Frequency Diverse Array With Discrete Fourier Transform for Single Target Estimation

Kai Wang 1, Zichuan Yu 1, Zhiyuan Jin 1, Feiyang Zhong 1, and Lu Tang 1

1Affiliation not available

December 7, 2023

Abstract

In order to overcome the limitations of phased array radar technology, the researchers proposed frequency diverse array technology to achieve accurate detection and tracking of target direction. In this paper, a method of distance and angle estimation based on DFT transform is proposed for frequency transform array. By cross-correlating the received data, a new received signal vector is established and solved by constructing an equation. Compared with other methods, this method overcomes the problem that FDA radar is prone to fuzzy estimation and insufficient resolution in target parameter estimation. It can not only directly estimate the distance and angle of the target, but also effectively improve the target resolution.
Frequency Diverse Array With Discrete Fourier Transform for Single Target Estimation

Kai Wang, Zichuan Yu, Zhiyuan Jin, Feiyang Zhong, Lu Tang

Abstract—In order to overcome the limitations of phased array radar technology, the researchers proposed frequency diverse array technology to achieve accurate detection and tracking of target direction. In this paper, a method of distance and angle estimation based on DFT transform is proposed for frequency transform array. By cross-correlating the received data, a new received signal vector is established and solved by constructing an equation. Compared with other methods, this method overcomes the problem that FDA radar is prone to fuzzy estimation and insufficient resolution in target parameter estimation. It can not only directly estimate the distance and angle of the target, but also effectively improve the target resolution.

Index Terms—frequency diverse array; Discrete Fourier Transform; cross-correlation.

I. INTRODUCTION

Phased array radar technology has always been one of the key technologies in radar systems, which can realize accurate detection and tracking of target direction. The main advantage of phased array radar technology is that it can provide directional gain and suppress strong sidelobe interference in other directions [1][2], thus achieving high-sensitivity detection of distant, weak signal targets. However, phased array radar technology also has certain limitations, most notably its inability to provide accurate range information for targets. This is because the phased array radar can only provide information related to the angle of the target, and cannot directly measure the distance between the target and the radar. Obtaining accurate distance information of a target is very important in many application scenarios, such as navigation, target tracking, and obstacle avoidance.

To overcome this limitation of phased array radar technology, Refs.[3, 4] introduced Frequency Diverse Array (FDA) technology. The core idea of FDA technology is to introduce tiny frequency shifts between adjacent antennas. This small frequency shift results in the formation of a range-dependent and time-dependent transmitted beam in the direction of the target. By analyzing the received echo signal, the distance information of the target can be inferred. This method extends the function of the radar system so that the radar can provide not only the direction information of the target but also the accurate distance information of the target. As a result, FDA technology is widely used in areas where target distance information is needed.

In addition to the ability to provide accurate distance information, FDA technology has several other advantages. It can achieve the acquisition of target distance information without adding additional hardware complexity. Also, FDA technology can achieve simultaneous positioning of multiple targets, improving the radar system's multi-target detection capability. Current FDA research efforts are focused on joint range and angle estimation [5–8], range-dependent interference suppression[9], beampattern synthesis[9], and their applications in radar [10] and communications[11].

However, while FDA technology has many advantages, it still faces some challenges in practical application. Firstly, how to use range-angle dependent beam patterns to improve the performance of radar and navigation applications is an important problem. In the FDA system, the beam pattern is related to distance and angle, so it is necessary to design corresponding signal processing algorithms to extract effective information and achieve high-precision target detection and tracking. Secondly, the problem of fuzzy estimation and insufficient resolution is often encountered in the target parameter estimation. This is because in the FDA system, due to the introduction of frequency offset, the spectral characteristics of the target echo signal have changed, which leads to the difficulty of parameter estimation. Therefore, how to improve the accuracy of parameter estimation is another key problem that FDA technology needs to solve.

DOA estimation can be regarded as a frequency estimation of a multi-frequency exponential signal. This relationship is critical in the field of radar and signal processing, directly affecting the performance and accuracy of applications such as radar systems. Many research results of multi-frequency estimation have been proposed, including Maximum Likelihood (ML), Root-MUSIC[12], ESPRIT[13] and MUSIC[14]. These methods are widely used to estimate the frequency of multi-frequency signals, but they have some limitations in dealing with sidelobe interference and spectrum leakage. To overcome these problems, Ref.[15]used particle swarm optimization method to optimize the frequency offset, so that the beam energy is concentrated on the interested points. Ref.[16] proposed a new FDA framework that employs symmetric and logarithmic frequency offsets as well as multi-carrier techniques to produce point-like beam pattern for target indication. The FDA framework with Taylor plus window frequency bias was proposed in Ref.[17], while Ref.[18] combined the IM concept to design a structure different from the traditional frequency division array, which can decouple the distance and angle dependence problems.

A novel approach based on CZT was proposed in Ref.[19]. In this new method based on CZT, the CZT bins are used to describe the relationship between the system parameters (amplitude, phase, frequency) and the observed spectrum. The characteristic of this method is to calculate the actual frequency and direction angle by using nonlinear equations with
unknown frequency parameters. The method fully considers sidelobe interference and spectrum leakage, and improves the estimation accuracy of azimuth angle. The application of this CZT based approach has also been extended to FDA radars. By introducing this new approach to FDA radar, it opens up more application possibilities for radar system performance, especially in complex environments where multiple targets need to be tracked simultaneously. In this paper, Ref.[19] is applied to the distance and angle estimation of single target in FDA radar, and the time parameter is eliminated by cross-correlation method. Through detailed simulation experiments, the algorithm has significant advantages in the actual distance and angle estimation. This not only provides an effective solution for radar applications, but also provides a new idea and method for the development of radar technology in the future.

II. FREQUENCY ESTIMATION METHOD

Consider the ULA (Uniform Linear Array) FDA radar model shown in Fig.1. The signal transmitted by the n-th antenna is:

\[ s_n(t) = e^{j2\pi f_0 t}, \quad n = 0, 1, \cdots, N - 1 \]  

(1)

where \( N \) is the number of antenna in the array. The carrier-frequency \( f_0 \) from the n-th antenna can then be expressed as:

\[ f_n = f_0 + n\Delta f, \quad n = 0, 1, \cdots, N - 1 \]  

(2)

where \( f_0 \) is the reference carrier frequency and \( \Delta f \) is the frequency offset, respectively. The transmitted signal from the n-th antenna will cover a distance \( R_n = R - nd \sin(\theta) \) to reach target, where \( d \) is element spacing. The signal received by the n-th antenna can be rewritten as:

\[ r_n(t) = e^{j2\pi f_0 t + n\Delta f} \left( e^{-\frac{j\pi 2nd}{R_n}} \right) \]  

(3)

where \( c \) is the speed of light. Due to \( \Delta f \ll f_0 \) and \( d = c/(2f_0) \), we have:

\[ r_n(t) = e^{j2\pi f_0 t - \frac{j\pi d}{c}} e^{j2\pi \left( \frac{\Delta f}{c} \right) n} \]  

(4)

After that, we can use a method of eliminating time terms based on correlation for Eq.4, cross-correlational processing of the received data, and establish a new receiving vector. Let the sending signal at time \( t \) be \( s_n(t) \), and cross-correlate \( s_n(t) \) with the receiving vector \( r_n(t) \). Thus, Eq.4 can be approximated as:

\[ r_n(t) = r_n(t) + \frac{s_n(t)}{R_n} \]  

(5)

\[ = e^{j2\pi f_0 t - \frac{j\pi d}{c}} e^{j2\pi \left( \frac{\Delta f}{c} \right) n} e^{-j2\pi \left( \frac{\Delta f}{c} \right) n}, n = 1, 2, \cdots, N - 1 \]

Let's denote \( \omega \) and \( \varphi \) as:

\[ \omega = 2\pi \left( \frac{f_0}{c} \right) \]  

(6)

In Eq.6, and are denoted as the known control parameters, and are the unknown parameters which should be estimated. We can rewrite Eq.5 as:

\[ r_n(t) = e^{j\omega t} e^{j\varphi n}, n = 1, 2, \cdots, N \]  

(7)

As show in Eq.7, the values of \( \omega \) are determined with the values of \( \theta \) and \( R \). The ranges and angles estimation for multitarget with FDA radar can be regarded as the frequencies estimation of the signal with several spectral components. By applying the DFT transformation to Eq.7, the superposition of N antenna can be written as:

\[ R(k) = \sum_{n=0}^{N-1} e^{j\omega n} W_N^{nk} \]  

(8)

\[ = e^{j\omega k} \frac{1 - e^{jN\omega}}{1 - e^{j\omega}} e^{-j\frac{k\pi}{N}} \]  

\[ = e^{j\omega k} e^{j\frac{k\pi}{N}} e^{j\frac{k\pi}{2}} e^{j\frac{k\pi}{2}} e^{-j\frac{k\pi}{2}} e^{-j\frac{k\pi}{2}} e^{j\frac{k\pi}{2}} e^{-j\frac{k\pi}{2}} e^{-j\frac{k\pi}{2}} \]  

(9)

Let \( \omega = \frac{k\pi}{2} \) \( (k' + \delta') \), where \( k' \) and are \( \delta' \) integers and decimals, respectively. \( k' \) is the value of \( k \) corresponding to the maximum value of \( R(k) \).

When we have the different DFT bins \( R(k') \) and \( R(k'+1) \), we can get:

\[ \frac{|R(k')|}{|R(k'+1)|} = 1 - \delta' \]  

(10)

where \( |*| \) represents the modulus operation on any complex number \( * \). Then we can calculate the estimate \( \delta' \) of \( \delta' \):

\[ \delta' = \frac{|R(k'+1)|}{|R(k'+1)| + |R(k')|} \]  

(11)

then \( \hat{\omega} = \frac{2\pi}{\delta'} \) \( (k' + \hat{\delta}') \).

The relation among \( \omega, R \) and \( \theta \) is shown in Eq.6 can be regarded as a linear equation with two unknowns parameters, which means we can calculate \( R \) and \( \theta \) when we have two different parameter combinations \((\omega^{(i)}, f_0^{(i)}, \Delta f^{(i)}), i = 1, 2, \cdots, \). We can calculate \( R \) and \( \theta \) in one snapshot. We estimate \( \omega^{(1)} \) from the received signal gotten by 1-th,2-th, \( \cdots, \frac{N}{2} \)-th antennas,
and estimate $\omega^{(2)}$ from the received signal gotten by 2-th, 4-th, \cdots, $N$-th antennas. $\omega^{(i)}$, $i = 1, 2$ can be regarded as the parameters generated by two FAD radars with $N/2$ antennas, whose frequency offsets are $\Delta f$ and $2\Delta f$, respectively. This strategy has good real-time performance. When we have two $\omega^{(i)}$, $i = 1, 2$, we can calculate the $\hat{R}$ and $\hat{\theta}$ according to:

$$\begin{align}
\hat{R} &= \frac{c(\omega^{(1)} - \omega^{(2)})}{2\pi df} \\
\hat{\theta} &= \sin^{-1}\left(\frac{c(\omega^{(1)} - \omega^{(2)})}{2\pi df}\right)
\end{align}$$ \hspace{1cm} (11)

However, this strategy is a little weak in anti-noise performance, since this strategy only use signals from $N/2$ antennas in each calculation of $\omega$.

## III. Simulation

This section shows the experimental simulation results. Unless stated otherwise, the fundamental carrier frequency is $f_0 = 5$GHz and the frequency offset is $\Delta f = -1.0$KHz. The number of elements is $N = 16$. The target is located at $(r_1, \theta_1) = (500\text{km}, -15^\circ)$. The accuracy of proposed method is compared with Root-MUSIC method[12], ESPRIT method[13], and Logarithmically Increasing Frequency Offset method. The signal-to-noise ratios (SNRs) is set $\text{SNR} = 2$dB, which is defined as follows:

$$\text{SNR} = 10 \cdot \log \frac{P_{\text{signal}}}{P_{\text{noise}}}$$ \hspace{1cm} (12)

where $P_{\text{signal}}$ is signal power, and $P_{\text{noise}}$ is noise power.

### A. Location distribution of estimation results

In this section, we mainly compare with LFO algorithm with $\xi = -1.8$KHz, so as to analyze the position distribution and resolution advantages of our algorithm estimation results. We consider single target scenario.

As shown in Fig.2(a), LFO-FDA uses logarithmically increasing frequency offset to achieve a beam orientation pattern with a maximum value at the target position. However, the beampattern based LFO-FDA method can only roughly determine the current position, and its accuracy depends on the step size of the beampattern. As mentioned above, our proposed method is equivalent to a further fine-grained estimation based on beampattern, and therefore, compared to the LFO-FDA method, our method has better positioning performance and can accurately differentiate target locations. Fig.2(b) shows the results of 200 operations. The red mark is the average value of the operations, whose value is $(499.83\text{km}, -15.1115^\circ)$. It can be found that our algorithm gives a more focused and accurate estimate of the distribution of results. The result of each operation can be close to the target, and the average result is consistent with the target position.

### B. Estimation performance analysis

As mentioned earlier, distance and angle estimates of multiple targets by FDA radar can be viewed as frequency estimates of signals containing multiple spectral components. Root-MUSIC method[12] and ESPRIT method[13] are two of the most classical and commonly used methods for frequency estimation of multiple spectral components, which are widely used in radar signal estimation. Therefore, in this chapter, we will compare with the Root-MUSIC method and the ESPRIT method to analyze the performance of our algorithm. We choose bias as the evaluation index of algorithm estimation accuracy, which is defined as follows:

$$\text{Bias} = \frac{|\hat{a} - a|}{|a|} \times 100\%$$ \hspace{1cm} (13)

Firstly, the distance parameters of the two targets are determined, and the performance of the three algorithms is tested by changing the angle difference between the two targets. The second target is located at $(500\text{km}, -15^\circ + \Delta\theta)$. From $-6.5^\circ$ to $6.5^\circ$, with $0.2^\circ$ as the step change of $\Delta\theta$ value, the estimation results of the three algorithms are shown in Fig.3. It can be seen that the Root-MUSIC method and ESPRIT method can not estimate the target location well in the case of single snapshot. In the single target scenario, $|\Delta\theta| > 4^\circ$, the Bias ($R$) and Bias ($\theta$) of the algorithm are 0.4 and 15, respectively. If converted to actual distance and angle, the accuracy is
about 2 km and 2.25. If $|\Delta \theta| < 4^\circ$, when the relative position difference between two targets becomes smaller and smaller, the estimation performance of the algorithm will deteriorate. When $|\Delta \theta| = 0.1^\circ$, the Bias ($R$) and Bias ($\theta$) of the algorithm are 4.5 and 100, respectively. If converted to actual distance and angle, the accuracy is about 22.5 km and 15.

IV. Conclusion

FDA can generate range-dependent and time-varying transmitting beams, overcoming the disadvantages of traditional phased arrays that can only generate angle-dependent beams. However, the beam pattern related to the range angle will affect the target detection and resolution performance. Based on the simulation results and analysis content, in the ULA-FDA radar scenario, the distance and angle estimation method of FDA radar targets proposed in this paper based on DFT can well estimate the distance and angle of targets, complete the parameter estimation task quickly and accurately, and effectively improve the target resolution.

References
