory kernel.

- **Type II: Exogenous-critical**, with \( p = 1 - \vartheta \) for \( c < t - t_c < t^* \). Here, the system is ripe (\( n \approx 1 \)), such that the exogenously induced displacement activity at time \( t_c \) cascades through the system of interconnected masses, triggering neighbouring masses that further trigger their own neighbouring masses and so on. The post-peak velocity relaxation is governed by the dressed memory kernel.

- **Type III: Endogenous-subcritical**, with \( p = 0 \) for \( |t - t_c| > t^* \). The displacement activity does not result from an exogenous event but instead from an endogenous
forcing. The system is not ripe \((n < 1)\) such that no cascade develops and the (small) peak is associated with no apparent precursory/recovery signatures.

- **Type IV:** Endogenous-critical, with \(p = 1 - 2\theta\) for \(|t - t_c| < t^*\). The displacement activity originates from endogenous growth/interaction within the ripe system \((n \approx 1)\), where the triggering cascades produce an approximately symmetrical power-law acceleration-deceleration behaviour around the peak.

This classification arises from the interplay of the bare long-memory process as embodied in equation (1) and the epidemic cascade throughout the system as captured by equation (2). It can be seen that the relaxation following an endogenous-critical peak (with a smaller exponent \(p = 1 - 2\theta\)) is slower than that following an exogenous-critical peak (with a larger exponent \(p = 1 - \theta\)). This longer-lived influence of an endogenous-critical peak results from the precursory process that impregnates the system much more than its exogenous counterpart (Sornette et al., 2004).

3. **Dataset and Analysis**

**Figure 1.** (a) Overview of the Preonzo landslide, Switzerland with the locations of five extensometers E1-E5, the boundary of this instability complex, and the headscars of historical failure events indicated. (b) Monitoring data of slope displacements by the five extensometers and recorded data of rainfall intensity by a pluviometer installed at the slope.

We test our theory based on the long-term monitoring dataset of a rainfall-induced landslide at Preonzo, Switzerland, which exhibited significant episodic movements over many years prior to a catastrophic failure in 2012 (Lei et al., 2023; Lei & Sornette, 2023; Loew et al., 2017). This active landslide has experienced multiple failures since the 18th century (Gschwind et al., 2019) (see the head scarp of historical events in Figure 1a). To closely monitor this instability complex that posed a great threat to the industrial and transport infrastructures located directly at the toe of the slope, five high-precision extensometers E1-E5 (see Figure 1a for their locations) were instrumented to measure the opening of tension cracks in the headscarp area. From 2008, a pluviometer was installed to monitor the local precipitation conditions. Figure 1b shows the time series of slope displacement measured by the five extensometers and of rainfall intensity recorded by the pluviometer between 2008 and 2012 (see the inset for the displacement time series from 2002 and Figure S1 in the Supporting Information for the time series of...
daily/cumulative rainfall amounts). One can see that this landslide exhibited a step-like deformation pattern over time as it progressively destabilized, leading up to a catastrophic failure on 15 May 2012. The displacement curve consists of numerous creep episodes (i.e., repeated cycles of accelerating-decelerating creeps) that often show a good coincidence with the occurrence of intense rainfall events.

Figure 2. Four categories of episodic landslide dynamics found in the velocity time series of the Preonzo landslide: (a) Type I, exogenous-subcritical; (b) Type II, exogenous-critical; (c) Type III, endogenous-subcritical; and (d) Type IV, endogenous-critical. The red arrow in (a) marks the timing of the local failure of a downslope northern sector of the slope on 9 May 2010. Insets show the post-peak relaxation of normalized velocity where dashed lines indicate the power-law fitting.

We compute slope velocities on a daily basis from the displacement time series recorded by the five extensometers. All the four types of episodic landslide dynamics, viz., exogenous/endogenous-subcritical/critical, can be found in the velocity time series (see Figures 2 and 3 for typical examples). We fit the data of normalized velocities to a power-law (see Text S1 and S2 in the Supporting Information for the normalized velocity calculation and fitting algorithm).
For the Type I peak on 7 May 2010 (Figure 2a), the velocity relaxation beyond ~8 days after the peak is characterized by an exponent of $p = 1.40 \pm 0.07$ (exogenous-subcritical) (Figure 2a, inset), whereas its short-term response within ~8 days after the peak is associated with a much smaller exponent of $p = 0.47 \pm 0.11$ (exogenous-critical), as expected from the prediction by equation (5). All five extensometers exhibit a similar two-branch power-law relaxation behavior with an exponent of $p = 0.46 \pm 0.10$ for the short-term response and an exponent of $p = 1.54 \pm 0.06$ for the long-term response (Figure 3a; see also Figure S2 in the Supporting Information for the power-law fitting for individual extensometers). Around this peak accompanied by mild precipitation (Figure 3a, left), the slope has experienced a localized failure in its northern sector downhill from the tension cracks where the extensometers are installed (see Figure 1a for the head scarp and section 4 for a discussion of the possible triggering mechanisms).

For the Type II peak on 9 August 2011 (Figure 2b), the post-peak velocity relaxation obeys a power-law with an exponent of $p = 0.55 \pm 0.02$ (exogenous-critical) (Figure 2b, inset). Prior to this peak, a heavy rainstorm has occurred (Figure 3b, left). All the five extensometers have captured this peak followed by a power-law relaxation with an overall exponent of $p = 0.63 \pm 0.03$ (Figure 3b; see also Figure S3 in the Supporting Information for the power-law fitting for individual extensometers).

In Figure 2c, we present a Type III endogenous-subcritical peak preceded by no rainfall event (Figure 3c). This peak is surrounded by an essentially time-independent velocity trajectory with $p \approx 0$ (Figure 3c), whereas most extensometers do not capture this peak and only show random fluctuations (Figure 3c and Figure S4 in the Supporting Information).

Lastly, we show a Type IV endogenous-critical peak (Figure 2d), which occurs after a progressively accelerating power-law growth of velocity followed by an approximately symmetrical power-law relaxation, with a common exponent of $p = 0.24 \pm 0.06$. It seems that the majority of the five extensometers has captured such an approximately symmetrical precursory-recovery dynamics with a small power-law exponent of $p = 0.21 \pm 0.04$ (Figure 3d), although the timing of the peaks recorded by individual extensometers is not fully synchronized (Figure S5 in the Supporting Information). One can notice that the time-dependent signatures of endogenous peaks are less apparent compared to exogenous ones (as reflected by the notable dispersion of the data in Figures 2d and 3d).

Interpreting these results in light of equations (5), (7) and (8), the obtained power-laws for these different peak types point to the existence of a single parameter $\vartheta \approx 0.45 \pm 0.10$, providing strong support for our theory.
Figure 3. Slope velocity time series measured by the five extensometers E1-E5 as well as rainfall intensity data recorded by the pluviometer (left panel) and post-peak velocity relaxation (right panel) for different types of peaks: (a) Type I, exogenous-subcritical; (b) Type II, exogenous-critical; (c) Type III, endogenous-subcritical; and (d) Type IV, endogenous-critical. The red arrow in (a) marks the timing of the local failure of a northern sector of the slope on 9 May 2010. In (c) and (d) right, pre-peak velocity data are also indicated (open markers) in addition to post-peak data (filled markers).

Figure 4. (a) Histogram of power-law exponents $p$ for post-peak velocity relaxation. The double arrows indicate the value ranges of $p = 1 - \vartheta$ (Type I peaks) and $p = 1 + \vartheta$ (Type II peaks), with $\vartheta \approx 0.45 \pm 0.1$. (b) Ensemble averaged velocity relaxation behavior for Type I and II peaks; error bars indicate the standard deviation associated with the ensemble average.

We implement a peak detection algorithm to automatically extract slope velocity peaks together with their surrounding time series from the 10-year long-term monitoring dataset. We qualify a peak in the velocity time series as a local maximum over a 20-day time window which is at least $k = 2.5$ times larger than the average velocity over a 2-month time window. The time window sizes and threshold value $k$ are chosen to give an effective detection of good-quality peaks (see Figure S2 in the Supporting Information), but the results do not significantly change by varying these parameters (see Figures S7-S10 and S13-S14 in the Supporting Information). In addition, we request that each peak has at least 10 days of post-peak data before reaching the next peak. In total, our algorithm detects 104 peaks from the entire dataset recorded by five extensometers. We then fit the post-peak velocity data of each detected peak to a power-law (see Text S2 in the Supporting Information) over a time window ranging from 10 to 30 days, with the “best” window chosen as the one giving the highest coefficient of determination $R^2$. We only keep the peaks with $R^2 > 0.8$ to extract unambiguous post-peak response functions, leaving 41 peaks. In Figure 4a, we show the histogram of their power-law exponents $p$, which cluster into two distinct groups, one with a median at $p \approx 0.59$ and the other with a median at $p \approx 1.52$. This result is compatible with our theoretical prediction based on $\vartheta \approx 0.45 \pm 0.10$, yielding $p \approx 1.45 \pm 0.10$ for Type I peaks and $p \approx 0.55 \pm 0.10$ for Type II peaks. It seems that Type III and IV peaks (with $p \approx 0$ or $0.1 \pm 0.20$) are absent in Figure 4a. This is because they usually
have small magnitudes and considerably fluctuating post-peak responses (Figure 2c-d and Figure 3c-d), making it difficult for them to pass the criteria of $k = 2.5$ and $R^2 > 0.8$. We then compute the ensemble average of the relaxation behavior for the two exogenous peak types (Figure 4b), with the fitted power-laws consistent with the existence of a single parameter $\vartheta \approx 0.45 \pm 0.10$. Our results in Figure 4 do not qualitatively change by varying the $k$ threshold from 1.5 to 3.5 and the $R^2$ threshold from 0.7 to 0.9 as well as the window sizes for peak detection (Figures S12-S14 in the Supporting Information), suggesting that our method and results are robust.

4. Discussion

We have presented a novel endo-exo theoretical framework to quantitatively classify episodic landslide movements into four fundamental types of distinct precursory/recovery signatures but related by a single common parameter $\vartheta$. All the four types of landslide dynamics have been observed in the Preonzo landslide with $\vartheta \approx 0.45 \pm 0.10$, which is different from the mean-field solution $\vartheta \approx 0$ for creep ruptures in heterogeneous materials (Nechad et al., 2005a, 2005b; Saichev & Sornette, 2005). Such a non-mean-field response reflects the intrinsic fluctuations and correlations resulting from triggered cascades of geomaterial mass motions in the landslide. This $\vartheta$ value close to 0.5 may be explained by the first-passage problem of an underlying random walk (Redner, 2001; Saichev & Sornette, 2010a), where a daughter mass surrounding a mobilized mother mass is only triggered to move when the fluctuating stress first reaches the strength level for sliding or fracturing.

Our results reveal that many rainfall-induced velocity peaks of the Preonzo landslide belong to the exogenous-critical type, meaning that the landslide dynamics in response to external perturbations is dominated by cascades involving high-order generations of mass movement triggering and the collective response of the entire mass population is slower and more persistent (governed by the dressed memory kernel with an exponent of $1 - \vartheta$) than the individual mass response (governed by the bare memory kernel with an exponent of $1 + \vartheta$). This implies that this landslide is operating around a critical state with the branching ratio $n$ intermittently increasing and receding close to 1, likely due to the competing damage and healing processes. This physical picture refines the concept of self-organized criticality stating that many crustal phenomena like earthquakes and landslides are evolving in a statistically stationary state of marginal stability (Bak & Tang, 1989; Hergarten & Neugebauer, 1998; Main, 1996; Sornette & Sornette, 1989; Sornette, 2006a; Turcotte, 1999). Such a paradigm explains why some rainfall events could trigger episodic landslide movements while others do not (Figure 3), which is simply due to the dynamically evolving nature of the system that is relaxed away (but not far) from the critical state after each peak and then attracted back to the critical state over time mediated by a continuous flow of external perturbations (e.g., rainfall, snowmelt, and diurnal temperature/humidity cycles). In addition, we have documented a unique exogenous-subcritical type of episodic landslide dynamics, which is related to the local failure of a downslope sector of the slope on 9 May 2010 (Loew et al., 2017). Before showing a rapid exogenous-subcritical relaxation characterized by a large exponent of $1 + \vartheta$, the landslide has actually experienced ~8 days of relatively slower exogenous-critical relaxation with a small exponent of $1 - \vartheta$ (see Figures 2a and 3a). Substituting this characteristic time $t^* \approx 8$ days together with $\vartheta \approx 0.45$ into equation (6) and the estimate $c \approx 1$ day, we obtain $n \approx 0.63$. This comparatively low $n$ value is consistent with the fact that this local failure-induced shock did not lead to a system-sized collapse since only a few generations of failure cascades have developed. Contrarily, the high $n$
value for rainfall-induced exogenous-critical shocks may be due to the fact that the disturbance by rainwater infiltration is likely to affect the entire slope and thus has a stronger spreading behavior. In our dataset, we also observe the presence of endogenous-critical landslide dynamics, indicating that cascading mass movements play a dominant role in triggering landslides through a kind of self-organized criticality. However, they are usually associated with small-magnitude peaks and weak time-dependence (governed by a relaxation exponent of $1 - 2\theta$ close to 0), making them sometimes difficult to be discriminated from the endogenous-subcritical dynamics driven by random fluctuations.

Up to now, we have mainly focused on the “endo-exo” regime where the landslide evolution is characterized by numerous accelerating-decelerating creep episodes driven by the interplay of exogenous perturbation and endogenous maturation. As the mass of the landslide progressively weakens, it could transition into the supercritical regime with $n > 1$ (Harris, 1963; Sornette, 2006a), where the number of triggering events in the system grows on average exponentially with time (Helmstetter & Sornette, 2002) or even faster (Sornette & Helmstetter, 2002). This critical transition is found to be often endogenously driven in different natural and social systems (Sornette, 2006b), which explains why many rainfall-induced landslides catastrophically fail in the absence of exceptional precipitation events (Eberhardt, 2008). If the supercritical regime is dominated by positive feedbacks with the slope acceleration behavior $\dot{v}(t) \propto v(t)^m$ characterized by $m > 1$, the system would exhibit a finite-time singularity and thus a catastrophic failure (Lei et al., 2023; Lei & Sornette, 2023; Sornette, 2002).

We fit the velocity time series of the Preonzo landslide prior to its catastrophic failure on 15 May 2012 to a power-law in the form of equation (9) with $p = 1/(m - 1)$, yielding a two-branch behavior with $p \approx 1.88$ ($m \approx 1.53$) for the early stage and $p \approx 0.49$ ($m \approx 3.04$) for the late stage (Figure S15 in the Supporting Information). This suggests that the system is indeed dominated by positive feedbacks which seem to become even stronger close to the final collapse. Our previous work showed that those late stage large velocities are “dragon-kings” (Lei et al., 2023) — a double metaphor for an event of a predominant impact/size like a “king” and a unique origin like a “dragon” (Sornette & Ouillon, 2012). This break in power-law scaling thus marks the transition of the system from the self-organised criticality regime where a catastrophic failure is unpredictable (the so-called “black-swan” regime) (Taleb, 2010) to the dragon-king regime where a catastrophic failure is predictable (Sornette & Ouillon, 2012). Interestingly, when entering the dragon-king regime, the system once experienced a temporary deceleration during 7-11 May 2012 just before the final collapse and such a precursory quiescence is consistent with the theoretical prediction for the supercritical regime with $n > 1$ and $\theta > 0$ (Helmstetter & Sornette, 2002). Substituting $t^* \approx 4$ days and $\theta \approx 0.45$ into equation (6) which also holds for the supercritical regime (Helmstetter & Sornette, 2002), we obtain $n \approx 7.46$, indicating an intense explosive branching process. Considering the analogue between landslides and earthquakes (Finnegan et al., 2022; Gomberg et al., 1995; Handwerger et al., 2016; Helmstetter et al., 2004; Lacroix et al., 2014), we postulate that the condition for this subcritical/critical-to-supercritical transition to occur is that the system shifts from $\alpha < \mu$ to $\alpha \geq \mu$ (Sornette & Helmstetter, 2002), where $\alpha$ is the exponent in the productivity law $p(E) \propto E^\alpha$ defining the number of daughter masses triggered by a mother mass of energy release $E$ and $\mu$ is the exponent in the Gutenberg-Richter-type density distribution of daily energy release of the landslide $f(E(t)) \propto E(t)^{-(1+\mu)}$. Given $E(t) \propto v(t)^2$, we derive $f(v(t)) \propto v(t)^{-(1+2\mu)}$ from the law of conservation of probability under a change of variable (Sornette, 2006a). Our previous work suggests that the
probability distribution of the \( v(t) \)’s of the Preonzo landslide follow an inverse gamma distribution (with \( \beta \) denoting its shape parameter) characterized by a power-law tail \( f(v(t)) \propto v(t)^{-(1+\beta)} \) (Lei & Sornette, 2023), with therefore \( \beta = 2\mu \). It is found that \( \beta \) progressively drops from 1.92 to 1.76 (correspondingly, \( \mu \) drops from 0.96 to 0.88) over ~1 month time (Figure S16 in the Supporting Information), suggesting an increased frequency of large velocities as the slope approaches the critical transition from the endo-exo regime (dominated by small velocities) to the dragon-king regime (dominated by large velocities) occurring at ~1 week before the final collapse (Lei et al., 2023). Thus, we would expect \( \alpha \approx 0.88 \), which is comparable to the typical value of \( \alpha \approx 0.8 \) for earthquakes (Helmstetter, 2003). This correspondence holds notwithstanding the fact that landslides happen in near-surface environments under low stress conditions, while earthquakes occur in deep subsurface regions subject to much higher stress levels. The decrease of \( \beta \) prior to catastrophic landslides is similar to the observed \( b \)-value decline prior to great earthquakes (Imoto, 1991; Nakaya, 2006; Smith, 1981), which is possibly due to increased differential stresses on rock bridges/asperities accommodating crack propagations (Scholz, 2015) and/or enhanced differential stresses on creeping fault patches promoting slip ruptures (Ito & Kaneko, 2023). It has also a natural interpretation within the physical picture of cascades of triggered events as described by self-excited conditional point processes (Helmstetter & Sornette, 2003). This observation points to the possibility to predict catastrophic landslides by monitoring the temporal evolution of the \( \beta \)-value.

Our novel conceptual framework points at the existence of a deep quantitative relationship between episodic landslide movements, external triggering events (e.g., rainfall, snowmelt, and seismicity), and internal frictional slip, damage, and healing processes within the landmass. Based on the well-documented dataset of the Preonzo landslide, we have provided a thorough validation of this framework, which can be further applied to many other landslides showing similar episodic movements (Agliardi et al., 2020; Bontemps et al., 2020; Cappa et al., 2014; Crosta et al., 2014, 2017; Finnegan et al., 2022; Handwerger et al., 2013; Lacroix et al., 2014). The results and insights obtained in this Letter have important implications for landslide hazard prediction and mitigation.

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Open Research

The slope displacement monitoring data of the Preonzo landslide are publicly available at the ETH Zurich Research Collection (https://doi.org/10.3929/ethz-b-000600495).

References


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Supporting Information for

Endo-exo framework for a unifying classification of episodic landslide movements

Qinghua Lei¹, Didier Sornette²

¹Department of Earth Sciences, Uppsala University, Uppsala, Sweden
²Institute of Risk Analysis, Prediction and Management, Academy for Advanced Interdisciplinary Studies, Southern University of Science and Technology, Shenzhen, China

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Introduction

This document provides supporting information to complement the theory, analysis, results, and discussions in the main Letter. Text S1 describes the method for calculating normalized slope velocities. Text S2 elaborates the method of least squares for power-law calibrations. Figure S1 gives the time series of slope displacements and daily/cumulative rainfall amounts. Figures S2-S5 show the power-law calibration to the velocity data of individual extensometers around different types of peaks. Figures S6-S10 show the time series of daily slope velocities recorded by five extensometers with the peaks and troughs identified based on different detection criteria. Figure S11 shows the histogram of determined slope residual velocities. Figures S12-S14 show the histogram of power law exponents for post-peak velocity relaxation and ensemble averaged relaxation of exogenous-subcritical and exogenous-critical peaks, with peaks selected based on different criteria of time window size, relative magnitude, and coefficient of determination of the fitting. Figure S15 shows the time series of slope velocity and rainfall intensity when the slope approaches a catastrophic failure as well as the variation of normalized velocity as a function of time to the failure. Figure S16 shows the temporal variation of the shape parameter of the inverse gamma distribution of daily velocities of the Preonzo landslide as it transitions from the subcritical/critical regime to the supercritical regime.
Text S1. Calculation of normalized velocities around a peak.

We compute normalized slope velocities \( \tilde{v}(t) \) around a peak based on the following equation:

\[
\tilde{v}(t) = \frac{(v(t) - v_0)}{(v(t_c) - v_0)}
\]  

(S1)

where the slope velocity \( v(t) \) reaches a peak value of \( v(t_c) \) at time \( t = t_c \) and \( v_0 \) is the residual velocity when the landslide system has fully recovered from external perturbations. However, the determination of this residual velocity for a rainfall-induced landslide (like the Preonzo landslide) is subject to significant uncertainties, because the landslide has very rare opportunities to completely recover from one rainfall event before the next one occurs. In this work, we estimate the residual velocity by first detecting troughs in the velocity time series (see Text S2). We qualify a trough in the velocity time series as a local minimum over a 20-day time window which is at least \( k = 2.5 \) times smaller than the 2-month average velocity. The time window sizes and the threshold value \( k \) are chosen to give an effective and reasonable detection of peaks and troughs from the data (see Figure S2), but the results do not significantly change by varying these parameters (see Figure S7-S10 and Figure S13). We then define the residual velocity associated with a given peak as the minimum of the two nearest troughs (with one before the peak and one after the peak). Note that this residual velocity tends to vary over time reflecting the nonstationary characteristic of the landslide. Figure S11 shows the probability density function of calculated residual velocities (associated with the identified peaks in Figure 2), which have a mean of 0.008 mm/day. We have also tested other possible approaches of determining the residual velocity, e.g., based on the average of the 10 nearest troughs around a peak or based on the minimum/average of the troughs located between the former peak and the latter peak. No significant changes in the results are found.

Text S2. Power-law calibration of velocity time series around a peak.

We fit the time series of normalized velocities \( \tilde{v}(t) \) around a peak to a power law:

\[
\tilde{v}(t) = A/|t - t_c|^p
\]  

(S2)

where \( t_c \) is the critical time chosen as the time of the peak, \( A \) is a constant, and \( p \) is the power law exponent. To estimate \( A \) and \( p \), we use the method of least squares to minimize the following quantity:

\[
s = \sum t_i r(t_i)^2,
\]  

(S3)

with

\[
r(t_i) = \log \tilde{v}(t_i) - \log A + p\log|t_i - t_c|.
\]  

(S4)

We then set the partial derivatives \( \partial s / \partial (\log A) \) and \( \partial s / \partial p \) to be both zero, leading to solve a linear system of two equations with the two unknowns \( A \) and \( p \).
Figure S1. Monitoring data of slope displacements by the five extensometers presented together with the data of (a) daily rainfall and (b) cumulative rainfall recorded by a pluviometer installed at the Preonzo slope.
Figure S2. Post-peak relaxation of Type I exogenous-subcritical peaks based on the monitoring data of the five extensometers E1-E5.
Figure S3. Post-peak relaxation of Type II exogenous-critical peaks based on the monitoring data of the five extensometers E1-E5.
Figure S4. Pre-peak (open symbols) acceleration and post-peak (colored symbols) relaxation of Type III exogenous-subcritical peaks based on the monitoring data of the five extensometers E1-E5.
Figure S5. Pre-peak (open symbols) acceleration and post-peak (colored symbols) relaxation of Type IV exogenous-critical peaks based on the monitoring data of the five extensometers E1-E5.
Figure S6. Time series of daily slope velocities recorded by the five extensometers E1-E5 (from top to bottom) instrumented at the Preonzo landslide, Switzerland. Peaks and troughs are marked by circles and squares, respectively. Each peak (respectively trough) is qualified as a local maximum (respectively minimum) over a 20-day time window which is at least $k = 2.5$ times larger (respectively smaller) than the average velocity over a 2-month time window.
Figure S7. Time series of daily slope velocities recorded by the five extensometers E1-E5 (from top to bottom) instrumented at the Preonzo landslide, Switzerland. Peaks and troughs are marked by circles and squares, respectively. Each peak (respectively trough) is qualified as a local maximum (respectively minimum) over a 20-day time window which is at least $k = 1.5$ times larger (respectively smaller) than the average velocity over a 2-month time window.
Figure S8. Time series of daily slope velocities recorded by the five extensometers E1-E5 (from top to bottom) instrumented at the Preonzo landslide, Switzerland. Peaks and troughs are marked by circles and squares, respectively. Each peak (resp, trough) is qualified as a local maximum (respectively minimum) over a 20-day time window which is at least $k = 3.5$ times larger (respectively smaller) than the average velocity over a 2-month time window.
Figure S9. Time series of daily slope velocities recorded by the five extensometers E1-E5 (from top to bottom) instrumented at the Preonzo landslide, Switzerland. Peaks and troughs are marked by circles and squares, respectively. Each peak (respectively trough) is qualified as a local maximum (respectively minimum) over a 40-day time window which is at least $k = 2.5$ times larger (respectively smaller) than the average velocity over a 4-month time window.
Figure S10. Time series of daily slope velocities recorded by the five extensometers E1-E5 (from top to bottom) instrumented at the Preonzo landslide, Switzerland. Peaks and troughs are marked by circles and squares, respectively. Each peak (respectively trough) is qualified as a local maximum (respectively minimum) over a 10-day time window which is at least $k = 2.5$ times larger (respectively smaller) than the average velocity over a 1-month time window.
**Figure S11.** Histogram of slope residual velocities plotted in (a) linear scale and (b) logarithmic scale.
Figure S12. Left: histogram of the power law exponents $p$ for post-peak velocity relaxation. Right: ensemble averaged relaxation of Type I (exogenous-subcritical) and Type II (exogenous-critical) peaks. Here, a peak is qualified as a local maximum over a 20-day time window which is at least $k = 2.5$ times larger than the average velocity over a 2-month time window, while the coefficient of determination for the fitting should meet (a) $R^2 > 0.9$ or (b) $R^2 > 0.7$. 
Figure S13. Left: histogram of the power law exponents $p$ for post-peak velocity relaxation. Right: ensemble averaged relaxation of Type I (exogenous-subcritical) and Type II (exogenous-critical) peaks. Here, a peak is qualified as a local maximum over a 20-day time window which is at least (a) $k = 1.5$ or (b) $k = 3.5$ times larger than the average velocity over a 2-month time window, while the coefficient of determination for the fitting should meet $R^2 > 0.8$. 
Figure S14. Left: histogram of the power law exponents $\rho$ for post-peak velocity relaxation. Right: ensemble averaged relaxation of Type I (exogenous-subcritical) and Type II (exogenous-critical) peaks. Here, a peak is qualified as a local maximum over a (a) 40-day or (b) 10-day time window which is at least $k = 2.5$ times larger than the average velocity over a (a) 4-month or (b) 1-month time window, while the coefficient of determination for the fitting should meet $R^2 > 0.8$. 
Figure S15. Left: Time series of the slope velocity measured by the five extensometers E1-E5 as well as rainfall intensity data recorded by the pluviometer for the period when the slope approaches a catastrophic failure on 15 May 2012. Right: variation of normalized velocity prior to the catastrophic failure as a function of time to the failure, which is fitted to a two-branch power law (indicated by the dashed line).
**Figure S16.** Temporal variation of the shape parameter $\beta$ (determined based on the maximum likelihood estimation) of the inverse gamma distribution of daily velocities of the Preonzo landslide which transitions from an endo-exo (subcritical/critical) regime to a dragon-king (supercritical) regime.