A novel integrated group interval BWM-TODIM method for food supplier selection

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Abstract—The selection of a food supplier involves multiple criteria, making the decision-making process complex and uncertain due to the presence of many unmeasurable and conflicting factors. To address this, this study proposes an integrated methodology that utilizes the advantages of interval numbers in dealing with complex and uncertain problems. Specifically, we suggest using the best-worst method (BWM) and the TODIM (an acronym in Portuguese for interactive and multiple attribute decision making) technique in an interval numbers environment. We introduce a new approach called interval best-worst method (IBWM) which employs interval preference degree and interval weight, and based on this, we propose the group interval BWM (GIBWM) method. Additionally, we present a novel interval TODIM (I-TODIM) method that is computationally simple, avoids the two paradoxes of the traditional TODIM method, and provides a clearer and more accurate interpretation of rankings. Furthermore, we suggest a geometric probability-based method for calculating preference and a preference-based method for calculating distance between two intervals. The method proposed in this paper has been validated by the food supplier selection problem based on real data and opinions from management experts and purchasing officer. We also conduct sensitivity and comparative analyses and discuss the effectiveness and advantages of our method.

Index Terms—Group interval best-worst method (GIBWM), multiple criteria decision making (MCDM), interval TODIM method (I-TODIM), food supplier selection (FSS)

I. Introduction

The food industry plays a critical role in the global economy, providing food for billions of people and making significant contributions to the livelihoods of those involved in food production, distribution, and consumption. The FSS consists of a diverse range of companies operating in different markets and offering various food products [21]. In recent years, the FSS has faced unprecedented challenges due to changing consumer preferences, environmental issues, and a growing global population, making supplier selection crucial in FSS [14]. FSS involves a complex stakeholder network, but all stakeholders share common goals of ensuring food quality, safety, and sustainability, as well as conflicting goals such as inventory management, i.e., low or high inventory levels, payment terms, i.e., extended or short-term, and commodity price issues [30]. Successful supplier selection can reduce procurement costs, shorten production lead times, improve customer satisfaction, and enhance organizational competitiveness [12].

Currently, the growing trend of outsourcing and offshoring, complex and tightening government and regional policies, and conflicting organizational and supply chain objectives have increased the importance and complexity of supplier selection decisions. The challenge is not only recognizing the role and practice of supplier management but also devising strategies and methods to address the supplier selection issues faced by procurement and supplier management professionals [3]. Green supplier selection (GSS) is an important aspect of these challenges, which is the process of selecting suppliers based on their environmental performance. As economic growth decouples from the corresponding environmental degradation caused by various pressures, including tightening environmental regulatory requirements and changes in consumer attitudes towards purchasing more environmentally friendly products [7], more and more companies are participating in GSS, aiming to reduce their environmental impact, enhance their reputation, and meet regulatory requirements [11]. Meanwhile, FSS also has specific factors such as food quality, safety, freshness, etc. These issues make the FSS process a complex decision-making problem and classify it as a multiple-criteria decision-making problem (MCDM) [37].

Decision analysis science have made significant contributions to the FSS field. In the current complex business environment, companies can use multi-criteria decision-making (MCDM) tools to evaluate and select suppliers based on a combination of traditional and green environmental standards. Various
MCDM models have been proposed for supplier selection in FSS, including fuzzy TOPSIS and AHP [13], extended TODIM method under type-2 interval fuzzy environment [19], fuzzy ELECTRE [24], artificial neural network (ANN), data envelopment analysis (DEA), and analytic network process (ANP) [15]. Additionally, a measure combining ranking based on optimal points (RBOP) and win-loss-draw (WLD) was developed to select the best cheese suppliers for supermarkets in Iran [37]. BWM was used to calculate the weights of criteria by [1], [5]. BWM is based on pairwise comparisons between the best and worst criteria and the other criteria. Compared with AHP’s pairwise comparisons, BWM requires fewer comparisons and has better consistency [20].

In most existing studies, one of the well-known and widely used methods for addressing uncertainty in MCDM problems is fuzzy numbers (FS) [34] and their extensions, such as interval fuzzy numbers [36], intuitionistic fuzzy numbers [2], type-2 fuzzy numbers [35], and linguistic FS that use linguistic terms to represent uncertainty [32]. Although these fuzzy numbers can well capture the uncertainty and fuzziness in practical problems, they have drawbacks such as complex mathematical forms that are difficult to manipulate mathematically, and difficult-to-quantify membership degrees that are not easily subjected to numerical analysis. In contrast, interval numbers have advantages such as simple expression forms, simple calculations, quantifiable degree of uncertainty, and ease of acceptance by decision-makers, while still being able to describe a certain degree of uncertainty and fuzziness in practical problems. Many MCDM methods have been extended using interval numbers theory, such as DEA [6], TOPSIS [31], VIKOR [22] and MOORA [25].

In this paper, we consider a case study where five frozen beef companies are evaluated as potential suppliers for a chain restaurant company based on ten criteria. Considering the advantages of interval numbers in dealing with uncertain and complex problems, inspired by [10], [17] based on group and individual decision combinations, we develop a complete interval number BWM method and TODIM method for evaluating criteria weights and selecting suppliers, respectively. Overall, the main contributions of this paper are as follows:

1. We propose a novel method for calculating interval preference based on geometric probability, which offers a clearer geometric interpretation and better differentiation of degrees of preference.

2. We suggest a preference-based method for measuring the distance between two interval numbers.

3. We recommend using the group interval BWM method in a fully interval number environment, as it can better capture the fuzziness of criterion weights by using interval numbers as weights, compared to the group interval BWM method proposed in [10]. Moreover, in our calculation model, we avoid reducing interval preferences to crisp numbers, which prevents any loss of information.

4. We introduce a consistency verification method applicable to individual interval BWM and group interval BWM methods.

5. The TODIM method proposed by [8], [9] can handle MCDM problems that require consideration of the subjectivity of DM behavior and interactions between criteria. In this study, we propose an interval number TODIM method inspired by [17], which avoids two paradoxes. We introduce the methods of interval number preference comparison calculation throughout the TODIM calculation process, making the calculation process simpler and more interpretable.

The rest of this paper is organized as follows. Section 2 introduces interval number theory and proposes a geometric probability-based interval preference calculation method, and a method to measure the distance between two interval numbers. Section 3 introduces the proposed relevant theories and their derivations, including the individual interval BWM approach, group interval BWM approach, and consistency ratio calculation approach. Section 4 introduce the proposed interval TODIM approach. Section 5 presents two examples, the first from example 1 to illustrate the advantages of our proposed group interval BWM approach over the group interval BWM approach proposed in [10], and the second example presents a real-world frozen beef supplier selection problem. Finally, section 6 concludes the paper and outlines some directions for future research.

II. Theories of interval numbers

In this section, we will present mathematical theories that are relevant to interval numbers.

A. Basic Definitions of Interval Numbers

An interval number \( a^* \) is generally denoted by \([a^-, a^+]\) [28]. Arithmetic operations are defined for two non-negative real interval numbers \( a^* = [a^-, a^+] \) and \( b^* = [b^-, b^+] \) and a real number \( r \):

\[
a^* + b^* = [a^- + b^-, a^+ + b^+] ,
\]

\[
a^* - b^* = [a^- - b^+, a^+ - b^-] ,
\]

\[
a^* \cdot b^* = [a^- \cdot b^-, a^+ \cdot b^+] ,
\]

\[
a^*/b^* = [a^-/b^+, a^+/b^-] , \text{ with } b^- \text{ and } b^+ \text{ is not } 0 ,
\]

\[
a^* = \left[\min(ra^-, ra^+), \max(ra^-, ra^+)\right] ,
\]

(II.1) (II.2) (II.3) (II.4) (II.5)
(α°)r = \left[ \min((α-)r, (α+)r), \max((α-)r, (α+)r) \right]. \quad (II.6)

B. Preference degree of interval numbers

In this section, we have given a new degree of preference calculation method using the approach of geometric probability. This method differs from preference [23] calculation methods in that it provides a better geometric interpretation and differentiation of degree of preference.

Assume that the contained region made up of \([a^-, a^+]\) and \([b^-, b^+]\) has a uniform probability distribution. Let \(a^* = [a^-, a^+]\) and \(b^* = [b^-, b^+]\) are any two interval numbers, the preference degree of \([a^-, a^+]\) over \([b^-, b^+]\) is defined as:

\[
P(a^* > b^*) = \begin{cases} 
1, & \text{if } a^- > b^+ \\
\frac{S_1}{S}, & \text{otherwise} \\
0, & \text{if } a^+ < b^-
\end{cases} \quad (II.7)
\]

Where \(S \neq 0\). \(S\) and \(S_1\) represent the total area formed by \(a^*\) and \(b^*\) in the Cartesian coordinate system and the area where \(a^*\) is greater than \(b^*\), respectively, as shown in Figure 1. They are given by the following form:

\[
S_1 = \frac{1}{2} \left[ \min(b^+, a^+)^2 - \max(a^-, b^-)^2 \right] + b^-[\max(a^-, b^-) - \min(b^+, a^+)] + \max(b^+ - b^-) (a^+ - b^+) (a^+ - b^+).0 \\
S = (a^- - a^+) (b^- - b^+) \quad (II.8)
\]

Additionally, if \(a^- = a^+\) or \(b^- = b^+\), we need to use the following formula provided by [23]:

\[
P(a^* > b^*) = \frac{\max\{0, a^+ - b^-\} - \max\{0, a^- - b^+\}}{a^- - a^+ + b^+ - b^-} \quad (II.10)
\]

If \(a^- = a^+\) and \(b^- = b^+\), to proceed with the calculation, we will make use of the following equation:

\[
P(a^* > b^*) = \begin{cases} 
1, & \text{if } a^+ > b^+ \\
0.5, & \text{if } a^+ = b^+ \\
0, & \text{if } a^+ < b^+
\end{cases} \quad (II.11)
\]

Property 3. \(a^*\) is considered to be superior to \(b^*\) if \(P(a^* > b^*) > P(b^* > a^*)\), represented as \(P(a^* > b^*) > b^*\).

Property 4. When \(a^* \cap b^* \neq \emptyset\): If \(a^* \geq b^*\), then \(P(a^* > b^*) \geq 0.5\); if \(a^* \leq b^*\), then \(P(a^* > b^*) \leq 0.5\).

Property 5. When \(a^* \cap b^* = \emptyset\): If \(a^* \geq b^*\), then \(P(a^* > b^*) = 1\); if \(a^* \leq b^*\), then \(P(a^* > b^*) = 0\).

C. Distance between two interval numbers.

Taking into account the limitations of existing distance measures between two interval numbers, this paper puts forward a method for measuring the distance between two interval numbers, which is based on the interval preference calculation method proposed earlier in the paper. The formula for this method is as follows:

\[
d(a^*, b^*) = |P(a^* > b^*) - 0.5| \quad (II.12)
\]

It is evident that the proposed distance satisfies the axiomatic definition of distance. The proposed distance is highly sensitive to changes in interval numbers, avoids counterintuitive results, and is computationally simple.
III. Interval best-worst method

A MCDM problem is typically represented as a matrix, as shown below:

\[
\begin{align*}
X^* &= \begin{bmatrix} C_1 & \cdots & C_j & \cdots & C_n \\
A_1 & [x_{11} & \cdots & x_{1j} & \cdots & x_{1n}] \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
A_m & [x_{m1} & \cdots & x_{mj} & \cdots & x_{mn}] 
\end{bmatrix}
\end{align*}
\]

where \(A_1, A_2, \ldots, A_m\) is a set of feasible alternatives, \(C_1, C_2, \ldots, C_n\) is a set of decision-making criteria, and \(i\) is the score of alternative \(i\) with respect to criterion \(j\).

A. Interval best-worst method

1) Goal programming based interval best-worst method: In MCDM, determining the criteria weights is crucial. The steps of interval BWM are as follows:

> Step 1: Determine the best and the worst criteria.

> Step 2: Determine the vector of the best criterion is preferred over all other criteria. The preference of the best criteria over the \(jth\) criteria is demonstrated as \(P_{Bj} = [p_{jW}, P_{jW}^*]\). The resulting vector of Best-to-Others preferences would be:

\[
P^*_B = ([P_{B1}, P_{B1}^*], [P_{B2}, P_{B2}^*], \ldots, [P_{Bn}, P_{Bn}^*]).
\]  

(III.1)

The above interval vector can be divided into two non-negative crisp vectors:

\[
\begin{align*}
P^-_B &= ([P^-_{B1}, P^-_{B2}], \ldots, P^-_{Bn}), \\
P^+_B &= ([P^+_{B1}, P^+_{B2}], \ldots, P^+_{Bn}).
\end{align*}
\]

> Step 3: Determine the vector of the other criteria is preferred over the worst criteria. The preference of the \(jth\) criteria over the worst criteria is demonstrated as \(P^*_W = [P^*_jW, P^*_jW]\). The resulting vector of Others-to-Worst preferences would be:

\[
P^-_W = ([P^-_{1W}, P^-_{1W}], [P^-_{2W}, P^-_{2W}], \ldots, [P^-_{nW}, P^-_{nW}])^T.
\]  

(III.2)

The above interval vector can be divided into two non-negative crisp vectors:

\[
\begin{align*}
P^-_W &= ([P^-_{1W}, P^-_{2W}], \ldots, P^-_{nW})^T, \\
P^+_W &= ([P^+_{1W}, P^+_{2W}], \ldots, P^+_{nW})^T.
\end{align*}
\]

> Step 4: Calculation of optimal weights. Assume the optimal weights vector is \(W^* = ([w^-_1, w^-_2], [w^+_2, w^+_2], \ldots, [w^-_n, w^+_n])^T\). According to [26], the interval weight vector \(W^*\) is considered normalized if and only if:

\[
w^+_1 + \sum_{j=1, j \neq i}^n w^-_j \leq 1, \quad i = 1, \ldots, n.
\]  

(III.4)

For each evaluation criterion, the best criterion has a weight of \(w^+_B\), and the worst criterion has a weight of \(w^-_W\). Consider the elements in the \(W^*_B\) and \(W^*_W\) vectors. If the preference is perfectly consistent, then it is crucial that \(W^*_B / W^*_W = P^*_B j\) and \(W^*_W / W^*_W = P^*_jW\) for all \(j = 1, \ldots, n\). As a result, \(P^*_B\) and \(P^*_W\) must be able to be written as:

\[
P^*_B = ([w^+_B, w^-_B], [w^+_B, w^-_B], \ldots, [w^+_B, w^-_B]),
\]

(III.5)

\[
P^*_W = ([w^-_W, w^+_W], [w^-_W, w^+_W], \ldots, [w^-_W, w^+_W])^T.
\]

(III.6)

According to (II.4), the interval vectors of preferences can be rewritten as:

\[
P^*_B = ([w^-_B - w^+_B], [w^+_B - w^-_B], \ldots, [w^+_B - w^-_B]),
\]

(III.7)

\[
P^*_W = ([w^-_W - w^+_W], [w^+_W - w^-_W], \ldots, [w^-_W - w^+_W])^T.
\]

(III.8)

which can be divided into the following four non-negative crisp vectors:

\[
P^-_B = ([w^-_B - w^+_B], [w^+_B - w^-_B], \ldots, [w^-_B - w^+_B]),
\]

(III.9)

\[
P^+_B = ([w^+_B - w^-_B], [w^+_B - w^-_B], \ldots, [w^+_B - w^-_B]),
\]

(III.10)

\[
P^-_W = ([w^-_W - w^+_W], [w^+_W - w^-_W], \ldots, [w^-_W - w^+_W])^T,
\]

(III.11)

\[
P^+_W = ([w^+_W - w^-_W], [w^+_W - w^-_W], \ldots, [w^+_W - w^-_W])^T.
\]

(III.12)

It can be easily proven:

\[
w^+_1 P^*_{Bj} = w^+_B, \quad j = 1, \ldots, n,
\]

(III.13)

\[
w^-_j P^*_{Bj} = w^-_B, \quad j = 1, \ldots, n,
\]

(III.14)

\[
w^+_w P^*_{Wj} = w^+_j, \quad j = 1, \ldots, n,
\]

(III.15)

\[
w^-_w P^*_{Wj} = w^-_j, \quad j = 1, \ldots, n.
\]

(III.16)

In practice, it can be difficult to achieve perfectly consistent criterion weights. Accordingly, the equations (III.13) to (III.16) may not be precisely accurate. Based on the above analysis, To enhance consistency, a creative solution is to minimize the maximum absolute gaps between \(|w^+_j P^*_{Bj} - w^-_B|, |w^-_j P^*_{Bj} - w^+_B|, |w^+_w P^*_{Wj} - w^-_j|, |w^-_w P^*_{Wj} - w^+_j|\), we can formulate
To solve model (III.17), we assume that the maximum absolute gap is $\xi$. Then, we can transform (III.17) into the following programming model:

$$\min \xi$$

subject to

$$w_i^+ + \sum_{j=1, j \neq i}^{n} w_j^+ \geq 1, \quad i = 1, \ldots, n,$$

$$w_i^- + \sum_{j=1, j \neq i}^{n} w_j^- \leq 1, \quad i = 1, \ldots, n,$$

$$|w_j^+ P_{Bj}^- - w_i^-| \leq \xi, \quad j = 1, \ldots, n,$$

$$|w_j^- P_{Bj}^- - w_i^-| \leq \xi, \quad j = 1, \ldots, n,$$

$$|w_j^+ P_{JW}^- - w_i^-| \leq \xi, \quad j = 1, \ldots, n,$$

$$|w_j^- P_{JW}^- - w_i^-| \leq \xi, \quad j = 1, \ldots, n,$$

$$w_j \geq 0, j = 1, 2, \ldots, n$$

(III.18)

It is evident that for a sufficiently large value of $\xi$, the solution space is non-empty, which means that there must exist a feasible region. Solving model (III.18), we obtain the optimal weights ($W^* = w_1^*, w_2^*, \ldots, w_n^*$) and $\xi$. It is worthwhile pointing out here that the model (III.18) also applies to crisp BWM since they may be viewed as a particular instance of interval BWM.

2) Consistency ratio for interval BWM: The consistency ratio (CR) is a widely used and effective index for measuring the degree of consistency in priority rankings (PRs). In this section, we introduce a consistency ratio for the proposed interval BWM method.

Based on the condition defined in [20], interval BWM is considered consistent when $P_{BJ}^* P_{JW}^* = P_{BW}^*$. To satisfy the equality relation $P_{BJ}^* P_{JW}^* = P_{BW}^*$, we need to subtract $\varepsilon^* = [\varepsilon, \varepsilon]$ from $P_{BJ}^*$ and $P_{JW}^*$, and add $\varepsilon^*$ to $P_{BW}^*$. Therefore, the following equation holds:

$$(P_{BJ}^* - \varepsilon^*) \times (P_{JW}^* - \varepsilon^*) = (P_{BW}^* + \varepsilon^*)$$

(III.19)

Obviously, the maximum inequality occurs when $P_{BJ}^*$ and $P_{JW}^*$ both have the maximum value, which is equal to $P_{BW}^*$. There is a condition for $P_{BW}^*$ that $P_{BW}^* < P_{BW}^*$, if we calculate the consistency index using $P_{BW}^*$, then all data associated with the BWM interval can utilize this $CI$ to preserve the effectiveness and feasibility of the CR. We can build an equation that reflects the minimal consistency as follows:

$$(P_{BW}^* - \varepsilon) \times (P_{BW}^* - \varepsilon) = (P_{BW}^* + \varepsilon)$$

(III.20)

$$(\epsilon)^2 - (1 + 2P_{BW}^*)\epsilon + ((P_{BW}^*)^2 - P_{BW}^*) = 0$$

(III.21)

We can calculate the consistency index (CI) by solving for different values of $P_{BW}^*$ ranging from 1 to 9, using the methodology presented in Table I. This table is similar to the one provided in [20].

Note that when the interval preferences have the least consistency, the $CI$ is the largest deviation value. As a result, the gap between the ideal solution in (III.18) and the $CI$ ought to be as wide as feasible. The deviation in (III.18) is the minimal value at the extreme, when $\xi = 0$. The interval preferences are therefore completely consistent. The interval preferences have the lowest consistency when $\xi = CI$ because the deviation in (III.18) has the highest value. As a result, we provide a $CR$ to evaluate the consistency and dependability of the calculated weights:

$$CR = \frac{\xi}{CI}$$

(III.22)

where $CR \in [0, 1]$, $CR = 0$ indicates greater consistency, and $CR = 1$ indicates less consistency.

<table>
<thead>
<tr>
<th>TABLE I: Consistency index</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{BW}^*$</td>
</tr>
<tr>
<td>CI</td>
</tr>
</tbody>
</table>

B. Group interval best-worst method (IGBWM)

1) Goal programming based group interval best-worst method: Since decisions are often made in groups, it is essential to develop a method that can collect the opinions of experts and present final weights. The following steps describe the suggested approach.

- Step 1: Determining the members of the decision team. The first step is to identify the members of the decision team who will participate in the decision-making process. Let there be $t$ experts in total.

- Step 2: Determining the weight of each decision member. In this step, if the importance of the members’ opinions and their expertise level are different, a
certain weight is assigned to each member. For this purpose, the BWM weighting method described in the previous section or other weighting methods can be used. Alternatively, a number between 0 and 1 can be selected as the expert weight vector, such that the sum of the member weights is 1. The importance of expert \(k\) is denoted by \(\lambda[k]\).

> Step 3: Each expert chooses the best and worst criteria. Each expert chooses the best and worst criteria from their perspective. The best criterion selected by expert \(k\) is denoted as \(C_{B[k]}\), and the worst criterion selected by expert \(k\) is denoted as \(C_{W[k]}\).

> Step 4: Determining Best-to-Other’s vectors. The preference of the best criteria \(C_{B[k]}\) over the \(j\)th criteria by expert \(k\) is demonstrated as \(P_{Bj[k]}\). Each expert evaluates the best criterion over all the other criteria, and then the BO matrix is constructed based on their evaluations.

> Step 5: Determining Others-to-Worst vectors. The preference of the \(j\)th criteria over the worst criteria \(C_{W[k]}\) by expert \(k\) is demonstrated as \(P_{wj[k]}\). Each expert evaluates the worst criterion over all the other criteria, and then the OW matrix is constructed based on their evaluations.

> Step 6: Calculating reference best and worst criteria. In this step, we combine the best and worst criteria selected by each expert to determine the reference best criterion \(C_B\) and the reference worst criterion \(C_W\). Assume that the score of \(C_j\) based on the best criteria is denoted by \(S_{best}(C_j)\), which is calculated as follows:

\[
S_{best}(C_j) = \sum_k \lambda[k] S_{best}^k(C_j). \tag{III.23}
\]

\(S_{best}^k(C_j)\) is obtained by

\[
S_{best}^k(C_j) = \begin{cases} 1 & \text{if expert } k \text{ selects } C_j \\ 0 & \text{otherwise.} \end{cases}
\]

The reference best criterion is determined by the criterion \(C_j\) that achieves the highest score \(S_{best}(C_j)\). In case of a tie between two criteria, one of them can be arbitrarily chosen as the reference worst criterion.

> Step 7: Calculation of optimal weights. Obtain the optimal weight vector of criteria \((W^* = w_1^*, w_2^*, \ldots, w_n^*)\) by the following model:

\[
\begin{aligned}
\min & \sum_{k=1}^t \lambda[k] \max_j \{ |w_j^+ P_{Bj[k]}^+ - w_j^-|, |w_j^+ P_{Bj[k]}^- - w_j^-|, |w_j^+ P_{wj[k]}^- - w_j^- + |w_j^+ P_{wj[k]}^+ - w_j^-| \} \\
\text{s.t.} & \begin{cases} w_i^- + \sum_{j=1, j \neq i}^n w_i^+ \geq 1, & i = 1, \ldots, n, \\
& w_i^- + \sum_{j=1, j \neq i}^n w_i^- \leq 1, & i = 1, \ldots, n, \\
& w_j^- \leq w_j^+, & j = 1, 2, \ldots, n \\
& w_j \geq 0, & j = 1, 2, \ldots, n \end{cases} \\
\end{aligned} \tag{III.25}
\]

Obove model can be converted into a linear programming form as follows:

\[
\begin{aligned}
\min & \sum_{k=1}^t \lambda[k] \xi[k] \\
& w_i^- + \sum_{j=1, j \neq i}^n w_i^+ \geq 1, & i = 1, \ldots, n, \\
& w_i^- + \sum_{j=1, j \neq i}^n w_i^- \leq 1, & i = 1, \ldots, n, \\
& |w_j^+ P_{Bj[k]}^+ - w_j^-| \leq \xi[k], & k = 1, \ldots, t, \\
& |w_j^+ P_{Bj[k]}^- - w_j^-| \leq \xi[k], & k = 1, \ldots, t, \\
& |w_j^+ P_{wj[k]}^- - w_j^-| \leq \xi[k], & k = 1, \ldots, t, \\
& |w_j^+ P_{wj[k]}^+ - w_j^-| \leq \xi[k], & k = 1, \ldots, t, \\
& w_j \geq 0, & j = 1, 2, \ldots, n \end{aligned} \tag{III.26}
\]

Solving model (III.26), we obtain the optimal weights \((W^* = (w_1^*, w_2^*, \ldots, w_n^*))\) and \((\xi[1], \xi[2], \ldots, \xi[t])\).

2) Consistency ratio for group interval BWM: Group consistency ratio can be calculated after solving the model. To evaluate the group consistency and dependability of the calculated weights, we provide a consistency ratio (CR):

\[
CR = \frac{\xi}{C_I} \quad k = 1, \ldots, t. \tag{III.27}
\]

where \(CR \in [0, 1]\), \(CR = 0\) indicates greater consistency, and \(CR = 1\) indicates less consistency. The consistency index (CI) can be searched in Table I.

IV. Interval TODIM

In this section, we introduce a new TODIM method based on interval numbers whose procedure is as follows.
The distance matrix is given by equation (IV.8). Let $\Phi$ be the preference degree matrix, given by equation (II.7) as follows:

$$ \Phi = \begin{bmatrix} \phi^1(A_1, A_1) & \cdots & \phi^1(A_1, A_m) \\ \phi^2(A_1, A_1) & \cdots & \phi^2(A_1, A_m) \\ \vdots & \ddots & \vdots \\ \phi^m(A_1, A_1) & \cdots & \phi^m(A_1, A_m) \end{bmatrix} $$

$\Phi$ is the overall preference matrix, where $\phi^j(A_i, A_k)$ represents the preference degree of alternative $A_i$ over $A_k$.

For each criterion $j$, where $j$ ranges from 1 to $n$, in the pairwise comparisons among the alternatives $i = 1, \ldots, m$ and $k = 1, \ldots, m$. The preference degree matrix is given by equation (II.7) as shown below:

$$ \Phi^j = \begin{bmatrix} \phi^1_j(A_1, A_1) & \cdots & \phi^1_j(A_1, A_m) \\ \phi^2_j(A_1, A_1) & \cdots & \phi^2_j(A_1, A_m) \\ \vdots & \ddots & \vdots \\ \phi^m_j(A_1, A_1) & \cdots & \phi^m_j(A_1, A_m) \end{bmatrix} $$

The distance matrix is given by equation (II.12) as shown below:

$$ D(A_i) = \begin{bmatrix} d(A_1, A_1) & \cdots & d(A_1, A_m) \\ \vdots & \ddots & \vdots \\ d(A_m, A_1) & \cdots & d(A_m, A_m) \end{bmatrix} $$

$D$ is the distance matrix, where $d(A_i, A_k)$ represents the distance between alternatives $A_i$ and $A_k$.

Step 1: Normalize the interval rating $X^* = (x^*_{ij})_{m \times n}$ as $\bar{X}^* = (\hat{x}^*_{ij})_{m \times n}$.

$$ \hat{x}^*_{ij} = \left[ \frac{\sum_{i=1}^{n} x_{ij}}{\sum_{i=1}^{n} \sum_{j=1}^{m} x_{ij}}, \frac{\sum_{j=1}^{m} x_{ij}}{\sum_{i=1}^{n} \sum_{j=1}^{m} x_{ij}} \right] \text{ if benefit criteria,} $$

$$ \hat{x}^*_{ij} = \left[ \frac{\sum_{i=1}^{n} \frac{1}{x_{ij}}}{\sum_{i=1}^{n} \sum_{j=1}^{m} \frac{1}{x_{ij}}}, \frac{\sum_{j=1}^{m} \frac{1}{x_{ij}}}{\sum_{i=1}^{n} \sum_{j=1}^{m} \frac{1}{x_{ij}}} \right] \text{ if cost criteria.} $$

Step 2: Calculating the preference and distance matrix. In this step, compute the preference degree and distance for each criterion $j$, where $j$ ranges from 1 to $n$, in the pairwise comparisons among the alternatives $i = 1, \ldots, m$ and $k = 1, \ldots, m$. The preference degree matrix is given by equation (II.7) as shown below:

$$ P(A_i) = \begin{bmatrix} p_1(A_i > A_1) & \cdots & p_1(A_i > A_m) \\ \vdots & \ddots & \vdots \\ p_n(A_i > A_1) & \cdots & p_n(A_i > A_m) \end{bmatrix} $$

$P$ is the preference matrix, where $p_j(A_i > A_k)$ represents the probability of preference of alternative $A_i$ over $A_k$.

$D$ is the distance matrix, where $d(A_i, A_k)$ represents the distance between alternatives $A_i$ and $A_k$.

Step 3: Calculating the dominance function. In this step, compute the dominance function for each criterion $j$, where $j$ ranges from 1 to $n$, in the pairwise comparisons among the alternatives $i = 1, \ldots, m$. We are inspired to propose the following dominance equation:

$$ \varphi_j^*(A_i, A_k) = \begin{cases} w_j^* d_j(A_i, A_k) \alpha, & p_j(A_i > A_k) > 0.5, \\ 0, & p_j(A_i > A_k) = 0.5, \\ -\lambda w_j^* d_j(A_i, A_k) \beta, & p_j(A_i > A_k) < 0.5 \end{cases} $$

where $\lambda > 1$ is the coefficient of loss aversion, $0 < \alpha, \beta < 1$ are estimable coefficients determining the convexity/concavity of the function, and $\alpha, \beta$ are not fractions with an even denominator.

It is worth noting that, for each $i = 1, \ldots, m$, the values $\varphi_j^*(A_i, A_k)$ can be arranged in an $m \times n$ matrix, as follows:

$$ \begin{bmatrix} C_1 & \cdots & C_j & \cdots & C_n \\ A_1 & d_1(A_1, A_1) & \cdots & d_j(A_1, A_1) & \cdots & d_n(A_1, A_1) \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ A_m & d_1(A_m, A_1) & \cdots & d_j(A_m, A_1) & \cdots & d_n(A_m, A_1) \end{bmatrix} $$

Step 4: Calculating the overall performance of each alternative.

$$ \varphi^*(A_i) = \sum_{j=1}^{m} \varphi_j^*(A_i, A_k), $$

which corresponds to the sum of all the elements of the matrix $\Phi^*(A_i)$.

Step 5: Order the alternatives according to the $\varphi^*(A_i)$ values.

The original TODIM method requires the computation of the total dominance ratios based on pairwise comparisons for each alternative $i = 1, \ldots, m$, which is then normalized to a value between 0 and 1 to facilitate the interpretation of the results. However, as the interval TODIM method proposed in this study yields interval-valued $\varphi^*(A_i)$, we can directly compute the preference degrees using equations (II.7) and (II.11) to obtain a ranking with a clear and accurate interpretability. The preference matrix for ranking of alternatives is calculated by equation (IV.5).

The score of alternative $A_i$ is obtained by aggregating the values in each row of the preference matrix $P_{\varphi}$:

$$ S(A_i) = \sum_{k=1}^{m} p(\varphi^*(A_i) > \varphi^*(A_k)) $$

The alternatives $A_i$ ($i = 1, \ldots, m$) can be sorted in descending order using the calculated value of $S(A_i)$:

$$ A_i | S(A_i) = \arg \max S(A_i) \succ \cdots \succ A_i | S(A_i) = \arg \min S(A_i) $$

To obtain a ranking that is clearer and more accurate interpretability based on the sorting results obtained from the above equation, the alternatives can be listed as a list according to the method used:

$$ A_i | i=1 \ Or \ 2 \ Or \ m \succ \cdots \succ A_i | i=1 \ Or \ 2 \ Or \ m $$

V. Numerical studies

A. Example 1

In this section, numerical examples presented through Hafezalkotob [10] will be used to verify
\[
P_{\varphi} = \begin{bmatrix}
A_1 & \cdots & A_k & \cdots & A_m \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
A_1 & \cdots & A_k & \cdots & A_m
\end{bmatrix}
\]

\[
\begin{align*}
A_1 & \quad p(\varphi^*(A_1) > \varphi^*(A_1)) \\
\vdots & \quad \vdots \\
A_m & \quad p(\varphi^*(A_m) > \varphi^*(A_m))
\end{align*}
\] (IV.5)

**TABLE II: Preference degree of the engines criteria**

<table>
<thead>
<tr>
<th>DMs</th>
<th>Best criterion</th>
<th>Worst criterion</th>
<th>C_1</th>
<th>C_2</th>
<th>C_3</th>
<th>C_4</th>
<th>C_5</th>
<th>C_6</th>
<th>C_7</th>
<th>C_8</th>
<th>C_9</th>
<th>C_10</th>
</tr>
</thead>
<tbody>
<tr>
<td>D_1</td>
<td>C_7</td>
<td>-</td>
<td>[3, 5]</td>
<td>[5, 7]</td>
<td>[1, 3]</td>
<td>[1, 3]</td>
<td>[5, 7]</td>
<td>[5, 7]</td>
<td>[1, 1]</td>
<td>[5, 7]</td>
<td>[7, 9]</td>
<td>[1, 3]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C_9</td>
<td>[1, 3]</td>
<td>[5, 7]</td>
<td>[1, 3]</td>
<td>[3, 5]</td>
<td>[5, 7]</td>
<td>[1, 3]</td>
<td>[7, 9]</td>
<td>[1, 3]</td>
<td>[1, 3]</td>
<td>[5, 7]</td>
</tr>
<tr>
<td>D_2</td>
<td>C_6</td>
<td>-</td>
<td>[5, 7]</td>
<td>[3, 5]</td>
<td>[5, 7]</td>
<td>[7, 9]</td>
<td>[1, 3]</td>
<td>[1, 3]</td>
<td>[1, 3]</td>
<td>[5, 7]</td>
<td>[5, 7]</td>
<td>[1, 3]</td>
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<tr>
<td></td>
<td></td>
<td>C_4</td>
<td>[3, 5]</td>
<td>[5, 7]</td>
<td>[1, 3]</td>
<td>[1, 1]</td>
<td>[5, 7]</td>
<td>[7, 9]</td>
<td>[5, 7]</td>
<td>[1, 3]</td>
<td>[1, 3]</td>
<td>[5, 7]</td>
</tr>
<tr>
<td>D_3</td>
<td>C_7</td>
<td>-</td>
<td>[1, 3]</td>
<td>[5, 7]</td>
<td>[3, 5]</td>
<td>[7, 9]</td>
<td>[5, 7]</td>
<td>[1, 3]</td>
<td>[1, 3]</td>
<td>[5, 7]</td>
<td>[5, 7]</td>
<td>[1, 3]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C_4</td>
<td>[3, 5]</td>
<td>[5, 7]</td>
<td>[1, 3]</td>
<td>[1, 1]</td>
<td>[3, 5]</td>
<td>[3, 5]</td>
<td>[7, 9]</td>
<td>[1, 3]</td>
<td>[5, 7]</td>
<td>[5, 7]</td>
</tr>
<tr>
<td>D_4</td>
<td>C_5</td>
<td>-</td>
<td>[3, 5]</td>
<td>[7, 9]</td>
<td>[3, 5]</td>
<td>[3, 5]</td>
<td>[1, 1]</td>
<td>[5, 7]</td>
<td>[3, 5]</td>
<td>[5, 7]</td>
<td>[1, 3]</td>
<td>[1, 3]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C_2</td>
<td>[3, 5]</td>
<td>[1, 1]</td>
<td>[3, 5]</td>
<td>[1, 3]</td>
<td>[7, 9]</td>
<td>[5, 7]</td>
<td>[5, 7]</td>
<td>[3, 5]</td>
<td>[1, 3]</td>
<td>[5, 7]</td>
</tr>
</tbody>
</table>

**TABLE III: Optimal weights of engine criteria**

<table>
<thead>
<tr>
<th>(\alpha)</th>
<th>(\xi)</th>
<th>C_1</th>
<th>C_2</th>
<th>C_3</th>
<th>C_4</th>
<th>C_5</th>
<th>C_6</th>
<th>C_7</th>
<th>C_8</th>
<th>C_9</th>
<th>C_10</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.455</td>
<td>0.079</td>
<td>0.079</td>
<td>0.079</td>
<td>0.061</td>
<td>0.079</td>
<td>0.101</td>
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<tr>
<td>0.25</td>
<td>0.517</td>
<td>0.081</td>
<td>0.081</td>
<td>0.081</td>
<td>0.064</td>
<td>0.081</td>
<td>0.103</td>
<td>0.174</td>
<td>0.081</td>
<td>0.081</td>
<td>0.174</td>
</tr>
<tr>
<td>IGBWM-I</td>
<td>0.50</td>
<td>0.576</td>
<td>0.083</td>
<td>0.083</td>
<td>0.083</td>
<td>0.066</td>
<td>0.083</td>
<td>0.104</td>
<td>0.166</td>
<td>0.083</td>
<td>0.083</td>
</tr>
<tr>
<td>0.75</td>
<td>0.631</td>
<td>0.085</td>
<td>0.085</td>
<td>0.085</td>
<td>0.068</td>
<td>0.085</td>
<td>0.104</td>
<td>0.160</td>
<td>0.085</td>
<td>0.085</td>
<td>0.160</td>
</tr>
<tr>
<td>1</td>
<td>0.685</td>
<td>0.086</td>
<td>0.086</td>
<td>0.086</td>
<td>0.070</td>
<td>0.086</td>
<td>0.105</td>
<td>0.155</td>
<td>0.086</td>
<td>0.086</td>
<td>0.155</td>
</tr>
<tr>
<td>IGBWM-II</td>
<td>0.222</td>
<td>0.065</td>
<td>0.057</td>
<td>0.061</td>
<td>0.031</td>
<td>0.067</td>
<td>0.074</td>
<td>0.178</td>
<td>0.048</td>
<td>0.056</td>
<td>0.156</td>
</tr>
</tbody>
</table>

**TABLE IV: Preference degree of the engines criteria**

<table>
<thead>
<tr>
<th>P_{B_1[k]}</th>
<th>P_{B_8[k]}</th>
<th>P_{W_1[k]}</th>
<th>P_{W_8[k]}</th>
</tr>
</thead>
<tbody>
<tr>
<td>D_1</td>
<td>D_2</td>
<td>D_3</td>
<td>D_4</td>
</tr>
<tr>
<td>[3, 5]</td>
<td>[5, 7]</td>
<td>[1, 3]</td>
<td>[3, 5]</td>
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<tr>
<td>[5, 7]</td>
<td>[5, 7]</td>
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<td>[1, 3]</td>
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<tr>
<td>[1, 3]</td>
<td>[1, 3]</td>
<td>[1, 3]</td>
<td>[3, 5]</td>
</tr>
</tbody>
</table>
the validity and advantages of our proposed interval group BWM. In this case, the four decision-makers are considering various criteria to select a hybrid vehicle engine. These criteria include the combined ultimate power ($C_1$), combined ultimate torque ($C_2$), fuel consumption ($C_3$), index of durability and maintenance ($C_4$), relative price ($C_5$), relative needed foreign fund ($C_6$), job creation index ($C_7$), carbon monoxide emission ($C_8$), total hydrocarbon emission ($C_9$), and nitrogen oxides emission ($C_{10}$).

The weight of DMs is considered as $\lambda = [0.132, 0.592, 0.197, 0.079]$. Table II presents the interval preferences of the linguistic terms considered by the designers, including the interval best-to-others and others-to-worst vectors of preference degrees for the engine criteria. In the following analysis, we compare the optimal weights of engine criteria and the objective function values obtained by our proposed interval group BWM (IGBWM-II) (III.26) with those obtained by the interval group BWM proposed by Hafezalkotob (IGBWM-I) [10] for different levels of uncertainty($\alpha$), as shown in Table III.

As can be seen in Table III, by employing IGBWM-I and IGBWM-II, different weights are obtained for criteria. The best weights are obtained using IGBWM-I as crisp numbers, but the best weights are obtained using IGBWM-II as interval weights, which better reflect the decision-maker’s subjective judgment and cope with ambiguous situations by providing a more appropriate range of weights. We can also observe that IGBWM-I produces varied weight results for various $\alpha$ , and how to select $\alpha$ again presents a challenge, unlike IGBWM-II, which does not encounter this issue. We note that in IGBWM-I, the weights assigned to attributes $C_1$, $C_2$, $C_3$, $C_5$, $C_8$, and $C_9$ are all equal for every $\alpha$ level, which contradicts the preference degrees assigned by the experts in Table II. For example, when comparing $C_1$ and $C_8$ in Table IV, it becomes apparent that $C_1$ should have a higher weight than $C_8$. IGBWM-II does not suffer from this issue. Additionally, the IGBWM-II proposed in this paper yields smaller errors, suggesting better consistency. Finally, we conduct a consistency analysis using Equation III.27, and the results are presented in Table V. The consistency ratio (CR) for IGBWM-II (CR = 0.042) is much smaller than the minimum consistent CR of 0.087 for IGBWM-I at different levels of $\alpha$. This finding suggests that IGBWM-II is a reliable and effective approach for solving multi-criteria decision-making problems.

B. Example 2

With the world’s population continually increasing and people adopting new lifestyles, there is a growing demand for a diverse range of dietary preferences. In a chain restaurant company, selecting an appropriate supplier is of utmost importance for achieving long-term development prospects, pursuing business strategies, and maintaining a competitive position in the market. Due to this, FSS has become a critical topic. This case study is conducted on the basis of the collected information and comments of management experts and purchasing officer.

1) Gibwm and I-TOMIM method: To obtain more reasonable evaluation results, we use the MCDM method proposed in this paper to evaluate the suppliers. The steps are shown in Figure 2.

The enterprise will evaluate five potential suppliers, $A_i$ (i = 1, 2, 3, 4, 5). In the selection process, ten criteria were considered based on literature research and expert opinions, as shown in Table 3. It is worth noting that some standards can be obtained by gathering information such as product prices, supply chain efficiency, etc., while others need to be evaluated by experts. Table VI presents the linguistic preferences and their corresponding interval preferences. In this example, DMs evaluate the preference degrees of the criteria based on linguistic preferences, with all experts given equal weight. Table VII shows the interval best-to-others and others-to-worst vectors of the criteria's preference degrees by expert evaluation.

The reference best and worst criteria are obtained by calculating equations (III.23) and (III.24):

\[ S_{best}(C_2) = 0.25, S_{best}(C_2) = 0.5, S_{best}(C_{10}) = 0.25, \]
\[ S_{worst}(C_3) = 0.25, S_{worst}(C_7) = 0.75. \]

Here, we only need to consider the criteria that have been identified by the experts as the best or worst, which are criteria $C_2$, $C_6$, and $C_{10}$ for the best, and criteria $C_3$ and $C_7$ for the worst. Therefore, the reference best and worst criteria is $C_6$ and $C_7$, respectively. Table VIII presents the optimal weights
Fig. 2: Theoretical flow chart of this study work.

of criteria obtained by solving (III.26), along with the corresponding error value and the CR calculated using (III.27).

The decision matrix is composed of scores of the suppliers on ten criteria, as shown in Table IX. Calculate the normalized interval ratings of the suppliers for the given problem using equation (IV.1) and list it in Table X.

Next, we calculate the preference matrix and distance matrix of each solution under each criterion. The preference degree matrix is given by equation (II.7) - (II.11) as shown below:

$$P(A_1) = \begin{bmatrix}
0.500 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 \\
0.892 & 1.000 & 0.007 & 0.284 & 1.000 & 0.720 & 0.996 & 0.692 & 0.676 & 0.000 \\
0.150 & 0.000 & 0.071 & 0.500 & 0.000 & 0.155 & 0.052 & 0.961 & 0.049 & 1.000 \\
0.500 & 0.000 & 0.000 & 0.916 & 0.000 & 0.500 & 0.140 & 0.692 & 0.002 & 1.000 \\
0.723 & 1.000 & 0.000 & 0.057 & 0.000 & 0.986 & 0.500 & 1.000 & 0.500 & 0.000 \\
0.108 & 0.000 & 0.993 & 0.716 & 0.000 & 0.280 & 0.004 & 0.308 & 0.324 & 1.000 \\
0.500 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 \\
0.000 & 0.000 & 0.728 & 0.716 & 0.000 & 0.047 & 0.000 & 0.879 & 0.012 & 1.000 \\
0.108 & 0.000 & 0.291 & 0.986 & 0.000 & 0.280 & 0.000 & 0.500 & 0.000 & 1.000 \\
0.259 & 0.000 & 0.062 & 0.168 & 0.000 & 0.912 & 0.004 & 0.983 & 0.324 & 1.000
\end{bmatrix}$$

$$P(A_2) = \begin{bmatrix}
0.108 & 0.000 & 0.993 & 0.716 & 0.000 & 0.280 & 0.004 & 0.308 & 0.324 & 1.000 \\
0.500 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 \\
0.000 & 0.000 & 0.728 & 0.716 & 0.000 & 0.047 & 0.000 & 0.879 & 0.012 & 1.000 \\
0.108 & 0.000 & 0.291 & 0.986 & 0.000 & 0.280 & 0.000 & 0.500 & 0.000 & 1.000 \\
0.259 & 0.000 & 0.062 & 0.168 & 0.000 & 0.912 & 0.004 & 0.983 & 0.324 & 1.000
\end{bmatrix}$$
<table>
<thead>
<tr>
<th>Indicators</th>
<th>objective</th>
<th>From</th>
<th>Descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply chain efficiency ($C_1$)</td>
<td>Maximum</td>
<td>[16]</td>
<td>The productivity and operational efficiency of the supply chain while meeting customer needs. (Measured on a verbal scale)</td>
</tr>
<tr>
<td>Customer satisfaction ($C_2$)</td>
<td>Maximum</td>
<td>[4]</td>
<td>The degree of satisfaction customers have with a product or service. (%).</td>
</tr>
<tr>
<td>Supplier relationship ($C_3$)</td>
<td>Maximum</td>
<td>[37]</td>
<td>The level of cooperation between a company and its suppliers, including the length of time working together and overall satisfaction. (Measured on a verbal scale)</td>
</tr>
<tr>
<td>Corporate social responsibility ($C_4$)</td>
<td>Maximum</td>
<td>[37]</td>
<td>Includes eco-labels, stakeholder relations and community recognition, wage standards, working conditions and environment, and compliance with international human rights. (Measured on a verbal scale)</td>
</tr>
<tr>
<td>Agility ($C_5$)</td>
<td>Maximum</td>
<td>[27]</td>
<td>The ability to provide variability in the products produced, i.e., providing product types in a timely manner based on customer requests. (Measured on a verbal scale)</td>
</tr>
<tr>
<td>Beef safety and quality ($C_6$)</td>
<td>Maximum</td>
<td>[16]</td>
<td>The quality of the fresh Beef is healthful and nutritious. Fresh Beef does not exceed the tolerable rate for foodborne disease risk. (Qualification rate)</td>
</tr>
<tr>
<td>Company size ($C_7$)</td>
<td>Maximum</td>
<td>Added by experts</td>
<td>Market share, performance track record, asset profile and infrastructure, financial health and stability, and technological competitiveness. (Measured on a verbal scale)</td>
</tr>
<tr>
<td>Product management ($C_8$)</td>
<td>Maximum</td>
<td>[37]</td>
<td>Product compliance procedures and regular audits, ongoing food safety training, and traceability. (Measured on a verbal scale)</td>
</tr>
<tr>
<td>Environment impact ($C_9$)</td>
<td>Minimum</td>
<td>[33]</td>
<td>Waste management, green manufacturing, green packaging and renewable energy, the number of ISO standards obtained, and geographical location. (Measured on a verbal scale)</td>
</tr>
<tr>
<td>Price ($C_{10}$)</td>
<td>Minimum</td>
<td>[37]</td>
<td>These elements are important in business operations: price display, discounts, product cost, logistics cost, tariffs and taxes, quantity discounts, payment terms, and shelving fees. (Yuan(s)/kg)</td>
</tr>
</tbody>
</table>

Fig. 3: Criteria for candidate frozen Beef suppliers

\[
P(A_3) = \begin{bmatrix}
0.850 & 1.000 & 0.929 & 0.500 & 1.000 & 0.845 & 0.948 & 0.039 & 0.951 & 0.000 \\
1.000 & 1.000 & 0.272 & 0.284 & 1.000 & 0.953 & 1.000 & 0.121 & 0.988 & 0.000 \\
0.500 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 \\
0.850 & 1.000 & 0.126 & 0.916 & 1.000 & 0.845 & 0.687 & 0.121 & 0.220 & 0.000 \\
0.957 & 1.000 & 0.004 & 0.057 & 1.000 & 1.000 & 0.948 & 0.763 & 0.951 & 0.000
\end{bmatrix}
\]

\[
P(A_4) = \begin{bmatrix}
0.500 & 1.000 & 1.000 & 0.084 & 1.000 & 0.500 & 0.860 & 0.308 & 0.998 & 0.000 \\
0.892 & 1.000 & 0.709 & 0.014 & 1.000 & 0.720 & 1.000 & 0.500 & 1.000 & 0.000 \\
0.150 & 0.000 & 0.874 & 0.084 & 0.000 & 0.155 & 0.313 & 0.879 & 0.780 & 1.000 \\
0.500 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 \\
0.723 & 1.000 & 0.171 & 0.000 & 1.000 & 0.986 & 0.860 & 0.983 & 0.998 & 0.000
\end{bmatrix}
\]
The distance matrix is given by equation (II.12) as shown below:

\[
P(A_5) = \begin{bmatrix}
0.277 & 0.000 & 1.000 & 0.943 & 1.000 & 0.014 & 0.500 & 0.000 & 0.500 & 1.000 \\
0.741 & 1.000 & 0.938 & 0.832 & 1.000 & 0.088 & 0.996 & 0.017 & 0.676 & 0.000 \\
0.043 & 0.000 & 0.996 & 0.943 & 0.000 & 0.000 & 0.052 & 0.237 & 0.049 & 1.000 \\
0.277 & 0.000 & 0.829 & 1.000 & 0.000 & 0.014 & 0.140 & 0.017 & 0.002 & 1.000 \\
0.500 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500
\end{bmatrix}
\]

The preference matrix of alternative supplier is

\[
\varphi^*(A_2) = \sum_{j=1}^{n} \sum_{k=1}^{m} \varphi_j^*(A_2, A_k) = [-1.162, -0.422],
\]

\[
\varphi^*(A_3) = \sum_{j=1}^{n} \sum_{k=1}^{m} \varphi_j^*(A_3, A_k) = [0.651, 1.460],
\]

\[
\varphi^*(A_4) = \sum_{j=1}^{n} \sum_{k=1}^{m} \varphi_j^*(A_4, A_k) = [0.204, 0.811],
\]

\[
\varphi^*(A_5) = \sum_{j=1}^{n} \sum_{k=1}^{m} \varphi_j^*(A_5, A_k) = [-1.207, -0.361].
\]

The preference matrix of alternative supplier is calculated by equation (IV.5) and the results are shown below:

\[
P_\varphi = \begin{bmatrix}
0.500 & 1.000 & 0.000 & 0.013 & 1.000 \\
0.000 & 0.500 & 0.000 & 0.000 & 0.490 \\
1.000 & 1.000 & 0.500 & 0.952 & 1.000 \\
0.987 & 1.000 & 0.048 & 0.500 & 1.000 \\
0.000 & 0.510 & 0.000 & 0.000 & 0.500
\end{bmatrix}
\]

The scores of alternative suppliers \(A_i\) are obtained
TABLE VII: Preference degree of suppliers

<table>
<thead>
<tr>
<th>DMs</th>
<th>Best criterion</th>
<th>Worst criterion</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
<th>C9</th>
<th>C10</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>C2</td>
<td>-</td>
<td>[1,3]</td>
<td>[1,1]</td>
<td>[1,3]</td>
<td>[5,7]</td>
<td>[3,5]</td>
<td>[1,3]</td>
<td>[7,9]</td>
<td>[1,3]</td>
<td>[5,7]</td>
<td>[1,3]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-</td>
<td>[5,7]</td>
<td>[7,9]</td>
<td>[5,7]</td>
<td>[1,3]</td>
<td>[3,5]</td>
<td>[5,7]</td>
<td>[1,1]</td>
<td>[5,7]</td>
<td>[1,3]</td>
<td>[5,7]</td>
</tr>
<tr>
<td>D2</td>
<td>C6</td>
<td>-</td>
<td>[5,7]</td>
<td>[1,3]</td>
<td>[5,7]</td>
<td>[5,7]</td>
<td>[3,5]</td>
<td>[1,3]</td>
<td>[1,1]</td>
<td>[7,9]</td>
<td>[3,5]</td>
<td>[3,5]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-</td>
<td>[3,5]</td>
<td>[3,5]</td>
<td>[3,5]</td>
<td>[1,3]</td>
<td>[1,3]</td>
<td>[5,7]</td>
<td>[1,1]</td>
<td>[3,5]</td>
<td>[3,5]</td>
<td>[7,9]</td>
</tr>
<tr>
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<td>C7</td>
<td>-</td>
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<td>[3,5]</td>
<td>[5,7]</td>
<td>[3,5]</td>
<td>[1,3]</td>
<td>[1,3]</td>
<td>[7,9]</td>
<td>[3,5]</td>
<td>[1,3]</td>
<td>[1,1]</td>
</tr>
<tr>
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<td></td>
<td>-</td>
<td>[7,9]</td>
<td>[1,3]</td>
<td>[7,9]</td>
<td>[3,5]</td>
<td>[1,3]</td>
<td>[3,5]</td>
<td>[1,3]</td>
<td>[3,5]</td>
<td>[7,9]</td>
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</tr>
<tr>
<td>D4</td>
<td>C6</td>
<td>-</td>
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<td>[5,7]</td>
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<td>[5,7]</td>
<td>[3,5]</td>
<td>[7,9]</td>
<td>[1,3]</td>
<td>[3,5]</td>
<td>[5,7]</td>
<td>[3,5]</td>
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</tbody>
</table>

TABLE VIII: Optimal weights of criteria

<table>
<thead>
<tr>
<th>ξ</th>
<th>CR</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
<th>C9</th>
<th>C10</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.212</td>
<td>0.041</td>
<td>0.056, 0.106, 0.048, 0.129</td>
<td>0.058, 0.056</td>
<td>0.106, 0.078</td>
<td>0.177, 0.023</td>
<td>0.078, 0.065</td>
<td>0.106, 0.129</td>
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<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TABLE IX: Decision matrix

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Benefit criteria</th>
<th>Cost criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C1</td>
<td>C2</td>
</tr>
<tr>
<td>A2</td>
<td>[3,5]</td>
<td>94.32</td>
</tr>
<tr>
<td>A4</td>
<td>[5,7]</td>
<td>97.51</td>
</tr>
<tr>
<td>A5</td>
<td>[4,6]</td>
<td>94.83</td>
</tr>
</tbody>
</table>

by aggregating the values in each row of the preference matrix $P_{r^i}$, as shown below:

$S(A_1) = 2.513, S(A_2) = 0.99, S(A_3) = 4.452, ~\text{(V.1)}$

$S(A_4) = 3.535, S(A_5) = 1.01.$

Based on the score values, we can determine the ranking order of the five alternative suppliers:

$A_3 \succ A_4 \succ A_1 \succ A_5 \succ A_2.$

Obviously, $A_3$ is the most desirable supplier.

2) Sensitive analysis: In this example, we set $\lambda$ to 1, which means that each loss will be weighted equally with its actual value in the calculation of the global value. We then conduct a sensitivity analysis on the variable $\lambda$ to investigate how changes in this parameter affect the ranking of alternative suppliers. Table XVI presents the ranking of the alternative suppliers for different $\lambda$ values. Table XVI indicates that as the value of $h$ changes, there is a slight variation in the preference degrees between alternative suppliers. However, only when $\lambda$ equals 1 and 1.5 did the suppliers ranked fourth and fifth exchange their positions, whereas the overall ranking remained unchanged for all other cases. This suggests that the proposed interval TODIM method exhibits...
### TABLE XI: Dominance degree matrix ($\Phi^*(A_1)$)

<table>
<thead>
<tr>
<th>FSs</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
<th>$C_6$</th>
<th>$C_7$</th>
<th>$C_8$</th>
<th>$C_9$</th>
<th>$C_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>[0.0,</td>
<td>[0.0,</td>
<td>[0.0,</td>
<td>[0.0,</td>
<td>[0.0,</td>
<td>[0.0,</td>
<td>[0.0,</td>
<td>[0.0,</td>
<td>[0.0,</td>
<td>[0.0,</td>
</tr>
<tr>
<td></td>
<td>0.0]</td>
<td>0.0]</td>
<td>0.0]</td>
<td>0.0]</td>
<td>0.0]</td>
<td>0.0]</td>
<td>0.0]</td>
<td>0.0]</td>
<td>0.0]</td>
<td>0.0]</td>
</tr>
<tr>
<td>$A_2$</td>
<td>[0.035,</td>
<td>[0.075,</td>
<td>[-0.039,</td>
<td>[-0.036,</td>
<td>[0.075,</td>
<td>[0.083,</td>
<td>[0.017,</td>
<td>[0.034,</td>
<td>[0.027,</td>
<td>[-0.092,</td>
</tr>
<tr>
<td></td>
<td>0.035]</td>
<td>0.092]</td>
<td>-0.034]</td>
<td>-0.027]</td>
<td>0.092]</td>
<td>0.149]</td>
<td>0.027]</td>
<td>0.057]</td>
<td>0.033]</td>
<td>-0.092]</td>
</tr>
<tr>
<td>$A_3$</td>
<td>[-0.033,</td>
<td>[-0.092,</td>
<td>[-0.036,</td>
<td>[0.0,</td>
<td>[-0.092,</td>
<td>[-0.187,</td>
<td>[-0.026,</td>
<td>[0.053,</td>
<td>[-0.052,</td>
<td>[0.075,</td>
</tr>
<tr>
<td></td>
<td>-0.033]</td>
<td>-0.075]</td>
<td>-0.031]</td>
<td>0.0]</td>
<td>-0.075]</td>
<td>-0.104]</td>
<td>-0.016]</td>
<td>0.088]</td>
<td>-0.044]</td>
<td>0.092]</td>
</tr>
<tr>
<td>$A_4$</td>
<td>[0.0,</td>
<td>[-0.092,</td>
<td>[-0.039,</td>
<td>[0.037,</td>
<td>[-0.092,</td>
<td>[0.0,</td>
<td>[-0.023,</td>
<td>[0.034,</td>
<td>[-0.055,</td>
<td>[0.075,</td>
</tr>
<tr>
<td></td>
<td>0.0]</td>
<td>-0.075]</td>
<td>-0.034]</td>
<td>0.05]</td>
<td>-0.075]</td>
<td>0.0]</td>
<td>-0.014]</td>
<td>0.057]</td>
<td>-0.046]</td>
<td>0.092]</td>
</tr>
<tr>
<td>$A_5$</td>
<td>[0.026,</td>
<td>[0.075,</td>
<td>[-0.039,</td>
<td>[-0.052,</td>
<td>[-0.092,</td>
<td>[0.123,</td>
<td>[0.0,</td>
<td>[0.055,</td>
<td>[0.0,</td>
<td>[-0.092,</td>
</tr>
<tr>
<td></td>
<td>0.026]</td>
<td>0.092]</td>
<td>-0.034]</td>
<td>-0.039]</td>
<td>0.075]</td>
<td>0.222]</td>
<td>0.0]</td>
<td>0.092]</td>
<td>0.0]</td>
<td>-0.075]</td>
</tr>
</tbody>
</table>

### TABLE XII: Dominance degree matrix ($\Phi^*(A_2)$)

<table>
<thead>
<tr>
<th>FSs</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
<th>$C_6$</th>
<th>$C_7$</th>
<th>$C_8$</th>
<th>$C_9$</th>
<th>$C_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>[-0.035,</td>
<td>[-0.092,</td>
<td>[0.034,</td>
<td>[0.027,</td>
<td>[-0.092,</td>
<td>[-0.149,</td>
<td>[-0.027,</td>
<td>[-0.057,</td>
<td>[-0.033,</td>
<td>[0.075,</td>
</tr>
<tr>
<td></td>
<td>-0.035]</td>
<td>-0.075]</td>
<td>0.039]</td>
<td>0.036]</td>
<td>-0.075]</td>
<td>-0.083]</td>
<td>-0.017]</td>
<td>-0.034]</td>
<td>-0.027]</td>
<td>0.092]</td>
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<tr>
<td>$A_2$</td>
<td>[0.0,</td>
<td>[0.0,</td>
<td>[0.0,</td>
<td>[0.0,</td>
<td>[0.0,</td>
<td>[0.0,</td>
<td>[0.0,</td>
<td>[0.0,</td>
<td>[0.0,</td>
<td>[0.0,</td>
</tr>
<tr>
<td></td>
<td>0.0]</td>
<td>0.0]</td>
<td>0.0]</td>
<td>0.0]</td>
<td>0.0]</td>
<td>0.0]</td>
<td>0.0]</td>
<td>0.0]</td>
<td>0.0]</td>
<td>0.0]</td>
</tr>
<tr>
<td>$A_3$</td>
<td>[-0.039,</td>
<td>[-0.092,</td>
<td>[0.023,</td>
<td>[0.027,</td>
<td>[-0.092,</td>
<td>[-0.214,</td>
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<td>[0.048,</td>
<td>[-0.054,</td>
<td>[0.075,</td>
</tr>
<tr>
<td></td>
<td>-0.039]</td>
<td>-0.075]</td>
<td>0.027]</td>
<td>0.036]</td>
<td>-0.075]</td>
<td>-0.119]</td>
<td>-0.017]</td>
<td>0.08]</td>
<td>-0.045]</td>
<td>0.092]</td>
</tr>
<tr>
<td>$A_4$</td>
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<td>[-0.092,</td>
<td>[-0.025,</td>
<td>[0.04,</td>
<td>[-0.092,</td>
<td>[-0.149,</td>
<td>[-0.027,</td>
<td>[0.0,</td>
<td>[-0.055,</td>
<td>[0.075,</td>
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<tr>
<td></td>
<td>-0.035]</td>
<td>-0.075]</td>
<td>-0.022]</td>
<td>0.054]</td>
<td>-0.075]</td>
<td>-0.083]</td>
<td>-0.017]</td>
<td>0.0]</td>
<td>-0.046]</td>
<td>0.092]</td>
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<tr>
<td>$A_5$</td>
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<td>-0.033]</td>
<td>-0.075]</td>
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</tr>
<tr>
<td>FSs</td>
<td>C₁</td>
<td>C₂</td>
<td>C₃</td>
<td>C₄</td>
<td>C₅</td>
<td>C₆</td>
<td>C₇</td>
<td>C₈</td>
<td>C₉</td>
<td>C₁₀</td>
</tr>
<tr>
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<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
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<td>--------</td>
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<td>[0.075, 0.092]</td>
<td>[0.104, 0.187]</td>
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<td>[0.044, -0.052]</td>
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<tr>
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<td>[-0.027, 0.036]</td>
<td>[0.075, 0.092]</td>
<td>[0.119, 0.214]</td>
<td>[0.017, 0.027]</td>
<td>[-0.08, -0.048]</td>
<td>[-0.045, -0.054]</td>
<td>[-0.092, -0.075]</td>
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<tr>
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<td>[0.104, 0.149]</td>
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<td>[0.04, 0.052]</td>
<td>[-0.092, -0.075]</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>FSs</th>
<th>C₁</th>
<th>C₂</th>
<th>C₃</th>
<th>C₄</th>
<th>C₅</th>
<th>C₆</th>
<th>C₇</th>
<th>C₈</th>
<th>C₉</th>
<th>C₁₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
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<td>[0.034, 0.039]</td>
<td>[0.05, 0.037]</td>
<td>[0.075, 0.092]</td>
<td>[0.0, 0.0]</td>
<td>[0.014, 0.023]</td>
<td>[0.057, 0.034]</td>
<td>[0.046, 0.055]</td>
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</tr>
<tr>
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<td>[0.017, 0.027]</td>
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<td>[0.029, 0.034]</td>
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<td>[0.092, 0.075]</td>
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<td>[0.017, 0.027]</td>
<td>[0.048, 0.041]</td>
<td>[0.034, 0.092]</td>
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<td>[0.0, 0.0]</td>
<td>[0.0, 0.0]</td>
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<td>[0.0, 0.0]</td>
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</tr>
<tr>
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<td>[0.055, 0.041]</td>
<td>[0.075, 0.092]</td>
<td>[0.123, 0.222]</td>
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<td>[0.054, 0.055]</td>
<td>[0.046, 0.075]</td>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>FSs</th>
<th>C₁</th>
<th>C₂</th>
<th>C₃</th>
<th>C₄</th>
<th>C₅</th>
<th>C₆</th>
<th>C₇</th>
<th>C₈</th>
<th>C₉</th>
<th>C₁₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td>[-0.026, -0.026]</td>
<td>[-0.092, -0.075]</td>
<td>[0.034, 0.039]</td>
<td>[0.039, 0.052]</td>
<td>[0.075, 0.092]</td>
<td>[-0.222, -0.123]</td>
<td>[0.0, 0.0]</td>
<td>[-0.092, -0.055]</td>
<td>[0.0, 0.092]</td>
<td></td>
</tr>
<tr>
<td>A₂</td>
<td>[0.027, 0.027]</td>
<td>[0.075, 0.092]</td>
<td>[0.032, 0.037]</td>
<td>[0.033, 0.045]</td>
<td>[0.075, 0.092]</td>
<td>[-0.204, -0.113]</td>
<td>[0.017, 0.027]</td>
<td>[0.0, 0.0]</td>
<td>[-0.045, 0.033]</td>
<td>[-0.092, -0.075]</td>
</tr>
<tr>
<td>A₃</td>
<td>[-0.038, -0.038]</td>
<td>[-0.092, -0.075]</td>
<td>[0.034, 0.039]</td>
<td>[-0.092, -0.052]</td>
<td>[0.075, 0.075]</td>
<td>[-0.225, -0.125]</td>
<td>[-0.092, -0.104]</td>
<td>[0.054, 0.044]</td>
<td>[0.092, 0.092]</td>
<td></td>
</tr>
<tr>
<td>A₄</td>
<td>[-0.026, -0.026]</td>
<td>[-0.092, -0.075]</td>
<td>[0.041, 0.032]</td>
<td>[-0.092, -0.055]</td>
<td>[-0.222, -0.123]</td>
<td>[-0.023, -0.014]</td>
<td>[-0.092, -0.054]</td>
<td>[-0.046, 0.044]</td>
<td>[0.092, 0.092]</td>
<td></td>
</tr>
<tr>
<td>A₅</td>
<td>[0.0, 0.0]</td>
<td>[0.0, 0.0]</td>
<td>[0.0, 0.0]</td>
<td>[0.0, 0.0]</td>
<td>[0.0, 0.0]</td>
<td>[0.0, 0.0]</td>
<td>[0.0, 0.0]</td>
<td>[0.0, 0.0]</td>
<td>[0.0, 0.0]</td>
<td>[0.0, 0.0]</td>
</tr>
</tbody>
</table>
TABLE XVI: Ranking orders of alternatives with different values of \( \lambda \)

<table>
<thead>
<tr>
<th>Different values of ( \lambda )</th>
<th>Ranking orders of alternative</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda = 1 )</td>
<td>( A_3 \succ A_4 \succ A_1 \succ A_2 \succ A_5 )</td>
</tr>
<tr>
<td>( \lambda = 1.5 )</td>
<td>( A_3 \succ A_4 \succ A_1 \succ A_2 \succ A_5 )</td>
</tr>
<tr>
<td>( \lambda = 2 )</td>
<td>( A_3 \succ A_4 \succ A_1 \succ A_2 \succ A_5 )</td>
</tr>
<tr>
<td>( \lambda = 2.5 )</td>
<td>( A_3 \succ A_4 \succ A_1 \succ A_2 \succ A_5 )</td>
</tr>
<tr>
<td>( \lambda = 3 )</td>
<td>( A_3 \succ A_4 \succ A_1 \succ A_2 \succ A_5 )</td>
</tr>
</tbody>
</table>

TABLE XVII: Ranking orders of alternatives with different methods

<table>
<thead>
<tr>
<th>Different methods</th>
<th>Ranking orders of alternative</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-TODIM</td>
<td>( A_3 \succ A_4 \succ A_1 \succ A_5 \succ A_2 )</td>
</tr>
<tr>
<td>I-TOPSIS</td>
<td>( A_3 \succ A_4 \succ A_1 \succ A_2 \succ A_5 )</td>
</tr>
<tr>
<td>I-VIKOR</td>
<td>( A_3 \succ A_4 \succ A_1 \succ A_2 \succ A_5 )</td>
</tr>
<tr>
<td>I-MOORA</td>
<td>( A_3 \succ A_4 \succ A_1 \succ A_2 \succ A_5 )</td>
</tr>
</tbody>
</table>

strong robustness.

3) Comparative analysis and discussion: In order to demonstrate the effectiveness of the proposed I-TODIM method, a comparative study was conducted with previously proposed I-TOPSIS, I-VIKOR, and I-MOORA methods. The ranking results of the alternative food suppliers obtained by these methods are listed in Table XVII.

From Table XVII, some important results can be inferred. The ranking results of the top priority and the two lower priority options remain the same according to the four methods, with \( A_3 \) being ranked the highest and \( A_2 \) and \( A_5 \) being the lower ones. This indicates a relatively high homogeneity between our proposed method and the other three methods, demonstrating its effectiveness. On the other hand, there are still some differences between our method and the other three methods, which can be explained by the following comparative analysis:

Table XVII shows that for the I-TOPSIS method, \( A_4 = A_1 \) and \( A_2 > A_5 \). This is because the method uses interval Euclidean distance to measure the difference between the best and worst alternatives and the candidate alternatives, but this distance measure does not consider interval width. As shown in Table XVI, the difference between \( A_2 \) and \( A_5 \) is very small, and the I-MOORA method that only considers the endpoint values also leads to the result \( A_2 > A_5 \). Our proposed method considers the difference in the number of intervals from a geometric perspective, which effectively addresses this deficiency.

The VIKOR method can balance the maximum group utility and the minimum individual regret, and may obtain a set of compromise results rather than a single result [18]. However, in most cases, decision-makers only want a single optimal result. Table XVII shows that the I-VIKOR method obtained the result \( A_3 = A_4 \), which resulted in two optimal choices. Compared with the other three methods, it can be seen that \( A_3 \) is superior to \( A_4 \), which is consistent with our proposed method’s conclusion, indicating its superiority over the VIKOR method.

The comparative results above indicate that our proposed I-TODIM method can obtain more reasonable ranking results. Finally, the model proposed in this paper combines the GIBWM and I-TODIM methods, considering the decision-maker’s psychological behavior characteristics and the interaction effects between attributes while dealing with fuzzy and complex problems, providing more accurate and reasonable results.

VI. Conclusion

Given the intense competition in the market, choosing the right food suppliers is crucial.

Since determining precise attribute values in MCDM problems is difficult or impossible, considering them as interval numbers is more appropriate. In this research work, we focus on using group BWM and TODIM methods in an interval number environment. BWM is a recent and effective MCDM method that constructs comparison systems in a structured manner and reduces inconsistency. We extend BWM to interval numbers to calculate standard weights. Considering that using interval weights does not lose the information provided by DMs, we construct a mathematical programming model to solve for interval weights. Although the mathematical programming solution process is cumbersome, it can be efficiently completed using tools such as MATLAB and Python. In addition, the advantages of TODIM technique are that it can solve the subjectivity problem of decision makers and consider the interdependence of alternative solutions. We propose a geometric probability-based method for calculating interval preference and a preference-based method for calculating distance, and introduce them into the interval TODIM method. Furthermore, the method we propose is not only applicable to interval numbers but also to mixed problems of interval and crisp numbers. Finally, two examples demonstrate that our method is not only reasonable, clear, and flexible but also highly robust.

Future research could consider applying BWM to various fuzzy environments. In addition, the GIBWM method can be combined with other MCDM methods.
to solve FSS problems. Finally, our proposed model can also perform well in many other fields, such as green building material selection, investment project selection, and evaluation of enterprise technological innovation capabilities.

References


