S-parameter Extrapolation for Improving Near-fmax Accuracy in 2x-thru Calibration

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December 7, 2023

Abstract

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Index Terms—2x-thru calibration, extrapolation, S parameters.

I. INTRODUCTION

The 2x-thru de-embedding has become a popular method for calibrating printed circuit board (PCB) components, connectors, and cables [1-4]. While its function is similar to the classical TRL method [5-6], specifically to move the reference plane of S parameter measurement from coaxial to non-coaxial interfaces, 2x-thru only requires two measurements (Fig. 1) which saves time and PCB area greatly. The accuracy of 2x-thru calibration has proven adequate for practical applications [7-8]. As such, some high-speed protocols like USB4 [9] officially accept it for component testing.

There is a problem in 2x-thru calibration that is quite common but has attracted little attention. As illustrated in Fig. 2, near the edge of the highest measurement frequency \(f_{\text{max}}\), the calibrated S parameters often exhibit abnormal ripples or spikes. The deviation from the “expected” value is sometimes small but occasionally it may reach 5 dB or larger. Examples in the literature include [7, Annex N], [8, Figs. 27, 28], and [10, Sec. IV]. In some cases like [8], the de-embedded \(S_{11}\) may exceed 0 dB at \(f_{\text{max}}\), violating passivity. Hence, this kind of responses is certainly an error, rather than the true response of the device-under-test (DUT). As this situation always occurs near \(f_{\text{max}}\), people usually ignore it at all. However, presenting calibrated S parameters with such spurious responses has two problems: (i) it makes the overall accuracy of the calibration questionable, especially at high frequencies close to \(f_{\text{max}}\), and (ii) it may affect the pass/fail test results of the DUT for protocols like USB.

In this paper, we will first provide an explanation for the source of such spurious response in Sec. II. Then, according to the underlying mechanism, a solution is proposed in Sec. III. Finally, measurement examples validating the proposed method are given in Sec. IV.

II. SOURCE

Refer to Fig. 1. The S parameters of the THRU is given by

\[
S_{11}^{\text{THRU}} = S_{11} + \frac{S_{21}^2 S_{22}}{1 - S_{22}^2} \quad (1a)
\]

\[
S_{21}^{\text{THRU}} = \frac{S_{21}^2}{1 - S_{22}^2} \quad (1b)
\]

where \(S_{11}\), \(S_{21}\), and \(S_{22}\) denote the fixture S parameters, and Fixture A and B are assumed mirror symmetric. The standard procedure in 2x-thru calibration is to take the inverse Fourier transform (FT) of (1a) to obtain the impulse response of \(S_{11}^{\text{THRU}}\),
truncating multiple-reflection term, and then take FT back to
frequency domain to get $S_{11}$, as illustrated in Fig. 3. Knowing
$S_{11}$, the $S_{21}$ and $S_{22}$ can be algebraically solved from (1).

In practice, the measurement bandwidth is limited. Denote
$S^\text{THRU}_{11}$ the band-limited version of $S^\text{THRU}_{11}$, i.e.,
$S^\text{THRU}_{11} = 0$ for $f > f_{\text{max}}$. Note that the time-gating process is equivalent to
multiplying the impulse response of $S^\text{THRU}_{11}$ with a rectangular
window $w(t)$ with width $T$. The theory of Fourier analysis reveals
that the FT of the gated impulse response is equal to the
convolution of $S^\text{THRU}_{11}$ with $W(f)$, the FT of $w(t)$:

$$
\text{FT}(S^\text{THRU}_{11} \cdot w(t)) = S^\text{THRU}_{11} \ast W(f)
$$

where $W(f) = e^{-j\pi f T} \text{sinc}(f T)$. Because the function $	ext{sinc}(x)$
decays as $1/x$ away from the origin, if the frequency of
observation $f$ is low, then the final integration in (2) will be
close to the ideal band-unlimited case:

$$
\int_{-\infty}^{\infty} S^\text{THRU}_{11}(f') W(f-f') df' = \int_{-\infty}^{f_{\text{max}}} S^\text{THRU}_{11}(f') W(f-f') df' = S_{11}
$$

On the other hand, if the frequency of observation is close to
$f_{\text{max}}$, then the approximation of (3) obviously does not hold. In
particular, if $f = f_{\text{max}}$, then only the left-half of the sinc function
is accounted for in the convolution integral. Therefore, the $S_{11}$
result at $f = f_{\text{max}}$ is in general wrong. As the fixture’s $S_{21}$, $S_{22}$,
and the DUT $S$ parameters are all derived subsequently, they
will all be affected by the $S_{11}$ error at $f_{\text{max}}$.

Quantitatively, if $T = 1$ ns (which corresponds to a 15 cm
THRU assuming $c_{\text{eff}} = 4$), then the first null of $	ext{sinc}(f T)$ occurs
at 1 GHz. This implies that within 1 GHz from $f_{\text{max}}$, the
calibration results are generally unreliable. Further away from
$f_{\text{max}}$, the error reduces due to the decay of sinc. The magnitude
of the error is difficult to quantify, because that also depends on
the values of $S^\text{THRU}_{11}$ near $f_{\text{max}}$. Intuitively, if $S^\text{THRU}_{11}$ happens to
be a null at $f_{\text{max}}$, the error might be smaller.

III. Solution

According to the mechanism described in Sec. II, the
simplest solution would be to conduct measurement to a higher
frequency, say $f'_{\text{max}}$; then the calibration results at the original
$f_{\text{max}}$ will not suffer from the problem of incomplete convolution,
and will thus be more accurate. However, it will require a VNA
with higher frequency which may not be available, and the calibration results at the new $f'_{\text{max}}$ still face the same problem.

The proposed solution is to do high-frequency extrapolation
on $S^\text{THRU}_{11}$ before IFT to time domain. This is a natural idea in
view of the explanation given above. Suppose the $S^\text{THRU}_{11}$ is
extrapolated from $f_{\text{max}}$ to $f'_{\text{max}}$ by some method; the convolution
integral of (2) will then be performed up to $f'_{\text{max}}$. Now, if the
distance from $f_{\text{max}}$ to $f'_{\text{max}}$ is larger than $nT$, then when observed
at $f_{\text{max}}$, at least the main lobe and up to the $(n - 1)^{th}$ side lobe of
the sinc function are accounted for in the convolution integral.
It can thus be expected that, as long as the extrapolation is
reasonable, the error at $f_{\text{max}}$ will be smaller than the original
case where $S^\text{THRU}_{11}$ is truncated to zero after $f_{\text{max}}$. The flow chart
of the complete method is shown in Fig. 4.

A. Extrapolation Method

There are several commonly used extrapolation methods
such as constant extrapolation [11-12], constant amplitude and
linear phase extrapolation [13], linear extrapolation [14],
polynomial extrapolation [14-16], and vector fitting [17]. In
this paper, we do not attempt to find out the best way of
extrapolation, because that will require a tremendous amount of
validation data. In addition, as pointed out in [14], the “best”
choice of extrapolation method is often case by case. Instead,
we aim at demonstrating the feasibility of pre-extrapolation for
reducing the spurious response near $f_{\text{max}}$ after 2x-thru
 calibration. Accordingly, the $S$ parameters to be extrapolated
are restricted to that of the THRU cal-kits.

In this paper, we propose the following Sun-Earth-Moon
(SEM) fitting method. Consider the model:

$$
M(j\omega) = c_S + r_M e^{-j(\omega T_E + \phi_M)} + r_M e^{-j(\omega T_M + \phi_M)}
$$

When viewed on the complex plane, $M(j\omega)$ resembles the trajectory of a Moon orbiting an Earth which in turn orbits a
Sun. There are seven parameters in (4), where $c_S \in \mathbb{C}$ while
the others are real. The parameter $c_S$ can be interpreted as the
position of Sun, $r_M$ the orbital radius of Earth, $T_E$ the orbital
speed of Earth around Sun, $r_M$ the orbital radius of Moon, and $T_M$ the orbital speed of Moon around Earth.

The method consists of three steps: (i) choose a frequency $f_M < f_{\text{max}}$ (ii) determine a set of the seven parameters such that (4)
fits $S^\text{THRU}_{11}(f)$ satisfactorily in the range $f_M$ to $f_{\text{max}}$; finally (iii)
extrapolate $S^\text{THRU}_{11}$ by extending (4) from $f_{\text{max}}$ to $f'_{\text{max}}$.
The rationale of this SEM method is the passivity and
causality of physical $S$ parameters. A passive $S$ parameter stays
within the unit circle, and a causal $S$ parameter is analytic and
therefore its trajectory on the complex plane will be continuous
and smooth. Also, an $S$ parameter with time delay will circle
around the origin clockwise. As a result, within a small
bandwidth, the trajectory of an $S$ parameter is likely to be circle
shaped. In a larger bandwidth, it may not suffice to use a single
circle to represent the trajectory. Therefore, two circles are used in (4) — that of Earth and Moon. More complex models, such as adding more circle terms and also allowing the orbital radii to vary with frequency, are possible. However, experience shows that the model (4) already provides a large degree of freedom for the purpose of fitting practical S parameters within several GHz bandwidth. As the objective here is to extrapolate S parameters for only a few GHz, the use of (4) should suffice.

Also note that the model (4) is causal by itself, because it is analytic and bounded in the right-half plane [17]. It is contrary to some other simple methods like constant extrapolation which is noncausal by construction. When the extrapolated portion of
the SEM model is concatenated to the in-band data, the overall response may not be causal. Nevertheless, if the trajectory at $f = f_{\text{max}}$ is smooth, the extent of noncausality should be smaller than the non-extrapolated case (i.e., $S_{11}^{\text{THRU}}$ truncated to zero at $f_{\text{max}}$).

To demonstrate the flexibility of the SEM method, Fig. 5 shows six examples of the fitting and extrapolation results of real measured $S_{11}$ and $S_{21}$. The details of the figures and circuits are described in the captions.

In our implementation, the Python optimization module scipy.optimize.minimize is used to obtain a local optimum of all the parameters. The objective function is the 2-norm fitting error of $M(j\omega)$ to the raw $S$ parameter in the range $f_{\text{fit}} < f_{\text{max}}$:

$$\Psi = \sqrt{\sum_{f_{\text{fit}}}^{f_{\text{max}}}|S(j\omega_k) - M(j\omega_k)|^2}$$  \hspace{1cm} (5)

As the objective function is nonconvex relative to the seven parameters, a good initial guess is critical. The determination of $f_{\text{fit}}$ and the initial guess is detailed in the Appendix.

The points to observe from the results in Fig. 5 are: (i) over a broad bandwidth, the trajectory of an $S$ parameter (viewed on the complex plane) is complicated; however, in a small bandwidth, $S$ parameters are indeed locally circle-like. (ii) In all six cases, the model (4) can provide a fairly good fitting of the raw $S$ parameters in $f_{\text{fit}} < f_{\text{max}}$. For case (b), although the fitting does not seem perfect, the scale of the error is only 0.1 dB. (iii) The SEM model provides a smooth extrapolation for several GHz beyond $f_{\text{max}}$. This is sufficient for reducing the spurious response near $f_{\text{max}}$ after 2x-thru calibration.

Note that for the purpose of this study, it suffices to extrapolate only the $S_{11}$. The $S_{21}$ results given in Fig. 5(b)(d) are merely to demonstrate the versatility of the model (4).

IV. Validation

A. Case 1

We use the test circuit shown in Fig. 7 to validate the effectiveness of pre-extrapolation on stabilizing the 2x-thru calibrated results at $f_{\text{max}}$. The details of the circuit are given in the caption.

Differential signaling is used here. Though not introduced in Sec. II, the differential version of 2x-thru de-embedding [1] is essentially to perform the single-ended version twice, one for the differential mode (DM) $S$ parameters and the other for the common mode (CM) $S$ parameters. The algorithm assumes the THRU cal-kit possesses good symmetry, so the mode conversions are neglected. The problem of spurious response at $f_{\text{max}}$ shows up in the differential version as well.

The mixed-mode $S$ parameters of the THRU are shown in Fig. 8. The SEM fitting and extrapolation of the $S_{cc11}$ and $S_{dd11}$
Fig. 7. The test circuit used for validation in Sec. IV. It consists of two differential microstrip lines. The first one is 4.45 in long, taken as THRU, whereas the other is 8.9 in long, taken as FDUT, so the DUT is a 4.45-in pure differential transmission line. Both cases use 2.92 mm vertical-launch connectors to attach to the VNA cables. The measurement is conducted from 10 MHz to 20 GHz with 2000 points and IF bandwidth 1 kHz. (Other circuits shown in the photo are not used here.)

Fig. 8. The mixed-mode S parameters of the THRU circuit in Fig. 7.

Fig. 9. The fitting and extrapolation of the SEM model for (a) $S_{cc11}$ and (b) $S_{dd11}$ of the THRU circuit, up to 22 GHz (110% of $f_{max}$).

are shown in Fig. 9. Observe that the SEM model provides fairly good fitting in $f_H \sim f_{max}$ and smooth extrapolation for 2 GHz beyond $f_{max}$. Note also that the extrapolation of $S_{cc22}$ ($S_{dd22}$) is similar to that of $S_{cc11}$ ($S_{dd11}$), and that the $S_{cc21}$ and $S_{dd21}$ need not be extrapolated for 2x-thru calibration purpose.

The mixed-mode S parameters of the Fixture A of THRU obtained from 2x-thru calibration are shown in Fig. 10. The points to observe from Fig. 10 are: (i) Without extrapolation, the fixture $S_{cc21}$ and $S_{dd21}$ exhibit a 4- and 2-dB spurious response at $f_{max}$, respectively. The spurious responses in $S_{cc22}$ and $S_{dd22}$ reach 0 dB at $f_{max}$, violating passivity. There are also
sharp drops in $S_{cc11}$ and $S_{dd11}$ at $f_{\text{max}}$. (ii) With extrapolation, the spurious responses at $f_{\text{max}}$ for all $S$ parameters disappear, and the resulting $S$ parameters are seemingly more reasonable. (iii) Extrapolation does not affect the low frequency calibration results. The $S$ parameters with and without extrapolation agree well with each other up to 19 GHz for $S_{cc21}$ and $S_{dd21}$, to 18.5 GHz for $S_{cc11}$ and $S_{dd11}$, and to 15 GHz for the other two.

The mixed-mode $S$ parameters of the de-embedded DUT are shown in Fig. 11. The observations are similar to that from Fig. 10. In short, with $S$ parameter pre-extrapolation, the 5–10 dB spurious responses at $f_{\text{max}}$ in the un-extrapolated results can be removed, giving seemingly more reasonable responses. In addition, the calibration results at low frequency are unaffected by extrapolation.

B. Case 2

In Figs. 10 ad 11, there are some small difference between the calibrated results with and without pre-extrapolation above 15 GHz. Close to $f_{\text{max}}$, the difference turns larger. Above 19 GHz, the un-extrapolated results exhibit abnormal jumps, or surge to 0 dB, which are clearly incorrect. Between 15 and 19 GHz, however, we cannot ascertain which results are more accurate.

To compare the relative accuracy at frequencies high but not very close to $f_{\text{max}}$, we resort to the following method. First, the original THRU and FDUT $s4p$ files are truncated at 18 GHz, i.e., $f_{\text{max}} = 18$ GHz now. Next, the same procedure as in Case 1 is repeated. On one side, the standard 2x-thru algorithm is carried out on the “new” THRU and FDUT. On the other side, the $S_{dd11}$ and $S_{cc11}$ of THRU are pre-extrapolated to 19.8 GHz (110% of $f_{\text{max}}$) and the 2x-thru algorithm applied. Finally, the calibrated results of these two routes are compared against the results in Case 1, i.e., that from the $f_{\text{max}} = 20$ GHz data (without pre-extrapolation to 22 GHz). The rationale of this comparison is that, if pre-extrapolation (to 19.8 GHz) causes the outcome to differ in the band 13–17 GHz, then the results of Case 1 can serve as a reference. Results that are closer to the Case 1 responses can be considered more accurate.

The extrapolation results of the $f_{\text{max}} = 18$ GHz data are shown in Fig. 12. The original data ($f_{\text{max}} = 20$ GHz) are also shown so that the accuracy of the SEM extrapolation can be examined. For $S_{cc11}$, it can be observed that the SEM model provides a very accurate prediction (extrapolation) above 18 GHz. For $S_{dd11}$, on the other hand, the prediction is less accurate. Even so, however, the extrapolation is still much more meaningful than directly truncating to zero, i.e., the un-extrapolated case.

The mixed-mode $S$ parameters of the Fixture A of THRU obtained from 2x-thru calibration are shown in Fig. 13. The points to observe are: (i) without extrapolation, the calibrated results exhibit clear spurious responses at the new $f_{\text{max}}$, 18 GHz. (ii) With extrapolation, the spurious responses disappear. (iii) In almost all cases where the results with and without extrapolation differ, the ones with extrapolation are closer to the original $f_{\text{max}} = 20$ GHz results.

The mixed-mode $S$ parameters of the de-embedded DUT are

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Fig. 11. The mixed-mode $S$ parameters of the de-embedded DUT. (a) $S_{cc11}$ and $S_{cc21}$, (b) $S_{dd11}$ and $S_{dd21}$. The reference impedances are 25 Ω for CM and 100 Ω for DM.

Fig. 12. The fitting and extrapolation of the SEM model for (a) $S_{cc11}$, and (b) $S_{dd11}$ of the THRU circuit truncated at $f_{\text{max}} = 18$ GHz, up to 19.8 GHz (110% of $f_{\text{max}}$).
shown in Fig. 14. The observations are similar to that from Fig. 13. In short, with S parameter pre-extrapolation, the 5–10 dB spurious responses at \( f_{\text{max}} = 18 \) GHz in the un-extrapolated results can be removed, and in almost all cases where the results with and without extrapolation differ, the ones with extrapolation are closer to the original \( f_{\text{max}} = 20 \) GHz results.

To sum up, the results of Case 2 indicate that doing S
parameter pre-extrapolation before 2x-thru calibration not only reduces the spurious responses at \( f_{\text{max}} \) but also slightly improves the accuracy at lower frequencies.

C. Comparison with State-of-the-Art Tools

In above, we compare the de-embedding results with and without pre-extrapolation based on our own implementation of the 2x-thru algorithm. In this subsection, comparison of the de-embedding results using three industry 2x-thru tools is presented. The used tools include: Keysight AFR [19, version 2022.1.0], Clearsig SFD [22, version 2022.08.17], and SnpExpert [21, version 2023.01.5771]. To avoid advertising, we use Tool A, B, C to denote the three tools (not to order).

We use the circuit of Case 1 (\( f_{\text{max}} = 20 \text{ GHz} \)) for testing. The mixed-mode S parameters of the extracted Fixture-A are compared in Fig. 15. From the results, it is observed that (i) Tool A&B present spurious response near \( f_{\text{max}} \). In particular, the \( S_{\text{cc}21} \) and \( S_{\text{dd}21} \) run to 0 dB at 20 GHz. (ii) Tool C gives nearly the same results as the proposed method up to 19 GHz. (iii) Tool A&C and the proposed method agree well below 15 GHz. The mixed-mode S parameters of the de-embedded DUT are shown in Fig. 16, from which we find the same observations as above. Both Tool C and the proposed method do not exhibit spurious response near \( f_{\text{max}} \).

Except for the spurious response near \( f_{\text{max}} \) which violates passivity and is surely incorrect, we do not comment on the difference between the tools because all three tools do not offer publicly accessible documents that detail their algorithms. What can be said from the results in Fig. 15 and 16 are: (i) the consistency of the proposed method with Tool C up to 19 GHz and also with Tool A up to 15 GHz verifies that our 2x-thru code is correctly implemented. Combining with the results of the previous sections, the effectiveness of the pre-extrapolation method in reducing near-\( f_{\text{max}} \) spurious response and also improving the overall accuracy is therefore validated. (ii) The existence of near-\( f_{\text{max}} \) spurious response in Tool A, B indicates that the proposed method has the potential of advancing the state-of-the-art. (iii) Tool C does not exhibit the near-\( f_{\text{max}} \) problem. Perhaps it has implemented some kind of extrapolation inside, but there is no document to support this conjecture.

V. CONCLUSION

In this paper, the problem of spurious response at the highest measurement frequency \( f_{\text{max}} \) in 2x-thru calibration was studied in detail. The source of the abnormality was identified to be the implicit convolution of the sinc window with the truncated S parameters. Specifically, when the frequency of observation is close to \( f_{\text{max}} \), the effect of truncation on the convolution results becomes more significant.

Based on the mechanism of the spurious response, a solution was proposed which is to pre-extrapolate the S parameters before 2x-thru calibration. In particular, only the return losses need to be extrapolated. To that end, a novel SEM fitting method was proposed. In this method, the highest frequency portion of the raw S parameter is taken out and fitted with an analytic model with seven parameters. Optimization packages in Python was used to find out the local best solution of the parameters. Six examples based on measured S parameters are presented to demonstrate the feasibility and flexibility of the SEM method. In all cases, the SEM model is capable of providing a smooth extension of the trajectory for several GHz above \( f_{\text{max}} \).

A differential test circuit was utilized to validate the performance of pre-extrapolation on reducing the spurious response after 2x-thru calibration. In the calibrated fixture and DUT responses, it is all observed that with extrapolation, the

![Fig. 15. The mixed-mode S parameters of the Fixture-A of the THRU in Case 1 obtained from 2x-thru calibration by various tools. (a) \( S_{\text{cc}11} \) and \( S_{\text{cc}21} \), (b) \( S_{\text{dd}11} \) and \( S_{\text{dd}21} \). The reference impedances are 25 \( \Omega \) for CM and 100 \( \Omega \) for DM. The curves labeled “proposed” are the same as in Fig. 10 (the one with pre-extrapolation).](image)
spurious response at $f_{\text{max}}$ can be effectively removed. In addition, calibrated responses at low frequencies are in general unaffected by the extrapolation. In some situations where small difference presents between extrapolated and un-extrapolated results at lower frequencies, we have used a second test case to show that the results with extrapolation could be the one that is more accurate. The reason is simple: although one never knows whether the extrapolated response beyond $f_{\text{max}}$ is close to the actual response or not, it is very likely that the extrapolated response is closer to the actual one than the un-extrapolated case, i.e., one that directly truncated to zero.

**APPENDIX**

A. **Determination of the SEM parameters**

The main proposal of this paper is to use the SEM model to fit the high frequency portion of the raw S parameters and then continue running the model for several GHz to serve as an extrapolation, so that the spurious response at $f_{\text{max}}$ after 2x-thru calibration can be removed. There are various possible ways to construct the SEM fit. Here, we briefly introduce our implementation.

The determination of $f_{\text{fit}}$ is based on two criteria: (i) for return loss and near-end crosstalk terms, the front delay is small so the speed of Earth $T_E$ is small. On the other hand, the reflection from the far end has a longer delay so the speed of Moon $T_M$ is faster. Viewed on the complex plane, we will see the Moon orbiting the Earth several rounds while the Earth slowly orbiting the Sun such as Fig. 5(c). For this case, we choose $f_{\text{fit}}$ such that in the range $f_{\text{fit}} \sim f_{\text{max}}$, the Moon orbits the Earth 1~5 turns (lunar months). The rationale is that it is unlikely the SEM model (4) can accurately fit the S parameter for more than five lunar months, as is evident from the second plot of each case in Fig. 5. (ii) For insertion loss and far-end crosstalk terms, the front delay is large so the speed of Earth $T_E$ is large. On the other hand, the multiple reflection term is usually small which implies the radius of Moon $r_M$ is small. Viewed on the complex plane, we will see the Earth orbiting the Sun rapidly, often in an inward spiral shape due to loss, as in Fig. 5(b). For this case, we choose $f_{\text{fit}}$ such that in the range $f_{\text{fit}} \sim f_{\text{max}}$, the Earth orbits the Sun for roughly 80% of a circle, (i.e., 10 months). The rationale is similar: since the radius terms ($r_E$ and $r_M$) in the model (4) do not vary with frequency, it is unlikely to accurately fit the trajectory for more than “one year”.

Once a proper $f_{\text{fit}}$ is determined, we use the SVD-based Pratt’s circle fit algorithm [18] to obtain an estimate of the center (i.e., location of the Sun, $c_S$) and the Earth’s radius $r_E$.

Knowing the Earth’s trajectory, the radius of Moon $r_M$ can be estimated by the RMS deviation of the S parameter from the Earth. The speed of Earth $T_E$ can be estimated by the averaged slope of the unwrapped phase of the S parameter in the range $f_{\text{fit}} \sim f_{\text{max}}$, because it corresponds to the front delay of the response. The speed of Moon $T_M$ can be estimated by counting the turns the Moon circles the Earth in the range $f_{\text{fit}} \sim f_{\text{max}}$. In practice, $T_M$ is rather difficult to estimate due to the inevitable measurement noise. Therefore, several different values of $T_M$ are attempted and the one that results in the smallest fitting error is used.

Finally, the phase term $\phi_E$ can be estimated by fixing the position of Earth at $f_{\text{max}}$, while $\phi_M$ do not need an initial guess.

With the initial guesses for all parameters, the optimization engine in Python is invoked several times, and the final best fitting is taken out. The extrapolation results in Figs. 5, 9, and 12 are all obtained in this manner.
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