Waveguide-Floquet Mapping Using Surface Susceptibilities for Passive and Active Metasurface Unit Cell Characterization

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Abstract—A simple waveguide-Floquet mapping is proposed and theoretically investigated that links the scattering response of a sub-wavelength metasurface unit cell under an infinite periodic array configuration and when measured inside a shielded rectangular waveguide environment. This mapping is based on extracting the effective surface susceptibilities as the fundamental constitutive parameters of the cells, via the Generalized Sheet Transition Conditions (GSTCs) of the unit cell using the waveguide measurements, and then reconstructing the unit cell response at an arbitrary incidence angles. Using two examples of a transmissive Huygens’ metasurface and a purely reflective unit cells. The proposed waveguide-Floquet mapping thus theoretically confirmed to determine the complete angular scattering of the unit cells. The proposed waveguide-Floquet mapping thus represents a simple technique to experimentally determine the angular scattering characteristics of the unit cell using a low-cost effective waveguide measurements.

Index Terms—Floquet modes, unit cell simulations, metasurface, surface susceptibilities, Generalized sheet transition conditions (GSTCs), rectangular waveguides.

I. INTRODUCTION

METASURFACES are electrically large 2D arrays of sub-wavelength resonators which are engineered geometrically and electrically to control the electromagnetic fields scattered from them [1]. These resonators are typically built using canonical dipolar structures, such as electric and magnetic dipoles, which are then engineered to pack within sub-wavelength surface/volumes, which practically may range from approximately $\lambda_0/10 - \lambda_0/2$, where $\lambda_0$ is the free-space wavelength. The overall metasurface on the other hand is tens of wavelengths large, and may easily consists of hundreds and thousands of resonating elements. By engineering the geometrical distribution of these resonators over the metasurface array, a variety of field transformations are unlocked across the electromagnetic spectrum, ranging from novel devices to field processing systems [2]–[5].

An important class of metasurfaces is a reconfigurable/tunable/dynamically engineered resonator arrays. In contrast to passive metasurfaces, these structures typically consist of tunable lumped circuit elements (e.g. capacitors and resistors) [6]–[10] or advanced dynamically tunable exotic materials, such as graphene, ferroelectrics or conductive oxides, to name a few [11]. Here, while the physical geometry of the resonators is fixed across the array, the reflection/transmission characteristics (i.e. the beam-forming functionality) of the surface are changed using an external electrical, mechanical or optical stimuli, for instance [12]–[14]. This further requires a well-developed control circuitry that controls the surface. Therefore, the development and prototyping of a complete final metasurface structure is both complex and expensive. Furthermore, practical uncertainties in lumped circuits and material parameters, typically lead to large phase/frequency shifts (in reflection and/or transmission) and losses, which makes a direct prototyping of a complete array a risky design approach [9], [10].

Another important aspect in metasurface design, is its field characterization. Being fundamentally a scattering device, it ideally requires a free-space bi-static-type measurement setup which is both expensive and bulky, and typically not a standard off-the-shelf system [15]. While this is an inevitable requirement to test a metasurface array, intermediate testing and debugging of the unit cells forming the metasurface array, may provide critical and practically useful information before the entire surface is built. Several works in the literature have been reported, focussing on developing novel unit cell configurations offering variety of field control mechanisms [9], [14].

For a cost effective demonstration, they have been measured inside rectangular metallic waveguides, where only few identical unit cells are sufficient to demonstrate their reflection/transmission properties [16]–[30]. However, these commonly reported waveguide-based unit cell measurements do not provide a complete characterization of the unit cells under test, and it is not clear as how this information can be used to build an entire metasurface array. The unit cells are typically designed in an infinitely periodic environment and under plane-wave excitations (i.e. Floquet boundary conditions and excitations). Performing the unit cell measurements inside rectangular waveguide thus does not represent the same scattering problem, and is therefore an incomplete characterization of the unit cell. More specifically, the scattering response measured at a frequency inside a certain rectangular waveguide corresponds to a particular incidence angle of the plane wave at that frequency. The incidence angle of the plane wave is a function of both frequency and dimension of the waveguide.

This was pointed out in an early work [31] but has not been followed seriously. Therefore, only one or a few measurements...
inside rectangular waveguides cannot directly correspond to the complete angular or broadband scattering behavior of a metasurface under plane wave excitation.

In this work, we will demonstrate that rectangular waveguides can indeed be used to completely characterize a unit cell, following certain constraints. We will present a systematic method that utilizes the effective surface susceptibilities of the unit cells and maps their reflection/transmission characteristics to their scattering response in an infinitely periodic environment. This forms an important intermediate link in the metasurface design stage, that can maximize the operational success of the overall metasurface structure when prototypeed. The basic idea was first presented in [32], however, the analysis was limited to normal incidence only. Here, this idea has been expanded to include variety of cases with significant generalization.

The paper is structured as follows: Sec. II presents the background theory of the sub-wavelength resonators and their effective surface susceptibility description. Sec. III presents the proposed waveguide-Floquet mapping process followed by a numerical demonstration of the proposed mapping using two examples of a transmissive unit cell and a purely reflective unit cell, respectively in Sec. IV. Sec. V discusses various aspects of this mapping and outlines salient points and precautions that must be taken along with recommendations for future extensions of this process. Finally, conclusions are provided in Sec. VI.

II. METASURFACE CHARACTERIZATION PROBLEM

A. Problem Statement

A typical metasurface is configured with a sub-wavelength period $\lambda \leq \lambda_0$, to support a single diffraction order when excited with a uniform-plane at an arbitrary angle. Such a unit cell is typically designed and modelled using Floquet boundary conditions in typical full-wave electromagnetic solvers, as shown in Fig. 1(a). The setup consists of a single unit cell surrounded by periodic or Floquet boundaries with assigned phase shifts corresponding to the angle of incidence of the uniform plane-wave excitations. The Floquet boundaries emulate an infinite periodic structure. Two Floquet ports can be then be assigned to capture the reflection and transmission characteristics of the unit cell, $R_{uc}(\omega, \theta)$ and $T_{uc}(\omega, \theta)$, which can be swept over incidence angle and frequency, to generate the complete angular scattering response of the cell.

This unit cell with its geometrical or electrical attributes is next used to construct a 2D array forming the metasurface, which in experiments is excited with a known source antenna, as shown in the left of Fig. 1(b). This field measurement system is operated under far-field conditions (i.e. TEM wave excitation), where the Tx source antenna is fixed at a certain angle illuminating the metasurface under test, and a receiving Rx antenna is swept over an angular range, also located in the far-field of the surface [9], [33]. This is then used to construct the complex $R(\omega, \theta)$ and $T(\omega, \theta)$ of the surface. This is also known in the literature as a fire-space bi-static measurement configuration. Naturally, due to far-field condition requirements, bi-static measurement is bulky and costly.

Moreover, due to finite size surface scattering, typically non-uniform metasurfaces and non-plane-wave excitation conditions in these systems, the fundamental constitutive parameters of the unit cell (i.e., effective surface susceptibilities) cannot readily be extracted experimentally.

A single (or a small collection of) unit cell(s) can also be measured inside a closed waveguide environment. Fig. 1(b) shows a typical setup on the right, where for instance, a WR90 coax-to-waveguide adaptor for X-band, can be directly placed on top of few cells to measure its reflection (or transmission if two adaptors are used on each side of the cell), i.e. $\{R_{wg}, T_{wg}\}$. Compared to the free-space bi-static system, this is a low cost and simple way to measure the unit cell response (active or passive). It is naturally not a replacement of the free-space bi-static system, which delineates the complete beam-forming capabilities of the surface. However, it has the potential to demonstrate unit cell behaviour and provide characteristics – which can form a look-up table to be used for subsequent metasurface design.

At this stage, one may wonder, if this waveguide measurement, $\{R_{wg}, T_{wg}\}$, can be used to extract the Floquet unit cell response, $R_{uc}(\omega, \theta)$ and $T_{uc}(\omega, \theta)$, and thus its complete angular scattering response? The answer to this question is, yes, it is possible under certain conditions. This can be understood from the electromagnetic mode that propagates inside a metallic rectangular waveguide. The fundamental mode $TE_{10}$ inside a metallic rectangular waveguide is a superposition of two uniform plane-waves [34], and thus has a similarity with the Floquet mode used in numerical solvers, as shown in Fig. 1(c). We will next show how this waveguide result can be rigorously mapped to the Floquet unit cell results, by utilizing their constitutive parameters.

B. Surface Susceptibility Description

A practical electrically thin metasurface with sub-wavelength period can be homogenized as long as $\Lambda \ll \lambda_0$. Such a surface can be viewed as a spatial discontinuity [35], whereby the fields $\{E, H\}$ scattered from it satisfy the Generalized Sheet Transition Conditions or the GSTCs, given by:

$$\hat{n} \times \Delta E_{||} = -j\omega\mu_0|\mathbf{M}_{||} - \hat{n} \times \nabla|| \left( \frac{P_{n}}{\epsilon_0} \right) \quad (1a)$$

$$\hat{n} \times \Delta H_{||} = j\omega|\mathbf{P}_{||} - \hat{n} \times \nabla|M_{n} \quad (1b)$$

where $\Delta\psi$ is the field difference across the metasurface, $\hat{n}$ is the surface normal, and $\psi_{||}$ and $\psi_{n}$ represent tangential and normal surface polarizations, respectively. More specifically, the electric and magnetic surface polarizations are given by;

$$\mathbf{P} = \epsilon_0\bar{\chi}_{ee}E_{av} + \sqrt{\mu_0\epsilon_0}\bar{\chi}_{em}H_{av} \quad (2a)$$

$$\mathbf{M} = \bar{\chi}_{mm}H_{av} + \frac{1}{\eta_0}\bar{\chi}_{me}E_{av} \quad (2b)$$

in terms of the average fields $\psi_{av}$, and the effective surface susceptibilities $\bar{\chi}$. The surface susceptibilities are the fundamental constitutive parameters of the metasurface unit cell, and each of them is a $3 \times 3$ tensor, capturing different physical functionality of the cell [36]–[38].
For a non-spatially dispersive metasurface, surface susceptibilities are independent of the incidence angle, $\theta$, and remain only frequency dependent capturing the temporal dispersion of the unit cells, i.e. $\tilde{\chi} = \tilde{\chi}(\omega)$. Therefore, susceptibilities being the constitutive parameters of the unit cell completely captures the angular scattering characteristics of the metasurface. They can be numerically extracted using finite number of Floquet unit cell simulations depending on the required number of susceptibility terms in the tensor to faithfully represent the unit cell. Our goal next is to determine them using waveguide measurements, which is the key to the sought-after waveguide-Floquet mapping.

### III. WAVEGUIDE-FLOQUET MAPPING

#### A. Proposed Mapping Concept

As illustrated in Fig. 1(c), the waveguide mode resembles the Floquet model used in unit cell simulations. The fundamental $\text{TE}_{10}$ mode of a rectangular waveguide of width $a$, has the form of [34]

$$
E_x(x,y) = A_{10} \sin \left( \frac{\pi x}{a} \right) e^{-j\beta_0 y} \hat{y}
$$

which can be expressed as a superposition of two plane-waves propagating at an angle $\theta$ and $-\theta$ in the $x-z$ plane, i.e.,

$$
E_x(x,y) = E_0 \left( e^{j\beta_x x} - e^{-j\beta_x x} \right) e^{-j\beta_0 z} \hat{y}
$$

where $E_0 = 2jA_{10}$, $\beta_x = \pi/a$. The angle of incidence, $\theta$ of these plane-waves are given by

$$
\theta(\omega) = \sin^{-1} \left( \frac{\lambda_0}{2a} \right).
$$

Therefore, we note that when a unit cell is measured inside a rectangular waveguide at a frequency $\omega$, its scattering response corresponds to the Floquet unit cell response at an angle $\theta(\omega)$. This is assuming that the unit cell has a symmetric angular response, i.e. $\{ R_{ee}(\theta), T_{me}(\theta) \} = \{ R_{me}(-\theta), T_{me}(-\theta) \}$.

This observation thus forms an important link between Floquet unit cell characterization and waveguide measurements. We can thus conclude that each waveguide measurement corresponds to a unit cell simulation at a fixed incidence angle $\theta$. If there are sufficient number of waveguide measurements corresponding to $n$ incidence angles, one can extract all needed surface susceptibility terms using (1) and (2). Once $\tilde{\chi}$ are known, the unit cell Floquet response can be calculated at an arbitrary angle. This forms the basis of the proposed waveguide-Floquet mapping. We remark here that this method thus represents the first reported systematic way to extract the surface susceptibilities of sub-wavelength unit cells in experiments.

#### B. Surface Susceptibility Extraction

A generic metasurface unit cell can be described using four $3 \times 3$ susceptibility tensors $\tilde{\chi}_{ee}$, $\tilde{\chi}_{mm}$, $\tilde{\chi}_{em}$ and $\tilde{\chi}_{me}$, i.e. 36 complex terms. To simplify the complexity of the subsequent susceptibility extraction, while showing the proposed waveguide-Floquet mapping in action, let us assume a reciprocal metasurface with no polarization rotation. This will help reduce the number of susceptibility unknowns in the examples to follow.

Consider next a uniform metasurface lying in the $x-y$ plane and $\hat{n} = \hat{z}$. Assuming, normal polarization for simplicity, only $E_y$, $H_x$ and $H_z$ field components exist. Furthermore, for a reciprocal surface, $\tilde{\chi}_{ee} = \tilde{\chi}_{me}$, $\tilde{\chi}_{mm} = \tilde{\chi}_{me}$ and $\tilde{\chi}_{em} = -\tilde{\chi}_{em}$, where $\{ \}^T$ represents a matrix transpose. This leads to the following unknowns and allowed susceptibility components only: $\chi_{y y}$, $\chi_{xx}$, $\chi_{z z}$, $\chi_{x y}$, $\chi_{xx}$, $\chi_{x z}$ and $\chi_{z y}$. Under these conditions, using (2), (1) reduces to:

$$
- \Delta E_y = -j k_0 \eta_0 \left\{ \chi_{z z} H_{x,av} + \chi_{y z} H_{z,av} \right\} - j k_0 \chi_{y x} E_{y,av} \tag{6a}
$$

$$
\Delta H_x = \frac{j k_0 \eta_0}{\epsilon_0} \chi_{y y} E_{y,av} - j k_0 \left\{ \chi_{y y} H_{x,av} + \chi_{z y} H_{z,av} \right\} - \nabla_x \left\{ \chi_{x z} H_{x,av} + \chi_{z z} H_{z,av} + \frac{1}{\eta_0} \chi_{y x} E_{y,av} \right\} \tag{6b}
$$

where the spatial derivative $\nabla_x = \partial / \partial x$. These unknown susceptibility components may now be extracted using Floquet unit cell simulations under plane-wave excitation, and using a finite number of incidence angles to generate multiple independent equations.
IV. Numerical Demonstration

A. Transmissive Unit Cell with Tangential Polarizabilities

Let us consider a commonly used transmissive unit cell with purely tangential surface susceptibilities, \( \chi_{ee}^{yy} \) and \( \chi_{mn}^{xx} \). This represents a co-located orthogonal electric and magnetic surface polarizabilities, forming the Huygens’ surface configuration [39]–[41]. For a uniform plane-wave incidenting at the angle \( \theta \), the transmission and reflection response of the unit cell can be obtained using (6) as [37],

\[
R = \frac{2j k_0 (\cos^2 \theta \chi_{mm}^{xx} - \chi_{ee}^{yy})}{(2j k_0 \cos \theta \chi_{mm}^{xx} + 2)(2j k_0 \chi_{ee}^{yy} + 2 \cos \theta)} \quad (7a)
\]

\[
T = \frac{2j k_0 \cos \theta \chi_{mm}^{xx} + 2)(2j k_0 \chi_{ee}^{yy} + 2 \cos \theta)}{(2j k_0 \cos \theta \chi_{mm}^{xx} + 2)(2j k_0 \chi_{ee}^{yy} + 2 \cos \theta)} \quad (7b)
\]

where \( k_0 = \omega/c \) is the wavenumber capturing the frequency dependence in \( R \) and \( T \). For this case, the two unknown surface susceptibilities can be extracted using a single incidence angle \( \theta \), as:

\[
\chi_{ee}^{yy} = \frac{2j \cos \theta}{k_0} \left( \frac{T + R - 1}{T + R + 1} \right) \quad (8a)
\]

\[
\chi_{mm}^{xx} = \frac{2j}{k_0 \cos \theta} \left( \frac{T - R - 1}{T - R + 1} \right) \quad (8b)
\]

This suggests that only a single 2-port waveguide measurement is sufficient to extract the susceptibilities in this case. Note that \( \theta \neq 0 \), as the waveguide cannot support a TEM wave. The following steps must be followed to implement the mapping:

(a) For each frequency, \( \omega \), the equivalent plane-wave angle, \( \theta_\omega \) is computed using (5), knowing the width of the waveguide.

(b) For each frequency, \( \chi_{ee}^{yy} \) and \( \chi_{mm}^{xx} \) are extracted using waveguide measurements \{\( R_{WG}(\omega), T_{WG}(\omega) \)\} and \( \theta_\omega \) in (8), i.e., \( \chi_{wg}(\omega) \).

(c) Finally, reconstruct the \{\( R(\omega, \theta), T(\omega, \theta) \)\} of the unit cell at any arbitrary incidence angle \( \theta \) using the waveguide extracted susceptibilities using (7). They will correspond to the Floquet unit cell simulations under infinite periodic conditions.

To illustrate this process, consider a three-layer (we should mention what are these three layers if it is three-layer) dog-bone structure as shown in Fig. 2(a). It consists of a dipole resonator as shown in the inset, with the cell being symmetric along the \( z \)-axis, so that the bi-anisotropic term \( \chi_{ee}^{yy} = 0 \). A waveguide simulation is then setup with a \( m \times n \) array with a waveguide TE\(_{10} \) mode input (with PEC boundaries all around the vacuum box) as also shown in Fig. 2(a). Care must be taken to design the waveguide to operate in the fundamental mode only within the frequency range of interest. The corresponding simulated waveguide response is shown in Fig. 2(b).

Following the steps outlined above, first the surface susceptibilities, \( \chi_{ee}^{yy} \) and \( \chi_{mm}^{xx} \), are extracted as shown in Fig. 2(c).
tangential magnetic: \( \chi_{mm}^{xx} = 0 \), tangential bianisotropic: \( \chi_{mm}^{xy} = \left( \frac{2j}{k_0} \right) \)

(9a)

tangential electric: \( \chi_{ee}^{yy} = \frac{4j}{k_0} \left( \frac{\cos \theta_2 (R_1 + 1)(R_2 - 1) \sin^2 \theta_1 - \sin^2 \theta_2 (R_1 - 1)(R_2 + 1) \cos \theta_1}{(\sin^2 \theta_1 - \sin^2 \theta_2)(R_2 + 1)(R_1 + 1)} \right) \)

(9b)

normal magnetic: \( \chi_{mm}^{zz} = -\frac{4j}{k_0} \left( \frac{(R_1 - 1)(R_2 + 1) \cos \theta_1 - \cos \theta_2 (R_1 + 1)(R_2 - 1)}{(\sin^2 \theta_1 - \sin^2 \theta_2)(R_2 + 1)(R_1 + 1)} \right) \)

(9c)

which are also compared to the ones extracted directly from the Floquet unit cell simulation. Clear electric and magnetic resonances are captured, as expected from a Huygens’ cell, with an excellent match between the unit cell and waveguide extracted susceptibilities. Once the susceptibilities are successfully extracted, the transmission response (reflection response not shown for brevity) is recreated for arbitrary incidence angles following (7), as shown in Fig. 2(d). At all angles, the Floquet unit cell response is successfully captured, including any small variations at higher angles. We should also note that these reconstructed responses are different from the basic waveguide response of Fig. 2(b), proving that the raw waveguide measurements are not directly indicative of the Floquet unit cell response.

B. Purely Reflective Cell

Next, consider another class of metasurface structures consisting of purely reflective unit cells, which are commonly used for making metasurface reflectors [10], [42]. Such unit cells consist of a sub-wavelength resonator on one side and a solid ground plane (modelled as PEC here) on the other. This implies \( T_{12} = T_{21} = 0 \) and back-reflection \( R_{22} = -1 \). Since \( R_{11} \neq R_{22} \), these unit cells feature a non-zero bi-anisotropic term \( \chi_{ee}^{xy} \) [38]. It can be shown using (6), that the tangential magnetic term \( \chi_{mm}^{xx} = 0 \), while \( \chi_{ee}^{xy} = 2j/k_0 = \text{const.} \). Consequently, the general angle dependent reflection coefficient, involving normal magnetic polarization (via \( \chi_{mm}^{zz} \)) and tangential electric polarization (via \( \chi_{ee}^{yy} \)) for a purely reflective cell can be obtained from (6) and is given by

\[
R = \left( \frac{4 \cos \theta + j k_0 \sin^2 \theta \chi_{mm}^{zz} - j k_0 \chi_{ee}^{yy}}{4 \cos \theta - j k_0 \sin^2 \theta \chi_{mm}^{zz} + j k_0 \chi_{ee}^{yy}} \right).
\]

(10)

For a lossless surface with purely real susceptibilities, \(|R| = 1\), as expected, i.e. an all-pass reflection response.

Unlike the Huygens’ cell of Fig. 2, only reflection response \( R \) is available in this case, while there are two unknown susceptibilities, \( \chi_{ee}^{yy} \) and \( \chi_{mm}^{zz} \), to be extracted. It suggests that we will need two independent measurements to find these two unknowns. From a Floquet unit cell simulation perspective, these susceptibilities can be extracted using the unit cell reflection response \( R_1 \) and \( R_2 \) from two different angles, \( \theta_1 \) and \( \theta_2 \). Using (6), these unknown susceptibilities can be found and are given by (9). It is clear in this case that a single waveguide measurement is not sufficient to fully characterize the unit cell. Therefore, we propose to use two waveguides with different widths \( a_1 \) and \( a_2 \), with two corresponding angle profiles \( \theta_1(\omega) \) and \( \theta_2(\omega) \) to generate two independent sets of measurements.

To illustrate this case, consider an example of a single-layer reflective dog-bone resonator structure, backed by an infinite ground plane. Also, consider that a small gap is introduced
in the main dipole arm, and a lumped capacitor element is added, as shown in Fig. 3(a). Such a cell may be used as a reconfigurable unit cell where its reflection response can be tuned by changing the capacitor value only while maintaining the unit cell geometry. Next, two waveguide configurations are built, consisting of $1 \times 4$ and $1 \times 5$ array, respectively. Again, the resulting waveguide widths (i.e. $a_1 = 4\lambda_L$ and $a_2 = 5\lambda_L$) are carefully chosen to ensure that they both operate in the fundamental TE$_{110}$ mode within the frequency range of interest. The frequency-dependent equivalent angle of plane-wave incidences inside the waveguides are also shown in Fig. 3(a). These angles and the two waveguide reflection responses will now be used to extract the surface susceptibilities following (9).

The two extracted surface susceptibilities are shown in Fig. 3(b) extracted from the two waveguide responses, and compared to the ones extracted directly from Floquet unit cell simulations. An excellent match between the two confirms that the extraction is successful across the entire frequency range. This is further validated by reconstructing the reflection response (both magnitude and phase) at various incidence angles following (10) and comparing it with the full-wave response obtained from Ansys HFSS. For comparisons, the reflection response of the two waveguides has also been superimposed with the reconstructed reflection for each angle, and it is clear that the intrinsic waveguide response is very different, and thus inaccurate, from the desired unit cell response in general.

V. DISCUSSIONS, PRECAUTIONS, AND RECOMMENDATIONS

A. Important Considerations

The two examples presented in the previous section successfully demonstrate the proposed waveguide-Floquet mapping. A general flowchart of this process is shown in Fig. 4. Several important considerations must be taken and constraints be followed utilizing this process in practical unit cell measurements.

1) The choice of the unit cell: The proposed waveguide-Floquet mapping process assumes a sub-wavelength metasurface unit cell that is homogenizable. The existence of an angle-independent effective surface susceptibility, $\chi(\omega)$, is a pre-requisite condition for this proposed mapping to work in this current form. This, therefore, excludes spatially-dispersive metasurfaces with angle-dependent susceptibilities [43]. The metasurface designers, thus, must carefully choose an appropriate unit cell configuration and first perform a Floquet unit-cell simulation to extract the susceptibilities. The designer must confirm that the susceptibilities are able to reconstruct the complete angular scattering response of the unit cell, i.e., $R'(\omega) = R(\omega)$ and $T'(\omega) = T(\omega)$, which represents the first decision making loop in Fig. 4.

2) The number of surface susceptibility terms: As seen in the example of the capacitor-loaded dog-bone structure, two different waveguides were needed to get independent measurement data for extracting multiple susceptibility terms. This illustrates that multiple independent measurements must be devised which will be proportional to the unknown susceptibility terms. This can potentially make the number of waveguides measurements large, and results sensitive to the differences between each measurement. However, thankfully, it is found that for determining a fully populated surface susceptibility tensor for the most general case, a maximum of six different 2-port waveguide measurements is needed [37]. Irrespective, the waveguide setups must be chosen in such a way that the waveguide-extracted susceptibilities match the ones obtained from Floquet unit cell, i.e., $\chi(\omega) = \chi_{wg}(\omega)$, which is represented by the second decision making loop in Fig. 4. This condition naturally ensures a successful reconstruction of the unit scattering parameters, i.e., $R_{wg}(\omega, \theta) = R(\omega, \theta)$ and $T_{wg}(\omega, \theta) = T(\omega, \theta)$. If this condition is not satisfied, the number of waveguide setups and corresponding conditions must be revised until this
condition is met. These steps must first be verified in simulations before proceeding to fabrication.

3) The choice of the waveguide width: The entire method used here assumes fundamental TE_{10} mode propagation inside the waveguide. This is controlled primarily by the waveguide width, a, and is given by f_{10} ∈ {c/2a, c/a}. If multiple waveguide measurements are required (as in the reflective dog-bone example), differing widths, will constrain the acceptable operation frequency range for the mapping. The waveguide widths must therefore be chosen carefully to maximize the overlap between the dominant bandwidths of each waveguide, while operating sufficiently far away from the TE_{10} mode cutoff of the lower width waveguide. To illustrate this point better, consider the example of a reflective loop resonator unit cell of Fig. 5(a). Similar to the reflective dog-bone, this loop resonator also requires χ_{yy} and χ_{zz}, and thus the two-waveguide method. The two waveguides here have widths of 27 mm and 36 mm respectively with the corresponding TE_{10} operation bandwidths of \{5.56, 11.12\} GHz, and \{4.16, 8.52\} GHz. Therefore, the second waveguide can support a higher-order mode beyond 8.32 GHz, which is not accounted for in the proposed mapping. Fig. 5(b-c) shows the extracted surface susceptibilities and the reconstructed reflection response at an example angle of 80°. While the mapping is still relatively successful, an error is clearly observed in both the extracted susceptibilities, as well as the reconstructed reflection response beyond about 8 GHz. This exemplifies that the choice of the waveguide widths is a critical consideration, and must be handled with care. This consideration is interrelated to the previous one regarding the number of susceptibility terms present, as a cell with a higher number of susceptibility terms will require more waveguide measurements. This will typically require the use of waveguides with larger cross sections, lowering the cutoff frequencies for the higher order modes, and thus lowering the bandwidth over which the unit cell can be characterized.

4) Polarization converting unit cell: The analysis and the example considered in this work has been limited to non-polarization converting and reciprocal unit cells only. If the unit cell of interest supports polarization conversion \cite{44}, \cite{45}, the proposed mapping can be extended to cover that scenario. In that case, the height of the waveguide, b, and the width of the waveguide a, must be carefully chosen to maximize the operation bandwidth. A good choice could be a square cross-section waveguide with dual-polarized feed inputs to measure cross-polarization.

B. Practical Use Case

As mentioned in Sec. II, while the waveguide-based unit cell characterization cannot replace a full bi-static RCS measurement system for characterizing the overall metasurface, it still represents an important and useful low-cost intermediate design tool. Before a complete metasurface with 2D array of resonators is constructed, the basic functionality of the underlying unit cell can be verified using the proposed waveguide-Floquet mapping method. The design iteration of the unit cell thus can be performed on the unit cell level first inside the waveguide before investing in a complete metasurface prototype. Moreover, once the metasurface is built, the response of the same unit cell (taking any practical non-idealities or parasitics in case of active surface, into account) obtained using the waveguide can serve as a reliable look-up table to generate the non-uniform reflection (or transmission) profile on the 2D array for achieving a desired beam-forming operation.

While the proposed waveguide-Floquet mapping provides a useful intermediate test bed for metasurface designs, it has three major drawbacks: 1) it requires proper through-reflect-line (TRL) waveguide calibration to bring the measurement reference plane to the metasurface boundaries. 2) The waveguide widths must be an integer multiple of the unit cell periods, i.e. a = nΛ. Moreover, if multiple waveguides are required, the constraints on the allowed unit cell period is further increased, where using standard waveguide dimensions may sometimes be no more feasible. In such cases, custom waveguide dimensions must be used and built, which can become a practical cost and prototyping consideration. 3) The proposed waveguide-Floquet mapping is only applicable for the TE_{10} mode, and thus, the operation frequency ranges may reduce as more susceptibility terms are needed to model the unit cell.
A simple waveguide-Floquet mapping has been proposed and theoretically investigated that links the scattering response of a sub-wavelength metasurface unit cell under an infinite periodic array configuration and when measured inside a shielded rectangular waveguide environment. Identifying that the fundamental TE\textsubscript{10} mode inside a rectangular metallic waveguide can be decomposed into two interfering uniform plane waves at oblique angles, a simple but useful mapping can be established with a Floquet plane-wave excitation under periodic boundary conditions surrounding the unit cell. This mapping is based on extracting the effective surface susceptibilities of the unit cell using the waveguide measurements, and then reconstructing the unit cell response at an arbitrary incidence angle. Using two examples of a transmissive Huygens’ metasurface and a purely reflective metasurface, the proposed waveguide-Floquet mapping has been confirmed to determine the complete angular scattering of the unit cells. Various important aspects of the mapping, limitations, and practical considerations have further been discussed. Beyond interesting theoretical aspects, the proposed waveguide-Floquet mapping represents a simple technique to experimentally determine the angular scattering characteristics of the unit cell using cost-effective waveguide measurements. This, therefore, provides a simple, yet effective, intermediate design stage to a metasurface designer, confirming the unit cell operation before a full resonator array forming the metasurface is built and tested in an otherwise expensive field measurement facility.

VI. CONCLUSION

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