Received Signal Modelling for Millimeter Wave and Terahertz Systems with Practical Impairments

Xiaojing Huang 1, Hao Zhang 2, Anh Tuyen Le 2, Andrew Zhang 2, and Jay Guo 2

1University of Technology Sydney
2Affiliation not available

December 7, 2023

Abstract

For wideband transceivers operating at millimeter wave and terahertz frequencies, the implementation of conventional digital predistortion for nonlinearity mitigation faces significant challenges due to the limited availability and/or complexity of high-speed digital signal processing. In this paper, a simple received signal model is proposed for wideband system with nonlinearity and other practical impairments, such as transmitter (Tx) and receiver (Rx) I/Q imbalances (IQIs), carrier frequency offset (CFO), and phase noise, to enable low-complexity impairment mitigation. An expanded memory polynomial (EMP) model is firstly proposed to capture Tx IQI and the nonlinearity over the entire transceiver chain. Exploiting the CFO and a novel transmission protocol, a blind Rx IQI estimation is also proposed. The noise enhancement after Rx IQI and CFO compensation is then evaluated as a noise factor related to the mean-square-error of the Rx IQI estimation. As a result, the received signal of the wideband system is finally modelled as an EMP plus additive noises followed by a band-limited noisy receiver filter. Simulation results using a millimeter wave system with 2.5 GHz bandwidth and 73.5 GHz carrier frequency are presented to verify the accuracy of the EMP modelling and validate the theoretical analyses.
Received Signal Modelling for Millimeter Wave and Terahertz Systems with Practical Impairments

Xiaojing Huang, Senior Member, IEEE, Hao Zhang, Anh Tuyen Le, Member, IEEE, J. Andrew Zhang, Senior Member, IEEE, and Y. Jay Guo, Fellow, IEEE

Abstract—For wideband transceivers operating at millimeter wave and terahertz frequencies, the implementation of conventional digital predistortion for nonlinearity mitigation faces significant challenges due to the limited availability and/or complexity of high-speed digital signal processing. In this paper, a simple received signal model is proposed for wideband system with nonlinearity and other practical impairments, such as transmitter (Tx) and receiver (Rx) I/Q imbalances (IQIs), carrier frequency offset (CFO), and phase noise, to enable low-complexity impairment mitigation. An expanded memory polynomial (EMP) model is firstly proposed to capture Tx IQI and the nonlinearity over the entire transceiver chain. Exploiting the CFO and a novel transmission protocol, a blind Rx IQI estimation is also proposed. The noise enhancement after Rx IQI and CFO compensation is then evaluated as a noise factor related to the mean-square-error of the Rx IQI estimation. As a result, the received signal of the wideband system is finally modelled as an EMP plus additive noises followed by a band-limited noisy receiver filter. Simulation results using a millimeter wave system with 2.5 GHz bandwidth and 73.5 GHz carrier frequency are presented to verify the accuracy of the EMP modelling and validate the theoretical analyses.

Index Terms—I/Q imbalance, nonlinear high power amplifier, memory polynomial, carrier frequency offset, wideband, millimeter wave, terahertz, and digital predistortion.

I. INTRODUCTION

Wideband millimeter wave (mm-wave) systems have been envisaged as the viable solutions to realizing high capacity wireless access for the sixth-generation (6G) mobile networks and providing multi-hundred gigabit wireless backhaul for future integrated space and terrestrial networks [1], [2]. With more bandwidth available in terahertz (THz) frequency band, THz systems are also able to provide high-speed space communications between satellites without atmospheric attenuation [3]. In such wideband applications, radio transceivers with direct-conversion architecture are favourable for reduced implementation cost. The high power amplifiers (HPAs) are also desired to operate near the saturation point to achieve high power efficiency.

However, there are various practical impairments that adversely impact on the wideband system performance if they are not properly mitigated. For example, the direct conversion architecture with in-phase(I)/quadrature(Q) up- and down-converters causes I/Q imbalance (IQI), the signal chain, including HPA, low noise amplifier (LNA), and analog-to-digital converters (ADCs), demonstrates nonlinearity [4], and a non-ideal local oscillator (LO) introduces carrier frequency offset (CFO) and phase noise. Numerous individual and/or joint solutions to the problem of practical impairment mitigation can be found in the literature for narrowband microwave systems, however, for wideband mm-wave and THz systems with nonlinearity, the problem becomes a significant technical challenge and is far from being solved completely.

Digital predistortion (DPD) is a widely adopted technique for HPA linearization and Tx IQI compensation in narrowband microwave systems, but it is neither necessary nor feasible for wideband mm-wave and THz systems. For example, according to the transmitter (Tx) radio frequency (RF) spectrum mask defined by ETSI for E-band mm-wave systems operating in the frequency bands 71-76/81-86 GHz [5], the adjacent channel interference is only required to be 30-40 dB lower than the center frequency component, which is satisfied with moderate HPA nonlinearity and/or after simple power back-off. Also, according to the authors’ experience, DPD may reduce the Tx power while linearizing the HPA, similar to the effect achieved by the power back-off strategy. Most importantly, even with the state-of-the-art digital technology, no ADC is available or affordable to sample the nonlinear HPA feedback signal with usually 4-5 fold faster speed [2] than the multi-gigahertz signal bandwidth, nor can the digital signal processing be implemented in real-time as the nonlinearity parameter estimation requires inversion of a large matrix with dimension equal to the total number of nonlinearity parameters and the indirect learning used in DPD requires many iterations. Although new techniques such as neural network and artificial intelligence [2], [6], [7] have been exploited recently to significantly improve the HPA linearization performance, the fundamental constraints of existing digital hardware capability make DPD not applicable to low-cost wideband mm-wave and THz systems.

A low-complexity alternative to DPD for wideband system is digital post-mitigation at receiver (Rx) with limited bandwidth and hence low sampling rate, if the nonlinearity over the entire signal chain can be modelled accurately and efficiently considering the impact of other practical impairments. As we all know, memory polynomial (MP) model is the most popular and effective nonlinearity model with lower complexity and better amenability than others such as the general Volterra model and Wiener model [8], [9]. To deal with the unavoidable IQI, a DPD structure is proposed for joint mitigation of HPA nonlinearity and Tx IQI in [10] with a conjugate MP model. IQI is also considered for predistorters

Xiaojing Huang, Hao Zhang, Anh Tuyen Le, J. Andrew Zhang, and Y. Jay Guo are with the Global Big Data Technologies Center, University of Technology Sydney, Australia (emails: [Xiaojing.Huang; Hao.Zhang; AnhTuyen.Le; Andrew.Zhang; Jay.Guo]@uts.edu.au).
used in concurrent dual- and tri-band transmitters with so-called two-dimensional (2D) MP models which involve cross-terms of the multi-band signals [11]–[13]. However, these models do not capture the full effect of IQI on the nonlinearity, as all the nonlinear components in the models only have linear signal indeterminate multiplied by higher order signal envelopes. Higher order signal indeterminates are included in some IQMP models [14]–[16], but they are actually the real and imaginary parts, which are real signals, of the I/Q imbalanced complex signal, such that these IQMP models lack the analyticity for nonlinearity analysis. If the signal chain has multiple nonlinear components, it is obvious that a cascade of them is still nonlinear, but how to formulate the overall nonlinearity parameters is still not found in existing literature.

In this paper, a more accurate and true 2D expanded memory polynomial (EMP) model is proposed for wideband system with Tx IQI, which is derived by applying binomial expansion to the I/Q imbalanced signal envelopes in the conventional MP model and selecting valid basis functions. With a cascade of multiple nonlinear components being modelled by a single MP model, the entire signal chain with Tx IQI can be modelled by the EMP model in principle. However, due to the impact of CFO which always exists in a practical system, the Rx IQI needs to be tackled separately [17], [18]. This is because the CFO leads to fast channel variation which may not be coped with by the signal processing for channel estimation and equalization. As such, making use of the CFO and exploiting a novel transmission preamble structure, a blind Rx IQI estimation technique is also proposed. After Rx IQI and CFO compensation, a wideband system with various practical impairments is finally modelled as an EMP plus additive noises followed by a noisy receiver filter. This simple band-limited received signal model will significantly reduce the complexity for next-step nonlinearity mitigation over the entire signal chain.

The contributions of this paper are three-fold. First, we formulate the MP parameters for a cascade of two nonlinear components, exemplifying that the entire transceiver chain can be modelled by a single nonlinear model. We then propose the EMP model with I/Q imbalanced input and derive the new EMP parameters given the Tx IQI and MP parameters. Simulation results show that the EMP model is more accurate than the conventional conjugate MP model.

Second, we propose a low-complexity blind Rx IQI estimation method in presence of CFO, enabled by a novel transmission protocol without increasing the signalling overhead. The phase rotation introduced by CFO is exploited for Rx IQI estimation without requiring the knowledge of the distorted training sequence at the receiver.

Third, we analyse the mean-squared-error (MSE) of Rx IQI estimation and determine the image reject ratio (IRR) after Rx IQI and CFO compensation, enabling a simple band-limited received signal model for a nonlinear system with practical impairments.

The rest of this paper is organized as follows to elaborate the above contributions in detail. In Section II, the conventional models of respective non-ideal components over the signal chain of a wideband system are introduced. The joint Tx IQI and nonlinearity model is then derived in Section III. In Section IV, the novel transmission protocol which enables low-complexity IQI and nonlinearity estimation at receiver is presented, followed by the descriptions of the Rx IQI estimation and compensation. The final simplified signal model is also given. To verify the accuracy of the EMP modelling and demonstrate the noise enhancement of the modelled receiver filter, simulation results are provided in Section V. Finally, conclusions are drawn in Section VI.

II. SYSTEM AND SIGNAL MODELS

We begin with the system description and the baseband signal modelling of each individual system component.

A. System Description

The considered direct-conversion wideband system with both Tx and Rx IQIs, nonlinear HPA at Tx, and nonlinear LNA, CFO, and phase noise at Rx is shown in Fig. 1 (a). The baseband signal after digital-to-analog converters (DACs), denoted as $x(t)$, is up-converted to RF signal with carrier frequency $f_c$, power amplified by the HPA, and transmitted over the wireless channel. The baseband equivalent of the transmitted RF signal is denoted as $y(t)$. The received RF signal after being amplified by an LNA, with baseband equivalent denoted as $z(t)$, is down-converted to baseband signal with carrier frequency $f'_c$. The received baseband signal, denoted as $r(t)$, is further converted to digital baseband via ADCs and processed to recover the transmitted data information.

Due to the mismatch of the two reconstruction filters, $h_{t,I}(t)$ and $h_{t,Q}(t)$, at the I and Q branches of the up-converter as well as the imperfect 90-degree phase shifter, represented by a phase imbalance $\theta_i$, Tx IQI will be introduced into the transmitted signal before HPA. Similarly, due to the mismatch of the two anti-aliasing filters, $h_{r,I}(t)$ and $h_{r,Q}(t)$, at the I and Q branches of the down-converter as well as the imperfect 90-degree phase shifter, represented by a phase imbalance $\theta_r$, Rx IQI will be introduced into the received signal after LNA. In addition, due to the nonlinearity of the HPA, the transmitted signal will be distorted with spectral regrowth. The nonlinearity of the LNA will cause further signal distortion. The non-ideal LOs at Tx and Rx as well as the difference between $f_c$ and $f'_c$ will also introduce phase noise and CFO into the received signal, causing signal phase variation and frequency shift.

B. I/Q imbalance and Nonlinearity Models

As has been well studied in the literature [18], given the reconstruction filters and the phase imbalance of the up-converter, the Tx I/Q imbalanced baseband signal can be expressed as

$$x'(t) = u_t(t) \ast x(t) + v_t(t) \ast x^*(t) \tag{1}$$

where $u_t(t) = \frac{1}{2}[h_{t,I}(t) + h_{t,Q}(t)e^{j\theta_i}]$ and $v_t(t) = \frac{1}{2}[h_{t,I}(t) - h_{t,Q}(t)e^{j\theta_i}]$ respectively, $x^*(t)$ denotes the conjugated version of the baseband signal $x(t)$, and $\ast$ denotes convolution.
Similarly, given the anti-aliasing filters and the phase imbalance of the down-converter, the Rx I/Q imbalanced baseband signal can be expressed as

\[ r(t) = u_r(t) \ast z'(t) + v_r(t) \ast z''(t) \]  

where \( u_r(t) = \frac{1}{2}[h_{r,I}(t) + h_{r,Q}(t)e^{-j\phi_0}] \) and \( v_r(t) = \frac{1}{2}[h_{r,I}(t) - h_{r,Q}(t)e^{j\phi_0}] \) respectively, and \( z'(t) \) is the received baseband signal impacted by phase noise and CFO.

Eqs. (1) and (2) represent the frequency-dependent IQI models in general. If \( u_1(t) \), \( v_1(t) \), \( u_r(t) \), and \( v_r(t) \) are delta functions with respective complex amplitudes, they become frequency-independent IQI models for the up-converter and down-converter respectively.

To produce sufficient transmitted signal power, the upconverted signal will go through a nonlinear HPA which can be described by the well-known MP model in baseband as

\[ y(t) = \sum_{p=1,p \text{ odd}}^{P} \alpha_p(t) \ast x'(t) \ast |x'(t)|^{p-1} \]  

where \( P \) is the maximum Tx nonlinearity order and \( \alpha_p(t) \) is the \( p \)-th order complex coefficient, i.e., the nonlinearity parameter, expressed as a function of time to introduce the memory. Similarly, the noise free LNA output at the receiver can be modelled as

\[ z(t) = \sum_{p=1,p \text{ odd}}^{P} \beta_p(t) \ast y'(t) \ast |y'(t)|^{p-1} \]  

where \( P \) is the maximum receiver nonlinearity order, \( \beta_p(t) \) is the \( p \)-th order nonlinearity parameter, and \( y'(t) = y(t) \ast h(t) \) is the received signal after the wireless channel, denoted as \( h(t) \).

### C. Complete Baseband Signal Model

The baseband signal models for the various components along the signal chain are summarized in Fig. 1(b) where an additive white Gaussian noise (AWGN) \( n(t) \) is present after LNA. The CFO and phase noise impacted baseband signal can be expressed as

\[ z'(t) = e^{j(2\pi f t + \varphi_0)} z(t) \approx e^{j(2\pi f t + \varphi(t))} z(t) + j\varphi(t) e^{j(2\pi f t + \varphi_0)} z(t) \]

where \( \Delta f = f_c - f'_c \) denotes the CFO, \( \varphi_0 \) is a constant initial phase, and \( \varphi(t) \) represents the phase noise which is a variation of carrier phase with time. As \( \varphi(t) \) is relatively small, we have used the approximation \( e^{j\varphi(t)} \approx 1 + j\varphi(t) \) in (5). This approximation also implies that the phase noise effectively introduces a signal dependent additive interference \( n_c(t) = j\varphi(t) z(t) \) with power proportional to the input signal power by a factor \( \sigma^2_\varphi = \int_{-\infty}^{\infty} S_{\varphi\varphi}(f) df \) where \( S_{\varphi\varphi}(f) \) is the power spectral density (PSD) of \( \varphi(t) \).

From this complete baseband signal model, we see that the received signal is the output of a cascade of various non-ideal components which introduce IQI and nonlinearity, corrupted by noise as well as frequency and phase variations. From (2)
and (5), it also becomes apparent that, if CFO exists, the overall system IQI and nonlinearity observed in the down-converted signal \( r(t) \) will vary with time very quickly. Hence, the Rx IQI needs to be estimated and compensated first at the receiver, followed by CFO compensation, before tackling the Tx IQI and nonlinearity.

III. NOVEL JOINT IQI AND NONLINEARITY MODEL

We then propose novel joint IQI and nonlinearity model to simplify the signal chain modelling.

A. Cascade of MP Models

In a practical system, there may be many nonlinear components along the signal chain, but it is possible to model the overall nonlinearity with a single model instead of modelling and linearizing them individually to reduce the signal-processing complexity. Exemplified by using the HPA and LNA separated by the wireless channel as enclosed in the dash-lined rectangle 1 in Fig. 1(b), we first show that a cascade of two MP modelled nonlinear components can be also modelled by an MP model.

Substituting \( y'(t) = y(t) + h(t) = \sum_{p=1}^{P_1} \alpha_p(t) * h(t) * x'(t) |x'(t)|^{p-1} \) into (4) and approximating \( \alpha_p(t) * h(t) \) as \( h_p \delta(t) \) where \( h_p = \int_{-\infty}^{\infty} \alpha_p(t) * h(t) dt \), we can formulate \( z(t) \) as a function of \( x'(t) \), i.e.,

\[
z(t) = \sum_{p=1}^{P_{max}} \lambda_p(t) * x'(t) |x'(t)|^{p-1}
\]

where \( P_{max} = P_1 P_r \) is the maximum signal chain nonlinearity order. The \( p \)-th order complex coefficient \( \lambda_p(t) \) can be derived as (see Appendix A)

\[
\lambda_1(t) = \beta_1(t) * \alpha_1(t) * h(t),
\]

\[
\lambda_p(t) \approx \beta_1(t) * \alpha_p(t) * h(t) + \min_{p'=3, p' \ odd} \beta_{p'}(t)
\]

\[
= \min \left[ \frac{p-\beta_{p'}}{2} \right]_{m=\max(0, \frac{(p-1)}{2} - \frac{(p-1)(p-1)}{2})} \cdot h_{2m+1} g_{p'}^{(m)} \left( \frac{2m}{2} - m \right)
\]

for \( 1 < p \leq P_1 \),

and

\[
\lambda_p(t) \approx \sum_{p'=3, p' \ odd} \beta_{p'}(t) \min \left[ \frac{p-\beta_{p'}}{2} \right]_{m=\max(0, \frac{(p-1)}{2} - \frac{(p-1)(p-1)}{2})} \cdot h_{2m+1} g_{p'}^{(m)} \left( \frac{2m}{2} - m \right)
\]

for \( P_1 < p \leq P_{max} \),

where \( g_{p'}^{(m)} \) is a real-valued coefficient associated with the term \( |x'(t)|^{2m} \) in the expansion of \( \sum_{m=0}^{(P_1-1)(p'-1)} g_{p'}^{(m)} |x'(t)|^{2m} \), and \( \min(a, b) \) denotes the minimum between \( a \) and \( b \). Note that, for memoryless HPA and Gaussian channel, Eqs. (8) and (9) are the exact model parameters.

In practice, we only need to model the nonlinearity with reduced order \( P << P_{max} \). As such, the nonlinearity of the entire signal chain can be modelled by a single MP model with nonlinearity order \( P \) as illustrated in the dash-lined rectangle 1 in Fig. 1(b).

B. Expanded Memory Polynomial Model

After modelling the nonlinearity along the entire signal chain with a single MP model, we can further incorporate the Tx IQI into the model, i.e., expressing \( z(t) \) in terms of \( x(t) \), for IQI and nonlinearity estimation and mitigation. However, as can be seen from Appendix B, \( z(t) \) depends nonlinearly on the IQI parameters \( u_1(t) \) and \( v_1(t) \), making it hard, if not impossible, to estimate the original IQI and nonlinearity parameters from \( z(t) \).

Nevertheless, with the new basis functions discovered in Appendix B, we can model \( z(t) \) in terms of \( x(t) \) as

\[
z_{EMP}(t) \approx \sum_{p=1}^{P} \sum_{q=1}^{P} \left[ a_{q,p}(t) * x_q(t) |x(t)|^{p-q} + b_{q,p}(t) * x^*(q) |x(t)|^{p-q} \right]
\]

where \( a_{q,p}(t) \) and \( b_{q,p}(t) \) are the complex coefficients, i.e., the new joint IQI and nonlinearity parameters, associated with the basis function \( x_q(t) |x(t)|^{p-q} \) and its conjugated version \( x^*(q) |x(t)|^{p-q} \) respectively. We refer to (10) as the EMP model\(^1\) since it is derived from the expansion of the MP model with I/Q imbalanced input. Now, instead of identifying \( u_1(t) \), \( v_1(t) \), \( a_{q,p}(t) \), \( h(t) \), and \( \beta_q(t) \) separately, we are able to estimate \( a_{q,p}(t) \) and \( b_{q,p}(t) \) with the simplified baseband signal model as enclosed in the dot-lined rectangle 2 in Fig. 1(b). Note that the EMP model is two dimensional, spanned by \( q \) and \( p \), with constraint \( p \geq q \). Also note that, with reduced nonlinearity order \( P \), residual nonlinear signal, i.e., the difference between the true and modelled nonlinear signals, will be introduced as additive interference, denoted as \( e_{f}(t) \).

Given the Tx IQI and MP parameters, the new EMP parameters can also be derived as (see Appendix B)

\[
a_{1,1}(t) = u_1(t) * \lambda_1(t),
\]

\[
b_{1,1}(t) = v_1(t) * \lambda_1(t),
\]

\[
a_{q,p}(t) \approx \lambda_p(t) \sum_{m=0}^{p-1} \left[ \frac{p+1}{2} \right] \left( \frac{p-1}{2} - m \right) |u|^{2m} u^{2m} v^{\frac{2p-2}{2} - m} v^{\frac{2p-2}{2} - m}
\]

\[
b_{q,p}(t) \approx \lambda_p(t) \sum_{m=0}^{p-1} \left[ \frac{p+1}{2} \right] \left( \frac{p-1}{2} - m \right) |v|^{2m} v^{2m} u^{\frac{2p-2}{2} - m} u^{\frac{2p-2}{2} - m}
\]

\(^1\)Alternatively, (10) can also be expressed as \( z_{EMP}(t) \approx \sum_{q=1}^{P} \sum_{p=q}^{P} [a_{q,p}(t) * x_q(t) |x(t)|^{p-q} + b_{q,p}(t) * x^*(q) |x(t)|^{p-q}] \).
for $p = 3, 5, \ldots, P$ and $q = 1, 3, \ldots, p$ where $u = \int_{-\infty}^{\infty} u_l(t) \, dt$ and $v = \int_{-\infty}^{\infty} v_l(t) \, dt$ so that $u(t) \approx u\delta(t)$ and $v(t) \approx v\delta(t)$.

### C. Simplified Signal Model with CFO and Rx IQI

With the up-converter, HPA, wireless channel, and LNA being jointly described by the proposed EMP model, the baseband signal model can be significantly simplified, though the CFO and Rx IQI have to be dealt with separately.

However, in case of no CFO presence, after absorbing the phase factor $e^{j\phi(t)}$ into the model parameters and ignoring the noise or modelling error for simplicity, the received signal can be expressed as $r(t) = \sum_{p=1}^{P} \sum_{q=1}^{P} \sum_{r=1}^{P} \{ [u_p(t) \ast a_{q,p}(t) + v_r(t) \ast b_{q,p}^*(t)] \ast x^p(t) \ast x^q(t) + \{ [u_p(t) \ast b_{q,p}(t) + v_r(t) \ast a_{q,p}^*(t)] \ast x^p(t) \ast x^q(t) \}$.

We see that the Rx IQI and nonlinearity combined with Tx IQI can be incorporated into one equivalent EMP model, making separate Rx IQI estimation or compensation unnecessary.

In practice, CFO is unavoidable especially for direct conversion wideband receivers. Therefore, efficient Rx IQI estimation and compensation are vital for low-complexity wideband receiver design.

### IV. BAND-LIMITED RECEIVED SIGNAL MODEL

We hence further derive the simplified baseband signal model after Rx IQI and CFO compensation with a uniquely designed transmission protocol.

#### A. Transmission Protocol with Reduced Nonlinearity Order

Targeting a reduced nonlinearity order $P$, the transmission protocol is shown in Fig. 2. Each frame has a preamble and a data payload sections, similar to any conventional one used in a wideband wireless system. The frame length is denoted as $T_f$ in time. The preamble is composed of a training sequence (TS) of length $N$ and a cyclic prefix (CP) with length longer than that of the overall channel response. For the system shown in Fig. 1, the overall channel is a cascade of those caused by the pulse shaping at Tx digital baseband, the up- and down-converters, the memories of HPA and LNA, and the wireless propagation channel. Denoting the sampling period as $T_c$ in digital baseband, the TS length is $NT_c$ in time. Note that, due to the precursor in the overall channel response (a precursor is the time period from the beginning to the peak of an impulse response), the timing reference point, i.e., $t = 0$, for the TS should be set a precursor ahead to prevent the interference of the random data payload to the TS.

In order to facilitate Rx IQI and, more importantly, nonlinearity estimation (not presented in this paper) at the receiver, two additional signalling methods are introduced into the transmission protocol, i.e., frame rotation and preamble power scaling, as exemplified in Fig. 2 with $P = 5$. More detailed descriptions are provided as follows.

**Frame Rotation:** With $\frac{P+1}{2}$ successive transmission frames as a group, all transmission frames in a group will rotate by an angle $\frac{2\pi}{P+1}$ group-by-group relevant to the previous group. $2(P+1)$ such rotations will complete a full $2\pi$ period and the rotation keeps going with the transmission. It is obvious that $(P+1)^2$ frames will be transmitted in a $2\pi$ frame rotation period. At the $m$-th frame, a phase shift caused by the frame rotation can be expressed as $\theta_m = \frac{2\pi}{P+1} \lfloor \frac{2m}{P+1} \rfloor$ where $\lfloor \cdot \rfloor$ denotes floor operation.

**Preamble Power Scaling:** In each transmission group, the powers of the preambles are scaled by factors $\rho_1$, $\rho_3$, $\cdots$, and $\rho_P$ respectively. To ensure a constant average power, the scaling factors should satisfy $\frac{2}{P+1}(\rho_1 + \rho_3 + \cdots + \rho_P) = 1$. Also, assuming a constant spacing $0 < \delta < \frac{\pi}{4k}$ between two adjacent scaling factors, we can formulate the scaling factors as $\rho_p = 1 - \frac{P-1}{4} \delta + \frac{P+1}{2} \delta$ for $p = 1, 3, \cdots, P$. At the $m$-th frame, the power of the $p$-th order nonlinear component in the received training signal will be scaled by a factor $\rho_p^{m(2m+1)(P+1)}$ where $(\cdot)^{(P+1)}$ denotes modulo-$(P+1)$ operation.

In the rest of the paper, both time and frequency domain signal processing techniques in digital baseband will be discussed. However, the signals will be presented as continuous functions of $t$ and $f$ respectively for convenience. Their discrete versions can be easily obtained by letting $t = nT_c$ and $f = kf_r$ respectively, where $n$ and $k$ are the discrete time and frequency indices respectively, and $f_r = \frac{1}{NT_c}$ is the frequency resolution as the frequency domain signals are all obtained through length-$N$ discrete Fourier transform (DFT).

#### B. Blind Rx I/Q Imbalance Estimation

As mentioned in Section II.C, with CFO present at the receiver, the Rx IQI should be separately estimated and compensated. However, after going through the I/Q imbalanced and nonlinear transmitter with unknown channel and CFO, the TS is distorted at the input of the receiver and hence conventional methods which require a known TS are not applicable for the Rx IQI estimation. Therefore, we propose a frequency-domain blind estimation technique without the knowledge of TS by exploiting only the phase rotations introduced by CFO and the transmission protocol.

From (2) and (5), the received training signal in the $m$-th frame can be expressed as $R^{(m)}(f) = e^{j\varphi_m U_r(t)}(f) \hat{Z}^{(m)}(f) - \Delta f] + N_{\varphi}(f - \Delta f] + e^{-j\varphi_m V_r(t)}(f) \hat{Z}^{(m)+}(f) - f - \Delta f) + \hat{N}_{\varphi}(m)_{+}(f - \Delta f)]$ in the frequency domain, where $U_r(t)$, $V_r(t)$, and $\hat{Z}^{(m)}(f)$ are the FTs of $u_r(t)$, $v_r(t)$, and the received training signal $\hat{z}^{(m)}(f)$ respectively, $\varphi_m$ is the initial phase at the beginning of the $m$-th frame, and $N_{\varphi}(m)_{+}(f)$ is the FT of the interference $n_{\varphi}(m)_{+}(t)$ due to phase noise.
frame rotation and preamble power scaling, $\hat{Z}^{(m)}(f)$ can be further expressed as $\hat{p}_{(2m+1)}^2 f_{(P\lambda+1)} [e^{j\pi\varphi_1} B_{11}(f) + e^{-j\pi\varphi_1} B_{11}(f) \tilde{X}_r(f)]$ where $A_{11}(f), B_{11}(f),$ and $\tilde{X}_r(f)$ are the FTs of $a_{11}(t), b_{11}(t)$ and pre-defined training signal $\tilde{x}(t)$ respectively, plus frequency-domain nonlinear signal $e_{1}^{(m)}(f)$ and noise $n^{(m)}(f)$ which are the FTs of residual nonlinear signal $e_{1}^{(m)}(t)$ and noise $\hat{n}^{(m)}(t)$ respectively.

Letting $m = \frac{P_l}{2} l + m'$ where $l$ is the index of a transmission group of $\frac{P_l}{2}$ frames with different preamble power scaling factors and $m' = 0, 1, \ldots, \frac{P_l}{2} - 1,$ $\varphi_m$ can be expressed as $\pi(P + 1) \Delta f T_l l + 2\pi \Delta f T_l m'$ + $\varphi_0$. Hence, for the $l$-th transmission group, defining $\hat{R}_l(f)$ as $\left( \hat{R}_l^{P_l-1}(f) \hat{R}_l^{P_l-1}(f) \ldots \hat{R}_l(0) \right)^T$ where $(\cdot)^T$ denotes matrix transpose, the received training signals can be finally expressed in matrix form as

$$\hat{R}_l(f) = \left( \begin{array}{cc} \mathbf{p} & \mathbf{p}^* \end{array} \right) \mathbf{D}_l \left( \begin{array}{cc} U_A(f) & V_A(f) \\ U_B(f) & V_B(f) \end{array} \right) + \hat{N}_l(f)$$

where $\mathbf{p} = \left( \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \sum e^{j2\pi f T_l} \ldots \sum e^{j2\pi f T_l} \right)^T, U_A(f), V_A(f), U_B(f),$ and $V_B(f)$ are defined as $e^{j\varphi_1} \sum_{l=1} B_{11}(f) \tilde{X}_r(f), e^{j\varphi_1} \sum_{l=1} B_{11}(f) \tilde{X}_r(f),$ and $e^{j\varphi_1} \sum_{l=1} B_{11}(f) \tilde{X}_r(f)$ respectively, $\mathbf{D}_l = \left( \begin{array}{cc} \mathbf{p} \mathbf{p}^* & \mathbf{p} \mathbf{p}^* \end{array} \right)$

where $\mathbf{p} = \left( \begin{array}{cc} \mathbf{p} & \mathbf{p} \end{array} \right)$

$\mathbf{D}_l = \left( \begin{array}{cc} \mathbf{p} \mathbf{p}^* & \mathbf{p} \mathbf{p}^* \end{array} \right)$

where $\mathbf{p} = \left( \begin{array}{cc} \mathbf{p} \mathbf{p}^* \end{array} \right)$

where $\mathbf{p} = \left( \begin{array}{cc} \mathbf{p} \mathbf{p}^* \end{array} \right)$

where $\mathbf{p} = \left( \begin{array}{cc} \mathbf{p} \mathbf{p}^* \end{array} \right)$

Step 1: With estimated CFO, find the least-squares (LS) solutions$^2$ of $U_A(f)$ and $V_A(f)$ from (15) as

$$(U_A^{(l)}(f) \quad V_A^{(l)}(f)) = \mathbf{D}^{-1}_l \left( \begin{array}{cc} \mathbf{p} \mathbf{p}^* & \mathbf{p} \mathbf{p}^* \end{array} \right)^{-1} \left( \begin{array}{cc} \mathbf{p} \mathbf{p}^* \end{array} \right) \hat{R}_l(f)$$

where $0 \leq \lambda \leq 1$ is selected according to $\Delta f$ to prevent possible ill-conditioned matrix inversion.

Step 2: Apply lowpass filtering to obtain the averaged $U_A^{(l)}(f)$ and $V_A^{(l)}(f)$ as

$$(U_A^{(l)}(f) \quad V_A^{(l)}(f)) = (1 - \mu) \left( \begin{array}{cc} U_A^{(l-1)}(f) \quad V_A^{(l-1)}(f) \end{array} \right) + \mu \left( \begin{array}{cc} U_A^{(l)}(f) \quad V_A^{(l)}(f) \end{array} \right)$$

where $0 < \mu < 1$ is a forgetting factor.

Step 3: An estimate of the Rx IQI parameter $B_t(f)$, defined as $\frac{\hat{V}_A(f)}{\hat{V}_A(f)}$, is obtained as

$$(B_t^{(l)}(f)) = \frac{\hat{V}_A^{(l)}(f)}{\hat{V}_A^{(l)}(f)}$$

As shown in Appendix C, denoting ensemble expectation as $E\{\cdot\}$, the MSE of the above estimation can be derived as

$$\text{MSE}(B_t^{(l)}(f)) = E\{|B_t^{(l)}(f) - B_t(f)|^2\}$$

$$\approx \left(1 - \lambda^2\right) \Omega^2 \left[ \left(1 - (1 - \mu) e^{-j2\pi f T_\lambda l} \right) \right]$$

$$\approx \left(1 - \lambda^2\right) \Omega^2 \left[ \left(1 - (1 - \mu) e^{-j2\pi f T_\lambda l} \right) \right]$$

where $\Omega = \frac{P_l}{P_l-1} \frac{\mathbf{p}^T \mathbf{p}}{P_l-1} \sum m' = 0 \rho_{2m' + 1} e^{j4\pi f T_\lambda m'}$.

$\text{SNR}(\delta)$ is a frequency-dependent equivalent signal-to-noise ratio (SNR) at the receiver. We assume that the CFO is relatively small such that its normalized value satisfies $|\Delta f T_\lambda| < \frac{1}{2}$. However, as shown in (31), at $\Delta f T_\lambda = 0$, we have $|\Omega|^2 = 1$ and there will be noise enhancement in $\hat{V}_A^{(l)}(f)$. Also, at $\Delta f T_\lambda = -\frac{1}{P_l-1} f T_\lambda$, we have $e^{-j2\pi f T_\lambda - \frac{1}{P_l-1} f T_\lambda} = 1$ and hence the residual $U_A(f)$ will become very large in $\hat{V}_A^{(l)}(f)$. To prevent the two conditions from happening, a practical strategy is to set $\lambda = 0$ when $|\Delta f T_\lambda| < \frac{1}{P_l-1}$ and $\lambda = 1$ when $|\Delta f T_\lambda| > \frac{1}{P_l-1}$.

$\text{MSE}(B_t^{(l)}(f))$ as functions of $\hat{\Gamma}(f, \Delta f)$ under different CFO conditions are shown in Fig. 3. Other parameters are selected as $P = 5, \delta = 0.83,$ and $B_t(f) = 0.1$. We see that, when $\lambda = 0$ at $\Delta f T_\lambda = 0$ and its close proximity ($\Delta f T_\lambda = \frac{1}{4(P+1)} f T_\lambda$ for example), an MSE floor occurs as SNR increases but it decreases as $\mu$ decreases.
except at $\Delta f T_f = 0$. When $\lambda = 1$ for a relatively larger CFO ($\Delta f T_f = \frac{1}{2}$ for example), the MSE decreases as SNR increases as well as $\mu$ decreases. Note that, in the absence of CFO, the Rx IQI can not be separately estimated as the MSE becomes very large (approximately $|B_{r}(f)|^2$). However, as discussed in Section III.C, in this case, the Rx IQI is combined with Tx IQI, demonstrating an overall effective IQI in the received signal. This effective IQI can be estimated and compensated at the signal detection stage along with the nonlinearity mitigation (not presented in this paper).

C. Adaptive Filtering for Rx I/Q Imbalance Compensation

With the estimated IQI parameter, we further propose the mitigation method which can be efficiently implemented at the receiver digital baseband.

Once the estimated $\hat{B}_{r}^{(l)}(f)$ is available, the Rx IQI can be compensated by subtracting $\hat{B}_{r}^{(l)}(f)R^{*}(-f)$ from $R(f)$, the frequency-domain received signal during the payload period, resulting in the frequency-domain output $R(f) - \hat{B}_{r}^{(l)}(f)R^{*}(-f)$ with signal component $e^{j\varphi_{u}U_{r}(f)}[1 - \hat{B}_{r}^{(l)}(f)B_{r}^{*}(-f)]Z(f - \Delta f)$ and image component $e^{-j\varphi_{u}U_{r}^{*}(-f)}[B_{r}(f) - \hat{B}_{r}^{(l)}(f)]Z^{*}(-f - \Delta f)$ due to the estimation error in $\hat{B}_{r}^{(l)}(f)$. The image rejection ratio can be evaluated, assuming that $1 - \hat{B}_{r}^{(l)}(f)B_{r}^{*}(-f)$ $\approx 1$ and after applying Jensen’s inequality [20], as

$$\text{IRR}(f) = E\{[1 - \hat{B}_{r}^{(l)}(f)B_{r}^{*}(-f)]^{2}\}$$
$$\approx E\{\frac{1}{B_{r}(f) - \hat{B}_{r}^{(l)}(f)}[1 - \hat{B}_{r}^{(l)}(f)B_{r}^{*}(-f)]^{2}\}$$
$$\approx \frac{1}{\text{MSE}\{\hat{B}_{r}(f)\}}, l \rightarrow \infty.\quad (20)$$

Though $\hat{B}_{r}^{(l)}(f)$ is estimated in frequency domain, the Rx IQI compensation is actually implemented by time-domain linear filtering. Denoting the inverse Fourier transform of $\hat{B}_{r}^{(l)}(f)$ as $\hat{B}_{r}^{(l)}(t)$, an estimate of $z(t)$, i.e., $\hat{z}(t)$, can be obtained as $r(t) - \hat{B}_{r}^{(l)}(t) + r^{*}(t)$ followed by CFO compensation, where $\hat{B}_{r}^{(l)}(t)$ is adaptively updated every transmission group of $\frac{T_s}{M}$ frames. The baseband signal model with signal processing flow graph for Rx IQI and CFO compensation is shown in Fig. 4 (upper).

D. Signal Model after Rx IQI and CFO Compensation

The received signal after Rx IQI and CFO compensation as $l \rightarrow \infty$ can be expressed in the frequency domain as

$$\hat{Z}(f)$$
$$\approx e^{j\varphi_{u}U_{r}(f + \Delta f)}[Z(f) + N_{\varphi}(f)] + e^{-j\varphi_{u}U_{r}^{*}(-f - \Delta f)}[B_{r}(f + \Delta f) - \hat{B}_{r}^{(l)}(f + \Delta f)]Z^{*}(-f - 2\Delta f)$$
$$= e^{j\varphi_{u}U_{r}(f + \Delta f)}Z_{\text{EMP}}(f) + e^{j\varphi_{u}U_{r}(f + \Delta f)}[E_{r}(f) + N(f) + N_{\varphi}(f)]$$
$$+ e^{-j\varphi_{u}U_{r}^{*}(-f - \Delta f)}[B_{r}(f + \Delta f) - \hat{B}_{r}^{(l)}(f + \Delta f)]Z_{\text{EMP}}^{*}(-f - 2\Delta f)\quad (21)$$

where $Z_{\text{EMP}}(f)$, $E_{r}(f)$, $N(f)$, and $N_{\varphi}(f)$ are the FTs of $z_{\text{EMP}}(t)$, $\epsilon_{r}(t)$, $n(t)$, and $n_{\varphi}(t)$ respectively. Defining the equivalent noise and equivalent SNR before Rx IQI and CFO compensation as $N'(f) = e^{j\varphi_{u}U_{r}(f)}[E_{r}(f + \Delta f) + N(f + \Delta f) + N_{\varphi}(f - \Delta f)]$ with variance $\sigma_{N}(f, \Delta f)$ and $\Gamma(-f, \Delta f) = \frac{|U_{r}^{*}(-f - \Delta f)|^{2}}{|\sigma_{N}'(f, \Delta f)|^{2}}$, respectively, we see that the equivalent noise power after Rx IQI and CFO compensation is enhanced by a factor $NF(f + \Delta f)$ compared with the power of $N'(f)$, where

$$NF(f) = 1 + \text{MSE}\{\hat{B}_{r}(f)\} \Gamma(-f, \Delta f).\quad (22)$$

From (21) and (22), the received signal of a wideband system with common practical impairments can be finally modelled by an EMP model corrupted by equivalent additive noise involving residual nonlinear signal, LNA noise, and LO phase noise, followed by a noisy receiver filter as shown in Fig. 4 (lower) where $U_{r}(f + \Delta f)$ is the frequency response of the receiver filter with frequency and SNR dependent noise factor $NF(f + \Delta f)$. Note that the phase factor $e^{j\varphi_{u}}$ has been absorbed into the EMP model parameters and the equivalent noise.

Further absorbing the receiver filter into the EMP model parameters, the EMP modelled $\hat{z}(t)$ can be expressed as

$$\hat{z}_{\text{EMP}}(t) = \sum_{p=1}^{P} \sum_{q=1}^{P} \left[ a_{q,p}(t) \ast x^{q}(t) \right] x^{p}(t) |p-q|^\eta + \hat{b}_{q,p}(t) \ast x^{p}(t) |p-q|^\eta\quad (23)$$

where the overall signal chain band-limited EMP parameters are $a_{q,p}(t) = e^{j\varphi_{u}(t)}e^{-j2\pi\Delta ft} \ast a_{q,p}(t)$ and $\hat{b}_{q,p}(t) = e^{j\varphi_{u}(t)}e^{-j2\pi\Delta ft} \ast \hat{b}_{q,p}(t)$.

V. SIMULATION RESULTS

We finally present the simulation results to verify the performance of EMP modelling and Rx IQI estimation.

A. Simulation Set-ups

A wideband mm-wave system operating at 73.5 GHz carrier frequency with 2.125 GHz bandwidth is simulated, adopting the EMP model and Rx IQI estimation to simplify the received signal modelling. The symbol rate is $\frac{1}{T_s} = 1.875$ Gsp and the sampling rate is $\frac{1}{T_s} = 2.5$ Gsp, resulting a sampling conversion ratio of 3/4. The spectral shaping pulse is selected as a root-raised-cosine (RRC) function with roll-off 0.013. The transmission frame contains a preamble section of length $2N_{c}$ with a frequency domain ZC sequence [21] of length $T_{c} = 9536\mu s$ and a payload section of length $8144\mu s$. With 16-quadrature amplitude modulation (16-QAM), the raw data rate can achieve 7.4 Gbps.

To simulate Tx IQI, we select $\theta = 5$ degrees, and $h_{r,1}(t)$ and $h_{r,2}(t)$ as a 2nd-order Butterworth lowpass filter with 3 dB cut-off frequency $0.468\frac{1}{T_s}$ and a 3rd-order Butterworth lowpass filter with 3 dB cut-off frequency $0.494\frac{1}{T_s}$, respectively, with further 1 dB gain mismatch on the Q branch. To simulate Rx IQI, we select $\theta = 5$ degrees, and $h_{r,1}(t)$ and $h_{r,2}(t)$ as two 4th-order Butterworth lowpass filters with 3 dB cut-off
\[ x(t) = \sum_{p=1}^{P} \sum_{q=1}^{P} \left[ a_{q,p}(t) \cdot x^q(t) |x(t)|^{p-q} \right] + b_{q,p}(t) \cdot x^q(t) |x(t)|^{p-q} \]

Fig. 4. Baseband signal model with Rx IQI and CFO compensation (upper) and final simplified signal model (lower).

B. Tx Signal PSD and EMP Modelling Error

Fig. 5 shows the simulated PSDs of the transmitted RF signal with nonlinear HPA and linear HPA respectively with 0 dBW effective isotropic radiated power (EIRP). The required Tx RF spectrum mask is also shown. We see that the nonlinear HPA does cause spectral regrowth, but it is well below the mask and hence a DPD is not necessary.

To demonstrate the accuracy of the EMP modelling for HPA, we let \( P = P_l = 5 \), \( \lambda_p(t) = \alpha_p(t) \) for \( p = 1, 3, \) and \( 5, \) and use Eqs. (11) to (14) to calculate the EMP model coefficients. The modelling errors, presented as error spectral densities, are shown in Fig. 6. With frequency-independent Tx IQI, the EMP accurately models the nonlinear HPA and hence the modelling error is zero. With frequency-dependent Tx IQI, modelling error (about \(-49.47\) dBm/MHz in the passband) occurs. However, the EMP model offers better performance than the conventional conjugate MP model (a special case of EMP with \( q = 1 \)) especially in the passband.

C. Estimated Rx IQI and IRR

With \( P = 5 \) and a preamble power scaling spacing \( \delta = 0.83 \), the blind Rx IQI estimation is simulated at 25 dB average SNR, i.e., signal power versus noise power of \( n(t) \), with \( \lambda = 1 \) and \( \mu = 0.1 \) after introducing a CFO \( \Delta f = 65.541 \) kHz at the receiver (such that \( \Delta f T_r = \frac{1}{4} \)), and the Rx IRR is then calculated after 400 transmission frames as shown in Fig. 7. We see that the estimated Rx IQI \( B_r(f) \) agrees with the true values over the signal bandwidth and the Rx IRR is above 30 dB in the signal passband, which conforms to the theoretically evaluated MSE shown in Fig. 3. The fluctuation of the estimated Rx IQI and IRR is due to the
notches of the channel frequency response, which results in different equivalent SNRs at different frequencies.

D. Rx Signal PSD and EMP Modelling Error

After propagation through the wireless channel and amplified by the nonlinear LNA, the received signal is further degraded, as seen from its PSD normalized by the signal power at the carrier frequency shown in Fig. 8. The PSD is fluctuated due to the wireless channel multipath reflection. Compared with the PSD without nonlinearity, the nonlinear spectral regrowth is more significant after LNA.

As $P_t = P_r = 5$, we would have the nonlinearity order $P_{\text{max}} = 25$ for the cascade of the HPA and LNA. To reduce the modelling complexity, we select the nonlinearity order as $P = 5$ for the entire signal chain and use Eqs. (7), (8), and (9) to determine the complex coefficients $\lambda_p(t)$ for $p = 1, 3$, and $5$. We then use Eqs. (11) to (14) to calculate the EMP model coefficients, resulting in the modelling errors as shown in Fig. 9. We see that, even with frequency-independent Tx IQI and Gaussian channel (i.e., only direct path), modelling error (about $-46.43$ dB in the passband) occurs for the reduced nonlinearity order of $P = 5$. With frequency-dependent Tx IQI and two-ray channel, the modelling error is increased to $-32.36$ dB in the passband. Compared with the conventional conjugate MP model (a special case of EMP with $q = 1$), the EMP model demonstrates obviously better modelling accuracy for the frequency-independent Tx IQI and Gaussian channel case and similar performance under the frequency-dependent Tx IQI and two-ray channel condition.

E. Noise Enhancement after Rx IQI and CFO Compensation

Finally, the normalized PSDs of the baseband signal and equivalent noise after blind Rx IQI and CFO compensation at 25 dB average SNR are shown in Fig. 10, which indicates that, under the simulated IQI, nonlinearity, phase noise, and CFO conditions, an equivalent SNR of above $20$ dB in the passband is achieved. To verify the simplified baseband signal model, i.e., an EMP model followed by a noisy receiver filter, we also show in Fig. 10 the modelled equivalent noise PSD normalized by the signal power at direct current, which is obtained as $N F(f + \Delta f) \sigma^2_{N'}(f, \Delta f)$. By assuming that $\Gamma(f, \Delta f) = \Gamma(-f, \Delta f)$, the noise factor can be determined as $NF(f) = 1 + \frac{\mu}{(2-\mu)(1-\lambda)([\lambda^2/(1-\lambda)])^2}$ from Eqs. (19) and (22).
where $\lambda = 1$, $\mu = 0.1$, and $\Omega = \frac{1}{3}$ for the selected $P$ and $\delta$ values. The normalized power of the equivalent noise $N(f)$, i.e., $\overline{\sigma_N^2}(f, \Delta f)$, is obtained as the sum of the normalized modelling error shown in Fig. 9, the normalized average noise power (i.e., $-25$ dBc), and the phase noise power (i.e., $-34.4725$ dBc) spectrally shaped by $|U_r(f + \Delta f)|^2$. We see that the simulated equivalent noise level is slightly higher than that of the modelled one. This is because the modelled equivalent noise power is obtained after the adaptive filtering for Rx IQI compensation is stabilized whereas the simulated equivalent noise power is obtained only from the first 400 transmission frames. Nevertheless, with the modelled equivalent noise, the complexity for wideband system analysis with various practical impairments can be significantly reduced.

**VI. Conclusions**

We have shown that the received signal of a wideband system with nonlinearity and other practical impairments can be efficiently modelled as the proposed EMP plus additive noises followed by a band-limited noisy receiver filter with noise factor related to the MSE of Rx IQI estimation. This simplified signal model will enable practical techniques to tackle the challenging nonlinearity mitigation problem at a wideband receiver with low complexity, removing the fundamental limitations of DPD. Simulation results verify the improved accuracy of the EMP model and validate the MSE and noise factor analyses.

**APPENDIX A: NONLINEARITY PARAMETERS OF CASCaded MEMORY POLYNOMIAL MODELS**

**TABLE I**

$g_{p'}^{(m)}$ FOR PRACTICAL NONLINEARITY PARAMETER CALCULATION

<table>
<thead>
<tr>
<th>$p'$</th>
<th>$m$</th>
<th>$h_0^{(0)}$</th>
<th>$h_1^{(1)}$</th>
<th>$h_2^{(2)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0</td>
<td>$h_0^{(0)}$</td>
<td>$h_1^{(1)}$</td>
<td>$h_2^{(2)}$</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>$2h_0^{(0)}h_1^{(1)}$</td>
<td>$h_1^{(1)} + 2h_1^{(1)}h_2^{(2)}$</td>
<td>$h_2^{(2)}$</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>$3h_0^{(0)}h_1^{(1)}$</td>
<td>$h_1^{(1)}$</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>$h_0^{(0)}h_1^{(1)}$</td>
<td>$h_1^{(1)}$</td>
<td></td>
</tr>
</tbody>
</table>

From (3) and (4), and with the approximation $\alpha_p(t) \cdot h(t) \approx h(t) \delta(t)$, $z(t)$ can be expressed in terms of $x'(t)$ as

$$z(t) = \sum_{p=1,p \text{ odd}} P_t \beta_1(t) \ast \alpha_p(t) \ast h(t) \ast x'(t) \ast x'(t) \ast |x'(t)|^{p-1}$$

$$+ \sum_{p'=3,p' \text{ odd}} P_t \beta_2(t) \ast \sum_{p=1,p \text{ odd}} P_t h_p x'(t) \ast |x'(t)|^{p+p' - 2}$$

$$- \sum_{p=1,p \text{ odd}} P_t h_p x'(t) \ast |x'(t)|^{p-1}$$

$$= \sum_{p=1,p \text{ odd}} P_t \beta_1(t) \ast \alpha_p(t) \ast h(t) \ast x'(t) \ast |x'(t)|^{p-1}$$

$$+ \sum_{p=3,p' \text{ odd}} P_t \beta_2(t) \ast \sum_{p=1,p \text{ odd}} P_t h_p x'(t) \ast |x'(t)|^{p+p' - 2}$$

$$- \sum_{p=1,p \text{ odd}} P_t h_p x'(t) \ast |x'(t)|^{p-1}$$

$$\cdot h_{2m+1} g_{p'}^{(m)}|x'(t)|x'(t)|x'(t)|^{p-1}$$

(24)

after further expanding the term $|x'(t)|^p$ as $\left( \sum_{p=1,p \text{ odd}} P_t h_p |x'(t)|^{|p-1|} \right)^{p-1}$ for a given $p'$ as $\left( \sum_{p=1,p \text{ odd}} P_t h_p |x'(t)|^{|p-1|} \right)^{p-1} = \left( \sum_{m=0}^{P_t-1} h(m) |x'(t)|^{2m} \right)^{p-1}$

max$(a,b)$ denotes taking the maximum between $a$ and $b$, and $g_{p'}^{(m)} = h(m) \otimes \cdots \otimes h(m)$, for $3 \leq p' \text{ odd} \leq P_t$

and $0 \leq m \leq \frac{(P_t-1-(p'-1))}{2}$, is a real-valued coefficient associated with $|x'(t)|^{2m}$ in the follow-up expansion of $\left( \sum_{m=0}^{P_t-1} h(m) |x'(t)|^{2m} \right)^{p-1}$. Note that $\otimes$ denotes discrete convolution and that the derivation of $g_{p'}^{(m)}$ is obtained after applying a $z$-transform property since $\sum_{m=0}^{P_t-1} h(m) |x'(t)|^{2m}$ is the $z$-transform of $h_{2m+1}$ for $z = |x'(t)|^2$. This $z$-transform property is also applied in deriving (24) such that the discrete convolution of $h_{2m+1}$ for $0 \leq m \leq \frac{P_t-1}{2}$ and $g_{p'}^{(m)}$ is involved.

Formulating (24) as an MP model of order $P_t P_r$, its nonlinearity parameters can be expressed in (7), (8), and (9).

For practical applications with reduced nonlinearity order $3 \leq P \leq 9$, the coefficients $g_{p'}^{(m)}$ in terms of $h(m)$ necessary for calculating the nonlinearity parameters are listed in Table I for convenience.

**APPENDIX B: EXPANSION OF MEMORY POLYNOMIAL MODEL WITH I/Q IMBALANCED INPUT**

Referring to Section III.A and Appendix A, a cascade of two nonlinear systems of orders $P_t$ and $P_r$ with parameters...
\[ z(t) = \sum_{p=1}^{P} \lambda_p(t) \ast [u_t(t) \ast x(t) + v_l(t) \ast x^*(t)] \]

We see that, though the nonlinearity parameter \( \lambda_p(t) \) is linearly convolved with a number of nonlinear basis functions composed of \( x(t) \) and the IQI parameters \( u_t(t) \) and \( v_l(t) \), the IQI parameters \( u_t(t) \) and \( v_l(t) \) themselves are nonlinearly involved in these basis functions. To facilitate parameter estimation at the receiver, it is desirable that the basis functions are purely composed of \( x(t) \) and that the joint IQI and nonlinearity parameters are linearly convolved with a set of such basis functions. These requirements are possible only if the IQI is frequency-independent, i.e., \( u_t(t) = u_b(t) \) and \( v_l(t) = v_b(t) \). For a given \( p \) and \( m, n = 0, 1, \cdots, \frac{p-1}{2} \), there are two types of basis functions which are only composed of \( x(t) \), i.e., \( x^p(t)x^{-(m+n)}(t)x^{-(m+n)}(t) = x^p(t)x^l(t) \) for \( l = m + n = 0, 1, \cdots, \frac{p-1}{2} \), \( p-1 \) and \( x^*(t)x^{p-1-(m+n)}(t)x^{-(m+n)}(t) = x^{p-l}(t)x^l(t) \) for \( l = m + n + 1 = 1, 3, \cdots, p \). Hence, all the possible basis functions for \( p = 1, 3, \cdots, P-2 \) can be found from both types of basis functions and are listed in Table II. These basis functions can be reformulated as

\[ x^p(t)x^l(t)^{p-q} \] and their conjugated versions \( x^*(t)x^l(t)^{p-q} \)

for \( p = 1, 3, \cdots, P \) and \( q = 1, 3, \cdots, p \).

Under frequency-dependent IQI condition, the cross terms in the basis functions, such as \( x^p(t)x^l(t) \) and its conjugated version, will appear, where \( \tau \) is a leading or lagging time offset in the exponentiated envelope. These cross terms will increase the number of basis functions considerably and hence the modelling complexity. However, the basis functions without any cross term can still be used to provide an efficient and effective modelling for a frequency-dependent I/Q imbalanced nonlinear system if we approximate \( u_t(t) \) and \( v_l(t) \) as \( u_b(t) \) and \( v_b(t) \) respectively for \( p > 1 \). Then, Eq. (26) can be expressed as

\[ z(t) \approx u_t(t) \ast \lambda_1(t) \ast x(t) + v_l(t) \ast \lambda_1(t) \ast x^*(t) \]

\[ + \sum_{p=3}^{P} \lambda_p(t) \sum_{m=0}^{\frac{p-1}{2}} \sum_{n=0}^{\frac{p-1}{2}} \left( \frac{\frac{p-1}{2}}{m} \right) \left( \frac{\frac{p-1}{2}}{n} \right) \]

\[ \cdot \left\{ u^m(t) \ast x^p(t) \ast x^l(t) \right\} \left\{ v^n(t) \ast x^p(t) \ast x^l(t) \right\} \]

\[ + \sum_{l=0}^{\min(l-1, \frac{p-1}{2})} \sum_{m=\max(0, l-\frac{p-1}{2})}^{\min(l, \frac{p-1}{2})} \left( \frac{\frac{p-1}{2}}{m} \right) \left( \frac{\frac{p-1}{2}}{l-m} \right) \]

\[ \cdot \left\{ u^l(t) \ast x^{p-l}(t) \ast x^l(t) \right\} \cdot \left\{ u^m(t) \ast x^p(t) \ast x^l(t) \right\} \]

(27)
where the binomial equality \( \binom{m}{m+n} + \binom{m}{n} = \binom{m+1}{n+1} \) is used for simplifying the derivation. Finally, introducing a new index 
\[ q = p - 2l \] for the summation over \( l = 0, 1, \ldots, \frac{p-1}{2} \) and 
\[ q = 2l - p \] for the summation over \( l = \frac{p+1}{2}, \frac{p+3}{2}, \ldots, p \), Eq. (28) becomes

\[
z(t) \approx u_0(t) + \lambda_1(t) \ast x(t) + v_1(t) \ast \lambda_1(t) \ast x^*(t) + \sum_{p=3}^{p=p+1} \lambda_p(t) \sum_{q=1}^{q=1} \sum_{m=0}^{m=0} \left( \binom{p+1}{m} \binom{p+1}{2} - \frac{m}{m} \right) \\
\cdot u^{m} x^{(m)}(t) |x(t)|^{p-q}
\]

\[
= u_0(t) + \lambda_1(t) \ast x(t) + v_1(t) \ast \lambda_1(t) \ast x^*(t) + \sum_{p=3}^{p=p+1} \lambda_p(t) \sum_{q=1}^{q=1} \sum_{m=0}^{m=0} \left( \binom{p+1}{m} \binom{p+1}{2} - \frac{m}{m} \right) \\
\cdot \left[ |u|^2 |x^{(m)}| |v|^{2m} u^{m} x^{(m)} x^*(t) |x(t)|^{p-q} + |v|^2 |x^{(m)}| |u|^{2m} v^{m} x^{(m)} x^*(t) |x(t)|^{p-q} \right]
\]

(29)

where the binomial equalities \( \binom{p+1}{m} = \binom{p+1}{p+1-m} \) and 
\( \binom{p+1}{m-1} = \binom{p+1}{m-1} \) are applied and the summation index 
\( m \) is substituted by \( \frac{p+1}{2} - m \) for the terms associated with the conjugated basis functions. From (29) we obtain the model parameters as expressed in Eqs. (11) to (14).

### Appendix C: MSE of Blind RX I/Q Imbalance Estimation

Letting \( \tilde{U}_{A}^{(l)}(f) = \tilde{V}_{A}^{(l)}(f) = 0 \) at \( l = 0 \), the solution of the difference equation (17) for \( l > 0 \) can be expressed as

\[
\begin{align*}
\tilde{U}_{A}^{(l)}(f) &= \mu \sum_{i=0}^{i=l-1} \left( 1 - \mu \right)^i \left( \tilde{U}_{A}^{(l-i)}(f) - \tilde{V}_{A}^{(l-i)}(f) \right) \\
\tilde{V}_{A}^{(l)}(f) &= \mu \sum_{i=0}^{i=l-1} \left( 1 - \mu \right)^i \left( \tilde{U}_{A}^{(l-i)}(f) + \tilde{V}_{A}^{(l-i)}(f) \right) \\
U_B(f) &= \mu \sum_{i=0}^{i=l-1} \left( 1 - \mu \right)^i \left( \frac{1 - \lambda |\Omega|^2}{1 - \lambda |\Omega|^2} e^{j2\pi (P+1) \Delta f T_i} + \frac{1 - \lambda |\Omega|^2}{1 - \lambda |\Omega|^2} e^{j2\pi (P+1) \Delta f T_i} \right) \\
V_B(f) &= \mu \sum_{i=0}^{i=l-1} \left( 1 - \mu \right)^i \left( \frac{1 - \lambda |\Omega|^2}{1 - \lambda |\Omega|^2} e^{j2\pi (P+1) \Delta f T_i} + \frac{1 - \lambda |\Omega|^2}{1 - \lambda |\Omega|^2} e^{j2\pi (P+1) \Delta f T_i} \right) \\
\tilde{N}_{i-l}(f) &= \left( \frac{1 - \lambda |\Omega|^2}{1 - \lambda |\Omega|^2} e^{j2\pi (P+1) \Delta f T_i} + \frac{1 - \lambda |\Omega|^2}{1 - \lambda |\Omega|^2} e^{j2\pi (P+1) \Delta f T_i} \right) \tilde{N}_{i-l}(f)
\end{align*}
\]

As \( l \to \infty \), we have

\[
\tilde{V}_{A}^{(l)}(f) \approx \frac{1 - \lambda |\Omega|^2}{1 - \lambda |\Omega|^2} V_A(f) + \frac{1 - \lambda |\Omega|^2}{1 - \lambda |\Omega|^2} U_A(f) + \frac{1 - \lambda |\Omega|^2}{1 - \lambda |\Omega|^2} U_B(f) + \frac{1 - \lambda |\Omega|^2}{1 - \lambda |\Omega|^2} V_B(f) + \frac{1 - \lambda |\Omega|^2}{1 - \lambda |\Omega|^2} \tilde{N}_{i-l}(f)
\]

(31)
where the term involving $V_B(f)$ is ignored as it is relatively small due to the product of $V_r(f)$ and $B_{1,1}^*(-f - \Delta f)$. Similarly, we have

$$U_A^*(f) \approx \frac{1 - \lambda |\Omega|^2}{1 - \lambda^2 |\Omega|^2} U_A(f), \quad l \to \infty$$  \hspace{1cm} (32)

after ignoring the terms involving $V_A(f)$, $U_B(f)$, $V_B(f)$, and noise. Finally, from (18) and assuming that $B_{1,1}^*(-f - \Delta f) \approx B_r(f)$ when $\Delta f$ is a small fraction of the bandwidth, we have

$$B_r^*(f) \approx B_r(f) + \frac{(1 - \lambda) \Omega}{1 - \lambda |\Omega|^2} e^{j2\pi [(P+1)\Delta f T_f + \frac{fT_f}{1 - \lambda |\Omega|^2}]} U_r(f) A_{1,1}(f - \Delta f) X^*(-f - \Delta f) \bigg( f - \Delta f \bigg)$$

$$+ \mu \frac{1 - \lambda |\Omega|^2}{1 - \lambda^2 |\Omega|^2} e^{j2\pi [(P+1)\Delta f T_f + \frac{fT_f}{1 - \lambda |\Omega|^2}]} U_r^*(f) A_{1,1}^*(-f - \Delta f) X^*(-f - \Delta f) \bigg( f - \Delta f \bigg)$$

$$+ \mu \sum_{i=0}^{l-1} \frac{(1 - \mu)^2}{1 - \lambda |\Omega|^2} e^{j2\pi [(P+1)\Delta f T_f + \frac{fT_f}{1 - \lambda |\Omega|^2}]} l \to \infty.$$  \hspace{1cm} (33)

Assuming that $|U_r(f) A_{1,1}(f - \Delta f) X^*(-f - \Delta f)|^2 \approx 1$ and $|U_r^*(f) A_{1,1}^*(-f - \Delta f) X^*(-f - \Delta f)|^2 \approx 1$ for simplicity, the MSE can be derived as shown in (19) after performing ensemble expectation over $|B_r^*(f) - B_r(f)|^2$. In deriving (19), (31), and (32), some exponential sum formulas\(^4\) are used.

REFERENCES


[5] European Telecommunications Standards Institute, “Harmonised standard for access to radio spectrum: fixed radio systems; characteristics and requirements for point-to-point equipment and antennas; part 2: digital systems operating in frequency bands from 1 GHz to 86 GHz,” ETSI EN 302 217-2, V3.3.1, October 2021.


\(^4\)The exponential sum formulas are $\sum_{n=0}^{N-1} e^{in\lambda} = \frac{1 - e^{iN\lambda}}{1 - e^{i\lambda}} = \frac{1 - e^{i\lambda}}{\lambda}$ and $\sum_{n=0}^{N-1} e^{in\lambda} = \frac{1 - e^{iN\lambda}}{1 - e^{i\lambda}} = \frac{2}{\lambda}$ for $0 < r < 1$ as $N \to \infty$. 


