Comments on Transient Magnetic Shielding of a Planar Conductive Thin Screen via Exact Image Theory?

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Abstract

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Index Terms—electromagnetic (EM) shielding, time-domain (TD) analysis, transient EM fields.

I. INTRODUCTION

The transient electromagnetic (EM) shielding effectiveness of a homogeneous thin sheet with (dispersion-free) conductive properties has been recently analyzed in [1] using a sophisticated joint-transform technique relying on space-time Fourier transforms. The main result of [1] is a time-domain (TD) analytical expression for the normal component (with respect to the planar layer) of the magnetic-field strength [1, Eq. (50)] that is subsequently employed to evaluate the TD shielding performance of the thin sheet.

It is next demonstrated that (a corrected version of) this result can be readily deduced from several previously published papers on the subject (not referenced in [1]). Indeed, the pulsed EM signal transmission via a conductive sheet is described in [2], where the plasmonic behavior of conduction electrons in the metal has been additionally accounted for. Closely relevant results with a focus on the close-range EM signal transfer via a conductive layer can also be found in [3] and [4, Sec. 16.1]. Furthermore, the TD transmission characteristics of a combined magneto-dielectric layer are described in detail in [5] (see also [6]).

II. PROBLEM DESCRIPTION AND ITS SOLUTIONS

The problem configuration (see Fig. 1) under consideration is essentially identical to the one analyzed in [1], [2], [3], for instance. It consists of a horizontal transmitting loop, $\mathcal{L}^T$, and a receiving loop, $\mathcal{L}^R$, that are separated by a thin highly conducting sheet of thickness $\delta$ and conductivity $\sigma$. The sheet can be characterized by its layer conductance ratio $Y_e = G^E/Y_0$ [4, Eq. (3.18)], where $G^E = \delta\sigma$ and $Y_0 = (\varepsilon_0/\mu_0)^{1/2}$ represent the layer’s conductance and wave admittance, respectively.

The time differentiation is denoted by $\partial_t$. The time convolution operator is represented by $\ast$. Consequently, the time integration operator can be expressed as $\partial_t^{-1}(f(t)) = f(t) \ast H(t) = \int_0^t f(\tau) H(t - \tau) d\tau$, where $H(t)$ denotes the Heaviside unit-step function.

The transient voltage induced in the receiving loop has been expressed as [2, Eq. (29)], [3, Eq. (1)], [4, Eq. (16.1)], [6, Eq. (2)]

$$V_R(t) = -\frac{A^T A^R}{2\pi Y_0} \frac{\partial_t^3 I^T(t)}{\eta_e + 2\tau/T} H(t - T),$$

(1)

where $T = Z/c_0$ with $Z = h^R + h^T > 0$ and $A^T A^R$ denotes the surface area of the transmitting ($T$) and receiving ($R$) loops, respectively. With the aid of Faraday’s law applying to the small receiving loop, $V_R(t) = -\mu_0 A^R \partial_t H_Z(0, -h^R, t)$, the (normal component of the) magnetic field follows at once

$$H_Z(0, -h^R, t) = \frac{\mu_0}{2\pi h^R} \frac{\ell}{Z^3} \hat{V}^T(s) \frac{\ell}{\eta_e + 2\tau/T} H(t - T).$$

(2)

To demonstrate the relation of (2) to (a correction of) [1, Eq. (50)], we shall introduce the magnetic-dipole, causal voltage excitation via $V^T(t) = \mu_0 A^T \partial_t H_Z(0, -h^R, t)$ (see [1, Eq. (1)]) and get

$$H_Z(0, -h^R, t) = \frac{\mu_0}{2\pi h^R} \frac{\ell}{Z^3} \hat{V}^T(s) \frac{\ell}{\eta_e + 2\tau/T} H(t - T).$$

(3)

In contrast to [1, Eq. (50)], (3) is expressed through a single time-convolution integral. Relying on Lerch’s uniqueness theorem [7, Appendix] that applies to $\{s \in \mathbb{R}; s > 0\}$, the TD expression (3) can be uniquely determined via its Laplace-transform counterpart

$$H_Z(0, -h^R, s) = \frac{\mu_0}{2\pi h^R} \frac{\ell}{Z^3} \hat{V}^T(s) \int_0^\infty \exp(-s\tau) F(\tau) d\tau,$$

where we use, for brevity, $F(\tau) = \tau (\tau^2 - T^2) / (\eta_e + 2\tau/T)$. Using integration by parts, $\int_0^\infty \exp(-s\tau) F(\tau) d\tau = s^{-2} F^{(1)}(T) \exp(-sT) + s^{-2} \int_0^\infty \exp(-s\tau) F^{(2)}(\tau) d\tau$, it is straightforward to rewrite (4) as

$$\dot{H}_Z(0, -h^R, s) = \hat{H}_Z(0, -h^R, s)$$

$$- \frac{Y_0}{2\pi} \frac{\ell}{Z^3} \hat{V}^T(s) \exp(-sT)$$

$$+ \frac{2\eta_e(1 - \eta_e^2/4)}{\pi\mu_0} \frac{\ell}{Z^3} \hat{V}^T(s) \int_0^\infty \exp(-s\tau) d\tau.$$

(5)
where \( \hat{H}_z^i \) denotes the magnetic field in the absence of the sheet

\[
\hat{H}_z^i(0, -h^R, s) = \frac{Y_0}{2 \pi Z^2} \frac{\ell}{T} \frac{e^{-sT}}{s} \exp(-sT) + \frac{1}{2 \pi \mu_0 Z^3} \frac{\ell}{s} \exp(-sT). 
\]

(6)

Upon transforming the result back to TD, we arrive at

\[
H_z(0, -h^R, t) = \frac{1}{2 \pi \mu_0 Z^2} \frac{\ell}{T} \frac{e^{-sT}}{s} \exp(-sT) + \frac{1}{\pi \eta_\text{E} + 2 Z^2} V^T(t - T) + \frac{2 \eta_\text{E}(1 - \eta_\text{E}^2/4)}{\pi \mu_0} \frac{\ell}{Z^3} V^T(t) * \frac{\hat{H}(t - T)}{(\eta_\text{E} + 2T/T)^3},
\]

(7)

where the time convolution integral can be explicitly expressed as

\[
V^T(t) * \frac{\hat{H}(t - T)}{(\eta_\text{E} + 2T/T)^3} = \int_{\tau = T}^{t} V^T(t - \tau) \frac{d\tau}{(\eta_\text{E} + 2\tau/T)^3} = \int_{\tau = 0}^{t - T} V^T(\tau) \frac{d\tau}{(\eta_\text{E} + 2(t - \tau)/T)^3}. 
\]

(8)

Using the latter form (multiplied by \( T^3/T^3 \)) in (7) we end up with the desired TD expression (cf., [1, Eq. (50)])

\[
H_z(0, -h^R, t) = \frac{1}{2 \pi \mu_0 Z^2} \frac{\ell}{T} \frac{e^{-sT}}{s} \exp(-sT) + \frac{1}{\pi \eta_\text{E} + 2 Z^2} V^T(t - T) + \frac{2 \eta_\text{E}(1 - \eta_\text{E}^2/4)}{\pi \mu_0} \frac{\ell}{c_0^3} \int_{\tau = 0}^{t - T} V^T(\tau) \frac{d\tau}{(\eta_\text{E}T + 2(t - \tau))^3}. 
\]

(9)

The correspondence with [1, Eq. (50)] can be seen by using \( \eta_\text{E} \equiv \hat{G}_s \), \( Y_0 \equiv 1/\mu_0 \), \( Z \equiv Z \), \( V^T \equiv v \) and \( \hat{\partial}_t^{-1} V^T \equiv \hat{V} \) (see [1, Eq. (46)]). Unfortunately, the convolution integral in [1, Eq. (50)] is lacking the \( 1/c_0^3 \) factor. The inconsistency that occurs also in other equations (e.g., [1, Eq. (43)]) can be revealed by taking the limit \( \hat{G}_s \to \infty \) in [1, Eq. (50)]. Under the limit, the second term on the right-hand side of [1, Eq. (50)] is vanishingly small. Consequently, to get the zero field below the perfectly conducting wall, the first term of [1, Eq. (50)] must cancel the third one as \( \hat{G}_s \to \infty \). Because of the omission of \( 1/c_0^3 \) in [1, Eq. (50)], that is apparently not the case.

REFERENCES


