Modelling and Analysis of Unbalanced Magnetic Pull in Permanent Magnet Synchronous Machine with Specialized Stator Winding

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Abstract

Electrical machines with eccentrically positioned rotors experience strong unbalanced magnetic pull directed towards the narrowest air gap. This unbalanced force increases the rotor eccentricity and degrades machine performance. Accurate computation and reduction of electromagnetic force are required for controlling electromagnetic vibration, and this can be achieved by employing a bridge-configured winding scheme. This paper presents a two-dimensional bridge configured permanent magnet synchronous machine model to investigates the force compensating strategy under different eccentricity of PMSM. Furthermore, the force components under rotor eccentricity are compared with Bridge ON and Bridge OFF conditions. The connection of bridge points allows the different currents to flow in different coil groups, reducing the unbalanced magnetic pull. The modelling and simulation of PMSM have been done using time-stepping finite element method. The finding indicates that connecting the bridge points can significantly reduce the amplitude of the electromagnetic force components induced by different rotor eccentricities.
Modelling and Analysis of Unbalanced Magnetic Pull in Permanent Magnet Synchronous Machine with Specialized Stator Winding

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Electrical machines with eccentrically positioned rotor experience strong unbalanced magnetic pull directed towards the narrowest air gap. This unbalanced force increases the rotor eccentricity and degrades machine performance. Accurate computation and reduction of electromagnetic force are required for controlling the electromagnetic vibration, and this can be achieved by employing a bridge configured winding scheme. This paper presents a two dimensional bridge configured permanent magnet synchronous machine model to investigate the force compensating strategy under different eccentricity of PMSM. Furthermore, the force components under rotor eccentricity are compared with Bridge ON and Bridge OFF conditions. The connection of bridge points allows the different currents to flow in different coil groups, reducing the unbalanced magnetic pull. The modelling and simulation of PMSM have been done using time-stepping finite element method. The finding indicates that connecting the bridge points can significantly reduce the amplitude of the electromagnetic force components induced by different rotor eccentricities.

Index Terms—Permanent magnet synchronous machine (PMSM), unbalanced magnetic pull (UMP), bridge configured winding (BCW), time-stepping finite element method (TS-FEM), Bridge OFF, Bridge ON, Fast Fourier transform (FFT).

I. INTRODUCTION

Permanent magnet synchronous machines (PMSMs) have been extensively used in automotive and aerospace applications because of their higher flux density and better performance compared to those having excitation winding. With their many advantages, the dynamic behaviour and stability of the PMSMs have been the main areas of research [1]. Typically, the stator and rotor axes are in alignment, hence the air gap is symmetric, and no unbalanced forces exist. However, the manufacturing and assembly faults cause axis misalignment, and the magnetic field becomes asymmetrical, resulting in an unbalanced magnetic pull (UMP) [2].

Rotor eccentricity is divided into static, dynamic and mixed eccentricity (combination of both) [3]. When the stator and rotor axes are not aligned, and rotor rotates on its own axis, this is called static eccentricity. Dynamic eccentricity occurs when the rotor axis is not aligned with the stator axis but rotates about it. Static eccentricity results in a constant pull in the direction of eccentricity, whereas dynamic eccentricity causes a rotating force vector [4]. Eccentricity results in UMP in the rotor and stator. UMP rises as eccentricity increases. Eccentricity in such machines is very important to research, as it is one of the causes of unwanted vibrations [5]. Machine research can be classified into two different categories. The first classification is the vibration, followed by the dynamic behaviour and rotor stability. Computation of UMP provides the basis of these two categories [1]. Several studies in the literature deal with the magnetic field distribution with rotor eccentricity and resulting unbalanced force in permanent magnet machines [5], [6], [7]. The vibration control of the rotor is primarily focused on mass imbalance and the impact of UMP has not been adequately explored [8]. An accurate estimation of the electromagnetic performance of a machine necessitates numerical modelling. The finite element analysis can provide insight into the magnetic field distribution in the motor at certain point or time. To make sure that the result approximates the actual-world characteristic of PMSM, lots of details in the geometry of the motor should be retained [9].

The stator winding with parallel paths reduces the UMP in the motors with asymmetric air gap. Considering the same asymmetric air gap condition, compared to a motor with series winding connection, one with parallel stator winding connection would operate more silently [10]. The asymmetric flux distribution creates equalizing currents in parallel paths of stator winding of induction machine. Differences in currents in parallel paths of circuit reduce distortion of the air gap magnetic flux and resulting force [11]. A small asymmetry in the magnetic flux distribution results in significant unbalanced force pulls the rotor in the direction of the maximum magnetic flux density. It turns out that a small quantity of extra flux can cause an asymmetry in magnetic flux distribution in air gap and result in a net transverse force acting on the rotor [12]. These generated lateral forces can be employed to control the vibration and to develop the self bearing machines. Eccentricity will produce two or six pole pair flux variations in a four-pole pair machine. Stator winding with parallel paths cause pulsating UMP vibration. The pulsating character of UMP is caused by winding connection that generates backwards rotating magnetic flux waves that interact with magnetic flux waves rotating forward with pole pair numbers differing by one, resulting UMP vibration of twice the supply frequency [13]. Earlier, a secondary set of winding is incorporated for passive and active suppression of the UMP. Machines with dual sets of winding have poor power ratings. In dual set winding arrangement, the primary winding produces the torque, whereas secondary winding creates lateral force across the air gap [14]. A three phase permanent magnet motor is rewound employing specialized single set of winding arrangement known as bridge configured winding (BCW). BCW arrangement is capable of both passive and active suppression of UMP. The currents that produce torque can be split into two parallel branches. A bidirectional supply located halfway down of each parallel path delivers the current that produces
lateral force. Every conductor in the BCW scheme carries both the torque producing current and current responsible for lateral force production [15]. The UMP decreases by increasing the parallel paths in stator winding of a permanent magnet motors but not as much as in induction machines [16]. The effect of different eccentricities on electromagnetic force production across the air gap in PMSM using the BCW scheme has not been investigated. Time-stepping finite element method (TSFEM) is most widely used to simulate the steady state and dynamic performance of electrical machines [17]. The magnetic field equation can be coupled with the circuit equation by TSFEM. Despite the fact the TSFEM is a more comprehensive model, modelling the rotation between the stator and the rotor is difficult [18].

This article presents modelling and numerical simulation of a 24 slots four pole PMSM using TSFEM. The novelty of this work is to demonstrate the passive control of UMP under different eccentricity conditions of PMSM equipped with specialized stator winding called bridge configured winding. A coupled field and circuit equation is derived for the simulation. The effect of rotor eccentricity in electromagnetic force production with BCW scheme across the air gap is analysed. A general circuit equation has been derived to implement the BCW scheme in the model. The same numerical model can be employed in creating the model, such as finite element mesh generation and weak formulation. Section IV presents and discusses the formulation in creating the model, such as finite element mesh generation and weak formulation. Section V, conclusions are drawn based on findings and recommendations for the future research are presented.

II. MATHEMATICAL MODEL

The proposed model is limited to two dimensional (2-D) field analysis. A conventional PMSM consists of a rotor, stator, permanent magnets, and coil. Maxwell’s equations are used to derive the formulation, which follows [19]

$$\nabla \times E = -\frac{\partial B}{\partial t}$$ \hspace{1cm} (1a)  

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$ \hspace{1cm} (1b)

where $E$ is the intensity of electric field, $B$ is the magnetic flux density, $H$ is the intensity magnetic field, $D$ is the electric field density and $J$ is the current density.

The displacement current density $\partial D/\partial t$ is minimal at low frequencies and can be ignored, (1b) reduced to

$$\nabla \times H = J$$ \hspace{1cm} (2)

The formulation begins by defining magnetic vector potential $A$ as

$$B = \nabla \times A$$ \hspace{1cm} (3)

The relation $B = \mu H$ can be used to express the effect of a particular material, where $\mu = \mu_0 \mu_r$ is the permeability of domain, $\mu_0$ is the permeability of air and $\mu_r$ is the relative permeability of domain.

A. Modelling of permanent magnets

The performance of permanent magnets in electrical machines is usually focused on the demagnetisation curve. Under normal working conditions, the permanent magnets are intended to be in the second quadrant of the $BH$ curve. The permanent magnets such as neodymium-iron-boron (NdFeB) are usually modelled by a linear slope [20]. The linear model relies on a constant remanent flux density $B_r$ of the permanent magnets and a slope represented by the permeability of the magnet as

$$B = \mu_0 \mu_m H + B_r$$ \hspace{1cm} (4)

where $\mu_0$ is the permeability of air and $\mu_m$ is the relative permeability of magnet material.

B. Governing equations

The governing equation for 2-D magnetic field approximation in electrical machines is represented as

$$\frac{\partial}{\partial x} \left[ \frac{1}{\mu} \frac{\partial A}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \frac{1}{\mu} \frac{\partial A}{\partial y} \right] = -J$$ \hspace{1cm} (5)

Current density $J$ comprises three components: one due to applied voltage, another due to the time variation of magnetic field and motion induced voltage. From the stationary frame of reference, the current density $J$ in rotating domains is expressed as

$$J = \sigma \left( -\frac{\partial A}{\partial t} + \frac{U}{l} + \omega_r r \times B \right)$$ \hspace{1cm} (6)

where $\sigma$ is the conductivity of material, $\mu$ is the permeability of the material, $U$ is the voltage applied, $l$ is the length of the conductor, $r$ is the radial distance from the stator axis and $\omega_r$ is the angular velocity. By using the fixed frame of reference, the relative angular speed becomes zero, and the current density is expressed as

$$J = \sigma \left( -\frac{\partial A}{\partial t} + \frac{U}{l} \right)$$ \hspace{1cm} (7)

All aspects of the motor have been considered. The complete formulation for the machine is expressed as

$$\frac{\partial}{\partial x} \left[ \frac{1}{\mu} \frac{\partial A}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \frac{1}{\mu} \frac{\partial A}{\partial y} \right] - \sigma \frac{\partial A}{\partial t} + \frac{U}{l}$$

$$= -\frac{\partial}{\partial x} \frac{1}{\mu} B_{ry} + \frac{\partial}{\partial y} \frac{1}{\mu} B_{rx}$$ \hspace{1cm} (8)
where $B_{rx}$ and $B_{ry}$ are, respectively, the $x$ and $y$ components of $B_r$. To couple circuit and magnetic field equations, total current flowing through each conductor must be calculated. The current $I_c$ in conductor of cross section $s$ is then obtained as

$$I_c = \int_s \sigma \left( -\frac{\partial A}{\partial t} + \frac{U}{L} \right) ds$$

(9)

The current of the conductor is coupled to the circuit by means of resistance $R_c$ and inductance $L_c$. The voltage applied at the terminal of the conductor is given as

$$U_c = R_c I_c + L_c \frac{dI_c}{dt} + R_c \frac{\partial A}{\partial t} ds$$

(10)

For a coil made of $N$ number of turns of thin conductors of cross section $s$, the total cross section area of the coil is $S = Ns$, and the circuit equation for the coil is expressed as

$$V = RI + L \frac{dI}{dt} + \frac{N}{S} \int_s \frac{\partial A}{\partial t} ds$$

(11)

where $I$ is the current in the coil, $R$ is the resistance and $L$ is the end winding inductance of coil.

The stator core is assumed to be made up of thin laminated sheets stacked together to minimise the eddy current loss. Thus, the eddy current term can be neglected. A permanent magnet retention band is present on the rotor to ensure that permanent magnets remain secured to the rotor during operation. However, not every term applies to a complete machine. The magnetic field equations in iron cores, band and air gap are given as

$$\frac{\partial}{\partial x} \left[ \frac{1}{\mu} \frac{\partial A}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \frac{1}{\mu} \frac{\partial A}{\partial y} \right] - \frac{\partial A}{\partial t} = 0$$

(12)

The magnetic field equation in the area of the stator conductor is

$$\frac{\partial}{\partial x} \left[ \frac{1}{\mu} \frac{\partial A}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \frac{1}{\mu} \frac{\partial A}{\partial y} \right] + \frac{N_{co}}{S} I = 0$$

(13)

The magnetic field equation in the permanent magnet domain can be expressed as

$$\frac{\partial}{\partial x} \left[ \frac{1}{\mu} \frac{\partial A}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \frac{1}{\mu} \frac{\partial A}{\partial y} \right] = -\frac{1}{\mu} B_{ry} + \frac{1}{\mu} B_{rx}$$

(14)

The governing equations derived constitute the strong form of the equations. A weak form is derived to develop a finite element model. The following section discusses the finite element formulations.

### III. Time Stepping Finite Element Modelling

The inputs to the time stepping finite element model are voltage supplies, machine geometry, and material properties, whereas the magnetic vector potential $A$ and currents are calculated. The problem to be analysed using the finite element method must be discretized in space. Galerkin’s method is used for finite element modelling, and magnetic vector potential $A$ can therefore be approximated as

$$A = \sum_{j=1}^{n} N_j A_j$$

(15)

where $N_j$ are the shape functions and $A_j$ are the approximation to the magnetic vector potential at nodes, and $n$ represents number of nodes in an element. The magnetic field equation for PMSM presenting the magnetic material, permanent magnets, and stator conductors is expressed as

$$\frac{\partial}{\partial x} \left[ \frac{1}{\mu} \frac{\partial A}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \frac{1}{\mu} \frac{\partial A}{\partial y} \right] - \sigma \frac{\partial A}{\partial t} + \frac{N_{co}}{S} I$$

$$= -\frac{1}{\mu} B_{ry} + \frac{1}{\mu} B_{rx}$$

(16)

Using Galerkin’s method, the discretised magnetic field equation for an element can be expressed as

$$\int \int \frac{1}{\mu} \left[ \frac{\partial N_i}{\partial x} \frac{\partial A}{\partial x} + \frac{\partial N_i}{\partial y} \frac{\partial A}{\partial y} \right] dx dy + \int \int \left[ \sigma N_i \frac{\partial A}{\partial t} \right] dx dy$$

$$- \int \int \frac{N_{co}}{S} I N_i dx dy = \int \int \frac{1}{\mu} \left( -B_{ry} \frac{\partial N_i}{\partial x} + B_{rx} \frac{\partial N_i}{\partial y} \right) dx dy$$

(17)

Alternatively, in matrix form

$$[k]_e \{A\}_e + [c]_e \frac{\partial \{A\}}{\partial t} = \{p\}_e \{I\}_e = \{b\}_e$$

(18)

Discretised circuit equation for an element can be expressed as

$$V = RI + L \frac{dI}{dt} + \frac{N_{co}I}{S} \int N_j \frac{\partial A_j}{\partial t} dx dy$$

(19)

In matrix form

$$V = RI + L \frac{dI}{dt} + \{g\}_e \frac{\partial \{A\}}{\partial t}$$

(20)

where

$$k = \int \int \frac{1}{\mu} \left( \frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} \right) dx dy$$

(21a)

$$c = \int \int \sigma N_i N_j dx dy$$

(21b)

$$p = \frac{N_{co}I}{S} \int N_j dx dy$$

(21c)

$$b = \int \int \frac{1}{\mu} \left( -B_{ry} \frac{\partial N_i}{\partial x} + B_{rx} \frac{\partial N_i}{\partial y} \right) dx dy$$

(21d)

$$g = \frac{N_{co}I}{S} \int N_j dx dy$$

(21e)

The discretized field and circuit equations for an element are now available. These equations must be assembled into a complete global system of equations describing the whole domain.

$$\begin{bmatrix} [K] \end{bmatrix} \{A\} + \begin{bmatrix} [C] \end{bmatrix} \frac{\partial \{A\}}{\partial t} - \begin{bmatrix} [P] \end{bmatrix} \begin{bmatrix} [T] \end{bmatrix} \{I\} = \{B\}$$

(22)

$$\begin{bmatrix} [G] \end{bmatrix} \frac{d\{A\}}{dt} + \begin{bmatrix} [R] \end{bmatrix} \{I\} + \begin{bmatrix} [L] \end{bmatrix} \frac{d\{I\}}{dt} = \{V\}$$

(23)
A double layer winding is used in a bridge configured winding scheme. Each slot will thus have two coil sides, and a transformation matrix \([T]\) is required to transform the circuit currents to coil side currents.

### A. Winding configuration

This paper examines a three-phase permanent magnet synchronous machine. Consisting of a four pole 24 slots PMSM with a specialized winding arrangement called bridge configured winding. Table I contains the complete specifications and dimensions of the machine. The currents that produce torque can be split into two parallel branches. A bidirectional supply located halfway down each parallel path delivers the current that produces lateral force. The bridge configured winding scheme requires no change to the primary torque generating supply [21]. Fig. 1 depicts the stator winding arrangement of PMSM with 24 stator slots according to the bridge configured winding scheme.

### TABLE I: Parameters and dimensions of the machine

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of stator slots</td>
<td>24</td>
<td>-</td>
</tr>
<tr>
<td>Number of poles</td>
<td>4</td>
<td>-</td>
</tr>
<tr>
<td>Axial length</td>
<td>100</td>
<td>mm</td>
</tr>
<tr>
<td>Outer diameter of stator</td>
<td>200</td>
<td>mm</td>
</tr>
<tr>
<td>Inner diameter of stator</td>
<td>95.50</td>
<td>mm</td>
</tr>
<tr>
<td>Outer diameter of rotor</td>
<td>91.50</td>
<td>mm</td>
</tr>
<tr>
<td>Inner diameter of rotor</td>
<td>42.40</td>
<td>mm</td>
</tr>
<tr>
<td>Thickness of air gap</td>
<td>1.75</td>
<td>mm</td>
</tr>
<tr>
<td>Thickness of PM</td>
<td>3.50</td>
<td>mm</td>
</tr>
<tr>
<td>Thickness of band</td>
<td>0.25</td>
<td>mm</td>
</tr>
<tr>
<td>Diameter of conductor</td>
<td>1.30</td>
<td>mm</td>
</tr>
<tr>
<td>Resistivity of conductor</td>
<td>1.68 \times 10^{-8}</td>
<td>\Omega m</td>
</tr>
<tr>
<td>End winding inductance</td>
<td>0.02</td>
<td>H</td>
</tr>
</tbody>
</table>

Fig. 1: Winding arrangement of permanent magnet synchronous machine.

B. Circuit equations when bridge points are open

The coils have been formed into 12 coil groups to illustrate the circuit’s modelling. Coils (A1, A3), (A2, A4), (A5, A7), (A6, A8), (B1, B3), (B2, B4), (B5, B7), (B6, B8), (C1, C3), (C2, C4), (C5, C7), and (C6, C8) have been referred to as coil groups (G1-G12) respectively. Fig. 2 shows a star-connected circuit diagram in the Bridge OFF condition of PMSM. The machine is powered by a three phase star-connected power supply. \(a1\) and \(a2\) are bridge locations of phase A, \(b1\) and \(b2\) are bridge locations of phase B, and \(c1\) and \(c2\) are bridge locations of phase C, respectively. Applying Kirchoff’s voltage law (KVL) in these five loops. Additionally, Kirchoff’s current law (KCL) needs to be applied about point O.

\[
\begin{align*}
[G_1 + G_4 - G_5 - G_8] \frac{\partial A}{\partial t} + 2Ri_1 - 2Ri_2 + 2L \frac{\partial i_1}{\partial t} & = V_A - V_B \\
2L \frac{\partial i_2}{\partial t} & = V_B - V_C \\
2L \frac{\partial (i_B - i_2)}{\partial t} - 2L \frac{\partial i_3}{\partial t} & = V_C - V_A \\
2L \frac{\partial (i_A - i_1)}{\partial t} - 2L \frac{\partial i_1}{\partial t} & = 0 \\
2L \frac{\partial (i_B - i_2)}{\partial t} + 2R(i_B - i_2) - 2Ri_3 & = 0 \\
2L \frac{\partial (i_A - i_1)}{\partial t} + 2R(i_A - i_1) - 2Ri_1 & = 0
\end{align*}
\]
Additionally, KCL needs to be applied about point O currents. Therefore, there are nine current bridge points are connected by a coil of inductance. The voltage source connected to these bridge points can provide the current, which helps to smooth out the distortion in the current flowing across the bridge points when these points are connected. The rotor will induce EMF, due to which a current flows across these bridge points when these points are connected. The number of coil sides required to transform the circuit current to air gap. An independent voltage source connected to these bridge points can provide the required lateral force for active control by injecting currents into the winding. A general circuit equation with a voltage source across the bridge points has been established. The bridge points are connected by a coil of inductance and resistance. The bridge voltage source, and are connected across the bridge. i_{bA}, i_{bB} and i_{bC} are the bridge currents, respectively. Therefore, there are nine current variables when bridge points are connected, including phase currents i_A, i_B, i_C, circuit currents i_1, i_2, i_3, and bridge currents i_{bA}, i_{bB}, i_{bC} respectively. The circuit shown in Fig. 3 comprises eight loops, applying KVL in these eight loops. Additionally, KCL needs to be applied about point O.

\[
\begin{align*}
(G_9 + G_{12} - G_{10} - G_{11}) \frac{dA}{dt} - 2R(i_C - i_3) + 2Ri_3 \\
- 2L \frac{di_C - i_3}{dt} - 2L \frac{di_3}{dt} &= 0 \\
i_A + i_B + i_C &= 0
\end{align*}
\] (24e)

Alternatively, in matrix form

\[
[Q] \frac{d[A]}{dt} + [R]{[I]} + [L] \frac{[I]}{dt} = \{V\}
\] (25)

C. Circuit equations when bridge points are close

The bridge configured circuit diagram for PMSM with voltage source connected to the bridge points is shown in Fig. 3. The distinctive feature of the bridge configured winding that makes it different from other active methods for reducing the unbalanced magnetic pull is that it provides passive control. Short-circuiting the bridge points allows for passive control of UMP. Any asymmetric magnetic field caused by the eccentric rotor will induce EMF, due to which a current flows across these bridge points when these points are connected. The current flowing across the bridge points is termed as equalising current, which helps to smooth out the distortion in the magnetic flux density across the air gap. A general circuit equation with a voltage source across the bridge points has been established. The bridge points are connected by a coil of inductance and resistance. The bridge voltage source, are connected across the bridge. i_{bA}, i_{bB} and i_{bC} are the bridge currents, respectively. Therefore, there are nine current variables when bridge points are connected, including phase currents i_A, i_B, i_C, circuit currents i_1, i_2, i_3, and bridge currents i_{bA}, i_{bB}, i_{bC} respectively. The circuit shown in Fig. 3 comprises eight loops, applying KVL in these eight loops. Additionally, KCL needs to be applied about point O.

\[
\begin{align*}
&[G_1 + G_4 - G_5 - G_8] \frac{dA}{dt} + 2Ri_1 - 2Ri_2 - Ri_{bA} \\
+ &Ri_{bB} + 2L \frac{di_{1}}{dt} - 2L \frac{di_{2}}{dt} - L \frac{di_{bA}}{dt} + L \frac{di_{bB}}{dt} = V_A - V_B \\
&[G_6 + G_7 - G_9 - G_{12}] \frac{dA}{dt} - 2Ri_2 - 2Ri_B - 2Ri_3 \\
+ &Ri_{bB} + Ri_{bC} - 2L \frac{di_{2}}{dt} + 2L \frac{di_{B}}{dt} - 2L \frac{di_{3}}{dt} + L \frac{di_{bB}}{dt} + L \frac{di_{bC}}{dt} = V_B - V_C \\
&[G_3 - G_1] \frac{dA}{dt} - 2Ri_1 + Ri_A - R_{ib_{A}} - 2L \frac{di_{1}}{dt} \\
+ &L \frac{di_{A}}{dt} - Lb \frac{di_{bA}}{dt} = -V_{ib_{a}} \\
&[G_2 - G_4] \frac{dA}{dt} - 2Ri_1 + Ri_A + 2Ri_{bA} + Ri_{b_{A}} \\
- &2L \frac{di_{1}}{dt} + L \frac{di_{A}}{dt} + 2L \frac{di_{bA}}{dt} + Lb \frac{di_{bA}}{dt} = V_{ib_{A}}
\end{align*}
\] (26a)

Alternatively, in matrix form

\[
[G] \frac{d[A]}{dt} + [R]{[I]} + [L] \frac{[I]}{dt} = \{V\}
\] (27)

If no external supply is provided across the bridge location, the bridge supply voltages are set to zero.

D. Transformation

A double layer winding is used in the bridge configured winding arrangement. Each slot of the stator corresponds to two different coil sides. The matrix [P] in equation (22) is \(n \times z\). The number of coil sides \(z\) is 48. The current vector I is of size \(6 \times 1\) in the Bridge OFF situation and \(9 \times 1\) in the Bridge ON situation. Thus, transformation matrices [T] of size \(z \times 6\) and \(z \times 9\) are required to transform the circuit current to coil side currents. The two full coils together form a coil...
group, and the coils of a coil group must have the same current as they are connected in series. The transformation from full coil currents to coil side currents is given as

\[
\begin{bmatrix}
i_{+A1} \\
i_{-A1} \\
\vdots \\
i_{+C8} \\
i_{-C8}
\end{bmatrix} = A \begin{bmatrix}
i_{A1} \\
i_{G1} \\
\vdots \\
i_{G12}
\end{bmatrix}
\]

Alternatively, in matrix form

\[
\{i_{cs}\}_{48\times1} = [T_0]_{48\times24} \{i_c\}_{24\times1}
\]

Coil group currents can be transformed into coil side currents as follows.

\[
\begin{bmatrix}
i_{A1} \\
i_{G1} \\
i_{G2} \\
i_{G3} \\
i_{G4}
\end{bmatrix} = \begin{bmatrix}
1 & 0 & \cdots & 0 \\
0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 1
\end{bmatrix} \begin{bmatrix}
i_{G1} \\
i_{G2} \\
i_{G3} \\
i_{G4}
\end{bmatrix}
\]

Alternatively, in matrix form

\[
\{i_c\}_{24\times1} = [T_1]_{24\times12} \{i_G\}_{12\times1}
\]

In the Bridge OFF condition, a transformation matrix to transform circuit currents to the coil group currents for phase A is given as

\[
\begin{bmatrix}
i_{G1} \\
i_{G2} \\
i_{G3} \\
i_{G4}
\end{bmatrix} = \begin{bmatrix}
1 & 0 & \cdots & 0 \\
-1 & 1 & \cdots & 0 \\
-1 & 1 & \cdots & 0 \\
1 & 0 & \cdots & 1
\end{bmatrix} \begin{bmatrix}
i_1 \\
i_2 \\
i_3 \\
i_A
\end{bmatrix}
\]

A similar transformation can be written for the other two phases. Therefore, the complete transformation from circuit to coil group currents in the Bridge OFF condition can be written as

\[
\{i_G\}_{12\times1} = [T_2]_{12\times6} \{i_{circuit}\}_{6\times1}
\]

For phase A, Circuit currents and bridge currents are transformed to coil group currents as follows.

\[
\begin{bmatrix}
i_{G1} \\
i_{G2} \\
i_{G3} \\
i_{G4}
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
-1 & 1 & 1 & 0 \\
-1 & 1 & 0 & 0 \\
1 & 0 & 0 & -1
\end{bmatrix} \begin{bmatrix}
i_1 \\
i_2 \\
i_3 \\
i_A
\end{bmatrix}
\]

A similar transformation can be derived for phase B and phase C. Therefore, the complete transformation from circuit to coil group currents in the Bridge OFF condition is given as

\[
\{i_G\}_{12\times1} = [T_2]_{12\times9} \{i_{circuit}\}_{9\times1}
\]

Therefore, the transformation matrix \([T]\) to transform currents in the circuit to coil side currents is given as

\[
[T]_{z\times m} = [T_0]_{z\times24}[T_1]_{24\times12}[T_2]_{12\times m}
\]

where value of \(m\) is 6 when bridge points are open and \(m\) is 9 when bridge points are connected.

### E. Temporal discretization

The derived problem is time dependent, and the governing equations are needed to discretize in both space and time domains. The time discretization employed here is given as [19]:

\[
\alpha \left( \frac{\partial A}{\partial t} \right)^t + (1 - \alpha) \left( \frac{\partial A}{\partial t} \right)^t = \frac{(A)^t + \Delta t - (A)^t}{\Delta t}
\]

The constant \(\alpha\) defines, whether the discretization is forward difference scheme \(\alpha = 0\), the backward difference scheme \(\alpha = 1\) or any other intermediate scheme \(0 < \alpha < 1\). If \(\alpha = 0.5\), the Crank-Nicholson scheme is implemented. By coupling (22) and (23) together, the coupled field and circuit equation in matrix form is

\[
\begin{bmatrix}
[K] & -[P][T] \\
0 & R
\end{bmatrix} \begin{bmatrix}
A \\
I
\end{bmatrix} + \begin{bmatrix}
C & 0 \\
G & L
\end{bmatrix} \begin{bmatrix}
\frac{dA}{dt} \\
\frac{dI}{dt}
\end{bmatrix} = \begin{bmatrix}
B \\
V
\end{bmatrix}
\]

Alternatively, equation (38) can be simplify as

\[
[M]\{X\} + [N]\{\frac{dX}{dt}\} = \{F\}
\]

The outcomes of (39) yields \(\{A\}\) and \(\{I\}\), \([M]\) and \([N]\) are the coefficient matrices, and \([F]\) is a vector representing excitation due to permanent magnet and input voltage. The Crank-Nicholson scheme is numerically stable. After time discretization, the final system equations will have the form.

\[
\begin{bmatrix}
[M] & \frac{2}{\Delta t} \{N\}
\end{bmatrix} \{X\}^{t+\Delta t} = -\begin{bmatrix}
[M] - \frac{2}{\Delta t} \{N\}
\end{bmatrix} \{X\}^{t+\Delta t} + \{F\}^{t+\Delta t} + \{F\}^{t}
\]

### F. Domain discretization and rotation

A simple and robust mesh generation process should be used for mesh generation of the machine. The finite element rotor mesh rotates at each time step in accordance with the rotation of the rotor. The finite element mesh of PMSM can be divided into stator and rotor, which are then created separately for these two parts. The stator and rotor mesh development has been carried out using quadrilateral element. The air gap comprises two layers, with one layer belonging to stator mesh. The air gap stitching approach accommodates the rotor movement and eccentricity [14]. The outermost layer of rotor mesh and upper layer of air gap, which belongs to the stator mesh, are connected by the air gap stitching approach as depicted in Fig. 4. A moving band distortion occurs during the rotation of the rotor with rectangular elements [19]. Therefore, modelling of air gap element was carried out using triangular elements. When the rotor rotates at each time step, it is possible to retain a constant mesh shape for both the stator and rotor, only the rotor mesh coordinates need to be changed, and air gap stitching must be performed. The finite element mesh of PMSM after rotation is shown in Fig. 5. This approach requires the stator and rotor mesh to be generated once, significantly reducing the time needed for each step in creating mesh [14], [17].
IV. SOLUTION OF COUPLED FIELD AND CIRCUIT EQUATIONS

At this point, the problem is ready for analysis. The mesh generation, assembly of the system, formulation implementation and saving of the solution were performed using MATLAB. However, before these stages, material properties and parameters for finite element simulation must be specified. Table II and III summarises the values that were used. The excitation of the coil and the permanent magnet is created using expressions (21c), (21d) and (21e) in field and circuit equations, respectively. These expressions defined the source based on the geometry of the machine. It is possible to update particular variables, allowing the three phase supply to be updated at each time step. The governing differential equations in the preceding section are used to derive the weak formulation. It is assumed that the electromagnetic field is negligible at the interior and exterior boundaries of the machine. Magnetic vector potential $A$ is set to zero at these boundaries. The residual is then minimised to obtain the solution. This results in a large set of equations with unknown node potentials. The coupled field circuit equation is time discretized using the Crank-Nicholson algorithm as given in (38). It is estimated that the mesh of the complete solution domain consists of 10368 elements and 31104 nodes. The stator and rotor region is discretized using quadrilateral elements, whereas the air gap region is discretized using triangular elements. The solver algorithm code is developed in MATLAB.

<table>
<thead>
<tr>
<th>Material</th>
<th>Relative Permeability</th>
<th>Conductivity (S/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>900</td>
<td>$2.00 \times 10^6$</td>
</tr>
<tr>
<td>PM</td>
<td>1</td>
<td>$6.25 \times 10^5$</td>
</tr>
<tr>
<td>Copper</td>
<td>1</td>
<td>$5.80 \times 10^7$</td>
</tr>
<tr>
<td>Air</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

A similar model is solved with a commercial electromagnetic simulation software package, COMSOL™, to under-
TABLE III: Simulation parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotational speed</td>
<td>1500</td>
<td>RPM</td>
</tr>
<tr>
<td>Supply frequency</td>
<td>50</td>
<td>Hz</td>
</tr>
<tr>
<td>Supply voltage</td>
<td>236.7</td>
<td>V</td>
</tr>
<tr>
<td>Time-step</td>
<td>1</td>
<td>ms</td>
</tr>
<tr>
<td>Total time</td>
<td>1</td>
<td>s</td>
</tr>
</tbody>
</table>

stand whether the results obtained by the developed code are accurate for the problem. With built-in functionality for rotation machines, it is easy to model motors and generators using COMSOL™. The verification model was created and solved using COMSOL™. There is almost no difference in crucial physical quantities evaluated with adequate mesh size. To validate the developed model, magnetic vector potential solution in the entire domain and air gap flux density were compared, a simple calculation using (3), whereas COMSOL™ output this quantity by default. The contour map of the magnetic vector potential in the cross section of the machine at \( t = 1 \) s is shown in Fig. 6. The outer corner of permanent magnets has the highest magnetic vector potential value for both models. In the majority of the stator part and inner part of the rotor, the value is very small. Fig. 7 depicts the air gap flux density for both the models to validate the model further. The developed model follows a pattern similar to the COMSOL™ model. The air gap flux density obtained by the developed code and COMSOL™ model is in good agreement. Therefore, it is possible to prove that the developed model is accurate.

V. SIMULATION RESULTS AND DISCUSSION

The coupled field and circuit equation discussed in section III has been used in the analysis. The distribution of electromagnetic fields in PMSM varies according to the state of the machine. In static condition, the rotor permanent magnets generate a magnetic field. The motor can drive the load when connected to the three phase sinusoidal supply. Meanwhile, the magnetic field in the entire domain is produced by rotor permanent magnets and current in the stator winding. Before analysing the electromagnetic force, the components of magnetic flux density in the air gap are first evaluated. The distribution of magnetic flux in the air gap is shown in Figs. 8 and 9. The permanent magnets appear to contribute the majority of the flux density in the air gap.

The finite element method has been used to calculate the currents in parallel branches of the stator winding and magnetic flux distribution in the air gap. In the absence of eccentricity, the forces along \( x \) and \( y \) direction will be zero. Eccentricity causes unbalanced forces, which is referred to as unbalanced magnetic pull. The unbalanced force around the air gap owing to eccentricity is determined using the Maxwell stress method from the magnetic flux density information.

\[
\sigma_r = \frac{1}{2\mu_o}(B_r^2 - B_t^2) \quad \text{and} \quad \sigma_t = \frac{1}{\mu_o}B_r B_t \tag{41}
\]

where \( B_r \) and \( B_t \) are the magnetic flux density components in radial and tangential directions, respectively and \( \mu_o \) is the air gap permeability. With reference to equation (39) the net force along \( x \) and \( y \) are

\[
F_x = L \int_0^{2\pi} (\sigma_r \cos \theta - \sigma_t \sin \theta) d\theta
\]

\[
F_y = L \int_0^{2\pi} (\sigma_r \sin \theta + \sigma_t \cos \theta) d\theta \tag{42}
\]

where \( L \) is stator core length and \( r \) is radial distance to the air gap.

The rotor permanent magnets in PMSM will generate symmetric magnetic flux density around the air gap however if the rotor is displaced off-centre, the distribution of the magnetic flux around the air gap will be asymmetric and will have additional harmonic field components with a difference of one pole pair [4]. These additional harmonic components induce voltages and cause equalizing currents to flow in parallel paths of windings. Equalising currents mitigate the UMP by reducing additional harmonics [22]. If the rotor is not centred, the inductance of the circuit where the air gap is higher than the average value is lower than the circuit where the air gap is lower. Therefore, a slightly higher current flow in the circuit facing a higher air gap. The difference in currents in the parallel path of the circuit tends to minimise asymmetry in air gap magnetic flux density at the opposite ends of the circuit [10]. A modified winding scheme that creates magnetic flux density similar to that of a motor with a dual layer winding scheme has been published in [23]. The main feature of bridge...
configured winding is that the currents that produce torque can be split into two parallel branches. A bidirectional supply located halfway down of each parallel path delivers the current that produces lateral force. The bridge configured winding scheme requires no change to the primary torque generating supply [12].

Simulations on PMSM were carried out for Bridge OFF and Bridge ON conditions. The influence of bridge configured winding on the reduction of UMP due to static, dynamic and a combination of static and dynamic eccentricity has been examined. Eccentricities are modelled by displacing the rotor off centre and constraining the rotation centre for static eccentricity to its own axis. In the event of dynamic eccentricity, the off-centred rotor is allowed to whirl around the stator centre at a synchronous speed. The actual machine often exhibits mixed eccentricity. To model static eccentricity, the rotor is displaced off-centre by 20% of the nominal air gap in the x-direction. In the event of dynamic eccentricity, the rotor centre is allowed to whirl about the stator centre with whirling radius of 20% of nominal air gap, whereas to model mixed eccentricity, the rotor has been shifted 10% of the nominal air gap in the x-direction and allowed to whirl around a point between rotor centre and stator centre with whirling radius of 10% of the nominal air gap.

**A. Bridge currents under static, dynamic and mixed eccentricity**

Due to the eccentricity, additional harmonic components are formed in air gap flux density. For each eccentricity condition, an analysis of bridge currents and forces applied to the rotor has been conducted. Fig. 10 show bridge currents for PMSM under static eccentricity. Bridge current signals are stored and analysed in the frequency domain. As can be seen from Fig. 11, static eccentricity generates harmonics in bridge currents. The FFT spectrum contains the fundamental frequency, $f$, and its odd multiples ($3f, 5f, \text{etc.}$) [24].

Fig. 12 and 13 show the bridge currents and frequency spectrum for PMSM in dynamic eccentricity condition, respectively. Dynamic eccentricity produced harmonics at frequencies $[1 \pm (2m - 1)/p]f$ [3], i.e., $25.39 \text{ Hz}, 74.22 \text{ Hz}, 125 \text{ Hz}$ etc., where $m$ is the integer number, and $p$ represents the pole pairs number. Dynamic eccentricity causes harmonics at rotor mechanical frequency because the electrical fundamental periods result in $1/p$ rotation of the rotor. Due to the rotation of minimum air gap towards the rotor, winding inductance and flux linkage change throughout the rotation, making them different in $1/p$ cycle [25].

Bridge currents and frequency spectrum for PMSM under mixed eccentricity condition are shown in Fig. 14 and 15. The FFT spectrum of bridge currents in case of mixed eccentricity contains harmonics at the fundamental frequency, $f$, its odd harmonics and harmonics at frequencies $[1 \pm (2m - 1)/p]f$.  

![Fig. 10: Bridge currents for PMSM with static eccentricity.](image1)

![Fig. 11: FFT spectrum of Bridge current in static eccentricity condition.](image2)

![Fig. 12: Bridge currents for PMSM with dynamic eccentricity.](image3)

![Fig. 13: FFT spectrum of Bridge current in dynamic eccentricity condition.](image4)
i.e., 25.39 Hz, 50.78 Hz, 74.22 Hz, 125 Hz, 150 Hz etc.

B. Unbalanced magnetic pull comparison in Bridge OFF and Bridge ON condition

The radial electromagnetic force in the air gap causes the UMP on the rotating shaft. The resultant electromagnetic force should be almost negligible in normal conditions. The balance of the rotating shaft is compromised due to the asymmetric air gap. Therefore, it makes perfect sense to examine and control the unbalanced radial electromagnetic force for PMSM with eccentricity. When the rotor is eccentric, the radial electromagnetic force density is maximum at the location of the shortest air gap and minimum at the location of the longest air gap, resulting in the UMP always pointing towards the minimum air gap. In case of static eccentricity, the electromagnetic force acting on the off-centred rotor is depicted in Fig. 16. When the inductance of each coil in the same phase is different, bridge connections allow the current of each coil group in the same phase to vary. The static eccentricity generates UMP with fixed magnitude and direction in the Bridge OFF condition. Bridge connection leads to electromagnetic force components along the y-axis with an average value of zero, as shown in Fig. 17, which means bridge connection causes UMP direction to shift from the eccentric position and provides a significant reduction of electromagnetic force along the x-axis. Fig. 18 depicts that in Bridge OFF condition, static eccentricity induces electromagnetic force components at 0, ±f and ±4f, where f represents the frequency of the rotor. The amplitude of the force components at 0 and ±f is significantly reduced in the Bridge ON condition, whereas the amplitude of force components at ±4f is increased.

As illustrated in Fig. 19, the electromagnetic force acting on the eccentric rotor with dynamic eccentricity significantly decreases in the Bridge ON condition. Fig. 20 depicts that dynamic eccentricity induces electromagnetic force components at ±f and ±3f in Bridge OFF condition. The amplitude of force components at ±f and ±3f is reduced in the Bridge ON condition, whereas an extra force component at ±5f is
VI. Conclusion

This article proposes a model for PMSM using TS-FEM. The circuit equation and electromagnetic field equations for PMSM are coupled and solved. In accordance with input voltages and permanent magnet magnetization, the stator current and magnetic field distribution can be calculated. A specialized stator winding scheme that incorporates bridge connection and has advantages of both single and double sets of coils arrangement in stator winding is used for electromagnetic analysis of PMSM. To verify the model, a similar model is developed using the simulation software package COMSOL™ to compare the magnetic flux density. The effect of bridge connection in an eccentric rotor PMSM has been demonstrated to minimise significantly the amplitude of unbalanced electromagnetic force components developed due to different eccentricities. Static eccentricity results in UMP with fixed magnitude and direction. The bridge connection causes the UMP direction to shift from the eccentric position, and the amplitude of force components at ±4fr is significantly reduced in the Bridge ON condition, whereas the amplitude at ±5fr is increased in case of static eccentricity. In the case of dynamic eccentricity, the minimum air gap rotates towards the rotor, resulting in a rotating force vector. The amplitude of force components at ±fr and ±3fr is reduced in the Bridge ON condition, whereas an extra force component at ±5fr is also induced in electromagnetic force. In the situation

also induced in the frequency spectrum of electromagnetic force. Fig. 21 depicts the trace of electromagnetic force acting on the rotor with mixed eccentricity. As can be seen from Fig. 22, mixed eccentricity induces all of the force components of static and dynamic eccentricity. The amplitude of force components at 0, ±fr, ±f and ±3fr is significantly reduced in the bridge ON condition, whereas the amplitude of force components at ±4fr and ±5fr is increased in frequency spectrum of electromagnetic force.
of mixed eccentricity, the bridge connection suppresses the amplitude of force components at $0$, $\pm f_r$, $\pm f$ and $\pm 3f_r$, whereas the amplitude of force components at $\pm 4f_r$ and $\pm 5f_r$ is increased. The major finding of this work is that the favourable effect of bridge connection exists even when the rotor remains eccentric. Bridge currents cause the current of each coil group in the same phase to vary. In this way, different currents in different coil groups of the same phase can considerably compensate for the asymmetric flux distribution in the air gap.

**REFERENCES**


