Motion Planners for Path or Waypoint Following and End-Effector Sway Damping with Dynamic Programming

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Abstract

Abstract: We propose two novel motion planners for a robotic manipulator with a passive end-effector that is free to sway during and after the robot’s motion. The planners utilize Dynamic Programming to generate trajectories that damp the end-effector’s residual sway while ensuring that the boom tip—the point to which the end-effector is attached—follows a collision-free path or time-dependent waypoints.

Our use case is a crane of a forwarder machine, a log-loading machine in the forestry industry, with a passive grapple. In the cluttered forest environment, accurate path following and grapple sway damping are critical to increase the operation efficiency and avoid harming the machine and environment.

The results of the simulation in a high-fidelity multibody-dynamics simulator showcase the effectiveness of our methodology in achieving exact path following or timed waypoints following and the residual sway damping. In particular, an average reduction of 75% in the residual sway is demonstrated, as compared to fifth, sixth, and tenth order polynomial trajectories, in six test cases, including common paths used by operators to pick and place logs.

Other merits of our Dynamic Programming trajectories are that they are smooth, computationally inexpensive, and result in reduced residual sway even for nonzero initial sway conditions.

Moreover, the generality of our methodology opens a new way to design anti-sway motion planners for construction cranes or quadrotors with a slung payload, in addition to serial manipulators with passive end-effectors.

Note to Practitioners: This work was motivated by the problems arising in the operation of log-loading cranes in the forestry industry: the problems of the end-effector’s large sway during crane reconfiguration and the collision between the crane and obstacles, which are detrimental to the efficiency of the operation. Similar issues arise, for example, in construction cranes transporting large hanging objects.

We propose a novel methodology to address both problems by generating smooth and computationally inexpensive trajectories for joint motion of the crane. The approach begins with the model of the sway motion and the definition of the collision-free path. Then, our Dynamic Programming algorithm generates anti-sway trajectories that satisfy the joint constraints. The results in a high-fidelity simulator show that our motion planners lead to precise path following and significant sway damping, and also confirm its superiority compared to polynomial trajectories, commonly used in industries.

Our methodology is also applicable to other dynamic systems with freely hanging objects, such as multi-degree-of-freedom robotic manipulators, construction cranes, and quadrotors carrying a slung payload. A possible limitation is that the methodology necessitates finding the sway dynamics model and payload properties of these systems.
**I. Introduction**

A. Background and Motivation

Forestry is a very important industry in the resource sectors of many countries, including Canada, Sweden, New Zealand and others. Despite this, the industry is severely lagging in the introduction of robotics, autonomy and AI-enabled technologies, when compared to its peer resource industries [1], such as mining [2] and agriculture [3]. The urgency to develop autonomy and intelligence of timber-harvesting machines, in particular, has become evident in the past decade because of the shortages of experienced operators in the industry [4], [5].

Operating forestry machines safely and efficiently requires extensive training for human operators to become proficient. The operators are also subjected to significant mental and physical strain in the challenging forest environment [6]–[8]. A typical forestry machine is effectively a mobile robot as it consists of a mobile base equipped with a crane-like manipulator with a specialized end-effector. Manipulating these large hydraulic cranes is a highly complex task, with studies indicating that operators spend most of their time (more than 80%) in the cabin solely focused on the crane manipulation [9]. Another study showed that a majority (73%) of crane accidents are attributed to operator error [10]. Consequently, even partial automation of forestry machines has a great potential for alleviating the workload and enhancing the safety of human operators in the forestry industry.

Researchers have been working to increase the autonomy of different types of forestry machines, such as feller bunchers, harvesters, and forwarders [11]–[15]. The forwarder machine is responsible for loading logs after they are cut in the forest onto its basket and transporting them to the roadside. Its essential components are a cabin, where the human operator is situated, a basket or trailer for log storage, a crane (or boom) with four actuated degrees of freedom (DOF), and a grapple attached to the tip of the crane (boom tip) via two passive DOF. The passive joints ensure the nominal vertical alignment of the grapple, facilitating the loading and unloading of logs. However, these passive joints cannot be directly controlled or commanded, resulting in undesired sway or swinging motion of the grapple during and after crane manipulation. Due to the heavy mass of the grapple, especially when it carries logs, its sway can be dangerous, potentially hitting trees in the vicinity of the machine or the cabin itself, thereby causing damage to the environment, machine, and even the operator. Therefore, accommodating this swaying motion of the grapple imposes an additional burden on the operator. In particular, the residual sway, which refers to the EE’s swinging motion that persists after the crane maneuver, adversely affects productivity, as the operator may be required to wait for the

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sway to diminish before proceeding with the next task in the operation. Furthermore, it may compromise other autonomy-related objectives, such as, environment perception with a crane-mounted camera [16], [17]. A trivial remedy for the sway problem is to manipulate the crane slowly; however, this is not a desirable solution since it also reduces the productivity of the machine.

Another important operational factor in the cluttered forestry environment is that the crane has to avoid obstacles while maneuvering, such as, trees, rocks, and the machine itself (the cabin and trailer), both for efficient operation as well as to prevent damage to the machine, the environment and the operator. Dealing with the obstacle avoidance issue presents another hindrance to the operator, increasing their mental and physical load. In a standard autonomy pipeline, a path planner would be used to generate an obstacle-free path, and the generated path should be followed accurately to ensure obstacle avoidance.

In this paper, we tackle both aforementioned problems by proposing motion planners which enable the forwarder crane to follow pre-defined collision-free paths or waypoints while at the same time, minimizing the EE’s residual sway for fast maneuvers.

B. State of the Art

1) Anti-Sway Approaches: The importance of anti-sway design approaches is not limited to forestry industry. Reducing the oscillation of a hanging object can be critical for construction cranes (like tower cranes and gantry cranes), quadrotors carrying a slung payload, or a robotic arm with a hanging payload. Researchers have employed different anti-sway approaches for trajectory design as well as at the controller design levels. Popular approaches include feedback control methods [18]–[20], parameter optimization using basis functions [21], input-shaped trajectories by impulse convolution [22]–[24], and dynamic programming which is based on Bellman’s principle of optimality [25]–[28].

Feedback control methods, in spite of the merits of feedback, have higher computational costs [20]. A drawback with the aforementioned parameter optimization method is the challenge of finding a global minimum of the objective function which may be non-convex [29]. The problem with the impulse convolution method is that it produces trajectories that can be non-smooth to the point of being unusable [29].

We select the Dynamic Programming (DP) approach [30]–[32] in this work. DP has been employed in different fields like reinforcement learning [33], control systems [34], [35], and optimization [36], [37]. Here, we utilize it to generate the anti-sway trajectories for the crane of the forwarder machine. Since the approach is used as an open-loop scheme, it is computationally less expensive compared to some closed-loop methods, such as the model predictive control (MPC). Moreover, it does not suffer from local minima or non-smooth trajectories, unlike the parameter optimization and input shaping methods. A standard computationally inexpensive DP algorithm can be applied to a discrete linear system; however, suitable linearization is required for nonlinear systems. We linearize the EE sway dynamics about the nominal trajectory (LNT), based on the demonstrated superiority of this linearization scheme compared to those about current states and inputs (LCSI) [28].

Another relevant observation about passive (hanging) payloads or EEs is that they are likely to have a non-zero initial sway at the beginning of the crane’s planned motion. Therefore, we require the anti-sway approach to also dampen the sway in these conditions, referred to here as non-rest-to-rest (NR2R) conditions, in addition to rest-to-rest (R2R) conditions. The authors of [38] and [39] designed NR2R trajectories by re-parameterizing the input shapers. We show that the DP approach adopted in our work can generate NR2R trajectories without any algorithmic changes nor additional computation cost, both further advantages of the proposed method.

2) Path Following Approaches: Path planning and path following are important and well-known subjects in robotics. Different methods for path planning, like potential field [40], [41], probabilistic roadmap [42], [43], and machine learning-based path planners [44], [45], as well as, various methods for path following, like arc-length parameterization [46], [47], waypoints following [48], and reinforcement learning [49], [50] have been used in the literature.

As noted earlier, it is important to follow the collision-free paths precisely in the harsh forestry environment. In this work, we assume that such paths have been generated, and we parameterize them with the arc-length parameter in order to follow them exactly, without error. As well, we consider a scenario where several intermediate waypoints need to be reached at certain times, called time-dependent waypoints, and design a planner for this purpose.

3) Forestry Machines Literature: In the context of forestry machines and operations, very few papers have addressed the sway problem while ensuring path following. The authors of [51] considered movement of the forwarder’s first (swelling) joint, and in [52], movement of second to fourth joints, which produce planer boom motions, were considered to design the anti-sway controllers. Therefore, in both cases, the sway was caused by the rotation of only one of the two passive joints supporting the end-effector. However, in a real scenario, all four crane joints contribute to the boom tip’s motion in 3D, which in turn causes the EE to sway in two directions. The anti-sway controllers in these papers were designed for point-to-point (P2P) movements. In [20], a nonlinear MPC was designed to follow pre-defined paths and to reduce the sway of a simplified grapple model, consisting of a rod and a connected disk-shaped object at its end, instead of the realistic model. Path tracking was achieved by including in the objective function the error of the boom tip position with respect to a reference. Thus, sway was reduced, and the paths were followed approximately, with non-zero error. In our previous work [28], anti-sway trajectories were designed for all four crane joints by employing the DP approach and with a realistic model of the grapple. However, the design was limited to the P2P maneuvers.
other contributions of this paper are as follows:

C. Contributions

In this paper, we propose two computationally inexpensive motion planners which generate anti-sway trajectories for the crane of a log-loading machine such that the boom tip follows either a prescribed collision-free path or timed waypoints. The other contributions of this paper are as follows:

- We tackle both problems of the EE sway and collision avoidance, jointly at the trajectory generation level, which has not been addressed previously for timber-harvesting machines.
- We incorporate an exact path follower into the anti-sway DP algorithm for the first time in the context of dynamic programming.
- We propose a new DP formulation, compared to the previous DP anti-sway solutions, for timed waypoint following.
- We account for crane joint constraints in the DP framework, not considered in previous DP anti-sway papers, by incorporating the inverse kinematics solver in our algorithm.
- We demonstrate that our motion planners can damp the residual sway and follow the desired path or timed waypoints for non-zero initial sway conditions as well at no additional cost.
- We validate the performance of our motion planners on the virtual model of the forwarder machine, by using a high-fidelity multibody-dynamics simulator.
- The proposed methodology for motion planning represents a general approach, applicable to any multi-DOF serial manipulators with a passive EE, as well as, cranes or quadrotors with a hanging object.

II. MODEL OF FORWARDER CRANE AND GRAPPLE

The forwarder crane, as depicted in Fig. 1, is a serial robotic arm possessing three revolute and one prismatic actuated joints. Two passive joints connect the EE—the grapple—to the boom. The sway of the EE is predominantly influenced by the movement of the boom tip which in turn, is directly affected by the motion of the preceding joints. Our forwarder model’s specifications and attributes are derived from the Tigercat 1075C forwarder.

1) Crane Kinematics: Employing the convention used in [53], the kinematics of the crane is defined by the frames assignment and DH parameters, as shown in Fig. 1. The boom tip Cartesian coordinates in the global frame ($F_0$) are as follows:

$$
 x_{bt} = \cos \theta_1 (-a_1 + a_2 \cos \theta_2 - a_3 \sin \theta_2 + \theta_3)
 + (d_4 + d_{4,c}) \cos (\theta_2 + \theta_3)
$$

$$
 y_{bt} = \sin \theta_1 (-a_1 + a_2 \cos \theta_2 - a_3 \sin \theta_2 + \theta_3)
 + (d_4 + d_{4,c}) \cos (\theta_2 + \theta_3)
$$

$$
 z_{bt} = d_1 + a_2 \sin \theta_2 + a_3 \cos \theta_2 + \theta_3
 + (d_4 + d_{4,c}) \sin (\theta_2 + \theta_3)
$$

2) Grapple Kinematics and Dynamics: A dynamics model of the EE sway is required in order to develop a sway-damping solution. The model explained here is based on the model proposed in [28]; it is formulated to define the motion of joints 5 and 6 as a function of the motion of the boom tip. Thus, the assembly attached to the boom tip is modeled with two revolute joints, corresponding two links and point masses positioned on these links, to represent the mass distribution of the different components, as shown in Fig. 2. The first point mass ($m_0$) is placed at the center of mass (COM) of the link connecting the upper passive joint (joint 5) to the lower passive joint (joint 6), and the second point mass ($m$) is at the COM of the remaining parts, including the next link, the rotator, claws, cylinder and piston. We introduce an auxiliary frame $F_{C1}$ with its origin at the boom tip, its $z$-axis directed upward parallel to the $z_0$-axis, and its $y$-axis aligned with the axis of the upper passive joint. Therefore, this frame rotates with frame $F_1$, i.e., through angle $\theta_1$ about the $z_0$-axis. We define reference frames $F_{G5}$ and $F_{G6}$ at the upper and lower passive joints attached to the corresponding links, respectively (see Fig. 2). The angle $\theta_5$ represents the rotation between $F_{C1}$ and $F_{G5}$ about $y$-axis, while $\theta_6$ represents the rotation between $F_{G5}$ and $F_{G6}$ about $x$-axis.

The global cartesian position of mass $m$ expressed in $F_0$ with the boom tip coordinates as per (1)-(3) is given by:
\[ x_m = x_{bt} + (x_{m/bt})_{F_{c1}} \cos \theta_1 - (y_{m/bt})_{F_{c1}} \sin \theta_1 \]
\[ y_m = y_{bt} + (x_{m/bt})_{F_{c1}} \sin \theta_1 + (y_{m/bt})_{F_{c1}} \cos \theta_1 \]
\[ z_m = z_{bt} + (z_{m/bt})_{F_{c1}} \]

where the position of \( m \) relative to the boom tip expressed in \( F_{c1} \) is:
\[ (x_{m/bt})_{F_{c1}} = l_1 \sin \theta_5 \]
\[ (y_{m/bt})_{F_{c1}} = l_2 \sin \theta_6 \]
\[ (z_{m/bt})_{F_{c1}} = -l_3 \cos \theta_5 \]

The lengths \( l_1 \) and \( l_2 \) are shown in Fig. 2, and \( l_3 \) is equal to \( l_1 + l_2 \cos \theta_6 \). The global position of \( m_0 \) can be found in a similar manner.

Employing the aforementioned information, the kinetic \((T)\) and potential \((V)\) energies of the grapple can be determined. Then, the dynamics equations can be found by following the Lagrangian approach:
\[ L = T - V \]
\[ \tau_i = \frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}_i} \quad i = 5, 6 \]

where \( \tau_i \) is the torque at joint \( i \), which for the two passive joints supporting the grapple is the damping torque at the joint, modeled as viscous friction with damping coefficient \( b_i \). The Lagrangian approach yields the grapple’s sway dynamics equations:
\[ -b_i \dot{\theta}_i = (m_0 l_0 + m_3) (\cos \theta_5 \cos \theta_1 \ddot{x}_{bt} + \cos \theta_5 \sin \theta_1 \dot{y}_{bt} + \sin \theta_5 (\ddot{z}_{bt} + g)) + m_0 l_0 (l_0 \ddot{\theta}_5 - l_0 \sin \theta_5 \dot{\theta}_5^2) + m_3 l_3 \ddot{\theta}_5 - 2l_2 \sin \theta_5 \dot{\theta}_5 \dot{\theta}_6 - l_2 \cos \theta_5 \sin \theta_5 \dot{\theta}_6 \]
\[ -b_6 \dot{\theta}_6 = m_2 l_2 (\ddot{x}_{bt} - \cos \theta_6 \sin \theta_1 - \sin \theta_5 \sin \theta_6 \cos \theta_1) + \dot{y}_{bt} (\cos \theta_6 \cos \theta_1 - \sin \theta_5 \sin \theta_6 \sin \theta_1) + l_2 \ddot{\theta}_6 + \cos \theta_5 \sin \theta_6 (\ddot{z}_{bt} + g) + \sin \theta_5 (l_1 \dot{\theta}_6 - l_2 \dot{\theta}_1) + \sin \theta_6 (l_3 - l_3 \cos^2 \theta_5) \theta_1^2 + l_3 \sin \theta_6 \theta_5^2 + 2l_3 \cos \theta_5 \cos \theta_6 \dot{\theta}_1 \dot{\theta}_6 \]

The accelerations \( \ddot{\theta}_5 \) and \( \ddot{\theta}_6 \) can be extracted from (12) and (13) as a function of other terms. Moreover, since our goal for the DP framework is to generate the trajectories for the boom tip, we substitute for \( \ddot{\theta}_1 \) and \( \ddot{\theta}_1 \) with the terms of the boom tip position, velocity, and acceleration by using \( \dot{\theta}_1 = \tan^{-1}(y_{bt}/x_{bt}) \). The grapple properties of the Tigercat 1075C forwarder are summarized in Table I.

### III. Dynamic Programming Framework

Dynamic Programming (DP) is an optimization algorithm that has its origin in Bellman’s “principle of optimality”. Based on this principle, if a series of decisions (policy) is optimal, then from any point in the series, the remaining decisions must constitute an optimal set (policy). Therefore, a series of optimal values can be calculated recursively backward, starting from the end state, which is beneficial for the residual sway-free goal.

#### A. DP Algorithm

First, we elucidate how DP solves a general quadratic optimization problem for a discrete piecewise linear system [30], [31]. The optimization problem structure is as follows:
\[ \min_{(u_1, \ldots, u_N)} \Gamma(x, u) \]

with the objective function expanded as
\[ \Gamma(x, u) = \sum_{k=1}^{N} \Gamma_k(x_k, u_k) \]

\[ \Gamma_k = \eta_k + x_k^T y_k + u_k^T z_k + \frac{1}{2} x_k^T Q_k x_k + 2 x_k^T R_k u_k + u_k^T S_k u_k \]

subject to
\[ x_{k+1} = A_k x_k + B_k u_k, \]
\[ x_1 = x_{in}, \]

where \( x_k, u_k, \) and \( x_{in} \) are the \( k \)-th states, \( k \)-th inputs, and initial states, respectively, and \( \Gamma_k \) is a quadratic function with \( \eta_k, y_k, z_k, Q_k, R_k \) and \( S_k \) as time-varying coefficients.

The optimal value function, which is the key ingredient of DP method, is defined in (19) as:
\[ \Lambda_i = \min_{u_i} \sum_{k=1}^{N} \Gamma_k(x_k, u_k). \]

The optimal value function is also quadratic in form, as stated in (20), because the objective function was quadratic:
\[ \Lambda_i(x_i) = \zeta_i + x_i^T v_i + \frac{1}{2} x_i^T W_i x_i, \]

where \( \zeta_i, v_i, W_i \) are coefficients to be determined as follows. Employing Bellman’s principle of optimality, the backward recursive relation is formed as:
\[ \Lambda_i = \min_{u_i} (\Gamma_i + \Lambda_{i+1}) \]

Substituting (16) and (20) into (21) and using (17) leads to:
\[ \zeta_i + x_i^T v_i + \frac{1}{2} x_i^T W_i x_i = \min_{u_i} \left\{ \zeta_{i+1} + y_i + x_i^T h_{4i} + u_i^T h_{5i} \right\} + \frac{1}{2} x_i^T H_{1i} x_i + 2 x_i^T H_{2i} u_i + u_i^T H_{3i} u_i \]

where
\[ H_{4i} = Q_i + A_i^T W_{i+1} A_i, \quad H_{2i} = R_i + A_i^T W_{i+1} B_i \]
\[ H_{3i} = S_i + B_i^T W_{i+1} B_i, \quad h_{4i} = y_i + A_i^T v_{i+1} \]
\[ h_{5i} = z_i + B_i^T v_{i+1} \]

By taking the derivative of the right-hand side of (22) with respect to \( u_i \) and setting it equal to zero, the optimal input is determined:

<table>
<thead>
<tr>
<th>Table I: The grapple’s properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l_0(m) )</td>
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<tr>
<td>0.14</td>
</tr>
</tbody>
</table>
\[ u^*_i = -H_{3i}^{-1}[H_{2i}^T x_i + h_{5i}] \]  

By replacing the optimal input \( u^*_i \) into (22) and aligning terms with the identical degree in \( x_i \), a set of recursive equations is derived:

\[
\begin{align*}
\zeta_t &= \zeta_{t+1} + \eta_t - \frac{1}{2}H_{5i}^T H_{5i}^{-1} h_{5i} \\
v_t &= h_{4i} - H_{3i}^T H_{5i}^{-1} h_{5i} \\
W_t &= H_{1i} - H_{2i}^T H_{5i}^{-1} H_{2i}^T
\end{align*}
\]  

(25)

where the values at the terminal (\( N \)-th) stage to initiate the backward recursion above are given by:

\[
\zeta_N = \eta_N \, , \, v_N = y_N \, , \, W_N = Q_N
\]  

(26)

Thus, the algorithm involves first evaluating the backward recursion, as per (25), followed by the forward recursion using (24) and (17) to compute the series of optimal input.

B. Path Following and Sway Damping in DP Framework

We now apply the previously presented basic DP framework to allow for the boom tip of the forwarder crane to follow a desired path in space, as well as, to dampen the residual sway of the grapple. The former objective is critical for obstacle avoidance which is typically enforced at the path-planning phase. To this end, we parameterize a given path by using the arc length parameter, denoted with \( s \in [0, L] \), where \( L \) is the length of the path, so that the cartesian coordinates of any point along the path can be expressed as a function of \( s \). Since the path must be followed by the boom tip, we write:

\[
\begin{align*}
x_{bt} &= f_x(s) \\
y_{bt} &= f_y(s) \\
z_{bt} &= f_z(s)
\end{align*}
\]  

(27)

where \( f_x, f_y, \) and \( f_z \) are functions of \( s \). Substituting (27) into (12) and (13), the sway dynamics equations can be re-written in terms of \( s \) and its derivatives. Therefore, the DP states and inputs to be used in the DP solution of the path following and sway damping problems are chosen as follows:

\[
\begin{align*}
x &= [s \, \dot{s} \, \theta_5 \, \dot{\theta}_5 \, \theta_6 \, \dot{\theta}_6]^T \\
u &= \dot{s}
\end{align*}
\]  

(28)\hspace{1cm}(29)

The desired final states, considering the residual sway-free goal, and the initial states are defined in (30) and (31), respectively as:

\[
\begin{align*}
x_f &= [L \, 0 \, 0 \, 0 \, 0 \, 0]^T \\
x_{in} &= [0 \, 0 \, \theta_{3in} \, \dot{\theta}_{3in} \, \theta_{6in} \, \dot{\theta}_{6in}]^T
\end{align*}
\]  

(30)\hspace{1cm}(31)

where the last four states in (31) are the initial sway conditions, which are set to zero in the absence of initial sway (R2R conditions).

We define an objective function such that it enforces a minimum-effort condition and the desired final condition:

\[
\Gamma = \left\{ \sum_{k=1}^{N} \frac{1}{2}u_k^T u_k \right\} + \frac{1}{2}(x_N - x_f)^T P_N (x_N - x_f)
\]  

(32)

where \( P \) is a 6x6 diagonal matrix for the penalty weight. We point out that our objective function does not include a path tracking error term, because the path following constraint is implicitly satisfied by formulating the sway dynamics in terms of the path parameter. Comparing (32) with (15) and (16), we deduce:

\[
\eta_N = \frac{1}{2}s_f^T P x_f \, , \, y_N = -P x_f \, , \, Q_N = P \, , \, S_k = 1
\]  

(33)

and the other coefficients of (16) are zero.

With the above state, input, and objective function definitions, the DP algorithm can be used to compute anti-sway trajectories for \( s \), thereby generating anti-sway trajectories for the boom tip and, through inverse kinematics, the anti-sway trajectories for the crane joints. In the following, we briefly explain how to parameterize different paths.

1) Arc-Length Parameterization: Since many paths can be decomposed into sequential circular and straight-line segments, here we demonstrate how to parameterize these two segments in Fig. 3 and Eq. (34) and (35):

- circular arc : \( p_0 + v_1 r (\cos \frac{s}{r} - 1) + v_2 r \sin \frac{s}{r} \)  
- line : \( p_0 + v s \)  

(34)\hspace{1cm}(35)

where \( p_0 \) is the position vector of the beginning point of the path. For the circular arc, \( v_1 \) is the unit vector from the center of curvature pointing to \( p_0 \), \( v_2 \) is perpendicular to \( v_1 \) pointing in the direction of motion, and \( r \) is the radius of curvature. For linear segments, \( v \) is the unit vector along the path, in the direction of motion.

C. Waypoint Following and Sway Damping in DP Framework

Instead of following a prescribed path, it may be desirable to have the boom tip of the crane to pass through a number of pre-specified waypoints at certain times, for instance, \( (x_{bt_{da}}, y_{bt_{da}}, z_{bt_{da}}) \) as the desired boom tip position at \( t_k \). We propose a new objective function to incorporate this goal into the DP framework by including a term for the time-dependent waypoint following. Thus, we define the DP states, DP inputs, and DP objective function as follows:

\[
\begin{align*}
x &= [x_{bt} \, \dot{x}_{bt} \, y_{bt} \, \dot{y}_{bt} \, z_{bt} \, \dot{z}_{bt} \, \theta_5 \, \dot{\theta}_5 \, \theta_6 \, \dot{\theta}_6]^T \\
u &= [\dot{x}_{bt} \, \dot{y}_{bt} \, \dot{z}_{bt}]^T
\end{align*}
\]  

(36)\hspace{1cm}(37)

\[
\Gamma = \left\{ \sum_{k=1}^{A} \frac{1}{2}u_k^T u_k \right\} + \left\{ \sum_{k=1}^{W} \frac{1}{2}(x_k - x_{da})^T P_k (x_k - x_{da}) \right\} + \frac{1}{2}(x_N - x_f)^T P_N (x_N - x_f)
\]  

(38)
The discrete linearized system, employing (45) and (46), is:

\[ \begin{align*}
\dot{x}_{k+1} &= A_k \delta x_k + B_k \delta u_k \\
\end{align*} \tag{48} \]

where \( \delta x_k \) and \( \delta u_k \) are the nominal states and inputs at \( t_k \), and:

\[ \begin{align*}
\delta x_k &= x_k - \bar{x}_k, \\
\delta u_k &= u_k - \bar{u}_k
\end{align*} \tag{49} \]

\[ \begin{align*}
A_k &= I + h \frac{\partial f}{\partial x} \bigg|_{x_k, \bar{u}_k}, \\
B_k &= h \frac{\partial f}{\partial u} \bigg|_{x_k, \bar{u}_k}
\end{align*} \tag{50} \]

The DP framework is applied to (48) to find \( \delta u_k \). Then, the optimal input for the real system is obtained by:

\[ u_k = \delta u_k + \bar{u}_k \tag{51} \]

In addition, the initial states, desired final states, and desired 0-th states used in the DP framework are:

\[ \begin{align*}
\delta x_{in} &= x_{in} - \bar{x}_1, \\
\delta x_f &= x_f - x_N, \\
\delta x_{dx} &= x_{dx} - \bar{x}_k
\end{align*} \tag{52, 53, 54} \]

IV. INVERSE KINEMATICS

As discussed earlier, our DP framework generates anti-sway trajectories for the boom tip. Since the forwarder arm is redundant with respect to positioning the boom tip, the corresponding inverse kinematics problem has an infinite number of solutions. Various existing methods available for redundant manipulators can provide an inverse kinematics solution for crane joint trajectories while minimizing a certain cost function. Here, we employ a locally optimal inverse kinematics method, which is computationally less expensive than globally optimal methods, to minimize the \( L_2 \)-norm of the crane joint velocities and to satisfy the joint velocity constraints. The following optimization problem must be solved at each time step:

\[ \min_{\dot{q}} \sum_{i=1}^{4} \dot{q}_i^2 \tag{55} \]

subject to

\[ \begin{align*}
\nu_{bt} &= J \dot{q}, \\
\dot{q}_{\text{min}} &\leq \dot{q} \leq \dot{q}_{\text{max}}
\end{align*} \tag{56, 57} \]

where

\[ \dot{q} = [\dot{q}_1, \dot{q}_2, \dot{q}_3, \dot{q}_4]^T = [\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3, \dot{\theta}_4]^T \tag{58} \]

\[ \nu_{bt} = [\dot{x}_{bt}, \dot{y}_{bt}, \dot{z}_{bt}]^T \tag{59} \]

and \( J \) is the Jacobian matrix of the crane. Furthermore, \( \dot{q}_{\text{min}} \) and \( \dot{q}_{\text{max}} \) are the minimum and maximum allowable velocities of crane joint \( i \), which are given in Table II.

<table>
<thead>
<tr>
<th>Joint</th>
<th>( \dot{\theta}_1 ) (rad/s)</th>
<th>( \dot{\theta}_2 ) (rad/s)</th>
<th>( \dot{\theta}_3 ) (rad/s)</th>
<th>( \dot{d}_4 ) (m/s)</th>
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</thead>
<tbody>
<tr>
<td>Min</td>
<td>-0.8</td>
<td>-0.45</td>
<td>-0.75</td>
<td>-0.9</td>
</tr>
<tr>
<td>Max</td>
<td>0.8</td>
<td>0.45</td>
<td>0.75</td>
<td>0.9</td>
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</table>

TABLE II: The minimum and maximum allowable velocities of the crane joints
V. SIMULATION RESULTS AND DISCUSSION

The implementation of the DP algorithm is performed in MATLAB. The numerical simulations to compute the nominal sway angles required to define the linear model in the DP methodology were carried out by using MATLAB’s ode45 solver. After solving the inverse kinematics in MATLAB, the anti-sway trajectory solutions are commanded to the virtual model of the Tigercat 1075C forwarder, created in a high-fidelity multibody-dynamics simulator, Vortex Studio, with Python scripts for interface to MATLAB, in order to validate the results. Vortex Studio is an advanced suite of real-time multibody-dynamics simulation software which has been employed for high-fidelity simulation of industrial mechanisms, vehicles, and cranes, including forestry cranes [54]. The Tigercat 1075C forwarder model in the Vortex Studio environment, used in this work, is shown in Fig. 4.

One important aspect to consider is the time step utilized to command the crane. In our virtual forwarder model, this time step is 0.0167s (equivalent to 60 Hz). In our MATLAB implementation, we employ a time step of 0.0001s to execute the DP algorithm and the inverse kinematics. Subsequently, we down-sample the points of the anti-sway crane joint trajectories to the 60 Hz rate to ensure compatibility with the virtual model. Thus, a simulation time step of 0.0167s is used for all the results, as presented next.

A schematic diagram representing our proposed methodology for both the path following and waypoint following motion planners is shown in Fig. 5. In the next sections, we begin with the results for the anti-sway path following and then for the anti-sway waypoint following, corresponding to the algorithm formulations in Sections III-B and III-C, respectively.

A. Anti-Sway Path Follower

We showcase the results for four maneuvers with their respective paths illustrated in Fig. 6, with four different initial sway conditions, as stated in Table III. The total maneuver time \( t_f \) is fixed to 6 s for all four path scenarios. The maneuver time and the path parameters are selected to yield an average path traversal speed, \( \bar{s} \), of approximately 1 m/s. The starting points for the paths are centered above the trailer, with cartesian coordinates \( [4.25, 0, 2.77] \) m for all paths, except for C-Common1, where the boom tip is initially placed deeper into the trailer, at \( [4.56, 0, 2.26] \) m. The nominal trajectories of \( s \) used for the linearization are 5-th order polynomials (5OP) satisfying the initial and final conditions for \( s \) and \( \dot{s} \) (as per Eq. (30)-(31)), and zero initial and final values for \( \ddot{s} \). We note that for all four maneuvers, the DP-generated trajectories yield exact path following of the nominal paths (red and blue paths overlap in Fig. 6.)

1) A-Circle: The first selected path is a circle with a radius of 0.75 m, placed in a plane parallel to the \( x_0 - y_0 \) plane, with zero initial sway condition. The nominal (blue) and designed DP (red) trajectories for \( s \) and \( \dot{s} \) are shown in Fig. 7. The joint velocities corresponding to the DP trajectories, shown only for this maneuver as an example, are plotted in Fig. 8. These figures confirm that our DP methodology produces smooth trajectories for the arc-length parameter, the boom tip motion, and the crane joint motions, the latter also satisfying the joint constraints. The sway angles for the nominal and DP trajectories are shown in Fig. 9.

![Fig. 4: Tigercat 1075C forwarder model in the Vortex Studio environment.](image)

![Fig. 5: Schematic diagram of our proposed methodology for the anti-sway motion planners.](image)

<table>
<thead>
<tr>
<th>( \theta_{s_{110}} ) (deg)</th>
<th>( \theta_{s_{110}} ) (deg/s)</th>
<th>( \theta_{s_{110}} ) (deg)</th>
<th>( \theta_{s_{110}} ) (deg/s)</th>
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<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>12</td>
<td>-3</td>
<td>-5</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>-10</td>
<td>23</td>
<td>10</td>
</tr>
<tr>
<td>E</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>F</td>
<td>4</td>
<td>-30</td>
<td>-9</td>
</tr>
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</table>

![Fig. 6: Boom tip paths for maneuvers A-Circle, B-Helix, C-Common1 and D-Common2. Desired paths (blue) and actual paths using DP trajectories (red) overlap.](image)
The sway angles are shown in Fig. 10 and Fig. 11, respectively, with the same legends as for the circular path. As can be seen from these figures, the smooth DP trajectories are capable of suppressing the residual sway for this complicated path, starting with the non-rest initial conditions. The highest values for $\theta_5$ and $\theta_6$ after $t_f$ when DP trajectories are utilized are 1.3° and 2.4°, respectively, representing the corresponding reduction of 86% and 72%, compared to using 5OP trajectories. Again, the boom tip follows the helical path precisely when utilizing the DP trajectories, as shown in Fig. 6-B.

3) C-Common1: The next two maneuvers were selected to represent common paths used by the operators to move the boom tip from above the trailer to above a log pile on the ground, next to the forwarder machine [15]. The first path considered starts from a slightly lower height, to represent the boom tip location deeper in the trailer (as would be the case at the start of the log-loading operation when the trailer is empty). The path, as shown in Fig. 6, is composed of three successive segments: first, a semicircle parallel to the $y_0$ plane, second, a line parallel to the $z_0$ axis, and third, a quarter-arc parallel to the $x_0-z_0$ plane. The grapple is initially at rest in this maneuver. Figure 12 shows the smooth DP-generated trajectories for $s$ and $\dot{s}$. DP trajectories lead to exact tracking of the path, as illustrated in Fig. 6-C. The sway angles shown in Fig. 13 confirm that the DP trajectories produce significant sway-damping relative to the 5OP motion: 86% and 92% reduction in maximum values of $\theta_5$ and $\theta_6$ after
B. Anti-Sway Waypoint Follower

We now demonstrate the performance of the DP formulation for sway damping, with the additional objective of timed waypoint following, for two maneuvers: Maneuver E contains three timed waypoints (in addition to initial and final points) on the same circular path as used in the A-Circle maneuver, while Maneuver F contains seven timed waypoints on a helical path, similar to that in the B-Helix test-case. The grapple’s initial sway conditions are noted in Table III. The maneuver time is $6 \text{s}$ and the maneuver starting point is $[4.25, 0, 2.77]$ m, as before. The nominal trajectories for the boom tip utilized to linearize the sway dynamics in maneuvers E and F are, respectively, the 6-th order polynomial (6OP) and 10-th order polynomial (10OP), to interpolate the respective waypoints, initial and final points, and the zero initial and final velocity constraints.

1) E-circular 3WP: Figure 16 displays the trajectories for $x_{bt}$ and $y_{bt}$ and also the boom tip path, confirming both the smoothness of the DP trajectories and satisfaction of the timed waypoints conditions. Here, $z_{bt}$ is considered constant. The maximum residual sway angles of the two passive joints, as seen from Fig. 17, are reduced by 89% and 46% when our DP motion planner is employed (respective angles are 1.6° and 2.4°).

2) F-helical 7WP: Here, 7 timed waypoints on a helical path, the axis of which is parallel to $z_0$-axis, and with radius and pitch of 0.75m and 2 m are specified for a more complex maneuver. Moreover, the EE has an initial sway stated in Table III. The trajectories and the path of the boom tip are depicted in Figs. 18. The DP trajectories are smooth and pass through
Fig. 16: Trajectories of $x_{bt}$, $y_{bt}$ (top) and path (bottom) for maneuver $E$-circular 3WP. The blue and red lines represent 6OP and DP results, respectively. The yellow points indicate the prescribed waypoints.

Fig. 17: Sway angles in maneuver $E$-circular 3WP. Solid lines and dotted lines represent $\theta_5$ and $\theta_6$, respectively, when employing 6OP (blue) and DP (red) trajectories. The yellow vertical line shows the maneuver’s end time.

In the objective function, the sway damping objective must be compromised against the added objective of satisfying the waypoint constraints. Furthermore, increasing the number of timed waypoints can affect the anti-sway performance negatively. If following many waypoints is required, we suggest dropping their time dependence, generating a path through them, and then using our proposed anti-sway motion planner for path following.

VI. CONCLUSION AND FUTURE WORK

In this paper, we proposed two novel motion planning solutions for manipulator arms, by utilizing the Dynamic Programming framework to achieve damping of the end-effector residual sway and, to follow collision-free paths or
time-dependent waypoints. We applied these motion planners to the crane of a forestry forwarder machine, for which both of the aforementioned goals are critical.

We began by formulating the grapple sway dynamics in terms of the boom tip coordinates. Then, we showed how to incorporate sway damping, as well as, path or timed waypoint following into the DP framework with a judicious choice of states, inputs, and objective functions. Since our DP methodology generates trajectories in task space, a locally optimal inverse kinematics solution was employed to obtain joint space motion, while satisfying the joint constraints. To evaluate the performance of the DP-generated motion against commonly used polynomial trajectories, we employed a virtual model of the forwarder created in a high-fidelity multibody-dynamics simulator, Vortex Studio. Six different maneuvers, based on circular, helical and multi-segment paths, with different initial sway conditions, were simulated. Our results confirmed the capability of the DP motion planners to track exactly either a prescribed path or timed waypoints, while suppressing the residual EE sway.

Table IV summarizes the maximum residual sway angles obtained with the DP planner and the corresponding percentage reductions compared to the polynomial trajectories. As can be seen, the DP trajectories lead to very small sway angles, on average 87% and 75% lower in path following and 78% and 45% lower in waypoint following, for $\theta_2$ and $\theta_5$, respectively. These results can be further improved by increasing the sampling rate of the simulator (or the hardware when implemented on a real crane).

In conclusion, our proposed motion planners produce computationally inexpensive and smooth trajectories, capable of following a path or timed waypoints and damping the residual sway in both R2R and NR2R conditions. Moreover, the methodology is general and can be applied to any manipulator with a passive EE, as well as to other dynamic systems with freely hanging payloads, such as cranes or quadrotors carrying a slug load. One potential avenue for future research involves optimizing the maneuver time while maintaining effective sway damping.

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REFERENCES


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<th>$\theta_{5\text{max}}$</th>
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<th>$\theta_{6\text{max}}$</th>
<th>$\text{reduction } \theta_6$</th>
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<td>79%</td>
<td>1.5°</td>
<td>79%</td>
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<td>B 1.3°</td>
<td>86%</td>
<td>2.4°</td>
<td>72%</td>
</tr>
<tr>
<td>C 2.6°</td>
<td>86%</td>
<td>0.8°</td>
<td>92%</td>
</tr>
<tr>
<td>D 0.3°</td>
<td>95%</td>
<td>0.6°</td>
<td>56%</td>
</tr>
<tr>
<td>E 1.6°</td>
<td>89%</td>
<td>2.4°</td>
<td>46%</td>
</tr>
<tr>
<td>F 4.5°</td>
<td>66%</td>
<td>2.9°</td>
<td>44%</td>
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