A Shape Function Order Approach to Accelerate the Computation Time of the J-A Formulation

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Abstract

The J-A formulation has been proposed as a way to model the electromagnetic behavior of superconducting devices in finite element simulations. It is based on the current density, J, and the magnetic vector potential, A. It has been found that, to ensure better stability of simulations results, the shape functions of J should be discontinuous Lagrange of constant order, whereas for A second order continuous Lagrange functions should be used. However, applying second order shape functions increase the computational complexity of the simulation, both of degrees of freedom and computation time. In this paper, an approach to solve this issue is proposed. The finite element domain is divided into two parts, superconducting and non-superconducting. For the superconducting parts, the shape function of A is second order continuous Lagrange to ensure numerical stability. For the non-superconducting parts, a first order continuous Lagrange is used for A. To couple the two types of domains, Dirichlet Boundary Conditions are used. Three case scenarios are investigated and their results are compared to those of the common J-A formulation. Full agreement is obtained.
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Index Terms—Keyword 1, Keyword 2, Keyword 3, Keyword 4, Keyword 5

I. INTRODUCTION

Simulations of superconducting devices have paramount importance to ensure the best quality of the designs of superconducting devices. These simulations often are done with the Finite Element Method (FEM). Several superconducting devices, such as fault current limiters [1], superconducting coils [2], superconducting magnetic levitation systems [3], superconducting electric machines [4], have been simulated with the FEM.

To build the electromagnetic models of the aforementioned devices, several formulations in FEM have been applied in the literature throughout the years, such as the H formulation [3], the H-ϕ formulation [5], the H-A formulation [6], the T-A formulation [4] and more recently the J-A formulation [7], [8]. S. Wang et al. [8] shows that with the use of the J-A formulation, the best choice to stabilize numeric results is using a discontinuous Lagrange function with a constant order for the J formulation, and a Lagrange function with a quadratic order for the A formulation. However, it implies an increase in the degrees of freedom and consequently the computation time rises.

In this article, a new approach is proposed to avoid the need to model the whole domain with a A formulation with a quadratic order. So, two A formulations are used to model the problem, where the quadratic order is just used inside the superconducting domain to create proper stabilization of the numeric computation and in all of the other domains an A formulation with a first order Lagrange function is applied. To couple both A formulations, a Dirichlet boundary condition is employed in the border of the superconducting domain. The results show the approach is feasible to be used and all comparisons present errors of less than 1%.

The article is divided as follows: section II presents the methodology focusing on the proposed approach, section III presents the four developed models and the application of the approach on them. Secton IV presents and discusses the results and secton V gives the conclusion of the article.

II. METHODOLOGY

In this section, the motivation and explanation for the proposed approach are delivered.

From the start, equation (1) presents the hybrid J-A formulation without coupling with an external circuit. This formulation has two state variables, the current density J, which is solved by the J formulation, and the magnetic vector potential A, solved with the A formulation. To couple those formulations, one uses the state variables themselves, where the magnetic vector potential computed in the A formulation enters the J formulation as a source. On the other hand, the J formulation computes the current density in the superconducting domain and enforces it in the A formulation as an external source [7]. As commonly applied, the resistivity of the superconductor is modeled based on the power-law as presented in equation (2), where $E_c$, $J_c$, $n$, $J$ and $E$ represent the critical electric field, critical current, transition index, current density, electric field, respectively.

$$\begin{cases}
\rho J = -\frac{\partial A}{\partial t} \\
\nabla \times \frac{1}{\mu} \nabla \times A = J
\end{cases} \quad (1)
$$

As presented in [8], the use of discontinuous Lagrange functions helps to stabilize the numeric results obtained with the J-A formulation. S. Wang et al. show that while using the discontinuous Lagrange function in constant order, the A shape functions must be in the second order to stabilize the numeric results. However, using variables modelled with second order functions implies an increase in computation time. This article proposes a new approach to keep the stabilization numeric result and avoid the need to model all domains using the magnetic vector potential in the second order.
Supposing the generic domains presented in the figure 1, two domains are specified, the superconducting domain $\Omega_{sc}$ and the non-superconducting $\Omega_{nsc}$ domain. The approach aims to model $A$ with second order functions in $\Omega_{sc}$ and with first order functions in $\Omega_{nsc}$. In this way, the numeric stabilization is kept without jeopardizing the computation time. For that, a Dirichlet Boundary condition in $\partial\Omega_{sc}$ is applied, where $A_{sc} = A_{nsc}$ in the boundary. Separating those domains, it is possible to consider different shape function orders for $A$. It is worth mentioning that the magnetic vector potential is approximated using a Lagrange function and just the current density is approximated using the discontinuous Lagrange function.

**III. CASE STUDIES**

This section explains the implemented case studies used to evaluate the proposed approach. Four models are implemented in different scenarios and conditions. All of those models are presented in the above subsections.

**A. Case 1: transport and induced current in a bulk**

The transport and the induced current models have the same geometry with one air domain and one superconducting domain. In the superconducting domain, a bulk is modeled with the following dimensions: 100 mm of width and 20 mm of height. The air domain is a circle with a radius equal to 1000 mm. Figure 2 presents the studied model. The transition index of the superconducting material is equal to 25 and the critical current density is equal to 500 A and the $E_c$ is $1\mu$V/cm.

In the transport case, a current is enforced in $\Omega_{sc}$ based on an integral restriction presented in equation (3), where $J_{sc}$ is the current density in the superconducting domain and $I_{ap}$ is the desired current to be applied. In this case study, a sinusoidal current equal to 450 peak and a frequency equal to 50 Hz are imposed in $\Omega_{sc}$.

$$\iint_{\Omega_{sc}} J \cdot d\Omega_{sc} = I_{ap} \quad (3)$$

In the induced case, using a Dirichlet boundary condition in $\partial\Omega_{nsc}$, a sinusoidal magnetic field with 0.01 T of amplitude and 50 Hz of frequency is imposed on the external boundary of the air domain.

In both cases, to couple the different $A$ formulations another Dirichlet boundary condition is used on $\partial\Omega_{sc}$ according to the equation (4).

$$A|_{\Omega_{sc}} = A|_{\Omega_{nsc}} \quad (4)$$

**B. Case 2: transport current in two coupled bulks**

In this case, two separated superconducting domains are considered and an integral restriction is implemented to couple them. The applied current is the same as the one used in case one with the same frequency and amplitude. Two Dirichlet Boundary Conditions are applied in boundaries $\partial\Omega_{sc1}$ and $\partial\Omega_{sc2}$ following the same way as case one and so equation (4) becomes (5). The dimensions of the air domain and the superconducting domains are kept the same as in the first case, where the center of both rectangles that represent the superconducting domains are placed at (0,-20) mm and (0,20) mm. Figure 3 presents the domains of the model. The transition index, the critical current, and the critical electrical field are the same as presented in case 1.

$$\begin{cases} A|_{\Omega_{sc1}} = A|_{\Omega_{nsc}} \\ A|_{\Omega_{sc2}} = A|_{\Omega_{nsc}} \end{cases} \quad (5)$$

To couple $\Omega_{sc1}$ with $\Omega_{sc2}$, where the current that passes through $\Omega_{sc1}$ is in the contrary direction of the current that passes through $\Omega_{sc2}$, one uses an integral restriction presented in equation (6). To impose the current in the superconducting domains, equation (3) is used.
Fig. 3. Transporte current case with two coupled superconducting domains.

\[ \iint_{\Omega_{sc1}} J d\Omega_{sc1} + \iint_{\Omega_{sc2}} J d\Omega_{sc2} = 0 \]  

(6)

C. Case 3: rotating machine case

In [5], R. Brambilla et al. proposed a A-H formulation to deal with rotating machine models. This model is used here as a benchmark problem, as it is in HTS modeling website [9]. The dimensions of the problem are kept the same, as well as the rotating frequency. In this case, both \( \Omega_{rot} \) and \( \Omega_{nsc} \) are modeled with a first order function for the magnetic vector potential. The magnetic vector potential shape function in \( \Omega_{sc} \) is in second order and the current density is in first order. A sinusoidal current is imposed in the superconducting domain \( \Omega_{sc} \) using equation (3). \( \Omega_{sc} \cup \Omega_{rot} \) compound the rotating domain that has a rotating frequency equal to 50 Hz. The applied current is a sine that has a peak equal to 450 A with a frequency equal to 50 Hz. The critical electric field is 1 \( \mu \)V/cm, the critical current density is \( 2.7 \times 10^7 \) and the transition index is set up 19. Figure 4 presents the studied case. Two couple the rotating and static domains, equation 7 is applied in the red boundary \( (\partial\Omega_{rot}) \) and to couple \( \Omega_{sc} \) and \( \Omega_{rot} \) one uses a continuity equation similar to equation 4 in the blue boundary \( (\partial\Omega_{sc}) \).

\[ A|_{\Omega_{nsc}} = A|_{\Omega_{rot}} \]  

(7)

IV. RESULTS AND DISCUSSIONS

This section presents the results of the four case studies. It is worth mentioning that to run all cases the current density variable was approximated by a discontinuous Lagrange function with a constant order, namely here J0 and the magnetic vector potential was solved in a Lagrange function with linear or a quadratic function, namely hereafter A1 or A2, respectively.

A. Case 1: Transport and induced current

Figure 5 presents the normalized current density \( J_z/J_c \) of the transport case for J0-A1-A2 system and for J0-A2 system. Comparing the figures, one can see the results match, generating an error of less than 1%. This fact is expected as both systems use the same Lagrangian approximation in the same order. Analyzing the graph, the current density is below \( J_c \), which is expected due to an applied current equal to 0.8 \( I_c \).

Fig. 4. Rotating machine case, with the same dimensions as those found in [5].

Fig. 5. Normalized current density of the transport case. Comparison between the two systems, the J0-A1-A2 and J0-A2.
In Figure 6, the AC losses results of the transport current case for the systems J0-A1-A2 and J0-A2 are compared and a error less than 0.5% is observed. It shows an agreement between the proposed approach J0-A1-A2 and the J0-A2.

![Fig. 6. HTS losses for transport current case comparison between J0-A1-A2 and J0-A2.](image)

For the induced case, Figure 7 shows the results of the normalized current density for the sinusoidal applied magnetic field with its maximum value and frequency equal to 0.01 T and 50 Hz, respectively. The agreement between both results is clear and it has an error of less than 1%. On the left side, the current density goes in xy plan of the figure and goes out on the right side, indicating the circulation of the current in the superconducting bulk as expected in induced case for superconducting applications.

Figure 8 presents the comparison between the results of the J0-A1-A2 and J0-A2 systems for the losses in W/m of the induced current case. Comparing both results, the error is less than 1%.

**B. Case 2: transport current in two coupled bulks**

Figure 9 presents the normalized current density for the two coupled superconducting bulk domains, where the current is enforced in the $\Omega_{sc1}$ and returns from $\Omega_{sc2}$, as indicated by the color graph in the figure 9. The error between the models is less than 1%. The normalized current density agrees with the enforced current, having a peak value equal to 0.8 $I_c$. The losses due to this applied current are presented in Figure 10. The peak value of the losses exceeds 4 W/m at around 12 ms. Comparison between the results show an error of less than 1%.

**C. Case 3: rotating machine**

The magnetic flux density can be observed in the rotating machine case as depicted in Figure 11. In the ferromagnetic material, the magnet flux density is approximately 3 T. Results between the A-H formulation and the J-A formulation with different shape functions, proposed in this work, agree. The applied current in the superconducting bulk, which is rotating, is equal to 0.8 $I_c$ with a frequency equal to 50 Hz. Figure 12 presents the normalized current density at 5 ms for both formulations. The approach presented here proves to have the potential to model a system with rotating domains, with ha
error less than 1% for the normalized current density.

Losses in the superconducting domain for the rotating machine case are shown in Figure 13, again the error between the formulation is less than 1%, which proves the capability of the approach to reduce the degrees of freedom and keep the trustworthiness of the results.

V. CONCLUSION

This article presents an approach to reduce the degrees of freedom of the finite element simulation of superconducting devices by means of shape function order choosing, where all domains that do not have a superconducting material can be modeled by the A-formulation with the magnetic vector potential variable approximated by a linear Lagrangian function. On the other hand, to keep the trustworthiness of the system in the superconducting domain, the A-formulation is kept as the quadratic Lagrangian function. In this article, it is shown that the approach is applicable to bulk systems in transport and induced current, in coupled bulk system and in rotating system as well. For the 2D cases, it is an interesting approach when there is a large air region or when there is a ferromagnetic material. This approach helps to reduce the degrees of freedom and consequently helps to decrease the computation time. It can be specially useful for systems in 3D that in future work will be explored.

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Fig. 12. Normalized current density for the rotating magnetic system.

Fig. 13. HTS losses for rotating machine case comparison.

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