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The Critical Latitudes of Jupiter and Saturn

*From Major Liabilities to Major Assets*

Timothy E. Dowling, *University of Louisville*

*Critical Latitude:*
Local max. or min. of $PV = \text{spin/depth ratio}$
The Critical Latitudes of Jupiter and Saturn

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Critical Latitude:
Where Rossby waves reverse direction

(The ice giants are not in this discussion because they do not have multiple critical latitudes.)
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**Critical Latitude:**
Where Rossby waves reverse direction

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Spin is proportional to depth:

\[(\zeta + f) = PV \times h\]

Depth reveals warps in the underlying geopotential
1988: Jupiter’s Great Red Spot (GRS) was discovered to be twice as deep on its poleward side as its equatorward side.

*Juno:* PJ7 MWR, Cheng Li (private comm.)

1989: Jupiter’s deep geopotential was mapped

Deep undulations are the tops of deep jets.

✓*Juno*

**References**

Dowling and Ingersoll, 1988, *J. Atmos. Sci.* 45, 1380–1396
The tops of Jupiter’s deep jets were discovered by:

1. Abandoning the established approach of assuming one form of interior-jet structure or another.

Instead, by:

2. *Inferring* interior jets from *Voyager* vorticity (spin) data.
1989:
The GRS straddles a critical latitude.

*Juno* has been quiet to date regarding critical latitudes.
1989:
The GRS straddles a critical latitude.

*Juno* has been quiet to date regarding critical latitudes.

✓ 2011:
*Marcus & Shetty*

Jet Stream Theorems

Unstable Jet

Critical latitude

Stable Jet

No critical latitude

Necessary Condition

Sufficient Condition

This makes it appear that critical latitudes are major liabilities.
But this is a common mistake. For example:

In the *Wizard of Oz*, Dorothy thinks “witches are old and ugly”...
but then she learns that “only bad witches are ugly.”

Just so, we need to fill in the blank:

Only _____ critical latitudes are unstable.
2006: Wind and temperature data from Voyager and Cassini revealed that Jupiter is striped with critical latitudes (Read et al.).

Read et al., 2006, Q. J. R. Meteorol. Soc. 132, 1577–1603
2006: Wind and temperature data from Voyager and Cassini revealed that Jupiter is striped with critical latitudes (Read et al.).

2009: Wind and temperature data from Voyager and Cassini revealed that Saturn is striped with critical latitudes (Read et al.).
Read et al., 2009, *Planet. Space Sci.* 57, 1682–1698

There is not one elephant witch in the room.
1995: Multiple critical latitudes shown to maintain stability by phase locking the fastest (longest) Rossby waves.  

2009: The longest Rossby waves lock onto the planet’s rotation period, yielding:


Jupiter: $9^h\ 55^m$  
Saturn: $10^h\ 34^m$

Multiple critical latitudes are major assets.

To use these assets, we need to understand how Rossby waves and critical latitudes govern jet stability.
Rossby Wave Example

absolute vorticity
Elapsed time: 1902-02-15 03:19

EPIC 5.22
Jupiter
0.25° grid

Long undulations are long Rossby waves

Keaveney, Lackmann & Dowling
1880: Shear stability theorems began appearing.

Rayleigh (1880)
Kuo (1949)
Fjørtoft (1950)
Charney & Stern (1962)
Arnol’d (1966)
Ripa (1983)

2014: Shear stability theorems were non-dimensionalized for the first time.

“Supersonic” critical latitudes are stable.
“Supersonic” critical latitudes are stable.

The key to understanding how critical latitudes control Rossby waves is the non-dimensional Rossby Mach number, $M_R$. 
Rossby waves are unidirectional, hence there are 2 “supersonic” cases:

2014: The reciprocal Rossby Mach number concatenates these into a single stability region.
Rossby waves are unidirectional, hence there are 2 “supersonic” cases:

- Stable: $M_R < 0$
- Stable: $1 < M_R$
- Stable: $M_R^{-1} < 1$

2014: The reciprocal Rossby Mach number concatenates these into a single stability region.
How “supersonic” critical latitudes work, $M_R^{-1} < 1$
How “supersonic” critical latitudes work, $M_R^{-1} < 1$

“Supersonic” critical latitudes are proven to be stable.

Notice the negative feedback when a PV eddy pivots with the shear.
How “subsonic” critical latitudes work, $M_R^{-1} > 1$
How “subsonic” critical latitudes work, $M_R^{-1} > 1$

“Subsonic” critical latitudes are fraught with instability.

Notice the positive feedback when a PV eddy pivots against the shear.
Jet Stream Theorems, Updated

<table>
<thead>
<tr>
<th>Necessary Condition</th>
<th>Unstable Jet</th>
<th>Stable Jet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical latitude</td>
<td></td>
<td>No critical latitude</td>
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Sufficient Condition
### Jet Stream Theorems, Updated

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<tr>
<td>“Subsonic” critical latitude, $M_R^{-1} &gt; 1$</td>
<td></td>
<td>No “subsonic” critical latitude</td>
</tr>
</tbody>
</table>

This makes it appear that “subsonic” critical latitudes are unstable.
Jet Stream Theorems, Sharp

To prove that a “subsonic” critical latitude is both necessary and sufficient for shear instability:

1. Prove that a “subsonic” critical latitude is necessary and sufficient for the existence of a neutral mode.

2. Prove that a neutral mode is necessary and sufficient for the existence of an unstable mode.
Jet Stream Theorems, Sharp

Unstable Jet

Necessary Condition

“Subsonic” critical latitude, $M_R^{-1} > 1$

Sufficient Condition

Stable Jet

No “subsonic” critical latitude

Deguchi, Hirota, and Dowling, Stability of alternating jets: necessary and sufficient conditions
To Do List

• Determine winds and temperatures from PV inversion (elliptic operator) as in Sun and Lindzen (1994), but with $M_R^{-1} = 1$ instead of $M_R^{-1} = 0$ (w/ Voyager vorticity paradigm shift).

• Add the shear stability constraint, $M_R^{-1} \leq 1$, to Juno gravity inversions everywhere there are jets (troposphere and interior).

• For systems with multiple stable critical latitudes (Jupiter, Saturn), apply the tight constraint (major asset), $M_R^{-1} \approx 1$:
  
  1989: tops of Jupiter’s deep jets (w/o Juno gravity data)
  2009: Saturn’s 10$^h$34$^m$ period (w/o Cassini ring-wave data)