Low Complexity Signature Estimation of Near-Field Spatial-Wideband Systems

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Abstract

This letter proposes a low-complexity, two-step angle, range and delay signature estimation algorithm for a sparse multipath near-field spatial-wideband system. With upcoming sub-THz systems expected to have a large number of antennas, the transmitted wideband signal is not only sensitive to the physical propagation delay across the array aperture but the resulting wavefront is no longer locally planar. We use a combination of discrete linear chirp transform and discrete Fourier transform based techniques to estimate the spatial wideband signature of such a system. Further, we propose an iterative neighbourhood search to acquire super-resolution estimates of the scatterer locations, delays and angle of arrival of the multipaths and show that they are asymptotically optimal approaching their respective Cramér-Rao bounds.
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Index Terms—Signature estimation, Near-field, Spatial-Wideband, sub-THz, massive MIMO, DFT, DLCT

I. INTRODUCTION

The mm-wave and sub-THz bands (30 GHz-300 GHz) promise an unprecedented bandwidth to support data communications reaching up to Terabits per sec (Tbps). This is realized through a combination of ultra-wideband channels at very high carrier frequencies [1]. A fundamental assumption underlying such systems is that the radiated waves are locally planar over the antenna array. Thus, wireless systems have conventionally operated in the far-field beyond the Fraunhofer distance. This assumption, however, breaks down as we move up the frequency spectrum and deploy more and more antennas at the transceivers [2]. Consider a sub-THz microcell with a radius of 150 m. A base-station using a 256-element uniform linear array (ULA) with half wavelength separation at 140 GHz has a Fraunhofer distance of almost 70 m resulting in a significant portion of the cell lying within its near-field. The Fraunhofer distance can extend to several hundred meters for larger arrays and higher frequencies. Hence, it becomes imperative that the far-filed approximation should be dropped and spherical wave propagation be considered while analysing such systems.

Existing communication systems can broadly be classified into the following three types: far-field narrowband [3], far-field wideband [4], [5], [6], and near-field narrowband [7], with little to no work available on near-field spatial wideband systems. In wideband massive MIMO systems, signals from multiple scatterers suffer a non-negligible propagation delay across the array aperture resulting in a spatially selective wireless channel [5]. The propagation delay and array geometry can be used to extract the spatio-temporal geometric signature of such a system which can be subsequently utilized for fast channel tracking, beamforming, spatial multiplexing and localization. Although far-field spatial wideband systems result in beam squint due to the frequency-dependent array steering vectors, the underlying waves are still assumed to be planar [5]. For a near-field spatial wideband system, the beam squinting effect is further coupled with the spherical wave propagation. Therefore, the delay of the arriving multipaths is no longer a linear function of the angle-of-arrival (AoA) but a quadratic function of the AoA coupled with the distance of the last-hop scatterer of the corresponding multipath.

The present work is motivated by the spatial-wideband characterization of far-field mm-wave massive MIMO systems in [5] and [6]. In summary, we aim to propose a combination of discrete linear chirp transform (DLCT) and discrete Fourier transform (DFT) based techniques to perform low-complexity channel signature estimation of near-field spatial wideband systems. The coarse estimates are refined through iterative neighbourhood searches to obtain asymptotically optimal estimates close to the Cramér-Rao bounds (CRB). Furthermore, the proposed method appears to be more attractive compared to subspace-based techniques that not only require multiple snapshots but suffer from prohibitively high computational costs since the involved eigenvalue decomposition has an approximate complexity of O(M^3), where M is the number of antennas [8].

II. NEAR-FIELD SPATIAL-WIDEBAND CHANNEL MODEL

Consider a base station (BS) with a M ≫ 1 element ULA serving a random user with a single antenna over a line-of-sight (LoS) multipath channel. Let τ_{p,m} denote the delay of the p-th path to the m-th antenna. Then, the received baseband signal at the m-th antenna is:

\[ y_m(t) = \sum_{p=0}^{P-1} \Gamma_p x(t - \tau_{p,m}) e^{-j2\pi \tau_{p,m}} \]  

(1)

where P denotes the number of individual paths with \( \Gamma_p \) being the corresponding complex channel gains, \( x(t) \) is the transmitted signal and \( f_c \) is the carrier frequency. For a nearfield spatial wideband channel, \( \tau_{p,m} \) can be expanded as

\[ \tau_{p,m} = \tau_p + \frac{md\sin\theta_p}{c} - \frac{m^2d^2\cos^2\theta_p}{2\tau_{0,p}c} \]  

(2)

where \( \tau_p \) is the multipath delay at the first receive antenna, \( d \) is the inter element separation, \( \theta_p \) is the corresponding AoA via the p-th path, \( \tau_{0,p} \) is the distance between the last-hop scatterer for the p-th path and the first receive antenna and \( c \) is the velocity of propagation. Note that \( \tau_p \geq \frac{d}{c} \) where equality holds only for the LoS component. Using (2) and (1), the spatial-time channel at the m-th antenna would be:

\[ h_m(t) = \sum_{p=0}^{P-1} \Gamma_p e^{-j2\pi \tau_{p,m}} \delta(t - \tau_{p,m}) \]  

(3)
The continuous-time Fourier transform (CTFT) of (3) gives
\[ H_m(f) = \sum_{p=0}^{P-1} T_p e^{-j2\pi f_p (1+\frac{j \theta_p}{\pi})} e^{-j2\pi f \tau_p} \]
where \( T_p = H_p e^{j2\pi f_p \tau_p} \) is the equivalent channel gain and (4) represents the spatial-frequency response of the channel at the \( m \)-th receive antenna. Assume that OFDM is used as the preferred multicarrier modulation scheme. Suppose the subcarrier spacing is denoted by \( \eta = W/N \), and \( f_n = n\eta, \ n \in [0, N-1] \) where \( W \) is the signal bandwidth and \( N \) is the number of subcarriers. Then, the spatial-frequency response of the \( m \)-th receive antenna for the \( n \)-th subcarrier is given by \( H_m(f_n) \) and the overall spatial-frequency channel matrix across the \( M \) receive antennas and \( N \) subcarriers may be expressed as:
\[ H = \{ H_m(f_n) \}_{m=0}^{M-1,n=0} = \sum_{p=0}^{P-1} H_p = \sum_{p=0}^{P-1} T_p \Lambda(\theta_p, r_{0,p}) B(\tau_p) \]
where \( H_p \in \mathbb{C}^{M \times N} \) is the near-field spatial-wideband channel matrix for the \( p \)-th path, and \( \Lambda(\theta_p, r_{0,p}) = [a(0, \theta_p, r_{0,p}), \ldots, a((N-1)\eta, \theta_p, r_{0,p})] \in \mathbb{C}^{M \times N} \) is the frequency dependent nearfield array steering matrix whose columns are given by:
\[ a(f_n, \theta_p, r_{0,p}) = \left[ 1, e^{-j2\pi f_n (1+\frac{j \theta_p}{\pi})} \left( \frac{\sin \theta_p}{\sin \theta_{0,p}} \right)^{-\frac{1}{2}}, \ldots, e^{-j2\pi f_n ((M-1)\eta-1)\eta \theta_p} \right]^T \]
\[ B(\tau_p) \in \mathbb{C}^{N \times N} \] is the diagonal path delay matrix accounting for the \( p \)-th path with delay \( \tau_p \), i.e., \( B(\tau_p) = \text{diag}(d(\tau_p)) \) where
\[ d(\tau_p) = [1, e^{-j2\pi \eta \tau_p}, \ldots, e^{-j2\pi (N-1)\eta \tau_p}] \]
is the \( p \)-th path delay vector.

Extracting the channel response over the \( N \)-th subcarrier from (5), we have
\[ h(f_n) = H(:,n) = \sum_{p=0}^{P-1} \tilde{T}_p(a(f_n, \theta_p, r_{0,p}), \ n = 0, \ldots, N-1 \)
where \( H(:,n) \) denotes the \( n \)-th column of \( H \), and \( \tilde{T}_p(f_n) = T_p e^{-j2\pi f_p \tau_p} \).

### III. A Primer on Discrete Linear Chirp Transform (DLCT)
Consider a finite support discrete-time chirp signal \( \phi(m) \) with base frequency \( \omega \) and chirp rate \( \xi \):
\[ \phi(m) = \exp \left( \frac{2\pi}{M} (\xi m^2 + \omega m) \right), \quad 0 \leq m \leq M - 1 \]
The discrete linear chirp transform (DLCT) of a discrete-time signal \( x(m) \), \( 0 \leq m \leq M - 1 \) and its corresponding inverse are defined as [9]
\[ X(k,l) = \sum_{m=0}^{M-1} x(m) \exp \left( -j2\pi \left( \frac{km}{M} + \frac{md}{2\lambda_r} \frac{\sin \theta_p (1 + \frac{l}{L})}{\lambda_c} \right) \right) \]
\[ x(m) = \sum_{k=0}^{L/2-1} \sum_{l=0}^{L/2-1} \frac{X(k,l)}{LM} \exp \left( j2\pi \left( \frac{km}{M} + \frac{md}{2\lambda_r} \frac{\sin \theta_p (1 + \frac{l}{L})}{\lambda_c} \right) \right) \]
\[ 0 \leq k \leq M - 1, \quad 0 \leq l \leq \frac{L}{2} - 1, C = \frac{2\lambda_c}{L} \]
Essentially, the DLCT decomposes a given signal using linear chirp basis functions of the form \( \phi_k(l) = \exp \left( -j2\pi \left( \frac{k}{M} (\xi C m^2 + km) \right) \right) \) characterized by the discrete frequency \( \frac{k}{M} \xi C \), where the chirp-rate \( \xi = 1/C \) is assumed to have finite support, i.e., \( |\xi| \leq \Lambda \).

It is emphasized that the DLCT does not represent a time-frequency transformation but rather a 2D chirp-frequency transform that can be obtained as a generalization of the discrete Fourier transform (DFT). A substitution of \( l = 0 \) in (9) confirms that \( X(k,0) \) is, in fact, the DFT of \( x(m) \).

### IV. Spatial-Wideband Parameter Estimation
Considering the spatial wideband channel model shown in (5), the noisy channel estimates over all the paths across all the receive antennas and subcarriers can be expressed as:
\[ \tilde{H} = H + \Delta \]
where \( \Delta \) denotes the estimation error and is modeled as zero-mean i.i.d. complex Gaussian and \( E\{|\Delta(m,n)|^2\} = \sigma_v^2 \).
We assume the channel estimates are known apriori through training-based Maximum Likelihood (ML) estimation [6]. For ML estimation, \( \sigma_v^2 = \frac{\sigma_n^2}{\pi^2 M} \), where \( \sigma_n^2 \) is the variance of the additive white Gaussian noise in the channel, \( N_c \) is the training sequence length, \( P_t \) is the power of the training symbols and \( N_c \ll N \). Subsequently, with (11), we perform a two-step estimation of the multipath delays followed by the joint estimation of the range of the last hop scatterers and their corresponding AoAs.

#### A. Joint Range and Angle-of-Arrival Estimation:
Let \( [x(m, f_n)] \) be the received signal at the \( m \)-th array element corresponding to the \( n \)-th subcarrier. Then, following (8) and (11),
\[ x(m, f_n) = \sum_{p=0}^{P-1} \tilde{T}_p(a(f_n, \theta_p, r_{0,p}), \ n = 0, \ldots, N-1 \]
where \( a(\cdot) \) is the \( m \)-th element of the array steering vector \( a(\cdot) \). Neglecting the error term, the DLCT of \( [x(m, f_n)] \) is:
\[ X_n(k,l) = \sum_{p=0}^{P-1} \sum_{m=0}^{M-1} \tilde{T}_p(f_n) \times \exp \left( -j2\pi \left( \frac{km}{M} + \frac{md}{2\lambda_r} \frac{\sin \theta_p (1 + \frac{l}{L})}{\lambda_c} \right) \right) \]
\[ \times \exp \left( -j2\pi \left( \frac{km}{M} - \frac{md}{2\lambda_r} \frac{\sin \theta_p (1 + \frac{l}{L})}{\lambda_c} \right) \right) \]
\[ 0 \leq k \leq M - 1, \quad 0 \leq n \leq N - 1, \quad -\frac{L}{2} \leq l \leq \frac{L}{2} - 1 \]
(13)
Noting (13) and following Appendix B, the DLCT of the $p$-th path for the $n$-th subcarrier will achieve a maximum if the following relations are satisfied:

$$
2k\pi \frac{d}{M} + 2\pi d \sin \theta_0(1 + \frac{r}{f_c}) = 0, \quad k \in [0, M - 1] \\
2\pi d \cos \theta_0(1 + \frac{r}{f_c}) = 0, \quad l \in [-L, L] \setminus \{0\} 
$$

(14)

The $\{(k_p, l_p)\}_{p=1}^P$ peaks of $|X_n(k, l)|$ corresponding to the $P$ paths for the $n$-th subcarrier can be obtained by a 2D peak search in the following domain:

$$
\{(k_p, l_p)\}_{p=1}^P 
$$

(15)

For localization problems with massive arrays, the chirp rate $\xi$ over all $[x(m, f_n)]$ may be approximately bounded as $\xi \leq \Lambda \approx \max(\xi_p) \approx \frac{M \delta - \eta_0}{\lambda_{\min}}$ for a given array size $M$, multi-path chirp rate $\xi_p$, wavelength $\lambda_p$, inter-element separation $d$ and $\eta_0 = \min(r_0, p)$, $\forall p \in [0, P - 1]$. The coarsely estimated localization parameters $\hat{\theta}_p, \hat{\eta}_p, \hat{r}_0, p$ for the $p$-th last-hop scatterer at the $n$-th subcarrier can now be obtained as:

$$
\hat{\theta}_p(f_n) = \sin^{-1}\left(-\frac{k_{p,\max} \lambda_c}{d M (1 + \frac{r}{f_c})}\right), \quad \hat{\eta}_p(f_n) = \frac{d \cos^2 \hat{\theta}_p(1 + \frac{r}{f_c})}{2 \lambda_c k_{p,\max} C/M} 
$$

(16)

where $\{(k_{p,\max}, l_{p,\max})\}_{p=1}^P$ are the indices for which $|X_n(k, l)|$ is maximized. The specific sample of far-field sources dictates that $\hat{r}_0, p \rightarrow \infty, \forall p \in [0, P - 1]$ and (13) is reduced to $[x(m, f_n)] = \sum_{p=0}^{P-1} \sum_{k=0}^{\infty} \sum_{l=-\infty}^{\infty} \hat{\Gamma}_p(f_n) e^{-j \frac{2\pi}{M}(1 + \frac{r}{f_c})} e^{j 2\pi k \rho n} e^{j 2\pi l \sigma n} + \Delta(m, n)$. Consequently, $l = 0$, and $X_n(k, l)$ simplifies to $X_n(0, k)$ which is the DFT of the far-field signal $[x(m, f_n)]$ with its corresponding DoAs given by [8]:

$$
\hat{\theta}_p(f_n) = \sin^{-1}\left(-\frac{k_{p,\max} \lambda_c}{d M (1 + \frac{r}{f_c})}\right), \quad k_{p,\max} \in [0, M - 1] 
$$

(17)

B. Iterative Neighbourhood Search: The resolutions of the localization parameters obtained in (16) are limited by the DLCT grid size $(M, L)$. A finer estimate can thus be obtained through a neighbourhood search around the coarse estimates of their DLCT spectra. Noting that the spatial-frequency localization function $|X(k, l)|$ is bivariate in $(k, l)$, the fine search method presented in Algorithm 1 is performed to obtain higher resolution estimates. The $i$-th neighbourhood search parameters around the coarse estimates of $\theta$ and $r$ for the $p$-th path $(k_{p,\max}, l_{p,\max})$ are denoted by $[\theta_0, \beta_0, \gamma_0, \delta_0]$ and $[\alpha_0, \beta_0, \gamma_0, \delta_0]$, respectively. Algorithm 1 is a modified 2D version of the classical Fibonacci search algorithm [10] when applied to unimodal bivariate functions. The search is performed over $\kappa$ iterations with initial parameters $[\phi, \alpha_{0,\kappa}, \beta_{0,\kappa}, \gamma_{0,\kappa}, \delta_{0,\kappa}]$ until a desired degree of precision of the finer estimates $(\hat{\theta}_p(f_n), \hat{\eta}_p(f_n))$ has been obtained. The super-resolution estimates of the localization parameters $(\hat{\theta}_p(f_n), \hat{\eta}_p(f_n))$ are subsequently acquired via the following relations:

$$
\hat{\theta}_p = \frac{1}{N} \sum_{n=0}^{N-1} \hat{\theta}_p(f_n), \quad \hat{\eta}_p = \frac{1}{N} \sum_{n=0}^{N-1} \hat{\eta}_p(f_n), \quad p = 0, \ldots, P - 1 
$$

(18)

C. Multipath Delay Estimation: Following (8) and (11), the channel vector over the $n$-th subcarrier may be expressed as

$$
\hat{h}(f_n) = \hat{H}(; n) = \hat{A}(f_n) \hat{F}(f_n), \quad n = 0, \ldots, N - 1 
$$

(19)

where $\hat{A}(f_n) = [a(f_n, \hat{\theta}_0, \hat{r}_0, 0), \ldots, a(f_n, \hat{\theta}_{P-1}, \hat{r}_{P-1}, 0)] \in \mathbb{C}^{M \times P}$, $\hat{F}(f_n) = [\tilde{f}_0 e^{-j 2\pi f_n \tilde{f}_0}, \ldots, \tilde{f}_{P-1} e^{-j 2\pi f_n \tilde{f}_{P-1}}]^T \in \mathbb{C}^{P \times 1}$, $\tilde{f}_p$ and $\tilde{r}_p$ are the corrupted versions of $\tau_p$ and $\tilde{r}_p$, respectively $\forall p \in [0, P - 1]$. Then, $\hat{F}(f_n)$ can be obtained as

$$
\hat{F}(f_n) = (\hat{A}(f_n) H(\tilde{f}_n), \quad n = 0, \ldots, N - 1 
$$

(20)

where $(\cdot)^H$ denotes the Hermitian. Let $\hat{D} = (\hat{F}(0), \ldots, \hat{F}((N - 1)\eta)) \in \mathbb{C}^{P \times N}$ be the extracted multipath delay matrix, and $\hat{F}$ be the $N \times N$ normalized DFT matrix, with $[\hat{F}]_{ij} = \frac{1}{\sqrt{N}} e^{-j 2\pi \frac{i j}{N}}$. Then, taking the DFT of $\hat{D}(p, :)$ yields:

$$
\hat{d}(\tau_p) = \hat{D}(p, :) e^{j \theta} 
$$

(21)

The $j$-th element of the noisy frequency domain delay vector is given by:

$$
[d(\tau_p)]_j = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} \hat{F}_p e^{-j 2\pi \eta \tilde{r}_i} e^{j 2\pi \frac{i j}{N}} 
$$

(22)

The delay estimates can now be readily obtained from the peaks of the delay magnitude spectrum $|\hat{d}(\tau_p)|$. Evidently, a coarse estimate of $[\hat{\tau}_p]_{p=1}^P$ is given by:

$$
\hat{\tau}_p = \frac{1}{\eta} \arg \max |\hat{d}(\tau_p)|^2, \quad p = 0, \ldots, P - 1 
$$

(23)

The resolutions of the delay estimates thus obtained are limited by the DFT grid size $N$. The finer delay estimates $\hat{\tau}_p$ can be obtained from $\hat{\tau}_p$ through an iterative neighbourhood search around the peaks of the delay magnitude spectrum $|\hat{d}(\tau_p)|$ as detailed in Algorithm 1.

Algorithm 1 2D Iterative Neighbourhood search

1: **Initialization:** $\phi = \frac{\sqrt{3} + 1}{2}, k_p = k_{p,\max}, l_p = l_{p,\max} (18)-(21)$
2: for $\kappa$ iterations do
3: \hspace{1em} $\alpha_0 = k_p - 1, \beta_0 = k_p + 1$
4: \hspace{1em} $\gamma_0 = \beta_0 = \hat{\theta}_0 = \alpha_0 + (\beta_0 - \alpha_0)$
5: \hspace{1em} while $|\beta_0 - \alpha_0| > 0$ do
6: \hspace{2em} $\beta_0 = \gamma_0$, if $|\hat{\theta}_0(l_p)| > |\hat{\theta}_0(l_p)|$
7: \hspace{2em} $\gamma_0 = \beta_0$, else if $|\hat{\theta}_0(l_p)| > |\hat{\theta}_0(l_p)|$
8: \hspace{2em} $\gamma_0 = \beta_0 = \alpha_0 = \alpha_0 = (\beta_0 - \alpha_0)$
9: \hspace{2em} $\gamma_0 = \beta_0 = \alpha_0 = \alpha_0 = (\beta_0 - \alpha_0)$
10: end while
11: \hspace{1em} $k_p = \frac{2\alpha_0 + \beta_0}{2}$
12: \hspace{1em} $\alpha_p = l_p - 1, \beta_p = l_p + 1$
13: \hspace{1em} $\gamma_p = \beta_p - \alpha_p = \gamma_p = \beta_p - \alpha_p$
14: while $|\beta_p - \alpha_p| > 0$ do
15: \hspace{2em} $\beta_p = \gamma_p$, if $|\hat{\theta}_p(l_p)| > |\hat{\theta}_p(l_p)|$
16: \hspace{2em} $\gamma_p = \beta_p$, else if $|\hat{\theta}_p(l_p)| > |\hat{\theta}_p(l_p)|$
17: \hspace{2em} $\gamma_p = \beta_p = \alpha_p = \alpha_p = (\beta_p - \alpha_p)$
18: \hspace{2em} $\gamma_p = \beta_p = \alpha_p = \alpha_p = (\beta_p - \alpha_p)$
19: end while
20: \hspace{1em} $l_p = \frac{\alpha_p + \beta_p}{2}$
21: end for
V. COMPLEXITY

The DFT of the \( P \) multipaths can be efficiently computed by FFTs in \( O(P\lambda^2) \). The DLCT can be efficiently implemented using FFT algorithms as well by leveraging the following observation on \( X_n(k,l) \):

\[
X_n(k,l) = \sum_{p=0}^{P-1} \sum_{m=0}^{M-1} |f_p(f_n)[x_p(m,f_n)]| e^{-j2\pi C(m^2/M)} e^{-j2\pi km/M} \tag{24}
\]

For each \( l \in [-\frac{M}{2}, \frac{M}{2} - 1] \), \( X_n(k,l) \) may be considered a DFT of the signal \( \sum_{p=0}^{P-1} f_p(f_n)[x_p(m,f_n)] e^{-j2\pi C(m^2/M)} \). The DLCT of \( X_n(\cdot) \) is performed for each of the \( N \) subcarriers and \( P \) paths. Then, an approximate complexity of the DLCT-DFT algorithm may be given by \( O(PLMN + P\lambda^2 M + P\lambda^2 N) \).

VI. RESULTS & DISCUSSIONS

This section evaluates the proposed algorithm vis-a-vis the wideband Cramér-Rao lower bound given in Appendix A. Let, \( M = 128 \) antenna elements with \( d = \frac{\lambda}{2} \), \( \lambda = 1 \) GHz divided over \( N = 128 \) subcarriers and \( f_c = 140 \) GHz. Fig. 1 shows the delay extracted DLCT spectra of the arriving multipaths at the zero-th sub-carrier for a combination of near and far-field scatterers assuming \( 1/\sigma^2 = 10 \) dB. A four-path sparse channel is considered where the arriving multipaths are presumed to have the following spectral characteristics - (333 ns, 60.1°, 100 m), (421 ns, 50.8°, 120 m), (13.2 ns, 40.8°, 1 m), and (12.5 ns, 70.4°, 1.1 m) respectively. The first two scatterers are beyond the Rayleigh distance \( \frac{2d}{\lambda} \) \( \approx 17 \) m of the receiver array and therefore, lie in its far-field while the last two scatterers are within its near-field. \((k,l)\) are chosen such that \( k \in [0,M-1], l \in [-M,M] \) and \( C = \frac{1}{2M+1} \). For the far-field scatterers, the DLCT spectra are seen to generate peaks at \( l = 0 \), reducing the DLCT to simply the DFT of these multipaths. The DLCT is, therefore, capable of handling both far and near-field characterizations of a spatial-wideband channel. Generally, all arriving multipaths can be identified without ambiguity through the peaks corresponding to their chirp-antenna-indices \((k,l)\) on the DLCT grid. Fig. 1: The DLCT spectra of the received multipaths for the zero-th subcarrier after delay extraction at \( 1/\sigma^2 = 10 \) dB

Fig. 2(a) compares the root-mean-squared error (RMSE) of the delay, angle and range estimates with their respective CRLBs over increasing \( 1/\sigma^2 \). While the AoA and delay estimates achieve their corresponding optimal lower bounds for estimation errors as high as \( 0 \) dB, the range estimate asymptotically converges to its Cramér-Rao bound as \( 1/\sigma^2 \) increases. The system parameters considered here are conservative for a sub-THz system. Fig. 2(a) therefore, provides a worst-case analysis of the DLCT-DFT algorithm and these estimates would improve with more practical system parameters. Intuitively, as the number of antennas increases, the angular and range estimates are expected to converge to their respective CRLBs even for high to moderate values of \( \sigma^2 \). This is reflected in Fig. 2(b). The delay estimates remain unchanged simply because they are independent of array size and dependent only on the number of subcarriers and more importantly on the operating bandwidth of the system which is assumed to be constant. Fig. 2(c) shows a minor improvement in the RMSE estimates with increased subcarriers. The improvements may be accorded to the fact that with additional subcarriers, higher frequency snapshots are available over the entire operating bandwidth. This is in accordance with (16) which shows that the angle and range estimates are a function of their respective subbands. However, more refined estimates can only be obtained either by increasing the number of antennas or improving the channel estimation error \( \sigma^2 \) when there is a constraint on the operating bandwidth.

VII. CONCLUSION

This work proposes a low-complexity DLCT-DFT based two-step approach towards estimating the joint angles-of-arrival and ranges followed by the multipath delays of a sparse near-field spatial wideband system. A low-complexity iterative neighbourhood search is then employed to acquire super-resolution estimates of the channel parameters that are shown to be asymptotically optimal to their respective CRLBs. To the best of the author’s knowledge, the proposed method aims to be one of the first approaches towards characterizing the spatial-wideband signatures of near-field massive MIMO channels for microcellular sub-THz communication.

APPENDIX A

The wideband stochastic CRLB for the joint angular-range estimation is given by [11]

\[
\text{CRLB}^{-1}(\theta, r) = \sum_{n=0}^{N-1} C^{-1}(\theta, r, n) \tag{25}
\]

\( C^{-1}(\theta, r, n) \) is the CRLB corresponding to the \( n \)-th subband. Then, following [12], we have,

\[
C^{-1}(\theta, r, n) = \frac{\sigma^2}{2} \left\{ \text{Re}(\text{D}^H\Pi_S^\perp\text{D}) \odot (\text{J} \otimes (\text{PS}^H\text{R}^{-1}\text{SP})^T) \right\}^{-1} \tag{26}
\]

where \( \sigma^2 \) is the noise power, \((\cdot)^H\) is the Hermitian, \( \odot \) denotes the Hadamard (Kronecker) product, and

\[
\text{J} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad \Pi_S^\perp = I - \Pi_S, \quad \Pi_S = (S^HS)^{-1}S^H \]

\[
\text{D} = [d_\theta(\theta, r_0, 0), \ldots, d_\theta(\theta, r_{P-1}, r_0, 0, \ldots, 0), \ldots, d_\theta(\theta, r_0, 0, \ldots, 0)]
\]

\[
d_\theta(\theta, r, 0, \ldots, 0, p) = \frac{\partial a}{\partial \theta}(f_n, \theta, r, 0, \ldots, 0), \quad p = 0, \ldots, P - 1
\]

\[
d_r(\theta, r, 0, \ldots, 0, p) = \frac{\partial a}{\partial r}(f_n, \theta, r, 0, \ldots, 0), \quad p = 0, \ldots, P - 1
\]

\[
\text{R} = SS^H + \sigma^2 \text{I}
\]
We evaluate the integral \( I = \int_0^\infty \sin^2 x \, dx \). Therefore, taking the limit as \( p \to 0 \), we have:

\[
\lim_{p \to 0} \frac{1}{p} \int_0^p \sin^2 x \, dx = \frac{1}{2} \int_0^\infty \sin^2 x \, dx = \frac{\pi}{4}.
\]

Therefore, following (31), \( |X(\Omega, \gamma)| \) is maximized when \((\Omega, \gamma)\) satisfy the following conditions:

\[
\Omega_0 - \Omega = 0, \quad \frac{1}{2}\xi_p - \gamma = 0
\]

This completes the proof.

### REFERENCES


