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Core Loss Modeling: Review and a New Approach Based on the Concept of Magnetic-flux resistance and the Square Law

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Abstract—this paper presents a new approach for linearizing the core loss calculation problem over wide range of frequency and magnetic flux density. The proposed approach is developed based on the fundamental square loss law and the concept of the magnetic-flux resistance ($R_{mag}$). Higher accuracy, less complexity and easier applicability are the main features of the derived model in comparison to Steinmetz and Bertotti models. The developed model is an alternative way to explain the hysteresis loss analogous to the AC resistance through the prediction of $R_{mag}$, It also gives better physics background and explanation than Steinmetz and Bertotti approaches to the core loss as it lies this phenomenon with the classic square loss law and the material propriety $R_{mag}$. The model is developed for several magnetic ferrite materials using measurement results from literature.

Index Terms—core loss, magnetic-flux resistance, square loss law, hysteresis, linear and nonlinear problem.

I. INTRODUCTION

Core loss calculation is a central research topic in power electronics and an indispensable engineering task in the design of magnetic devices. It is increasingly gaining interest because of the importance of the magnetic devices and their significant impact on the efficiency and the power density of power electronic converters. Core loss is a complex phenomenon and the basic physics theory characterizing the phenomenon is yet not fully formulated. Several dependent and independent factors, such as the excitation shape, the frequency, the stress, the temperature, the core shape, the material microstructure, are contributing to the generation of the core loss. The investigation of the core loss phenomenon has early started after the formulation of Maxwell’s equations. It was characterized using the concept of hysteresis introduced by Ewing [2]. Core loss calculation was firstly formulated in a quantitative and empirical manner by Steinmetz in 1908 [1]. With the intention to derive a general law of the magnetic loss, Steinmetz showed that the magnetic loss follows equation (1) where $\beta$ is a constant equal to 1.6 and $k$, a coefficient depends on the chemical structure of the magnetic material.

$$P_c = k f B_m^\beta$$  \hspace{1cm} (1)

Scientists’ view to Steinmetz work showed that his equation is limited to pure ferromagnetic materials [2]. It was also shown that the equation does not hold true at low and high (near the saturation region) magnetic flux density [2-3]. Although, the limitations found in the Steinmetz equation and the scientists’ criticism of his equation considering it as a fundamental physics law for magnetic loss, the equation has found extensive utility from an engineering perspective. Steinmetz equation was generalized to form (2) in order to account the well-known eddy currents effect. The three parameters in equation (2) are known as the Steinmetz parameters, but unfortunately, $k'$ has deviated from its original definition, initially introduced by Steinmetz as magnetic resistance in his original paper [1]. This change in the definition of $k$ leads to a misunderstanding of the original Steinmetz equation resulting in significant limitations in the developed Steinmetz-like equations such as the Generalized Steinmetz equation (GSE), the Improved GSE and the improved$^2$ GSE as we will discuss in section II of this paper. Steinmetz equation is a standard method to calculate core loss given in manufacturers’ datasheet. Loss curves are commonly given in decimal logarithmic graphs making its utilization a little complex and a source of error. The calculation of ($k$, $\alpha$, $\beta$) by curve fitting techniques and the use of (2) is more practical and efficient. On the other hand, curve fitting techniques require the use of high number of parameters in order to reduce the error. In general, the average error Steinmetz equation for sinusoidal excitation using fitting technique can be set as lower as 10% when suitable and accurate curve fitting methods are applied over narrow and limited $f$ and $B$ ranges. It is necessary to express the Steinmetz parameters as function of $f$ and $B$ to increase the model accuracy over wide range of $f$ and $B$, however, this process makes the model not physically meaningful. In such case, we obtain a mathematical model in which we have a variable expressed as a power of the same variable which is of no physical sense.

$$P_c = k f^\alpha B_m^\beta$$ \hspace{1cm} (2)

As previously stated, in power electronics research, Steinmetz equation is regarded as a fundamental reference to deal with core loss of non-sinusoidal excitations. Several models have been developed on the basis of Steinmetz equation mainly to solve this issue. Research on this subject were carried out to solve three main problems. The first problem, which has been deeply broaden in the literature, is the core loss of an alternating non-sinusoidal excitation. It is applied to transformers used in several DC-DC and resonant converters and to inductors in resonant converters.
second issue is related to the unsymmetrical excitations known as the effect of the DC bias on the core loss. This problem is a complex task and accurate models to physically qualify and quantify the phenomenon are not yet developed. This issue is encountered in DC inductors used in DC-DC converters. The last issue is related to the zero-voltage loss known as the relaxation loss and this one is much more complicated than the previous. The sources affecting the core loss go beyond the three former issues to include many other effects such as temperature, stress, material permeability, core size and geometry. A detailed discussion of these effects and their complexity can be found in [4].

From a point of view classification that is commonly adopted by researchers, Steinmetz empirical approach is considered as one of three principal approaches presented in the literature. The second approach to calculate core loss is the mathematical modeling of the hysteresis loop. Hysteresis is a phenomenon that describes the lag of the magnetic flux density (B) to the magnetic field intensity (H). This lag leads to the formation of a nonlinear loop known as the hysteresis loop. The area of the loop is by definition the core loss of a complete magnetization cycle [2-3]. Preisach and Jiles-Atherton models are the most referred models in the literature [5-6]. This approach has several drawbacks. The first drawback is the complexity of the mathematical equations, which require the calculation of several parameters. The second drawback is that they only account for the static loss and they do not take into consideration the dynamic loss resulting from eddy currents effect. Some attempts were performed to account the dynamic loss, however, the results still missing the verification with experiments [7-8]. Furthermore, in this approach, it is not possible to calculate the relaxation loss as this type of loss are generated after the removal of the excitation and cannot be manifested in the hysteresis loop. In our paper, we will not review the works based on this approach as it has a limited application in the design of high frequency magnets.

The third approach used for loss calculation is the loss separation technique. The loss model of this approach is the sum of three loss terms known as the hysteresis (static) loss, the eddy current loss and the excess loss. The original idea to express the total core loss as a sum of three kinds of losses refers to Steinmetz. In Steinmetz paper, the dynamic loss term was derived empirically in a similar way to the static loss from the measured loss. The elaboration and the set of the theory of this concept was established by Bertotti. Because of the stochastic behavior of the hysteresis phenomenon, the theory was first developed in a statistical manner. The solution was given in microscopic scale for a magnetic objects (MO) representing a group of domain walls. This group is a Bloch wall which contains large domains. Later on, a deterministic solution was derived in order to make easy its application. The theory is usually known the Statistical Theory of Losses (STL). It was verified and intensively used for very low frequency (few hundreds Hz and for magnetic materials with less than 5% of alloy). The application of this technique is widely used in calculating the core loss of the AC-drive electric machines where the eddy currents plays a major role in increasing the magnetic loss. Although the derivation of Eq (3) is based on a deep theoretical investigation of the eddy current loss and, as reported in the literature, it give more understanding of the physics of core loss, it is not a common approach for high frequency magnetic devices and Steinmetz approach is more favored.

\[ P_c = P_h + P_d + P_{ex} \]  
\[ P_d = \frac{\pi d^2 B^2 f^2}{6 \rho} \]  
\[ P_{ex} = 8 \left( \frac{6.5 V_a}{\rho} \right) B^{1.5} f^{1.5} \]

The primary goal of this paper is to develop a loss model that, like the Steinmetz equation in accuracy and user-friendliness, is also capable of providing a physical interpretation of the core loss phenomenon—an aspect relatively absent in the Steinmetz model. In our approach, we introduce a new concept of resistance called the magnetic-flux resistance \( R_{mag} \) derived empirically by application of the classic physics square loss law. It is a magnetic propriety characterizing the core loss of the magnetic material.

Prior to that, a review of the different Steinmetz-like loss models for non-sinusoidal excitations and the loss separation techniques are presented. We limit to review these models because they are directly applied to the design of the high frequency magnetic components and their parameters can be fully or partially derived from the loss curves in manufacturers’ datasheet. This review has two main purposes and one reason. First it could be used as a useful reference for researchers working on the subject of core loss calculation showing what potential contributions can be achieved in the future. It could be also considered as a guide for the designers of magnetic components to choose the suitable model for their applications as the paper will highlight the pros and cons of the existing models. The reason of the review is because in the last decade there have been many contributions on core loss modeling, however, the research database is still missing a deep review to discuss and analyze the advantages and the drawbacks of these works. In this paper, we try to analyze the most important contributions of the first approach which have been given a considerable attention and satisfaction is in the scientific community. We will also investigate some works of the last approach that we found them interesting and in deep connection to the first approach and to the design of high frequency magnetic components.

The paper is structured as follows. In the second part, we present the review of the empirical loss models under non-sinusoidal excitation. The loss separation technique is reviewed in the third part. The principle and the fundamental basis of the proposed approach and the analysis of the results with experiments are developed in the fourth part. Finally, the main results of the paper are synthesized in the conclusion.
II. EMPIRICAL LOSS MODEL FOR NON-SINUSOIDAL EXCITATION

Empirical loss models for non-sinusoidal excitations were developed in order to match the sinusoidal loss data given by manufacturers with the non-sinusoidal excitations existing in power electronics converters. The loss data given in the datasheet can be computed using equation (2) with an approximate average error of 10%. The change in the excitation waveform leads to a significant change in the core loss which requires more research efforts to firstly understand the physical effects of this change and then to develop a mathematical model that compute the corresponding loss with sufficient accuracy. In the following, we present the models for triangular and trapezoidal flux excitation and we discuss their pros and cons.

A. Core loss under triangular flux excitation
The rectangular voltage excitation is used in many power conversion circuits such as the full-bridge and the forward converters. Flux waveform which includes a DC component is not the subject of this paper. An asymmetric triangular flux waveform (Duty cycle \( \neq 50\% \)) causes an increase in the core loss. The core loss tends to increase as the charging and discharging time asymmetry increases. This asymmetry is expressed as a function of the duty cycle (D). Different models have been published in the literature in order to accurately calculate the core loss under this excitation shape. The difficulty in modeling core loss for this waveform is how mathematically includes the duty cycle in the Steinmetz equation without losing the consistency and the physical interpretation of the effect of each variable and parameter. In the following, we discuss the principles, the advantages and the drawbacks of the main models developed in the literature.

1) Modified Steinmetz equation
The Modified Steinmetz Equation (MSE) is the first model developed to calculate core loss under non-sinusoidal excitation [9]. Its mathematical formula is given in the following expression (6):

\[
P_c = (k \cdot f_{eq} \cdot B_m^\beta) \cdot f
\]

where \( f_{eq} \) is the equivalent frequency to the average change of the magnetic flux density during a hysteresis cycle. The concept of the equivalent frequency for the core loss calculation was used in several other models such as the works in [10] and [15]. The MSE for the triangular flux waveform is expressed as follows:

\[
P_c = W_{rec} \cdot k \cdot f^\alpha \cdot B_m^\beta
\]

\[
W_{rec} = \left( \frac{2f}{\pi^2 D - \pi^2} \right)^{\alpha - 1}
\]

The Steinmetz parameters are calculated at the switching frequency \( f \) and not \( f_{eq} \). As priory said, the core loss increases as the duty cycle decreases, however, the implementation of (7) given in Fig.1 shows the opposite expected change for \( \alpha \) lower than 1 (\( \alpha = 0.9 \) in the figure) which makes the model inefficient and not suitable for such case. Parameter “\( \alpha \)” is usually higher than 1, however, since there are infinity of solutions of Steinmetz parameters \( \{k, \alpha, \beta\} \) for the same loss curves, it is possible to have “\( \alpha \)” lower than 1. This case can be encountered when using advanced technique such as the least square fitting technique. In order to avoid such issue, the Steinmetz parameters need to be calculated with \( \alpha \) being imposed higher than 1 which can also reduce the model accuracy and limits its physical meaning. The MSE accuracy was verified in [5] with a flux waveform of two harmonically related sinusoids and the results shows that its prediction is much lower than the experimental results.

Fig. 1. Effect of \( \alpha \) on the loss evolution of MSE

From a point of view consistency with the Steinmetz equation and the core loss physics, the MSE has several issues which are related to the basic of its derivation. First, the use of two variables for the frequency \( f \) and \( f_{eq} \) in the same equation is confusing from a physics perspective and complicates the understanding of the physical interpretation of the model. In addition to that, expressing the hysteresis loss as a function of \( f_{eq} \), which in turn is function of the switching frequency (8), is misleading because the hysteresis energy loss is, by definition, independent on the frequency. It would be more convenient and reasonable that the additional term, to account the asymmetry of the waveform, to be independent on the frequency.

In summary, the MSE model lacks the accuracy of loss prediction and the absence of a meaningful physics interpretation of the model.

2) Generalized Steinmetz Equation

The Generalized Steinmetz Equation (GSE) is based on the hypothesis that the core loss of a non-sinusoidal excitation can be modeled as a function of \( B(t) \) and its time derivative \( dB(t)/dt \) [10]. The three parameters are determined by identification to the Steinmetz equation for the case of a sinusoidal excitation. The parameters \( a \) and \( b \) correspond to \( \alpha \) and \( \beta \) respectively, while \( k_1 \) is given in equation (10).

\[
P_c = \frac{1}{k_1} \int_0^T \frac{|dB(t)/dt|^a}{k} \cdot |B(t)|^b \cdot dt
\]

\[
k_1 = \frac{(2\pi)^{a-1} \int_0^\pi |\cos \theta|^a |\sin \theta|^b \cdot d\theta}{k}
\]

The GSE shows better results than the MSE, however, it has some disadvantages and limitations. The first limitation is the difficulty in calculating \( k_1 \) which requires numerical tools and this can be accompanied with a lack of accuracy. The second drawback is the inconsistency with the Steinmetz equation.
equation. Although, it has been claimed by the GSE authors that the model is consistent with the Steinmetz equation, it is clear that this is not correct as explained in the following. First $k$ in the Steinmetz equation was defined as a propriety of the magnetic material independent on $B$ and $f$ at low frequency. But, in the GSE model, as given in (10), it is dependent on $\alpha$ and $\beta$ which has no physical meaning or interpretation. Second, $\beta$ in the Steinmetz equation was found to be close or slightly higher than 2 in most of ferrite materials, which quietly agrees with the hysteresis loss law in which the losses are proportional to $B^2$. However, in the general case where $B(t)$ is non-linear, it can be seen in the GSE model (9) that $\beta$ corresponds to $\beta-\alpha$ making the hysteresis loss nearly proportional to $B$ instead of $B^2$. Furthermore, the dependence between the model parameters increases the error and reduces its accuracy. GSE has other limitations such as its low accuracy at low duty cycle. On the other hand, the advantage of the GSE is it offers the possibility to be implemented in transient spice simulation [11].

3) Improved Generalized Steinmetz Equation

This model is an improved version of the GSE and it is based on the composite waveform hypothesis (CWH) [12]. The CWH is principally a direct application of the superposition technique if we consider $db/dt$ as the source. The superposition technique is applicable in this case even that the problem is nonlinear because the different sources $db/dt$ are acting at different time instants. The CWH states that the average loss during one cycle is the sum of all the loss of each portion of the flux waveform. A portion is defined as the time interval during which the flux density is linearly dependent on or constant with time. Such concept can hold true for any waveform. In the IGSE, the instantaneous magnetic flux density in the GSE model is replaced by its peak-to-peak value for two reasons. First, it simplifies the integral calculation and second it improves the accuracy. This simple change allows also to account the core loss due to minor loops and gives better consistency to Steinmetz equation in comparison to the GSE. The mathematical expression of the IGSE is written as follows:

$$P_c = \frac{1}{T} k_i \int_0^T \left| \frac{dB(t)}{dt} \right|^a \left| \Delta B \right|^b dt \tag{11}$$

$$k_i = \frac{(2\pi)^{\alpha-1} 2^{\beta-\alpha} \beta^2}{k_c \cos \theta} \left( e^{-\alpha} \right) \tag{12}$$

In addition to its easy application, the IGSE has shown better accuracy than the GSE as $k_i$ calculation has lower error to its equivalent in the GSE. On the other side, The IGSE account one drawback which is the parameter $k_i$ which needs to be calculated numerically. An approximate formula was given in [12]. Similarly, to the GSE, the main limitation of the IGSE is its error increases significantly at very low duty cycle. As explained for the GSE, the dependency between the Steinmetz parameters is obviously a significant source of error that reduces the accuracy. For instance, the error of $k_i$ due to 10% error in $\alpha$ and $\beta$, can reach up to 30% as shown in Fig. 2. The total error of the model is obviously much higher which decreases its accuracy. In addition to that, the IGSE loses significantly the accuracy as the duty cycle moves away from 0.5. Its inaccuracy can reach more than 100% at $D=0.05$ [9].

Finally, although the IGSE has gained a bit of consistency with the Steinmetz equation by using $B_m$ compared to the GSE, the model is considered as inconsistent with Steinmetz equation because the Steinmetz parameters in the IGSE are dependent on each other’s which contradicts with what is defined in [1].

4) Waveform-Coefficient Steinmetz Equation

The Waveform-coefficient Steinmetz equation (WcSE) supposes that the increase in the core loss for triangular flux waveform is due to the flux shape [13], a waveform factor ratio (FWC) between the triangular and sinusoidal shapes was calculated to improve the accuracy of the model.

$$P_c = FWC \ k \ f^{\alpha} B_m^{\beta} \tag{13}$$

Unfortunately, the factor (FWC) does not include the effect of the waveform asymmetry. The WcSE is only valid for a duty cycle of 0.5. The foundation WcSE, based on a symmetric flux waveform, failed to account the real physics causes of the core loss for asymmetric flux waveform such as the time derivative of the magnetic flux density. Its accuracy is expected to be worse than the IGSE. The later core loss cause is overcome by the ISE described in the following section.

5) Improved Steinmetz Equation

The Improved Steinmetz Equation (ISE) was proposed in [14]. The principle of the core loss calculation of this method is based on the CWH. Its application for the case of the triangular flux waveform excitation is described as follows. The average core loss generated during the first half-cycle (on-time) is equal to the half of the core loss that is generated due to an effective frequency equals to $\frac{1}{DT}$ while the average core loss of the second-half is due to an effective frequency of $\frac{1}{(1-D)T}$. Both amounts of loss are calculated using the Steinmetz equation. The main difference with the previous models is that the Steinmetz parameters of each case are determined at the corresponding effective frequency and not at the switching frequency. For the case of a rectangular excitation, the ISE is simply written as follows:

$$\langle P_c \rangle = \frac{\pi}{4} (k_{e1} D \ f_{e1} a_{e1} B_{\beta e1} + k_{e2} (1-D) f_{e2} a_{e2} B_{\beta e2}) \tag{14}$$

$(k_{e1}, a_{e1}, \beta_{e1})$ and $(k_{e2}, a_{e2}, \beta_{e2})$ are the Steinmetz parameters for the corresponding effective frequency $f_{e1} = \frac{f_1}{D}$ and

![Fig. 2. K error due its dependence on $\alpha$ and $\beta$, x-axis is the error of $\alpha$ or $\beta$](image-url)
\( f_{eq} = \frac{f_1}{1-D} \) \( \pi \) is a constant characterizing the loss factor between the core loss of sinusoidal and square excitations for some materials [13]. The ISE has several advantages over the previous models which can be summarized as the following. First and unlike the previous models which are expressed in an integral form, the ISE has exactly the same form as the Steinmetz equation and the same definition of the Steinmetz parameters. This gives more simplicity and easiness to the user and it conserves the same physical meaning of the Steinmetz equation. The ISE can be physically interpreted since the parameters are independent on each other. \( "k" \) remains the coefficient that depends on chemical proprieties of the magnetic material which in turn depend on the frequency and the temperature and slightly on \( B \). From a point of view, the ISE has shown better accuracy than other models which results from the consideration of the effective frequency effect on the Steinmetz parameters. Its efficacy is more manifested at the very low duty cycle at which the GSE and the IGSE have both shown a significant failure.

One major disadvantage of the ISE is the calculation of the Steinmetz parameters when the effective frequencies \( f_1 \) and \( f_2 \) are not within the range of tested frequency of the datasheet and this may happen at high switching frequency and extreme duty cycle. Its error might be worse than other models. It is difficult to predict an accurate value of the Steinmetz parameters in such cases and it is more useful to use the first model of the ISE developed in [14]. More discussions about this issue is described in the section II.C.

6) Stenglein effective frequency loss equation (SEFLE)
The concept of this method is similar to the MSE equation and its principle is to express the core loss of non-sinusoidal excitation as a function of an equivalent frequency [15]. Unlike the MSE where \( f_{eq} \) is derived from the average change of the flux, the equivalent frequency of the SEFLE is derived from the second derivative of the magnetic flux density. The given reason is to account the fast-dynamic change of the flux within the turn-on and turn-off instants. The phenomenon was described as like the core is excited with a Dirac pulse at the extrema of the flux waveform. The SEFLE model for rectangular excitation is given by the following equation:

\[
P_c = f \cdot k \cdot B_{pk}^\beta \cdot f_{eq}
\]

\[
f_{eq} = \left( 1 + c \left( \frac{2f}{\gamma (1-D)} \right)^\gamma \right)
\]

Where \( c \) and \( \gamma \) are two parameters which can determined from loss data in datasheet. Fig. 3 shows the loss ratio between a triangular and sinusoidal flux waveform using MSE and SEFLE models. In general, the MSE and the SEFLE have similar convergence at low duty cycle. Similarly, to the MSE, the SEFLE shows a good accuracy within the duty cycle range [0.1-0.9]. It can be also observed that the MSE fails for 3C90 as the material parameter \( \alpha \) approaches to 1, however this limitation is solved for the SEFLE as it can be understood from eq(16). The verification of the SEFLE model at 0.05 and 0.95 duty cycle clearly shows a significant error which can reaches more than 100%. It was shown in [14] that the loss ratio varies from 2 to 4 in ferrite materials.

This later drawback is an indication of the weakness of the model basics and the concept of the equivalent frequency. From a point of view practicability, the SEFLE model needs four parameters which can be extracted from the loss curves given in the manufacturer’s datasheet. Parameters \( k \) and \( \beta \) can be derived from low frequency loss data while \( c \) and \( \gamma \) are derived from high frequency loss data. This requires to perform additional experiments at low frequency to fully determine the model.

![Fig. 3. Loss ratio between the rectangular and sinusoidal excitation for the MSE and SEFLE models, f=100 kHz.](image)

7) Improved Generalized Composite Calculation
The improved Generalized Composite Calculation (IGCC) applies the CWH and inherits its main advantage (accuracy) from the basic concept of the ISE. The equation of the IGCC is given by (17-19) [16],

\[
P_c = f \sum_{i=1}^{n} p_{sym}(\tilde{f}(t),B_{pk}) \Delta t_i
\]

\[
P_{sym} = \lambda(f)B_{pk}^\beta(t)
\]

\[
\tilde{f}(t) = \frac{\frac{dr}{dt}}{2B_{pk}^\beta}
\]

\( \tilde{f}(t) \) represents the equivalent frequency to the local change of the flux. \( k \) and \( \alpha \) are embedded in one equation called \( \lambda \) which is a function of the equivalent frequency. \( \beta \) and \( \lambda \) are dependent on the equivalent frequency and they can be fitted using a third order polynomial. Replacing \( k \) and \( f \) by a third order equation makes the model of no physics background as the effect of the frequency is no longer clear and the \( k \) parameter is no longer existing. In addition to that, there is no explanation of the basics and the background of \( \tilde{f}(t) \) which is a weak spot of the model as explained in the following. First, the dependence of \( \tilde{f}(t) \) on \( B_{pk} \) has no physics foundations. It appears that the factor \( 1/B_{pk,\lambda} \) is used to compensate the difference between the local \( B \) of each time segment and which is the actual field for generating the core loss of that segment and the total \( B_{pk} \) which is the main field of the average loss. This solution will have a significant error because \( B \) and \( \tilde{f} \) have unequal effects on core loss. This error can be manifested in irregular trapezoidal flux waveform in which there is no constant flux. The IGCC shows a significant improvement in the accuracy for the triangular excitation and a specific trapezoidal waveform (not including zero-switching time) compared to the IGSE, however, its accuracy is not yet verified for the general case of the trapezoidal excitation including relaxation loss [22-23]. The maximum registered error mentioned in [16] for the IGCC and the IGSE in the case of a triangular flux waveform is about 10% and 20% respectively, tested for N87 material.
This improvement in the IGCC is resulting from the application of the ISE.

B. Core loss under Trapezoidal flux excitation

The trapezoidal excitation is generated in numerous power electronics circuits such as the Phase-shift FB converter, the Dual-active-bridge converter. It was shown that core loss under this excitation is higher than the one of the rectangular excitations of same frequency [18]. Taking into consideration this loss increase in the design process of the transformer is of great importance.

The reason of the loss increase is due to the relaxation effect which is generated during the zero-voltage time. The impact of this effect on the core loss was investigated in the literature and there have been few attempts to calculate its loss [2] [12]. In order to fairly assess the accuracy of each method and the consistency of its mathematical model with the physical behavior, a good understanding of the fundamental physics of the relaxation loss is required. In the following, we shortly remind the principle of the relaxation loss and thereafter, we examine the different models.

1) Relaxation loss in magnetism

The relaxation loss is a well-known phenomenon in magnetism. It characterizes the magnetic loss behavior of a magnetic core upon the removing of a changing magnetic field acting on the core. It depends on the chemical composition of the material, the temperature, the relaxation time and the historic loss [3] and [17-18]. The characterization of the relaxation loss was investigated in power electronics research in few research papers [19-23]. The former references have explored this kind of loss for the case of a trapezoidal excitation. The phenomenon process is described as follows. For a typical trapezoidal excitation as given in Fig.4 and Fig.5 during the on-time [0, DT], the instantaneous magnetic flux density increases with time resulting in an increase in the instantaneous loss. At the instant DT, the voltage becomes zero giving a constant magnetic flux density during the zero-voltage time. In principle, the instantaneous loss is also zero during this time as there is no changing magnetic field, however, due to the relaxation phenomenon, there is still dissipation of loss.

In the following, we discuss the main models developed in the literature.

2) Improved Generalized Steinmetz Equation

The IIGSE is the first model developed to account the relaxation loss in the trapezoidal excitation for power electronics applications. It includes two separate terms: the first term is the IGSE and the second term is the relaxation loss [20].

\[
(P_r) = \frac{1}{\tau} \int_0^T k_r \left[ \frac{dB}{dt} \right] \frac{\Delta B}{\tau} \Delta B + \sum_{n=1}^{\infty} P_{rt} Q_{rt} 
\]

(20)

\[
(P_{rt}) = \frac{1}{\tau} k_r \left[ \frac{dB}{dt} \right] \frac{\Delta B}{\tau} \Delta B \left( 1 - e^{-\frac{t}{\tau}} \right)
\]

(21)

\[
Q_{rt} = \begin{cases} 
\text{e}^{-q(\frac{t}{\tau})^\mu}, & \text{Triangular waveform} \\
1, & \text{Trapezoidal waveform} 
\end{cases}
\]

The IGSE has better accuracy than the IGSE in the prediction of the core loss for trapezoidal flux waveform as it includes a term for the relaxation loss, however, it degrades the accuracy for the triangular flux waveform. Its drawbacks are listed as follows. First, it obviously inherits the limitations of the IGSE discussed previously. Second, it requires four additional parameters for the trapezoidal version and five parameters for the triangular version. The necessity that these parameters are determined by experiments renders the model impractical and not useful. Furthermore, the fact that the simple case, which is the triangular waveform, requires 5 additional parameters while the complex case (trapezoidal waveform) requires only 4 is not reasonable and is a major shortcoming of the model. This issue comes from the basis of the model which supposes at small duty cycle there is a manifestation of relaxation loss during the off-time. This hypothesis contradicts with the fundamental definition of the relaxation phenomenon defined by the complete removal of the excitation. The implementation of model (20) shows that the relaxation loss continuously increases with the zero-voltage time, an evolution that conflicts with the physical behavior of the relaxation loss and the results found in [22]. The evolution of the relaxation loss with the off-time was investigated in [22] and it has been shown that the relaxation loss exhibits a maximum at a specific time called the critical relaxation time. Another important issue of the IIGSE is that the results was based on experimental test performed on a trapezoidal excitation with constant on-time and variable off-time (Fig. 5) which is not the same excitation that exists in the FB or DAB converters (Fig. 4). The chosen waveform to compute the loss of the trapezoidal flux waveform is not the suitable one because this later depends on its historic loss (on-time loss). In other terms, a change in the on-time loss due to a change of the off-time, leads to a change in the relaxation loss. This point was not considered in the modeling process of the IIGSE.

3) Filtered GSE (FGSE)

The filtered GSE is given by equation (23) [21].

\[
P_r = \frac{1}{\tau} \left[ \int_0^T k_r \left[ \frac{dB}{dt} \right] \frac{\Delta B}{\tau} \Delta B + k_r \left[ \frac{dB}{dt} \right] \frac{\Delta B}{\tau} \Delta B + k_e \left( \frac{dB}{dt} \right)^2 \right ] dt
\]

(23)

The FGSE has three terms. The first term is the IGSE. The third term is the eddy current loss from the loss separation technique. The second term is the term to account the relaxation loss developed by the IIGSE. Despite that the authors state that the FGSE has better accuracy than IGSE and the MSE, the model has several issues. A fundamental issue is the use of the Steinmetz approach and the loss separation approach together in one equation. Steinmetz equation was developed to accommodate the static hysteresis loss and the eddy current loss using the power function (2). In the loss separation technique (Bertotti model), the losses are separated into three parts: hysteresis loss, dynamic loss and excess loss. Therefore, the use of eddy current loss term from Bertotti model and the Steinmetz equation is a clear duplication of the eddy current loss. Eventually, the term of the relaxation loss is underestimated and not accurately
predicted. The FGSE has also the same disadvantages of the IIGSE such as the required additional parameters and more importantly the used approach to compute the relaxation loss.

4) Improved* Steinmetz Equation
This method was developed in [22] and [23]. It is referred here as the Improved* Steinmetz equation. The principle of the IISE is based on the superposition technique. In this case, the use of the superposition technique is valid as the losses are generated at different time slots. The IISE method supposes that the total loss due to the trapezoidal excitation is the sum of two terms of loss: the on-time loss, which can be accurately calculated using the ISE, and the off-time loss also called the zero-voltage loss. This later can be known by extracting the on-time loss from the total loss obtained from measurements. The experimental tests in [22] were performed using a trapezoidal excitation having a fixed on-time and variable off-time while the experimental tests in [23] were performed using a trapezoidal excitation having a variable on-time, variable off-time and fixed period. Fig. 4 and Fig. 5 highlights the main differences between the two waveforms. The objective is to firstly investigate the dependence of the relaxation loss on the on-time and the previous historic loss (on-time loss) from the waveform in Fig.5 and then makes its general application in the second waveform in Fig.4.

![Fig. 4. Typical waveform of the voltage and the flux density in full-bridge converter, operating under constant frequency [23].](image)

![Fig. 5. Typical waveform of the voltage and the flux density operating under variable frequency [22].](image)

The obtained results showed that the relaxation loss slightly increases as the off-time increases until it reaches a maximum at a specific relaxation time and then it significantly decays to very low value. This evolution was proved by the fundamental theory of the relaxation loss [3]. The ratio between the critical relaxation time and the switching period was found to be constant for a given material, for instance, it is equal to 1/2.65 for N87 ferrite. The main advantage of the model in [22] is it showed the physical behavior of the relaxation loss and its dependence on the off-time and the switching period. However, the limitation of the model is it cannot be applied for the practical trapezoidal excitation used in power electronics circuits (Fig.4). This limitation was investigated in [23] which is an extension of [22]. Unlike the case in [22] where the core loss shows a maximum at a specific time, the total core loss for the investigated trapezoidal excitation exhibits a continuous increase as the duty cycle decreases. The theoretical analysis shows that the on-time and the off-time losses, both, increases as the duty cycle decreases. We have mentioned above that the off-time loss depends on the on-time loss (prior or historic loss). This later is the reason for the continuous loss increase of the off-time loss. The experimental results of the total loss showed also the same evolution with respect to the duty cycle.

A simplified model as function of the ISE model and the duty cycle was proposed to approximately calculate the core loss. The advantage of this model it only requires the Steinmetz parameters and the operating duty cycle. It was mentioned that the model still needed some improvements to increase its accuracy, however, it is a useful and straightforward model with acceptable accuracy.

C. Discussion
In this part, we summarize the advantages and the drawbacks of some models for the triangular and trapezoidal flux waveforms. To give a useful assessment and criticism, the reviewed models are examined using the following comparison criteria:

1) Criterion Nº1
Physics background and consistency with the Steinmetz equation: this criterion measures the fundamental basics of the model and the consistency between the mathematical equations and the physics phenomenon of core loss.

2) Criterion Nº2
Accuracy: the accuracy is measured by the error over wide range of frequency, magnetic flux density and duty cycle. We use the results reported in [14], [34] and the results performed in this paper.

3) Criterion Nº3
Simplicity and practicability: the simplicity is measured by the number of parameters used in the model and the practicability is measured by the method applied to determine these parameters.

4) Criterion Nº4
Generality or possibility to be generalized: it measures the number of effects taken into consideration in the model and the model flexibility to be improved and upgraded.
Tab. I and II summarize the advantages and the drawbacks according to the former criteria. In the following, we highlight the main results and we give a brief summary to what it has been concluded.

For the triangular flux waveform, the ISE can be considered as the best model in terms of accuracy, consistency with the Steinmetz equation, practicability and physics background than the IGSE, the IIGSE and the FGSE. The consistency to the Steinmetz equation is an important feature because it gives the model the potential to be generalized to account other effects without affecting its basics. It uses the same independent parameters \((k, \alpha, \beta)\) as Steinmetz equation, however, other models use dependent parameters which make their physical meaning a little ambiguous and vague. In terms of practicability, the IIGSE and the FGSE have included 5 extra-parameters which require additional measurement data to be identified and this complicates its use in the design of magnetic components. The ISE and the IGSE are two straightforward models to apply, however, the ISE performs much better in terms of accuracy as previously stated. This result was reported in some papers such as [14] and [34] and in the following we present more comparison results using the loss data available in the MagNet database [4] and we highlight their major issues to be solved in future works.

Fig. 6 and Fig. 7 show the error of the ISE and the IGSE models for N87 ferrite over the magnetic flux density range [20-300] mT and the frequency range [56-177] kHz at \(D=0.2\) and 0.3 for a triangular flux waveform. In this test, the Steinmetz parameters for IGSE are calculated with high accuracy at each frequency to reduce its error. As it can be seen, at low \(B (20 \text{ mT} \text{ to } 70 \text{ mT})\), both models have significant error higher than 50%. This issue is expected because the Steinmetz parameters are also B dependent at low excitation. It is recommended to treat this range of flux separately to increase the accuracy because within this range the permeability is nonlinear. It is the transition from the initial permeability to the amplitude permeability. It might also be better to use Rayleigh model. For \(B\) higher than 70 \(mT\), the ISE shows better accuracy especially for \(D=0.2\). It was proven that the IGSE accuracy degrades as \(D\) moves away from 0.5. The main drawback of the ISE at low duty cycle is that the effective frequency becomes bigger than the maximum tested frequency of the datasheet which leads to an inaccurate prediction of the Steinmetz parameters and therefore the model accuracy significantly degrades (Fig. 8).

The average error of the ISE and the IGSE for \(D=0.2\) is about 18% and 38% respectively. For \(D\leq 0.3\), the ISE and the IGSE have approximately the same average error as similarly shown is the previous studies. The study in [34] shows that the average 95 percentile error is about 36%. The study was carried on 10 materials and 55816 test points. For 0.9 and 0.1 duty cycle, the average 95 percentile reaches up to 54%. In general, the study in [34] and this study are in good agreement about the limitation of the IGSE at low duty cycle. The IGCC could be a best alternative to the IGSE, however, it does not give a better explanation the physics background of the core loss.

For the trapezoidal flux waveform, the IISE has accurately model the relaxation loss than the IIGSE. The evolution of the IISE relaxation loss model with the off-time showed a maximum at a specific off-time called the critical relaxation time which agrees with the basic theory of the relaxation loss [22-23]. The IIGSE might have accurate prediction of the total loss, however, as it is based on the IGSE, it can be understood that the individual prediction of the relaxation loss and the on-time loss is not accurate. The IISE is also more practical as it only need the Steinmetz parameters and the authors of the IISE have recommended an additional correction factor for each duty cycle. Using a look-up table, the model can be utilized in complex optimization problems. In contrast, the IIGSE and the FGSE requires 4 additional parameters which are not available in the datasheet and which requires extra measurement data. The shortcoming of the IGSE to accurately characterize the relaxation loss and the on-time loss reduces the potential of the model to upgrade the model to accommodating other effects. The high number of independent parameters is not preferred to be used in complex optimization problems as it increases the error and time consuming.

![Fig. 6. Error of the IGSE for N87 ferrite and for D=0.2 and D=0.3, T=90ºC](image)

![Fig. 7. Error of the ISE for N87 ferrite and for D=0.2 and D=0.3, T=90ºC](image)

![Fig. 8. Effective frequency f1 and f2 for a triangular excitation, model (14)](image)
<table>
<thead>
<tr>
<th>Criterion Nº1</th>
<th>IGSE</th>
<th>IIGSE</th>
<th>ISE</th>
<th>MSE/SEFLE</th>
<th>IGCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Frequency and flux effects are included.</td>
<td>• Frequency and flux effects are included.</td>
<td>• Frequency and flux effects are included.</td>
<td>• Frequency and flux effects are included.</td>
<td>• Frequency and flux effects are included.</td>
<td></td>
</tr>
<tr>
<td>• Dependence between parameters has no physics meaning.</td>
<td>• Consistent with Steinmetz equation.</td>
<td>• Consistent with Steinmetz equation.</td>
<td>• Concept of equivalent frequency is not justified.</td>
<td>• Concept of equivalent frequency is not justified.</td>
<td></td>
</tr>
<tr>
<td>• Not consistent with Steinmetz equation.</td>
<td>• Steinmetz parameters are unchanged.</td>
<td>• Steinmetz parameters are modified.</td>
<td>• Steinmetz parameters are modified.</td>
<td>• Steinmetz parameters are modified.</td>
<td></td>
</tr>
<tr>
<td>• Second term of IIGSE is unnecessary.</td>
<td>• Not consistent with Steinmetz equation.</td>
<td>• Not consistent with Steinmetz equation.</td>
<td>• Not consistent with Steinmetz equation.</td>
<td>• Not consistent with Steinmetz equation.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Criterion Nº2</th>
<th>IGSE</th>
<th>IIGSE</th>
<th>ISE</th>
<th>MSE/SEFLE</th>
<th>IGCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Error is less than 10% for symmetric waveform [14] and [34].</td>
<td>• accuracy is not verified.</td>
<td>• Good accuracy</td>
<td>• Good accuracy in the D range of [0.1-0.9]</td>
<td>• Good accuracy</td>
<td></td>
</tr>
<tr>
<td>• Dependence between parameters increases the error [this study].</td>
<td>• It inherits all the limitations of IGSE.</td>
<td>• Accurate over D range [0.05-0.5] [14] and [22-23].</td>
<td>• Error can be higher than 100% for D lower than 0.1 [this study].</td>
<td>• Error less than 10% [16].</td>
<td></td>
</tr>
<tr>
<td>• Error varies between 30% and 60% for D=0.05 [14].</td>
<td>• Second term of the model adds error</td>
<td>• Accuracy degrades for effective frequency not within the datasheet frequency range [this study]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• 95 percentile Error reaches an average of 54% for D=0.9 tested for 10 materials [34].</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• 95 percentile Error reaches an average of 36% for D∈[0.1-0.9] and 55816 test points [34].</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Error is higher than 70% at very low B [this study].</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Criterion Nº3</th>
<th>IGSE</th>
<th>IIGSE</th>
<th>ISE</th>
<th>MSE/SEFLE</th>
<th>IGCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Easy application</td>
<td>• Complex and not applicable.</td>
<td>• Easy application</td>
<td>• 3 parameters for MSE</td>
<td>• Easy application</td>
<td></td>
</tr>
<tr>
<td>• 2 parameters derived from datasheet.</td>
<td>• Three parameters derived from datasheet and 5 parameters from extra measurements.</td>
<td>• 3 parameters for ( M_{SE} )</td>
<td>• 4 parameters for ( SEFLE )</td>
<td>• 8 parameters.</td>
<td></td>
</tr>
<tr>
<td>• Parameter ( k ) requires numerical tool</td>
<td></td>
<td>• extra-measurement at low frequency required by ( SEFLE )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Criterion Nº4</th>
<th>IGSE</th>
<th>IIGSE</th>
<th>ISE</th>
<th>MSE/SEFLE</th>
<th>IGCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Valid for triangular waveform.</td>
<td>• Valid for triangular waveform.</td>
<td>• Valid for triangular waveform.</td>
<td>• Valid for triangular waveform.</td>
<td>• Valid for triangular waveform.</td>
<td></td>
</tr>
<tr>
<td>• Not valid for unsymmetrical waveform.</td>
<td>• Can be generalized trapezoidal waveform.</td>
<td>• Not valid for trapezoidal waveform.</td>
<td>• Not valid for trapezoidal waveform.</td>
<td>• Valid for triangular waveform.</td>
<td></td>
</tr>
<tr>
<td>• Not valid for trapezoidal waveform.</td>
<td></td>
<td>• Not valid for unsymmetrical waveform.</td>
<td>• Not valid for unsymmetrical waveform.</td>
<td>• Valid for trapezoidal excitation with no zero-switching time.</td>
<td></td>
</tr>
</tbody>
</table>
Tab. II summary of the pros and cons of Steinmetz-like equations for trapezoidal flux waveform

<table>
<thead>
<tr>
<th>Criterion Nº1</th>
<th>IIGSE</th>
<th>ISE</th>
<th>FGSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>BF100</td>
<td>• Relaxation loss included but underestimated.</td>
<td>• Relaxation loss dependent on the historic loss.</td>
<td>• Relaxation loss included but underestimated.</td>
</tr>
<tr>
<td>BF101</td>
<td>• Relaxation loss independent on the historic loss.</td>
<td></td>
<td>• Relaxation loss independent on the historic loss.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Criterion Nº2</th>
<th>IIGSE</th>
<th>ISE</th>
<th>FGSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>BF110</td>
<td>• Testing data are limited.</td>
<td>• Testing data are limited.</td>
<td>• Testing data are limited.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Criterion Nº3</th>
<th>IIGSE</th>
<th>ISE</th>
<th>FGSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>BF120</td>
<td>• Complex and not applicable.</td>
<td>• Easy application</td>
<td>• Complex and not applicable.</td>
</tr>
<tr>
<td>BF121</td>
<td>• Three parameters derived from datasheet and 4 parameters from extra measurements.</td>
<td>• Three parameters derived from datasheet and one parameter from a look-up-table.</td>
<td>• Three parameters derived from datasheet and 4 parameters from extra measurements.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Criterion Nº4</th>
<th>IIGSE</th>
<th>ISE</th>
<th>FGSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>BF130</td>
<td>• Valid for trapezoidal waveform in Fig.5.</td>
<td>• Valid for trapezoidal waveform in Fig.4 and Fig.5.</td>
<td>• Valid for trapezoidal waveform in Fig.5.</td>
</tr>
</tbody>
</table>

III. LOSS SEPARATION TECHNIQUE

The loss separation technique supposes that the total loss is the sum of three independent kinds of loss. The first term is the static loss known as the hysteresis loss or the original Steinmetz equation which can be determined from loss measurement at low frequency. Its energy loss is frequency independent and directly proportional to \( B^0 \) and linearly dependent on \( k \) defined as the material propriety and called the magnetic resistance in Steinmetz paper [1]. The second term is the dynamic loss resulting from the eddy currents effect. The third term is called the excess loss which is proportional to \( f^2 \) and \( B^2 \).

\[
P_c = f \left( k_h \beta_m + k_e f B_m^2 + k_e f^{0.5} B_m^{1.5} \right) \quad (24)
\]

The loss separation model was proposed by Steinmetz in 1908 in his famous paper “on the law of hysteresis” as an empirical model without giving any explanation of its physics theory [1]. The original equation of Steinmetz does not include the excess term and the dynamic loss term is same as the second term given in equation (24). The analytical expression of the eddy currents loss term was firstly given by solving Maxwell’s equations in [25]. The solution was derived with the assumptions of a homogenous distribution of the magnetic field and perfect structure of the domain walls (uniform spacing and same length) for sheet laminations. The eddy currents loss was shown to be proportional \( f^2 \) and \( B^2 \) and depending on the length to the spacing ratio of the sheet laminations. The main limitation of [25], which is the negligence of the inhomogeneity in the magnetic materials structure and in the distribution of the magnetic field, was solved by Bertotti using a statistical approach [26-28]. The excess loss was determined by Bertotti using his statistical approach considering the effects of the material microstructure and the lamination geometry. The experimental verification of Bertotti’s formula for several ferromagnetic materials shows a good accuracy of the model. Bertotti model was adopted by most of researchers especially for the loss calculation of electric machines where the excitation voltage carrying high frequency harmonics.

The first version of Bertotti model is a microscopic model. In the second version, the model was upgraded to a macroscopic level using a deterministic method [29]. In order to improve the accuracy of the model for various materials and applications, the model parameters require some alteration using loss measurements of magnetic materials. This process makes of Bertotti model a semi-empirical model. The eddy currents term includes factors from experiments and theory.

Several semi-empirical models based on the loss separation technique were developed in the literature, mainly to account the effect of the excitation waveform and the magnetic material composition. As an example, in [30], Fourier Series was applied to Bertotti model to account the effect of harmonics generated by non-sinusoidal excitations in ferromagnetic laminations. In general, the application of Fourier series for loss prediction in non-sinusoidal excitations does not give accurate results as it is only valid for linear problem while the core loss is a complex nonlinear problem.

In [31], the effect of the duty cycle for triangular and trapezoidal flux waveform was integrated in the classic eddy currents and excess losses terms as given in eq (25) and eq (26) respectively.

\[
P_c = k_h \beta_m + k_c \left( \frac{d}{2} \right)^2 + \left( 1 - \frac{3}{5} \right)^2 \left( \frac{d}{2} \right)^2 + k_e \left( \frac{d}{2} \right)^{1.5} + \left( 1 - \frac{3}{5} \right)^{1.5} \left( \frac{d}{2} \right)^{1.5} \quad (25)
\]

\[
P_c = k_h \beta_m + k_c \left( \frac{d}{4} \right)^2 + k_e \left( \frac{d}{4} \right)^{1.5} \quad (26)
\]
Where $k_c$ and $k_e$ are duty cycle dependent and determined by curve fitting from measurements. As it can be seen in the equations, the peak flux density in Bertotti model is replaced by $dB/dt$. It is clear that the developed models are erroneous because the effect of the duty is cancelled by $1/\mu dt$. The claimed experimental results showed that the duty has a very negligible effect on the core loss of nanocrystalline and amorphous material while its effect is significant on silicon steel material. On contrary, in [32], the effect of the duty cycle was found to be significant on nanocrystalline material. For instance, it was reported that the loss increases at $D=0.2$ was about 160% and 120% for $f=5$ kHz and $f=1$ kHz respectively for 1K107B nanocrystalline material. The difference in results on the duty cycle effect on nanocrystalline between studies [31] and [32] shows the need for further investigation on this subject, however, we think that the study of [31] lacks the accuracy on the loss measurements as the duty cycle effect was verified by so many studies such as [32]. One other drawback of model (26) is it does not account the relaxation loss which can be very significant at low duty cycle as shown in [19-22]. In [33], the loss separation technique was used to characterize the loss for nanocrystalline and ferromagnetic materials under triangular flux waveform in which the permeability is highly frequency-dependent. The effect of the change in the permeability is considered by applying complex permeability obtained by curve fitting technique from the datasheet and integrated in the magnetic field solution of Maxwell’s equations. To account the duty cycle effect, a technique called loss decomposition and the use of the effective frequency is applied similarly to what is proposed by the ISE method and the principle of the CWH [14]. The results show that the duty cycle has a significant effect on core loss in various materials (nanocrystalline, MnZn, Fe-Co). However, the effect of the duty cycle between materials is different with the frequency change. As an example, Finemet nanocrystalline material has a capability to keep the effect of the duty cycle constant over the frequency range [1 kHz- 1MHz]. In contrast, it was mentioned that the effect of the duty cycle tends to increase with the frequency for N87 ferrite material.

In general, the loss separation technique has found wide applications and it was adopted by researchers equally to Steinmetz approach. From our point of view, this approach has some limitations related to its basic foundations. In the following, we shortly remind its theory, then we present the reasons of our view.

As its mentioned above, the loss separation technique is a principle stating that the total loss is the sum of three independent kinds of loss. Its foundation was based on this hypothesis by Steinmetz, however, the author did not give any physical explanation or a theoretical proof for this idea. The use of the summation technique and the application of the stochastic method by Bertotti was explained by the discontinuous aspect of the hysteresis known as the Barkhausen jumps. Although the theory of this principle was solved by a rigorous mathematical development, the approach does not reach to give an insightful and a correct explanation of the core loss phenomenon. The first point to argue on this approach is that it gives the impression that there are different acting magnetic fields ($B$) in the loss generation. As it can be noticed in eq (24), the weight of the used power-parameters of the magnetic flux density $B$ and the frequency changes from one loss term to another. In principle, the magnetic flux density and the frequency, each of them as physics variables causing the loss generation, cannot have more than one power exponent. The magnetic flux density $B$ is by definition the force that causes the loss. At a given instant of time, this force has only one and a unique parameter to measure its power. In fact, the presence of more than one parameter for $B$ or for $f$ is confusing from a physics point of view when all kinds of loss happen at the same instant of time. The differing exponents assigned to $B$ can be interpreted physically as follows: at a specific moment in time, the force exerted by $B$ can exist in two distinct spatial positions, each with varying power. It is more plausible to consider the force to have a consistent exponent while exhibiting different effects contingent on its spatial position, which pertains to the material in question, specifically the magnetic material in our case. Adhering to this perspective, the impact of eddy currents manifests as a modification in the property of the magnetic material and not the force $B$ itself.

We can also add the following argument to explain the limitation in Bertotti model. How the eddy currents effect is manifested as a change in the AC resistance of the winding, however, their effect manifests as a change in the magnetic loss in the magnetic core, while both cases are matter phenomenon? This asymmetry in accounting the two types of loss opposes the nature of electric and magnetic energy formulated by Maxwell’s equations. From a mathematical view, it is possible to model as (24), however, such model does not correctly reflect the physical phenomenon despite that it can be accurate. Nevertheless, there is no doubt that the effect of the eddy currents on core loss is a fundamental physical and experienced fact that should be characterized and calculated by a mathematical expression, however, its effect could be manifested on a change on the magnetic material. In other words, we believe that is possible and more convenient to see the effect of the eddy currents on the material in a similar way to the AC resistance and not on the loss. The alternative approach that we propose is based on this idea and it is developed in the following section.

IV. PROPOSED APPROACH

The proposed approach is developed using analogy with the classic physics loss theory such as the conductive loss which is proportional to the square of the electric current and the friction loss of fluids which is proportional to the square of the fluid velocity. In our point of view, we think that there is no reason that the magnetic loss to be an exception of the classic physics laws in a way it does not follow the square law.

It is possible to generalize the square loss law to the following law by assuming that the magnetic loss is proportional to the magnetic energy as given in (27).

$$P_m \propto E_m = \frac{\mu_0 m^2}{\mu_a}$$

(27)

The nature of energy loss implies that it is a function of energy consumed. As magnetic energy is proportional to $B^2$,
it logically follows that energy loss is also $B^2$-dependent. This hypothesis is also valid for the calculation of the AC resistance and can be simplified to the square law because the electric field is a conservative field. Equation (27) gives the following fundamental consequences:

- Core losses are function of the square of the circulating magnetic flux density “$B$” within the core. On this basis, the physics square loss law is also conserved.
- Core losses are linearly proportional to the frequency.
- Core losses are inversely proportional to the amplitude permeability of the magnetic material. This later depends on the magnetic flux density and the temperature. We assume, based on the general definition of $\mu = \frac{\Delta B}{\Delta H}$, that the permeability is by definition independent on the frequency as long as the switching frequency is lower than the self-resonant frequency of the magnetic material. The choice of the amplitude permeability is made as it is the suitable one for the chosen operating region of $B$. The mathematical model of the amplitude permeability could be determined by curve fitting technique from the datasheet as follows:

$$\mu_a = K (T - 25) + \mu_a(25^\circ C)$$  \hspace{1cm} (28)  
$$\mu_a(25^\circ C) = s B^2 + r B + t$$  \hspace{1cm} (29)  
$$K = (a B^3 + b B^2 + c B + d)$$  \hspace{1cm} (30)  

equation (28) supposes that the amplitude permeability is linearly dependent on the temperature. The linearity with the temperature is assumed to hold true in a similar way to what is observed for the initial permeability. This later is linear to the temperature within a wide range of temperature in most ferromagnetic materials. In case the linearity is not verified for some materials, the equation could be altered to account the nonlinear behavior. Although the mathematical definition of the two types of permeability ($\mu$ and $\mu_a$) are different, the physical nature of these types of permeability is the same which is the capability of the material to conduct a magnetic field. Each one is defined at a specific magnetic flux range. The permeability slope ($K$) is non-linearly dependent on $B$ and can be accurately fitted with a third order equation as given in (30). Fig.10 shows the variation of the permeability slope ($K$) of the tested materials.

One reason of the nonlinearity of the slope $K$ as function of $B$ is due to the inhomogeneity of the chemical compound of the magnetic material. It is essential to emphasize that equation (30) is not physically meaningful because it incorporates distinct exponents to the physical quantity (B). Instead, Equation (30) can be understood as the aggregation of various functions ($a \cdot B^3$, $b \cdot B^2$ and $c \cdot B$ etc.). Each of these functions characterizes the effect of $B$ on a self-effective permeability associated with the individual chemical element of the ferromagnetic material. A similar interpretation can be applied to equation (29).

- Core losses are function of a physical quantity called the magnetic-flux-resistance ($R_{mag}$) determined by compensation of the magnetic energy term from the measured loss. It depends on the frequency, the temperature and the magnetic flux density. It is by definition a unitless quantity.

Finally, the equation of the core loss per unit volume is given as follows:

$$P_L = R_{mag} \frac{f B_m^2}{\mu_a}$$  \hspace{1cm} (31)  

The magnetic-flux resistance is a concept that characterizes the core loss and it does not have any role on the magnetic energy. The electromagnetic energy is basically determined by the reluctance, defined as the analog of the electric resistance $R$, however, the energy loss is characterized by the magnetic-flux-resistance as defined in this paper. In fact, the reluctance could be understood as the electric resistance from energy perspective and the magnetic-flux-resistance could be understood as the electric resistance from core loss perspective.

The concept of the magnetic resistance is not new. It was proposed by Steinmetz as the name for $k$ parameter in (2). In this paper, it is proposed as a true unitless physical quantity to characterize the core loss.

In the following, we proceed with the analysis of the proposed approach with experimental results.
A. Loss data to calculate the Magnetic-flux-resistance

The magnetic resistance can be determined from core loss measurement. It is equal to:

\[ R_{\text{mag}} = \frac{\mu_B p_e \text{(measured)}}{f B_m^2} \]  

(32)

\( R_{\text{mag}} \) is tested for eight ferrite materials (N87, N27, N49, 3C90, 3C94, 3F4, fair-rite 78, fair-rite 77), however, we limit to shows the results of the six first materials to shorten the paper’s content. The measured loss data are openly available in [34]. They are measured under a symmetric triangular flux waveform (50% duty cycle) and under the following temperatures: 25, 59, 70 and 90 °C. More details about the used measurement technique and the test set-up can be found in [4].

B. Frequency, flux density and temperature effects on \( R_{\text{mag}} \)

The results of \( R_{\text{mag}} \) are given in Figs.12-13. In general, the proposed approach has several benefits in comparison to the existing approaches. The first remarkable advantage of the proposed approach is it simplifies the nonlinear problem to a linear one over wide range of frequency and magnetic flux density especially at high temperature. Fig.11 gives an insightful picture about the main difference between the Steinmetz approach and our approach to calculate core loss.

Several mathematical laws could be applied to model \( R_{\text{mag}} \). In our work, \( R_{\text{mag}} \) can be calculated using the equation below. At a given frequency, it is a linear function of \( B \).

\[ R_{\text{mag}}(f, B) = R_{\text{mag0}}(f) + S_0(f) B \]  

(33)

\[ R_{\text{mag0}}(f) = b_1 + b_2 f + b_3 f^2 \]  

(34)

\[ S_0 = a_1 f^3 + a_2 f^2 + a_3 f + a_4 \]  

(35)

For 3C90, using model (36) is more accurate and easier to model the magnetic-flux resistance over the full \( f \) and \( B \) ranges:

\[ R_{\text{mag}}(f, B) = \frac{a \exp(r_1 f)}{f} \frac{\exp(r_2 B)}{f} \]  

(36)

For the first model (33), a third and a second order equations are enough to accurately model the effect of \( f \). The complete model has 7 parameters. The problem could be simplified more if we limit the frequency range. In Steinmetz approach, 9 parameters are necessary and the accuracy is still to be improved. For the second model (36), it has 3 parameters only. In the following, we discuss the terms of the first model.

1) \( R_{\text{mag0}} \)

\( R_{\text{mag0}} \) is the magnetic resistance of the material at negligible flux, approximately close to zero. We call it the zero-flux magnetic resistance. At low frequency, its value is negligible compared to its value at high frequency as it can be seen in Fig.14. As the frequency increases, \( R_{\text{mag0}} \) increases. Between 56 kHz and 200 kHz, \( R_{\text{mag}} \) has a nonlinear dependence on the frequency, but the changing rate is very small. Beyond 200 kHz, the dependence with the frequency becomes linear, however the changing rate increases significantly.

\( R_{\text{mag0}} \) could be a good metric to compare the loss performance of materials with respect to the frequency at low \( B \). N87 and N27 have the biggest and approximately the same \( R_{\text{mag0}} \). \( R_{\text{mag0}} \) for 3C94 is smaller than the previous materials and the difference between them increases as the frequency increases. These three materials are suitable for frequency lower than 500 kHz. In contrast, N49 and 3F4 are suitable for MHz operation. They have negative optimum \( R_{\text{mag0}} \) around 200 kHz. This characteristic is very preferred at MHz operation, because at this frequency range, the operating \( B \) is relatively small and \( R_{\text{mag0}} \) effect dominates the effect of \( S_0 \).

2) term \( S_0 \)

Unlike the results found for \( R_{\text{mag0}} \), 3F4 and N49 have higher \( S_0 \) than the rest of materials. As it can be seen, 3F4 has an approximate constant \( S_0 \). Its loss due to high flux density with respect to the frequency is stable within the tested interval [56-500] kHz. On the other hand, N49 has \( S_0 \) which increases at a constant rate with the frequency. The impact of \( S_0 \) for these materials on the total loss is much greater than \( S_0 \) over the full frequency range. The dominance of \( S_0 \) over \( R_{\text{mag0}} \) increases with \( B \) and for that reason it is better to operate this kind of materials at low \( B \) (Fig.17 shows the example of N49).

For the remaining materials, their \( S_0 \) tends to have a maximum at a specific frequency. This can be clearly seen for 3C94 (250 kHz) and N27 (200 kHz). In contrast to what is found for N49 and 3F4, the impact of \( S_0 \) is lower than the impact of \( R_{\text{mag0}} \) at high frequency, however it is bigger at low
frequency which make them suitable for high flux-low frequency operation (Fig.16).

C. Physical interpretation

$R_{mag}$ can be interpreted as the magnetic material propriety. It depends on the frequency and the magnetic flux density. For a given frequency, it has a linear dependence on B. the linearity is expected as B is a non-conservative energy-force which depends on the magnetic path. The dependence on the frequency is found to be non-linear which is a common behavior for all materials with the frequency. In this paper, we have used polynomial and exponential functions to characterize $R_{mag}$. It worth to mention that these mathematical models are not unique. By knowing the exact chemical composition of the magnetic material, we think it will be possible to find better interpretation of the different coefficients of the mathematical models and hence the possibility to converge to a unique mathematical law. This objective can be easily verified with pure magnetic materials.

D. Model accuracy

Fig.18 to Fig.23 show the model error for N87, 3C94, 3F4, N49, 3C90 and N27 respectively for the magnetic flux density range [0.024-0.27] T and the frequency range [56-446] kHz. The average error is 6.48%, 5.8%, 6.87%, 3.85%, 7.03% and 7.07% for dataset of 401, 349, 75, 74, 219 and 90 respectively. In comparison to the existing approaches, the proposed method has lower number of parameters, more straightforward and its accuracy is much better.

![Fig. 12. $R_{mag}$ variation with respect to B, f for N87 at 25, 50, 70 and 90°C](image)

![Fig. 13. $R_{mag}$ variation with respect to B, f of 3C90, N49, 3F4 and 3C94 materials at 90°C](image)
Fig. 14. $R_{mag0}$ variation with respect to $f$ at 90ºC

Fig. 15. $S_0$ variation with respect to $f$ at 90ºC

Fig. 16. Ratio between $S_0*B$ and $R_{mag0}$ terms for N87.

Fig. 17. Ratio between $S_0*B$ and $R_{mag0}$ terms for N49.

Fig. 18. Ramg Model absolute Error for N87, mean error=6.48

Fig. 19. $R_{mag}$ Model absolute Error for 3C94, mean error=5.8
Fig. 20. $R_{mag}$ Model absolute Error for 3F4, mean error=6.87. For this material a cubic fit function of $B$ is used as $R_{mag}$ is approximately independent on $f$.

Fig. 21. $R_{mag}$ Model absolute Error for N49, mean error=3.85

Fig. 22. $R_{mag}$ Model absolute Error for 3C90, mean error=7.07

Fig. 23. $R_{mag}$ Model absolute Error for N27, mean error=7.03

E. $R_{mag}$ optimization

In most materials, we can see that $R_{mag0}$ and $S_0$ have a strict increase with respect to the frequency and the magnetic flux density. Therefore, a solution of an optimal $R_{mag}$ by optimizing $f$ or $B$ is not possible. Several design methods have been published in the literature to minimize the core loss such as the area product, the core geometry coefficient and the works published in [36-37]. These design methods show that there exist an optimal $B$ and an optimal $f$ for given design. However, these methods did not account that these two variables are also contributing to the magnetic energy which is required to be maximized. So, looking for an optimal solution for the core loss using $f$ or $B$ necessarily leads to reduce the maximum magnetic energy and therefore the power density will also decrease.

On another perspective, if we consider the temperature rise constraint, then it is possible to look for the optimal solution which is the maximum ($B$ and $f$) that gives higher energy without thermal issue. In other terms, minimizing the core loss is not a correct step to design and optimize magnetics. In fact, the designer needs only to achieve the maximum energy that also guarantees the thermal equilibrium. This conclusion is an intuitive result of the hypothesis given by eq (27). More details about this subject and the effect of the design variables on core loss and power density is discussed in [38].

F. Discussion

In this paper, we have proposed a radical approach to calculate core loss in magnetic components. The concept of the magnetic-flux resistance is introduced to characterize the loss performance of magnetic materials in easier way instead of using the core loss curves given in the datasheet and the Steinmetz or Bertotti models. Magnetic manufacturers can use this concept as an alternative approach to characterize their products. The proposed approach has several benefits compared to Steinmetz and Bertotti approaches. The first benefit is it converts the problem from nonlinear (as solved by Steinmetz and Bertotti) to a linear problem. As shown previously, for a given frequency, the magnetic-flux resistance can be expressed by a linear equation and this advantage allows to reduce the mathematical complexity in
determining the required parameters and can improve the accuracy. The results show that the average error of the tested materials is about 5% for a B range of [0.024-0.27] T and f range of [56-446] kHz. In addition to that, the proposed approach enables to give a useful explanation and a physical interpretation of the frequency and the magnetic flux density effects on $R_{mag}$. In Steinmetz approach, it was difficult for researchers to understand the physical meaning of the Steinmetz parameters or to understand why a given material is suitable for such frequency range or magnetic flux density interval. The decomposition of the total loss into static and dynamic losses in Bertotti model is also confusing as explained in section III. By analogy to the AC resistance and the classic physics square loss law, this paper presents a linear model for the magnetic-flux resistance, that is composed of two terms: one term is uniquely caused by the frequency and the second term is due to $B$ and $f$. The impact of each term was found to be dependent on the material. This approach can also help to classify and to choose the magnetic materials according to $R_{mag}$. And $S_p$.

In addition to that, the proposed approach highlights a significant aspect about the magnetic loss optimization. This study shows that the optimal (f, B) solution is one that maximizes the power before reaching thermal runaway. Previous studies which are based on Steinmetz equation, suggested an optimal solution (f, B) to minimize loss. However, given the inherit link between the energy and energy-loss as well as the linear relationship between $R_{mag}$ and $B$, it becomes apparent that an optimal solution (f, B) for minimum core loss does not exist. In essence, the optimal (f, B) solution is the solution that maximizes the energy regardless the energy-loss.

Future works will focus on investigating the physical interpretation of the developed model and the relationships with the magnetic domain theory of the hysteresis. Thermal modeling, the dependence of $R_{mag}$ on the temperature and the optimization methods of magnetic components accounting also the winding loss will also be investigated. In addition to that, the use of the proposed approach to characterize the core loss of non-sinusoidal excitation is certainly one of the main reasons to develop this approach and a principal research topic in the future.

V. CONCLUSION

This paper discussed the limitations and the advantages of the existing models for core loss calculation of non-sinusoidal excitations. It also summarizes the principles of Steinmetz and Bertotti models and their main limitations to quantify and characterize the core loss phenomenon. The present paper proposed an alternative approach which eventually gives a simple linear solution to compute core loss. The approach is based on the concept of the magnetic-flux resistance and the square loss law. The magnetic flux resistance is a physics quantity which can be modeled by a simple linear equation over wide range of frequency and magnetic flux density and it can be used to characterize and calculate core loss. The linearity feature of our approach allows to increase the accuracy and reduce the complexity of core loss calculation problem. Future works will focus on the application of this approach for non-sinusoidal excitation, the thermal characterization of the magnetic-flux resistance and the physical interpretation of this later and the link with the hysteresis loop and the domains theory.

VI. REFERENCES


