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Abstract

The four-port couplers are the fundamental building blocks of large-scale passive beam-forming network design. They find application in the design of antenna feed networks, power combiners and dividers, balanced mixers and amplifiers. Conventional branch-line couplers (BLCs) were designed for quadrature-phase imbalance using four quarter-wavelength transmission lines (TLs). On the other hand, conventional rat-race couplers (RRCs) are designed for in-phased/out-of-phased imbalance using six identical quarter-wavelength TLs. To obtain a non-quadrature phase-shift, external phase shifters are required. Recently, BLCs with inherent non-quadrature phase imbalance properties have received attention among microwave researchers. There are several types of implementation such as lumped, distributed, and mixed with different topologies, that have been proposed by several researchers. It is quite challenging for young researchers to extract all possible design data from the design equations and simulate them to understand their figure of merits. This motivates the author to provide solutions for non-quadrature unequal power division BLCs.
I. INTRODUCTION

The four-port couplers are the fundamental building blocks of large-scale passive beam-forming network design. They find application in the design of antenna feed networks, power combiners and dividers, balanced mixers and amplifiers. Conventional branch-line couplers (BLCs) were designed for quadrature-phase imbalance using four quarter-wavelength transmission lines (TLs). On the other hand, conventional rat-race couplers (RRCs) are designed for in-phased/out-of-phased imbalance using six identical quarter-wavelength TLs. To obtain a non-quadrature phase-shift, external phase shifters are required. Recently, BLCs with inherent non-quadrature phase imbalance properties have received attention among microwave researchers. There are several types of implementation such as lumped, distributed, and mixed with different topologies, that have been proposed by several researchers. It is quite challenging for young researchers to extract all possible design data from the design equations and simulate them to understand their figure of merits. This motivates the author to provide solutions for non-quadrature unequal power division BLCs.

Conventionally, the BLCs were designed for a quadrature phase difference [1]. Recently, Wong et al. proposed the idea of the non-quadrature BLC [2]. The design equations of the non-quadrature BLC were further improved by M. J. Park [3], and Wu et al. [4] for better performance. The design proposed in [4] comprises two 90° TLs, which leads to potentially unrealisable characteristic impedances of TLs. Sinha and De [5] further generalised the design equations of [4], which provides more design flexibility. Previous design [2], [3] and [4] can easily be explained using [5]. A multi-section version of the non-quadrature BLC is reported in [6], [7], [8]. In recent years non-quadrature type BLCs have found application in beam forming matrix design [9], [10], [11]. In the design of decoupling networks [12], non-quadrature BLCs are also used.

An alternative to an uncoupled line ring BLC configuration, a coupled line-based non-quadrature BLC with coupling between two 90° TLs has been proposed by Y. Wu et al. [13], and other TLs of [4] are replaced by C-section lines. This kind of design is proposed for quadrature coupler in [14]-[15]. The design of [13] is further generalised by Sinha and De in [5]. An optimal design of a quadrature coupler with coupled line topology is proposed in [16]-[17]. A modified tunable version of coupled line non-quadrature coupler was proposed by Kalantari et al. in [18].

In recent years, tunable and reconfigurable couplers [6], [18], [19], [20], [21], [22], [23], [24], [25], [26], [27], [28], [29] are investigated by several researchers. The tunability of these couplers is obtained using tunable TLs. These tunable TLs consist of variable reactors and distributed TLs. Conventional quadrature-BLCs are also used in conjunctions with the variable TLs to obtain tunability.

In [30], Wu et al. provide a solution for non-quadrature BLC with unequal resistive ports. The work done by Sinha in [31] develops computer-aided design algorithms for multi-port networks with unequal complex port impedances. Using algorithms, one can easily design lumped-Π based non-quadrature BLCs with unequal complex port impedances. Chen et al. [32] proposed a lumped-Π based non-quadrature BLC with unequal resistive port. The designs of [32] can easily be obtained using [31].

For miniaturization and harmonic suppression or dual-band operation of BLCs, equivalent TL like symmetric T-structure and Π-structure [33]-[34] were used. The problem with the symmetric-T is that if the stubs are placed inside the coupler, they may intersect. To solve this, asymmetric-T structures were proposed in [35]-[37]. The design equations had some theoretical limitations as they tried to equate an electrically symmetric and asymmetric network. Ahn pointed out these limitations [38].

Sinha et al. [39]-[40] proposed a technique to replace the 90° TLs of the 3 dB rat-race coupler (RRC) with 90° phase shift using the mirror symmetry property of the RRC. Conventional even-odd mode decomposition [41] based design technique does not work for asymmetric four port couplers (RRC and BLC). Sinha and De [5] proposed a port-decomposition method to design multi-port networks irrespective of the eclectically or physical symmetry of the network. The port decomposition technique reduces an extensive network to a smaller one by terminating some ports with short circuit loads and analyzing the S-matrix of the smaller network. The port decomposition technique has similarities with the Y-matrix technique [31], [42], [43], [44]. However, the port decomposition technique is more robust than the Y-matrix method, as the Y-matrix of a network does not always exist. It is convenient to use the Y-matrix technique if the Y-matrix of the network exists. An alternative to the decomposition technique, one may use the ABCD-matrix technique proposed recently by Levine and Matzner [45]-[46].

Sinha and De provided design equations of the non-quadrature phase difference BLC with unequal power division...
II. DESIGN EQUATIONS OF BLC

A. Arbitrary Phase Unequal Power Division BLC using TL

The BLC divides the input signal into two output signals with a phase difference of \( \varphi \) or \( 180^\circ - \varphi \) with \( 0^\circ < \varphi < 180^\circ \) depending upon the input port. The amplitudes of the output signals may be equal or unequal depending upon the power division ratio. Following the definition of the BLC, the scattering parameters of generalized BLC can be written as [5]

\[
[S_{BLC}] = \frac{1}{\sqrt{n+1}} \begin{bmatrix}
0 & e^{-j\varphi} & -\sqrt{n} & 0 \\
e^{-j\varphi} & 0 & 0 & -\sqrt{n} \\
-\sqrt{n} & 0 & 0 & -e^{j\varphi} \\
0 & -\sqrt{n} & -e^{j\varphi} & 0
\end{bmatrix}.
\] (1)

The desired S-matrix of the BLC can be realised using a four TL-based ring structure, as shown in Fig. 1a. If the phase shift \( \varphi \), power division ratio \( n \) and port impedance \( Z_0 \) are given, one can use \( \theta_\beta \) as a control variable and determine other design parameters using the following equations [5]

\[
\cos \theta_\alpha = \frac{1}{\sqrt{n+1}} (\cos \varphi - \sqrt{n} \cos \theta_\beta) \quad (2a)
\]

\[
\cos \theta_\gamma = -\frac{1}{\sqrt{n+1}} (\cos \varphi + \sqrt{n} \cos \theta_\beta) \quad (2b)
\]

\[
Z_\beta = \frac{1}{\sqrt{n}} Z_0 \sin \varphi \sin \theta_\beta \quad (2c)
\]

\[
Z_\alpha = \frac{1}{\sqrt{n+1}} Z_0 \sin \varphi \sin \theta_\alpha \quad (2d)
\]

\[
Z_\gamma = \frac{1}{\sqrt{n+1}} Z_0 \sin \varphi \sin \theta_\gamma \quad (2e)
\]

For example, a BLC designed for \( n = 2 \) and \( \varphi = 75^\circ \) using \( \theta_\beta = 75^\circ \) as input, the other design parameters are determined using (2), and the design schematic is shown in Fig. 2a. On the other hand, if we chose \( Z_\beta = 50 \Omega \) as input, the design schematic is obtained as in Fig. 2b.

The symmetric-TLs can be replaced by equivalent symmetric two-port networks (TPN) for size reduction and harmonic suppression or to achieve dual-band operation [33]-[34]. Recently efforts have been directed to replace the symmetric-TLs with asymmetric-T [35]-[37] to obtain even better figure-of-merits than their symmetric counterparts. The initial works [35]-[37] equated network parameters of symmetric TL with asymmetric T-structure, which has some theoretical limitations. It has been shown in [5] that a symmetric RRC or BLC can be designed using asymmetric structures, and the design equations were obtained using the port-decomposition technique.

B. Asymmetric-Structure based BLC

The asymmetric two-port network (TPN) based BLC topology is shown in Fig. 1b. The TPNs are the subnetworks of the BLC. The transmission or ABCD parameter \( [A] \) of the TPNs are defined in such a way that the BLC have s-parameters as in (1). We can write the transmission or ABCD parameter of the TPNs as,

\[
[A] = \begin{bmatrix}
A & B \\
C & D
\end{bmatrix} = \begin{bmatrix}
a & jb \\
je & d
\end{bmatrix}
\] (3a)

\[
ad + bc = 1
\] (3b)

where, \( a, b, c \) and \( d \) are real numbers.
The ABCD parameters or A-matrices of the TPN blocks can be calculated using the port-decomposition technique [5] or Y-matrix technique [31] and given as

\[
[A_\beta] = \frac{1}{\sqrt{n}} \left[ \begin{array}{c} jZ_0 \sin \varphi \\
\tan \varphi_1 \tan \varphi_2 \\
\end{array} \right] (4a)
\]

\[
[A_\alpha] = \frac{1}{\sqrt{n+1}} \left[ \begin{array}{c} jZ_0 \sin \varphi \\
\cos \varphi - \tan \varphi_2 \\
\end{array} \right] (4b)
\]

with, \(c_\alpha = \frac{(n+1) - (\cos \varphi + \tan \varphi_2)(\cos \varphi + \tan \varphi_2)}{Z_0 \sin \varphi}\) (4c)

[C. BLC using Π topologies]

In [31], a systematic algorithm is developed to design multiport networks with lumped-Π topologies. Here we will develop the design process of the BLC using distributed-Π-type topology. The distributed-Π topology of the network is shown in Fig. 5a. Using Y-matrix method [31], we obtain the design equations of distributed-Π based BLC as

\[
Z_\alpha = \frac{Z_0 \sin \varphi}{\sqrt{n+1} \sin \theta_\alpha} (5a)
\]

\[
Z_\beta = \frac{Z_0 \sin \varphi}{\sqrt{n} \sin \theta_\beta} (5b)
\]
Figure 6. GUI of Branch Line Coupler Designer

\[
Z_\gamma = \frac{Z_0 \sin \varphi \tan \theta_\gamma}{\sqrt{n + 1} \cos \theta_\alpha + \sqrt{n} \cos \theta_\beta - \cos \varphi} \quad (5c)
\]

\[
Z_\delta = \frac{Z_0 \sin \varphi \tan \theta_\delta}{\sqrt{n + 1} \cos \theta_\alpha + \sqrt{n} \cos \theta_\beta + \cos \varphi} \quad (5d)
\]

For desired \( n \) and \( \varphi @ f_0 \), and given electrical lengths \( \theta_\alpha, \theta_\beta, \theta_\gamma \) and \( \theta_\delta \), we can find characteristic impedances \( Z_\alpha, Z_\beta, Z_\gamma \) and \( Z_\delta \) using (5a)-(5d). For example, a distributed-II BLC is designed for \( n = 1 \) and \( \varphi = 75^\circ \) with input electrical lengths \( \theta_\alpha = \theta_\beta = \theta_\gamma = \theta_\delta = 60^\circ \), the characteristic impedances are determined as \( Z_\alpha = 39.4 \) \( \Omega \), \( Z_\beta = 55.8 \) \( \Omega \), \( Z_\gamma = 88.2 \) \( \Omega \) and \( Z_\delta = 57.1 \) \( \Omega \). On the other hand, if we know the characteristic impedances \( Z_\alpha, Z_\beta, Z_\gamma \) and \( Z_\delta \), we can easily be obtained the value of electrical lengths \( \theta_\alpha, \theta_\beta, \theta_\gamma \) and \( \theta_\delta \) (5a)-(5d). For example, the same coupler is designed for \( n = 1 \) and \( \varphi = 75^\circ \) with input characteristic impedances \( Z_\alpha = Z_\beta = Z_\gamma = Z_\delta = 50 \) \( \Omega \), the electrical lengths are calculated as \( \theta_\alpha = 43.1^\circ \), \( \theta_\beta = 75^\circ \), \( \theta_\gamma = 46.9^\circ \) and \( \theta_\delta = 58.1^\circ \).

The capacitive stubs of the coupler can also be replaced by equivalent capacitors, leading to the mixed-II topology as shown in Fig. 5b. The lumped-II topology of BLC, shown in Fig.5c, can easily be obtained using the automated algorithm [31], proposed by Sinha. The design equations are given as

\[
X_\alpha = \frac{Z_0 \sin \phi}{\sqrt{n + 1}} \quad (6a)
\]

\[
X_\beta = \frac{Z_0 \sin \phi}{\sqrt{n}} \quad (6b)
\]

\[
B_\gamma = \frac{\sqrt{n + 1} + \sqrt{n - \cos \varphi}}{Z_0 \sin \varphi} \quad (6c)
\]

\[
B_\delta = \frac{\sqrt{n + 1} + \sqrt{n + \cos \varphi}}{Z_0 \sin \varphi} \quad (6d)
\]

The reactance \( X_\alpha \) and \( X_\beta \) and susceptance \( B_\gamma \) and \( B_\delta \) can be converted into inductor or capacitor at the design frequency \( f_0 \) using [31, eq. (4)-(5)].

D. BLC As A Power Divider

A four-port coupler can always be used as a three-port power divider by terminating the isolation port by its port impedances. Considering this fact, if we terminate port-1 of the BLC with a matched load and use port-4 as the input port, the BLC can be used as a three-port power divider as shown in Fig.7. The scattering matrix of the proposed power divider is

\[
[S_{PD}] = -\frac{1}{\sqrt{n + 1}} \begin{bmatrix} 0 & \sqrt{n} & e^{j\varphi} \\ \sqrt{n} & 0 & 0 \\ e^{-j\varphi} & 0 & 0 \end{bmatrix}, \quad (7)
\]

where \( n = \left| S_{21}\right|^2 \) is the power division ratio and \( \varphi = \angle \left( \frac{S_{21}}{S_{31}} \right) \) is the phase difference between the output ports. The topologies discussed for BLCs in the previous section are also applicable to the power divider. The corresponding design equations remain unchanged.

III. DESIGN EXAMPLES USING BLC DESIGNER

A computer-aided design tool: “Branch Line Coupler Designer” [53] has been developed for the reader for a
Figure 8. Lumped-Π non-quadrature equal power division BLC design for $Z_0 = 50 \, \Omega$, $f_0 = 0.9 \, \text{GHz}$, $n = 1$ and $\varphi = 60^\circ$: (a) schematic diagram, (b) amplitude response (c) amplitude imbalance (d) phase imbalance

Figure 9. Distributed-stepped quadrature unequal power division BLC design for $Z_0 = 50 \, \Omega$, $f_0 = 1 \, \text{GHz}$, $n = 2$, $\varphi = 90^\circ$, $\phi_1 = -15^\circ$, $\phi_2 = 20^\circ$ and $Z_1 = 70 \, \Omega$: (a) schematic diagram, (b) amplitude response (c) amplitude imbalance (d) phase imbalance

Figure 10. Mixed-T non-quadrature unequal power division BLC design for $Z_0 = 50 \, \Omega$, $f_0 = 1 \, \text{GHz}$, $n = 1/2$, $\varphi = 105^\circ$, $\phi_1 = -10^\circ$, $\phi_2 = 20^\circ$ and $Z_1 = 70 \, \Omega$: (a) schematic diagram, (b) amplitude response (c) amplitude imbalance (d) phase imbalance

better user experience of the proof of concept. This CAD tool is a Windows software application, where engineers can interactively design and analyse different branch-line couplers with desired design constraints. The application window of “Branch Line Coupler Designer” is shown in Fig. 6. The application is downloadable from Zenodo https://zenodo.org/record/8172705. It is free software, and available under Creative Commons Attribution 4.0 International. The software is developed using python.

Now, we will demonstrate some practical designs of BLCs using three different types of implementations. The first one is the lumped-II implementation. The BLC is designed for $Z_0 = 50 \, \Omega$, $f_0 = 0.9 \, \text{GHz}$, $n = 1$ and $\varphi = 60^\circ$. Fig. 8 shows the design schematic and analysis results. The coupler provides the desired response at the design frequency $f_0 = 0.9 \, \text{GHz}$.

One example of a distributed-stepped implementation is shown here. The design parameters are chosen as $Z_0 = 50 \, \Omega$, $f_0 = 1 \, \text{GHz}$, $n = 2$, $\varphi = 90^\circ$, $\phi_1 = -15^\circ$, $\phi_2 = 20^\circ$ and $Z_1 = 70 \, \Omega$. Fig. 9 shows a schematic diagram and analysis results of the distributed-stepped type BLC. The coupler shows the desired quadrature imbalance and unequal power division at the design frequency with a good amount of operating bandwidth.

The third example is the mixed asymmetric-T type implementations of a non-quadrature unequal power-division ratio coupler. The user input are chosen as $Z_0 = 50 \, \Omega$, $f_0 = 1 \, \text{GHz}$, $n = 1/2$, $\varphi = 105^\circ$, $\phi_1 = -10^\circ$, $\phi_2 = 20^\circ$ and $Z_1 = 70 \, \Omega$. The analysis is carried out in the frequency range of 0.7-1.3 GHz. The results are shown in Fig. 10. The analysis shows that the designed coupler provides desired performance at the designed frequency.

Eight different topological combinations can be implemented using the BLC designer. Only three examples are shown here. It is recommended that the reader can explore the software with different types of implementation.

IV. CONCLUSION

In this paper, the design of branch line couplers as power dividers has been simplified using closed form solutions.
These closed form solutions form the basis of a CAD tool to help young designers with BLC design. The BLC designer allows the user to choose various implementations of the same coupler. Users can choose the best-performing coupler for their requirements. Because of design and analysis flexibility, engineers can design non-quadrature couplers as per the design requirement and engineers can explore various options regarding the practical feasibility of the design.

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