Application of Aperiodic 'Einstein' Monotile in Limited Field of View Phased Arrays

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Abstract—The discovery of the 'Einstein' monotile represents one of the most significant advancements in geometry in 2023. Research based on this monotile has been initiated across various fields. This paper introduces a limited field-of-view (LFOV) phased array based on the 'Einstein' monotile (Hat polykite) to address grating lobes. The proposed phased array demonstrates reduced implementation complexity compared to aperiodic phased arrays constructed from periodic or conditionally aperiodic tiles. It also exhibits increased engineering practicality compared to aperiodic phased arrays made from non-'Einstein' aperiodic tiles, particularly in assembly with loadbearing lattice structures. Two examples of Hat polykite-based phased arrays are presented in this paper. In Example A, a phased array is introduced where each subarray consists of a single antenna element. The proposed phased array is optimized to achieve a maximum grating lobe level (MGL) of -15 dB. In Example B, a subarray based on the Hat polykite comprises 8 antenna elements. The optimized phased array achieves an aperture efficiency of 90% and maintains a flat grating lobe level within a beam scanning range of 18°.

Index Terms—‘Einstein’ monotile, LFOV, aperiodic arrays, aperiodic tiling, grating lobe reduction, phased array.

I. INTRODUCTION

PHASED antenna arrays have been widely adopted in various communication systems, including radar systems, wireless communication systems, and measuring/controlling systems, among others. Progress associated with phased antenna arrays has been recorded in terms of reduced sidelobes, enhanced flexibility of beamforming, an improved range of beam scanning angles, the capability of generating multi-beams, as well as increased gain and aperture efficiency [1]. On the other hand, reducing the cost of phased antenna arrays is regarded as an essential research question. Specifically, the cost of an active phased array is mainly attributed to the transmit/receive (T/R) modules they employ. Typically, the number of T/R modules adopted in an antenna system depends on the active channels this antenna system possesses. For an active phased array that has a wide range of beam scanning angle, the space between antenna elements is small, the density of active antenna channels is consequently high. Namely, a sufficient number of T/R modules to be assembled in this phased array is mandatory. The most feasible solution for reducing the overall cost of this phased antenna array is to decrease the price of each T/R module, for instance, the price of the chips embedded in a T/R module and the expenses associated with the manufacturing and packaging of a T/R module. However, for the phased antenna array that have the same area but possesses a narrow range of beam scanning angle, the density of active antenna channels required in this antenna array is considerably decreased. A more attractive solution for reducing the overall cost of this antenna array is to incorporate high gain radiating element and optimize the array framework and arrangement, allowing the number of T/R modules to be reduced while maintaining the aperture efficiency of the antenna array [2].

The phased antenna arrays used for space-borne applications typically possess a limited field of view (LFOV). For example, for the phased antenna array assembled on a geosynchronous orbit (GEO) satellite, a relatively small beam scanning range (i.e., <10°) is enough to cover all the targets on Earth [3]-[5]. Considering that high-gain subarrays have been commonly used in space-borne phased antenna arrays, there is an increased demand for reducing the number of T/R modules assembled in those arrays. Nonetheless, if too few T/R modules are adopted, the space between antenna elements/subarrays will be too large and will generate grating lobes. In such cases, additional approaches for eliminating grating lobes will need to be implemented.

Frequently used approaches for eliminating grating lobes including subarray overlapping and aperiodic array factor generating. The former approach is typically implemented by modified impedance matching and feeding networks, e.g. Butler matrix feeding network, intervolve feeding network, lens feeding network, etc. [6]-[8]. Nevertheless, the complexity of feeding networks reduces the adaptability of this approach in large subarrays. The grating lobe elimination obtained from this approach also possesses some strict preconditions. These issues restrict the effectiveness of this approach in achieving an overall cost reduction for the phased antenna array. The latter approach, i.e., aperiodic array factor generating, is typically implemented by breaking the periodicity of phase center in a phased array. This can be realized by interleaving/rotating/ translating subarrays, combining subarrays with different sizes, or incorporating...
amplitude weighting [9]-[14]. Two critical issues associated with this approach are low aperture efficiency and a massive variety of subarrays. These issues also result in a high cost or low feasibility of engineering implementation of this approach.

Aperiodic subarrays demonstrate a high potential in increasing the aperture efficiency as well as reducing the variety of subarrays. Phased arrays based on octomino subarray was firstly announced in [15]. Lately, many aperiodic subarrays based on periodic tiles were displayed, including but not limited to L-shaped tile, polyhex-shaped tile, diamond tile, [16]-[22]. To ensure that these subarrays to be completely tiled a plane whilst having an aperiodic phase center, customized geometrical designing rules [22] or additional position optimization algorithms were adopted [23]-[29]. With the increasing area and scale of the phased array, the number of design rules and the varieties that need to be optimized are dramatically increased. This results in a significantly increased design or optimization time and may even render the phased array undesignable or the position optimization algorithm unable to converge.

The utilization of aperiodic tiling in phased array applications began to attract attention around 2005 [30]. The evolution of aperiodic tiling is introduced in Section II of this paper. Generally speaking, an unconditional aperiodic tiling allows a complete plane (a phased array) to be fully tiled without geometric features evaluation [22], [31]-[32], as well as position optimization algorithms. The position optimization of antenna elements assembled on an aperiodic tiling-based phased array can proceed without considering the combination and truncation of tiles, leading to significantly reduced complexity [33].

The 'Einstein' aperiodic tile is a specific aperiodic tile that only consists of a monotile. The first two sets of 'Einstein' tiles, i.e., Hat polykite and turtle polykite, were found in April 2023 [34]. In this paper, a LFOV phased array based on a 'Einstein' monotile, i.e., Hat polykite, is proposed. In Section II, we will explain how the engineering practicality of an aperiodic tile-based subarray can experience a significant improvement, especially in assembling a loadbearing lattice, with the adoption of the Hat polykite. In the subsequent sections, we will also demonstrate the procedures for constructing a Hat polykite based subarray and provide two different examples of placing and optimizing the antenna elements on that subarray.

II. PERSPECTIVES ON ANTENNA SUBARRAY BASED ON “EINSTEIN” APERIODIC TILING

In this section, the evolution of aperiodic tiling has been introduced. Its capability of realizing phased arrays was also discussed. Afterwards, a brief description of the 'Einstein' tile was presented. The unique engineering practicality it possesses, i.e., the capability of the 'Einstein' tile in assembling load-bearing lattice, has been analyzed.

A. Evolution of aperiodic tiling in antenna subarray applications

Although phased array implemented from aperiodic tiling show intuitive strengths in complexity reduction, only a few aperiodic tiles possess engineering practicality. Early aperiodic tilings, for instance, Wang's tiling [35], typically comprise numerous shapes of different tiles (i.e. 20424 shapes), posing challenges in practical implementation due to complex local matching rules [36]. Moreover, these rules fail to ensure the completion of a tiled plane. Some may even lead to a situation where no additional matching can proceed before completing a plane. In other words, aperiodic tiling that consists of numerous tile types is not unconditionally tiled for a complete plane. ‘Cut and project’ normally serves as a
simplified methodology for constructing an ‘approximate complete’ plane from aperiodic tiling that consists of numerous types of different tiles [37]. The core idea involves projecting aperiodic units onto a quasi-periodic sequence, such as a 1D-Fibonacci-type sequence, enabling the estimation of their positions. Nonetheless, it is crucial to acknowledge that the resulting aperiodic tiling from cut and project is susceptible to leaving overlaps or gaps, as indicated by the term "approximate complete." On the other hand, to implement a phased array, the imperative for a genuinely complete plane is non-negotiable. There are also some aperiodic tiles consisting of a limited number of units, while tiling a complete plane from these units often requires adherence to additional specific conditions [38]. For example, a Taylor-Socolar tile comprises only one unit, yet the edges of this unit are uniquely symbolized. The complexity of tiling a complete plane using conditional aperiodic tiles is therefore comparable to tiling a complete plane with periodic tiles.

Before the discovery of 'Einstein' tiles, Penrose tiles were considered as a relatively close solution for adoption in phased array applications [39]. In Penrose tiling, only two types of tiles are used: either kite and dart or thick and thin. These two tiles are fully unconditionally tiled for a complete plane, i.e., without additional matching rules. Moreover, Penrose tiling adheres to Berger’s law (i.e., a larger element, e.g., kite and dart in Penrose tiling, can only be constructed by combinations of kites or darts). Berger’s law supports the aperiodicity of Penrose tiling. It also means that Penrose tiling can be constructed using the substitution method [40], allowing a completed plane to be strictly tiled without overlap or gaps.

Commonly, for most large-scale phased arrays consisting of multiple subarrays, especially for air-borne or space-borne antennas where an all-metal antenna element is recommended [41]-[42], they need to be assembled with a load-bearing structure. However, Penrose tiling is fully unconditionally constructed, implying that every unit in Penrose tiling appears without a predictable order. Considering a typical load-bearing structure is simply constructed as a lattice, a group of points on such a lattice cannot be found which can overlap at the same or fixed positions on Penrose tiles. On top of that, at least three of these points can form engineering-stable triangles. When this is the case, subarrays based on aperiodic tiles need to be assembled with a custom-designed load-bearing structure. This may result in the overall cost of designing and manufacturing the load-bearing structure exceeding the overall cost of designing and manufacturing the subarray. Such a cart-before-the-horse result is strongly discouraged in engineering applications.

In the next subsection, we will demonstrate that a subarray based on the ‘Einstein' tile can be assembled with a conventional loadbearing lattice, resulting in a remarkable improvement in engineering practicality.

**B. Phased array based on Hat polykite.**

The ‘Einstein' tile is an aperiodic monotile, consisting solely of a single type of tile. The ‘Einstein’ tiling is also fully unconditionally. The aperiodicity of ‘Einstein’ monotile is proved by Berger’s law as well as “translating and scale” analysis. They also indicate that the ‘Einstein’ monotile is

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**Fig. 4.** Aperiodic tiling-based subarray comparison tree.
capable of tiling a complete plane [34]. The ‘Einstein’ tiling
thus possesses every strength that Penrose tiling has that can
be potentially adopted in a phased array.

The analysis of phased array in this paper is based on Hat
polykite, i.e. one of the first announced ‘Einstein’ monotile. It
should be noted that, although other ‘Einstein' tiles share the
same topology as the Hat polykite, they could possess entirely
different characteristics in the fields of physics, chemistry, and
engineering. This means that a phased array based on a
different type of ‘Einstein’ monotile needs to undergo a
complete analysis rather than adopting the results given in this
paper.

Different from Penrose and many other aperiodic tiles, the
Hat polykite is derived from a strongly periodic tile, i.e. Laves
[3,4,6,4] tile [43] (in the following sections, we refer Laves
[3,4,6,4] tile as a Kite). Fig. 1 shows a Hat polykite on a grid
of Laves [3,4,6,4] tiling. For Laves [3,4,6,4] tiling, angles
around each vertex are the same. Consequently, a Hat polykite
unit can be divided into different regular polygons, e.g. regular
triangles. This is an essential precondition for Hat polykite to
be assembled with a lattice.

Fig. 2 illustrates the Hat polykite configuration. According to
David’s paper [34], a typical Hat polykite consists of edges
with two lengths. If we determine the length of the short edge
as 1, the length of the long edge is \( \sqrt{3} \). These edges can be
regarded as the half edge length and the height of an
equilateral triangle. Note that a Hat polykite consists of three
vertices derived from the ‘6-vertex’ in a Laves [3,4,6,4] tiling
(i.e., with an angle of 60° in Laves [3,4,6,4] and a 2×60° angle
in the Hat polykite). A Hat polykite, therefore, contains an
equilateral triangle with an edge length of \( 2 \times \sqrt{3} \) (this is the
distance between each neighboring ‘6-vertices’). One possible
lattice loadbearing structure that can be assembled with the
Hat tiling is a \( 3 \times 3 \) lattice, as shown in Fig. 3. When this
lattice is placed underneath the Hat polykites, it ensures that
the aforementioned ‘6-vertices’ always overlap the vertices of
the lattice. In other words, a Hat polykite will have at least
three vertices that overlap the lattice vertices. (Note: In fact,
we found that a Hat polykite has at least five vertices that
overlap with the vertices of the lattice. We observed that the
remaining two vertices are ‘4-vertices’ in a Laves [3,4,6,4]
tiling, forming a 90° angle. However, we have yet to find a
rigorous and intuitive way to prove the existence of these two
vertices). As these three vertices construct an equilateral
triangle, they provide engineering triangular stability.

Namely, the phased antenna array based on a Hat polykite can
be supported by a \( 3 \times 3 \) lattice loadbearing structure. Taking
practical manufacturing into consideration, a potential solution
is to leave drill holes at the positions of overlapping vertices.
These drill holes can be pre-set and the Hat polykite with these
drill holes can therefore to be mass-produced.

Fig. 5. (a) Kite (Laves [3,4,6,4]) tile, (b) unreflected “hat” polykite, (c)
reflected “hat” polykite.

The Hat polykite, denoted as \( T [\sqrt{3}, 1] \) in [34], is only a
specific case of the 'Einstein' monotile. The authors have
claimed that for any positive values of \( a \) and \( b \), \( T [a, b] \) can
construct an 'Einstein' monotile. However, considering the
embedding of the lattice supporting structure, a ratio of \( \sqrt{3} \)
between \( a \) and \( b \) is recommended. This is because when a
different ratio is applied, the 'Einstein' monotile \( T [a, b] \) will
not be derived from the Laves [3,4,6,4] tiling. The above
method for constructing the lattice loadbearing structure will
not be applicable (we do not preclude the existence of
additional methods for constructing the lattice loadbearing
structure, but those methods are likely to be more complicated,
as they are no longer based on calculations involving regular
polygons).

In addition to the essential engineering practicality that the
Hat polykite possesses, antenna element optimization is also
relatively easier to proceed with on a Hat polykite based
subarray, as it consists of only one tile compared to other
aperiodic tiles. Fig. 4 compares antenna subarrays based on
aperiodic tiling constructed from different tiles. From the
perspective of engineering applications, a subarray based on
the Hat polykite is undoubtedly the most appealing.

III. NUMERICAL RESULTS AND DISCUSSION

In this section, the radiation properties of a LFOV phased
array based on Hat polykite is analyzed. A Hat polykite
consists of 8 Kite (Laves [3,4,6,4]) tiles. As the Hat polykite is
aperiodic and is not chiral [45], a reflected Hat polykite exists
(See Fig. 5). In order to differentiate their radiation properties,
in this paper, both unreflected and reflected Hat polykite has
been studied.

For planar an antenna array located in the x-y plane, the
radiation pattern of the antenna array can be expressed by the
following function:

\[
F(\theta, \phi) = \sum_{n=1}^{N} E_{F_n}(\theta, \phi) A_n \exp(jk(x_n* u + y_n* v))
\]

(1)

where \( N \) refers to the number of antenna elements in this
array, \( E_{F_n}(\theta, \phi) \) refers to the radiation pattern of a single
antenna element (also named as element factor), \( A_n \) is the
amplitude of the excitation voltage for the nth element. \((x_n, y_n)\)
is the coordinate value of the nth element.
Seen from Equation (1), the radiation pattern of an antenna array is determined by the element factor $\text{EF}_n(\theta, \phi)$ as well as the array factor $A_n \exp(jk(x_n^*u+y_n^*v))$. This implies that the grating lobe level could be reduced either by optimizing the element factor or optimizing the array factor [46]. In the next two subsections, we will present two examples of arranging antenna elements on a Hat polykite. In the first example, the subarray based on a Hat polykite consists of only a single antenna element. The array factor was optimized in this example. In the second example, a subarray consisting of multiple antenna elements is proposed. The element factor is, in turn, optimized in this example.
A. Example A: A single antenna element in one Hat polykite

In Example A, we propose a phased array constructed from Hat tiling. Each Hat polykite subarray comprises only one antenna element. Firstly, we compare the radiation pattern of the Hat polykite based subarray with a periodic hexagon tile-based subarray. Following this, the Hat polykite subarray is optimized using a genetic algorithm (GA). The radiation pattern and simulation results indicate a significant reduction in grating lobes.

For a phased array based on aperiodic tiling, one important parameter that need to be considered is the minimum distance between each element, which can be written as:

\[ d_{\text{min}} = \min (d_n), n=1,2,\ldots N \quad (2) \]

where \(d_n\) refers to the minimum distance between the \(n\)th element and its nearest element.

The equivalent distance between each element can be written as:

\[ d_{\text{eq}} = \sqrt{\frac{S_{\text{array}}}{N}} \quad (3) \]

where \(S_{\text{array}}\) is the total area of the antenna array.

As introduced in Section II, the Hat polykite is derived from a kite tile, i.e., Laves [3,4,6,4] tile. This is to say, the area of a Hat polykite should be the same as the area of a hexagon. Therefore, one persuasive way to demonstrate the grating lobe reducing ability of a Hat polykite based phased array is to compare it with a phased array that based on periodic hexagon tiles. Fig. 6 presents the antenna array layouts of a phased array based on Hat polykites as well as hexagon tiles. In both phased arrays, the only antenna element is placed at the center of the tile. The element factor is defined as \(\cos(\theta)\). Both phased arrays consist of 238 antenna elements. According to equation (3), their \(d_{\text{eq}}\) is the same. If we define the area (denoted as \(A\)) of both tiles as \(2L_0\), the distance between adjacent elements in Hat polykite (denoted as \(d_{\text{eq}}\)) can be calculated as \(\sqrt{\frac{A}{2L_0}}\). The distance between adjacent elements in periodic hexagon tiles (denoted as \(d_{\text{eq}}\)) can be calculated as \(\sqrt{\frac{A}{\sqrt{3}L_0}}\).

Fig. 7 displays the radiation patterns between a hexagon tile-based phased array and a Hat polykite-based phased array. Since the distance between adjacent elements in both phased arrays is larger than \(1\lambda_0\), obvious grating lobes can be observed in the radiation pattern for the hexagon tile based phased array. The maximum grating lobe level (MGL) is simulated as -3.7 dB (0°). On the other hand, the MGL for the Hat polykite-based phased array is simulated as -9.5 dB (0°). The MGL has reduced significantly by 5.8 dB.

The MGL of the Hat polykite based phased array can be further decreased by optimizing the array factor, which reflects the optimization of the position of the elements. This optimization allows energy within the grating lobe to be further dispersed and more uniformly distributed throughout the sidelobes. To achieve this, a genetic algorithm (GA) has been introduced. The methodology of implementing the GA is recorded in [33]. The coordinates of the antenna elements, i.e., \((x_n, y_n)\), were selected as optimization parameters. The goal of the optimization was to minimize the peak grating level. Fig. 8 illustrates the layout of the Hat polykite-based phased array after optimization and the corresponding radiation pattern. The simulated MGL of which is further reduced to -15 dB (0°), which has a 11 dB reduction comparing to hexagon tile based phased array.

TABLE I compares the geometric features and electrical characteristics of the phased array with different layouts.

A LFOV phased array is expected to have reduced number of active channels. This could be realized by increasing the space between elements and reducing the grating number level, as proceeded in Example A. On the other hand, a LFOV phased array is also expected to have a high aperture efficiency. The aperture efficiency of an antenna array could be calculated by:

\[ \text{Eff}_{\text{aperture}} = \frac{10 \log_{10} D_\theta}{4\pi L_0^2} \times 100\% \quad (4) \]

\(D_\theta\) indicates the maximum directivity of the antenna array in the normal direction. The \(D_\theta\) for optimized Hat polykite based phased array is simulated as 31.52 dB. The \(\text{Eff}_{\text{aperture}}\) for this phased array is calculated as 23.7 %. The relatively low aperture efficiency is attributed to the sparse arrangement of antenna elements, resulting in underutilization of the antenna element aperture. One way to improve the aperture efficiency is to use the high gain antenna element. For instance, if we increase the element factor from \(\cos(\theta)\) to \((\cos(\theta))^4\), the aperture efficiency of the optimized phased array could be significantly improved, reaching 72.5 %. However, considering that a high-gain element typically possesses a regular geometric feature, if a Hat polykite-based subarray consists solely of a high-gain antenna element, the electrical field is not uniformly distributed on the aperiodic subarray. In other words, the aperture of the Hat polykite cannot be fully utilized. Another approach to enhance the aperture efficiency of the proposed phased array is to increase the number of elements placed on a Hat polykite. This ensures the full utilization of aperture of the Hat polykite, thereby achieving a high aperture efficiency.
In general, a scanning angle range of ±10° is sufficient for LFOV phased arrays applied to GEO satellites. However, expanding the scanning angle range for a phased array can broaden its scope of application. To achieve this, the MGL needs to remain lower than the FSLL within this extended scanning range. In this paper, we also demonstrate an optimized phased array. The beam scanning range of which has been extended to 18°.

Recall that in Example B, a Hat polykite based subarray consists of eight antenna elements. These antenna elements are interconnected with passive networks and equally power divided. If we regard the subarray as a ‘big element’, the element factor of this subarray can be expressed as:

$$SAF(\theta, \varphi) = \sum_{n=1}^{8} |E_n(\theta, \varphi)| \times A_n \exp \left(jk(x_n u + y_n v)\right)$$  \hspace{1cm} (5)$$

Equation (5) indicates that the element factor of this subarray can be tuned by optimizing the coordinates of the antenna elements. The authors in [6] have announced that by optimizing the element factor of the subarray, the energy within the grating lobe can be dispersed and then interwoven within the main lobe. This, in turn, broadens the width of the main lobe, and the MGL is consequently kept relatively low within a wider range.

Following above ideology, a GA has been introduced to optimize the element factor of the proposed subarray. The coordinates of the antenna elements, i.e., (x_n, y_n), were selected as optimization parameters. The goal of the optimization was...
Fig. 10. Layouts and Radiation patterns for optimized Hat polykite-based array where eight elements are optimized in the kite-tiles respectively (antenna element positions are marked with solid dots) (a) antenna layout, (b) b), (c) and (d) are the corresponding simulated transmission patterns for different scanning angles ($\theta_{\text{scan}}=0^\circ$, $\theta_{\text{scan}}=10^\circ$ and $\theta_{\text{scan}}=15^\circ$).

Fig. 11. Comparison of the first sidelobe level (FSLL), maximum grating lobe level (MGL), and aperture efficiency (AE) between the proposed regular array and the proposed optimized array.

to maximum the beamwidth of the main lobe. Fig. 10 (a) presents the layout of the proposed phased array after optimized. Figs. 10 (b)-(d) displays the radiation patterns of the optimized phased array among scanning angle of 0°, 10° and 15°. The FSLL of the optimized phased array is akin to the FSLL of the regular phased array, whilst the MGL of the optimized phased array is raised much slower. Shown in Fig. 11, the MGL remains lower than -20 dB when the scanning angle is lower than 15°. The MGL is same as FSLL when the scanning angle is 18°. Namely, the beam scanning range of the optimized phased array is 18°.

The simulated aperture efficiency of the optimized phased array is approximately 90%. This is attributed to the non-uniform distribution of antenna elements within a subarray, leading to the underutilization of the Hat polykite's aperture. Nevertheless, the aperture efficiency remains considerably high and satisfies the requirements of an LFOV phased array.

Please note that, in Example B, the array factor for the entire phased array is not optimized. As a result, the maximum grating lobe (MGL) of the phased array is close to -9.5 dB and -10.5 dB when the scanning angle is 20° (similar to the MGL of the original phased array in Example A). It is reasonable to predict that this MGL can be further reduced after optimizing.
both the array factor and element factor of the subarray in Example B.

### C: Geometric features and electrical characteristic comparison

In TABLE II, the geometric features and electrical characteristics of the proposed phased array are compared with some representative designs. It can be observed that the maximum grating lobe (MGL) of the proposed phased array is lower than that of most other aperiodic phased arrays except [22]. However, in [22], the geometric features of the tiles vary with the changing subarray numbers, making mass production challenging. Subarrays in [16] and [19] also consist of a single tile each, but these tiles are periodic, requiring additional position optimization algorithms for tiling a plane. [47] presents an aperiodic phased array based on Penrose tiling. The maximum sidelobe level (MSLL) in [47] is close to the optimized MGL in Example B. However, the design in [47] lacks engineering practicality as it cannot be assembled with a load-bearing lattice.

### IV. CONCLUSION

In this paper, a LFOV phased array is proposed for the exploration of an 'Einstein' aperiodic monotile (i.e., Hat polykite). Akin to aperiodic arrays based on other 'non-Einstein' aperiodic tiles, no additional design or optimization of the geometrical/positioning is required when using Hat polykite-based subarray to compose a complete planar array. On the other hand, compared to aperiodic arrays made from 'non-Einstein' tiles, the proposed array allows a load-bearing lattice to be assembled underneath the radiator. The engineering practicality of the proposed array is thus significantly improved. Two examples of phased arrays based on the Hat polykite are studied in this paper. In Example A, a phased array is introduced where each subarray consists of a single antenna element. The proposed phased array is optimized to achieve an MGL of -15 dB. In Example B, a subarray based on the Hat polykite comprises 8 antenna elements. The optimized phased array achieves an aperture efficiency of 90% and maintains a flat grating lobe level within a beam scanning range of 18°.

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