The graphene application for the Reconfigurable Intelligent Surface (RIS) design for the NOMA-RIS-MIMO system. Phenomenological view.

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Keywords

Graphene metasurface, RIS, PSWF eigenbasis, RIS Controller.

I. Introduction

Phenomenological modeling [1] of any system object instead of its physical modeling means that the object, as it is, might be reproduced as a “phenomenon” (i.e. how it is seen by an external “observer”) through its output characteristics, both statistical and deterministic. Sure, it might be named mathematical modeling as well.

As it can be seen from [2-6], NOMA-RIS-MIMO transmission system design was proposed as a completely based on phenomenological models. From several statements outlined there, it follows:

- **First**: an application of the General Kronecker Channel Model (GKCM) for the MIMO channel with fading and dispersion (see details in [2-3] and references therein).

Note that, in the framework of this model the matrix of the MIMO channel impulse response \( H(t,\tau) \) is represented in the following formal way:

\[
H(t,\tau) = V_{r_t} \left[ \tilde{\Omega} \cdot G(t,\tau) \right] V^H_{r_t} , \quad (1.1)
\]

where “H” indicates the Hermitian transpose; \( \tilde{\Omega} \) element wise square root of the coupling matrix of the channel (CM) (see [2-6, 13]); \( V_{r_t,rc} = \left[ U_1, ..., U_{n_{rc}} \right] \) orthogonalization matrixes.
taken at transmit and receive side [5], formed by the so-called Prolate Spheroidal Wave Functions (PSWF) as an “Universal Eigenbasis”; \( \text{reff} \) is and isolated group of eigenvalues corresponding to this eigen basis [5]; \( G(t, \tau) \) – matrix of identical and normalized Gaussian processes (see [5] and references therein for details). Note, that for known MIMO arrays, the universal PSWF eigenbasis is predefined (see straightforward analogy with the Fourier series) and completely represents the MIMO channel impulse response (see details in [5]). From the engineering perspective the PSWF eigenbasis is nothing else but artificial (virtual) trajectories of propagation phenomena.

As it is seen from (1.1) the GKCM MIMO model is absolutely “phenomenological” according to its description. Moreover, the orthogonalization approach applied for the presentation of \( V_{\text{TX,RX}} \) matrixes by means of the virtual (artificial) trajectories, formalized by the universal eigen basis (PSWF) [1-6], additionally illustrates this issue.

Another comment to phenomenological approach: to reproduce the model (object) only by its external characteristics, the input (excitation) signals must be fixed (or predefined). Then, the “connection” between the input and the output signals of the object might be characterized by means of the so-called “system functions”. Particularly for channel modeling (GKCM, etc.), the so-called P. Bello system functions (see, for example the impulse response function in (1.1)), are broadly applied [7] (see also [2-6], etc.).

- **Second**: chaotic modeling of the \{UE’s\}-NOMA users, after they pass the MIMO Channel with fading and dispersion. Particularly for the scenarios when the channel shows double selectivity features, it might be represented as Gaussian stochastic processes (see [2-6]). The latter is rather common for the dense networks of high-speed vehicles (HSV channels), typical for 5G+, 6G, etc. wireless communications. Chaotic modeling in these scenarios is rather opportunistic for identification, classification, etc. at the incoherent processing algorithms (see [2-4]) in the Rx terminals.

- **Third**: invariant (incoherent) methods of signal modulation and demodulation for users in order to achieve “robustness” for their noise immunity characteristics to the MIMO channel distortions such as Doppler shifts, frequency offsets, etc. Note, that such impairments are very harmful mainly in the double selective wireless channels such as HSV channels (see [2, 3, 6] and references therein). It is worth also stressing here that, all above mentioned concepts are formulated on the “phenomenological basement” [1] and are related to the physical phenomena, taking place in the NOMA-MIMO Channels.

Next: to significantly improve the main characteristics of the NOMA-MIMO Wireless transmission it was recently proposed the application of the “Reconfigurable Intelligent Surfaces” (RIS) to artificially “improve” the wave propagation features of the MIMO channel by creating a set of “beamforming” trajectories, which drastically improve the propagation features. Details can be found in [2-4] and in the references therein.

One must notice that, the current material is dedicated to the digital metasurface, based on graphene implementation for the RIS design, with strong application of the phenomenological (not physical) “ideology”. So, it is assumed that the previously mentioned issues and references are somehow “familiar” to the Reader.

Though, it might be clear now, that the following material will be completely dedicated to the digital formulation of the graphene implementation of the RIS, but based on the
phenomenological view. The latter is mainly motivated by the material published in [9, 11] (see also references therein) and reformulated hereafter according to the phenomenological concept. More concrete material will be presented in the next sections.

Though, the rest of the paper is organized as follows. Section II is totally dedicated to the digital representation of the metasurface. Section III presents the description of the multitrajectory radiation of the digital metasurface, excited by the input artificial trajectories in the form of Prolate Spheroidal Wave Functions (PSWF). Section IV is devoted to the RIS representation as a Planar Array (PA) model. Section V is dedicated to comments about future work and to concluding remarks.

II. Digital metasurface based on graphene implementation [10,11].

Application of metasurfaces in communication problems has a rather long history [9-11] and is usually defined as an artificial structure with subwavelength thickness and, as already mentioned, modeled as a Planar Array (PA) of periodic or quasi periodic subwavelength elements, certainly adjusted to determine the PA radiation properties. In [10, 11] it was stated that, to achieve reconfigurable properties of the metasurfaces it is reasonable to apply metamaterials, such as graphene with local tuning of its unit cells. For this purpose the “states” of a unit cell must be discretized to “convert” the metamaterial (graphene) into a digital metamaterial and the local states of each unit cell can be locally switched, introducing this metamaterial as almost ideal for RIS implementation, particularly for the terahertz frequency band [9-11]. The latter is rather prospective for 6G Wireless Communications [2, 3, 10, 11]. Taking then into account that, 6G communication networks will require large amounts of RIS blocks incorporated, for example, into the urban or rural buildings, and that graphene, as a material, is rather cheap for production, so, it is a good motivation to consider graphene implementation for RIS in the framework of the Incoherent Paradigm for NOMA-RIS-MIMO system design.

Based on the graphene, the sketch of the digital metasurface design is presented in the following. The interested reader can find more details in the already mentioned references [10, 11].

At the same, time one must notice that, the digital metasurface design needs to be “adjusted” to the earlier proposed [2, 3, 4] algorithms for RIS based on the concept of the MIMO Channel orthogonalization: virtual (artificial) trajectories in the form of PSWF, GKCM MIMO modeling, etc. [2, 4, 5] (see next section).

So, the unit cell of the graphene is a subwavelength thickness graphene layer located on the silicone base in different ways: full layer fusion, single (or dual) patch fusion etc. and it can be certainly characterized by different “equivalent circuit” models, formed by RLC circuits in the terahertz frequency band.

On the base of those RLC circuits as the equivalent circuits for unit cells (see figure 1 and figure 2 in [10]) one can see them, as a special case of the equivalent circuit of the general “transmission line” model of the media for wave propagation with the corresponding amplitude-phase characteristics. The latter can be applied obviously to make a digitalization of the graphene metasurface (see [10, 11]).

Another attempt (see the following) to make a digitalization is an application of antenna PA model for the metasurface, i.e. represent the array model of the PA graphene metasurface
through its partial arrays or antenna “lobs” or “partial diagrams” in the form of PSWF which follows from the orthogonalization of PA (see [5, 6], etc. and see also Figure 1). Each partial diagram can be digitalized and represented by a certain number of bits and therefore it coincides totally with the digital representation of unit cells and its states presented in [10, 11].

Though, if the metasurface is described by $p=1,P$ cells and $k=1,K$ states for each cell, then the Matrix $Q(p,k)$ is its formal description.

\[
\begin{align*}
\{ \theta_j, \phi_j \} & \quad \text{Incidental beams} \\
\{ \theta, \phi \} & \quad \text{Radiation beams} \\
\Sigma & \quad \text{Beam Addition}
\end{align*}
\]

Figure 1.

Remind that, as it was shown in [2-4], the RIS is completely characterized by the so-called Coupling Matrix (CM) and controlled in a specific way to obtain a set of required SNR’s values for the NOMA users {UE’s} in the Rx terminals to achieve their successful identification [2-4]. So, the $Q(p,k)$ matrix of the graphene metasurface of the RIS and the CM Matrix for the NOMA-RIS-MIMO system design must be “adjusted” by the digital controller of the RIS in the way, that the element of the CM associated with the required value of the $h_j^2$ (SNR) must be “produced” by certain state of the appropriate unit cell.

Finally, it must be commented that, for the NOMA-RIS-MIMO scenarios it must be considered the case of numerous excitations (incidental radiations) on the RIS metasurface and therefore many reflections (radiations) from the RIS [11].

The proposal how to do this is the principal “target” of the current material and it will be presented in the next section.
III. “Multitrajectory” description of graphene metasurface for RIS implementation.

Once more invoking the ideas of the phenomenological approach for graphene RIS model (see Figure 1), one can consider the following. The incident waves (radiations) coming to the RIS are excitations or activation signals for the Digital Graphene Surface collocated on the silicon layer. Being formalized through the $Q(p,k)$ matrix those waves activate certain unit cells, while each of them are characterized by a finite number of states, which are producing the radiation beams to the Rx terminal.

Though, how to establish those states and what might be its number? If the GKCM is applied as a NOMA-RIS-MIMO channel model, the incidental waves are a finite number of PSWF’s artificial trajectories beams from the Tx terminal “falling” on the RIS metasurface (see details in [2, 3]). Assuming, as also above, in the following double selectivity of the corresponding channels, all those beams might be described by means of Gaussian Random Processes [2, 3, etc.]. Each beam is accumulated by the signals of different NOMA users {UE}. Therefore, they are characterized by different statistical parameters. The same model might be applied for radiation beams from the RIS to the Rx terminals as well.

Hereafter it is not assumed that, only “activated” unit cells are “obliged” to generate radiation “beams” to the Rx terminal, as unit cells of the metasurface are somehow “connected” to perform as the Planar Array (see below) and radiation might be generated by another unit cell. So, generally different artificial trajectories are activated by different unit cells (see Figure 1). Moreover, being modeled as Gaussian Random Processes it is reasonable to associate the number of stages for each unit cell characterization to the Gaussian Process properties. Hereafter, it is proposed to associate the states with the numerical characterization of the level crossing numbers, for example, by three levels: 10%, 50% and 90% probabilities of crossing. Their definition is well known and tabulated [8]:

$$ F_G(x_{\alpha}) = \alpha, \quad (3.1) $$

where $F(x)$ is the Laplace function, $x_{\alpha}$ - is a level of the stage and

$$ \alpha = \begin{cases} 0.9 \\ 0.5 \\ 0.1 \end{cases} $$

It was already mentioned [10, 12] that, the incidental waves, as well as radiated ones (particularly, all of them in the form of PSWF), are supposed to be codified, for example by Alamouti OSTBC code for MIMO transmission, but this issue is apart from the present material.

Next, it is time to remind, that the considered graphene metasurface might be modeled as a Planar Array (PA). It was shown earlier [3, 4] that the PA radiation pattern can be successfully represented through the orthogonalization approach (with predefined mean square error (MSE)) by means of the universal PSWF eigenbasis. For simplicity, it is
convenient to apply, the PA approximation by non-uniform (or uniform) Linear Array (n-ULA, or ULA) in the covariance sense (see [5] for details).

Certainly, this PSWF representation must be associated with the closest unit cell together with their codes for the digitalized graphene metasurface of RIS. Somehow in advance, it is worth mentioning, that the methodology established in [10, 11] for unit cell design seems to be almost directly “shifted” to PSWF eigenbasis (by straightforward analogy of physical waves and artificial beams), but this is a subject of another material. Some “hints” corresponding to this are illustrated in the next section.

IV. Graphene RIS modelling as PA, approximated by n-ULA, or ULA.

As it was already pointed out (see [9-11], etc.) the metasurface might be defined for modelling, as a Planar Array (PA). Though it seems rather opportunistic to apply this model for the ongoing phenomenological description of the RIS based on graphene. In other words, it seems reasonable to get from the metasurface properties the adequate parameters for its PA model [9]. Actually, if the PA model is properly established, then the orthogonalization procedure for the PA together with its approximation by n-ULA → ULA is also well established (see [2, 3, 5]); So, it might be applied as a good option in the following.

To illustrate the basic principles of the PA orthogonalization approach, the material of this section will start with the necessary “extractions” from [5].

As one of the basic statements for such kind of problems (see also [5], for example), an arbitrary configuration of PA is assumed, that each element has a radiation pattern $G(\varphi, \theta)$, where $\varphi, \theta$ – are azimuth and elevation angles for the pattern. Its statistical characterization of the spatial covariance matrix element is $r_{k,l}$, where $k,l = 1,M$, and $M$ is the number of antenna elements. It is also assumed (see [5, 9]), that the elements of the PA behave themselves as electrically large aperture antennas, which in other words means, that, when considering them as “scatters”, they are much “bigger” than the angular dimensions of the antenna array.

With the help of these assumptions, it was shown (see details in [5]) that, the $r_{k,l}$ element of the covariation matrix of the PA can be presented as:

$$r_{k,l} = \text{const} \Re \left\{ \int \left| G(\theta) \right|^2 W(\theta) \exp \left\{ -j \frac{2\pi}{\lambda} (R_k \sin \theta_k - R_l \sin \theta_l) \sin \theta \right\} d(\sin \theta) \right\}, \quad 4.1$$

where $\theta = \left\{ \theta \left| G(\theta) \right|^2 W(\theta) \neq 0 \right\}$ is a limited set, $R_k, R_l$ are the distances to the $k,l$ elements of the PA.

So, the following main properties can be noted:

- $r_{k,l}$ of PA depends only on the $z$-projection (in rectangular coordinates) of the “antenna’s” position.

- Element $r_{k,l}$ might be seen as incomplete Fourier transform of the block $\left| G(\theta) \right|^2 W(\theta)$ where $W(\theta)$ is the PDF for $\theta$. In this regard, the latter means that its orthogonalization might be done with PSWF basis (see details in [5] and references therein).
- The covariance matrix element $r_{kj}$ of any PA coincides with the covariance matrix element of n-ULA with the same geometric parameters [5].

Though, the following proposition was formulated in [5]: the orthogonal decomposition of any PA can be done through the statistically equivalent n-ULA with the geometrical characteristics of the initial PA.

A further simplification, mainly for modeling purposes, might be done by introducing the **ULA equivalent** to the n-ULA by equalizing the 3db main lobe widths of n-ULA and ULA. One can see the above material as an illustration of the application of the phenomenological approach to a metasurface modeling, particularly for the RIS design, as the number of PSWF accepted for the orthogonalization are nothing else but “virtual” radiations from the RIS, which have to be adjusted to the basis $\{\text{PSWF}\}_{Ri}$ through the RIS Coupling Matrix (CM) [2,3,13].

Though, it might be once more emphasized, that the above material is mainly adequate for phenomenological modeling of the incoherent paradigm for NOMA-RIS-MIMO system. For the “physical” implementation of the system it is necessary to propose an implementation “bridge” between the phenomenological GKCM and the real MIMO channel. The way to achieve this is already proposed (see [6], for example).

Now, it is time to present the way to calculate the equivalent parameters of the PA model for the graphene metasurface.

For this matter it is reasonable to apply the results presented in [9] for the calculus (estimation) of the radiated field from the RIS, based on antenna theory. According to them, all the RIS surface is subdivided into elements $\{\Delta S_i\}$, which might be considered as antenna apertures of multi antenna system (PA), each with radiation pattern $f_i(\theta)$ (see also examples for them in [9]).

Certainly, if $\Delta S_i$ is an antenna aperture, the effective aperture $A_i$ with directivity $D_i$ is:

$$D_i = \frac{4\pi}{\lambda^2} A_i \leq \pi$$

where $\lambda$ is a working wavelength and $A_i \leq \Delta S_i$. In the following it is assumed that for all “$i$” $\forall \Delta S_i = \Delta S = (\Delta l)^2$, where $\Delta l$ is an element size, or longitude (estimated in terms of unit cell).

To neglect the “grating” lobes it might be assumed, that $\Delta S \leq \frac{\lambda^2}{4}$ [9]. So, finally one has the following equation system:

$$D_i = \frac{4\pi}{\lambda^2} A_i \geq 1$$
$$A_i \leq \Delta S$$
$$\Delta S \leq \frac{\lambda^2}{4}$$
$$\Delta S = (\Delta l)^2$$

$$\Delta S = (\Delta l)^2$$
From (4.3) it obviously follows that \( \Delta l \leq \frac{\lambda}{2} \), i.e. \( \Delta l = \delta \lambda \) where \( \delta_{\text{max}} \leq \frac{\lambda}{2} \) and \( \delta \) is [9]:

\[
0.28 \leq \delta \leq 0.5
\]  
(4.4)

From the examples of the radiation patterns, suggested in [9], it follows that \( \delta \) must be chosen as \( 0.5 \geq \delta \geq 0.4 \), and though \( D_i \leq \frac{\lambda}{4} \). The latter is rather close to \( \{D_i\} \) mentioned in SCMO 3 for MIMO antennas [12].

So, for the calculus of the PSWF set for the radiation of the RIS, it might be possible to apply the geometrical parameters of the ULA approximation from the PA. Finally, let us choose the following ULA data, which will be applied for the calculus of the PSWF radiations in Figure 2.

It was found (see also [12]) that, for \( D_i \sim 30^\circ \), it might be chosen a ULA model for PA-RIS with \( L = 15 \) elements, \( \frac{\lambda}{d} = 1 \), where \( d \) is the distance between elements. Next, assuming (only for simplicity) that (ULA)_Rx is the same as the “ULA model” for the RIS and assuming the Laplace approximation for PAS (Power Azimuth Spectrum) of the channel, to simplify the calculus of the “mutual” connections at the CM, the algorithm for the RIS controller is:

\[
\Omega_{n,n} = \frac{h_j^2}{(\varphi_n^2)^2 \lambda_n^2 \theta_{n,n}}
\]  
(4.5)

where \( h_j^2 \) – is a required SNR value for the successful identification of the \( j \)-th NOMA user UE, \( \varphi_n(\cdot) \) – are both PSWF \( n \)-th radiation and \( n \)-th virtual trajectories of the Rx antenna terminal, \( \theta_{n,n} \) – PAS value for the \( n \)-th beam, and \( \{\lambda_n\} \) – are eigenvalues for \{PSWF\}.

The simplified illustration, mentioned above, shows, that for the RIS design by means of graphene metasurface it is easy to concretize the general algorithm for RIS [2, 3]. To achieve the requirements for \( \{h_j^2\} \) from (4.5), it might be reasonable to apply (up to \( \Delta \theta_{\text{max}} = 25^\circ \)) the diversity addition of the radiations from different cells, which can improve the beams directivity for the \( j \)-th UE and certainly increase \( h_j^2 \).  

\[\text{see also comments from experiments in [10] as well}\]
For the digitalized (encoded) metasurface (see details in [10, 11] for angle $\theta$), identification of the PSWF beam, which depends on $\theta$ as well (see Fig. 2), with the “nearest unit cell” is trivial and can be easily applied in the RIS controller [2, 3, 13].

Now, it is time to mention one important statement, which directly follows from the material of the Sections II and III: the methodology for unit cells design, presented in [10, 11] might be almost straightforward applied in the framework of the phenomenological approach by “shifting” the “physical” radiations to PSWF artificial beams!

Moreover, to fulfill the requirements for the SNR’s $\{h_j^2\}$ [4] for {UE’s} identification, incoherent quadratic addition of the different unit cells radiations can be applied, which corresponds to different PSWF beams of the unit cell. So, it makes the RIS controller (see (4.5)), not only simple, but effective and flexible as well.

V. Comments for the future and some concluding remarks.

As it follows from the presented material, the graphene implementation of the RIS metasurface can be successfully analyzed and completed in the framework of the phenomenological approach to the general view of the incoherent NOMA-RIS-MIMO transmission system design. It sounds rather opportunistic, as RIS for the future 6G and beyond systems needs to be a broadly demanded device, distributed along the service zone, both rural and urban.

In [2, 3] it was already pointed out that, the MIMO channel orthogonalization method applied for the incoherent paradigm of the system design allows mainly not only to
“compensate” the energetic losses due to the “incoherency” of the processing algorithms but, significantly diminish their computational complexity, which might be seen as an important issue considering the remote RIS applications.

Processing the virtual trajectories in a separate (independent) fashion together with their “diversity addition” in the graphene metasurface controller (see the experiment in [9], for example) provides a flexible, simple and effective evaluation of the \( \{ h_i^2 \} \), required for \{UE’s\} identification (classification) (see [2, 4] for details).

Sure, all these statements, which look rather attractive for practice, need to be proved experimentally in the future.

Hereafter, several conclusions will be presented, which follow directly from the presented material and might be attractive for the reader.

- Discretization and coding of the graphene metasurface of RIS provides effective processing of the virtual PSWF trajectories (beams) both at Rx terminals and at RIS, which forms a **unique** dissipated control system, but certainly divided in the space.

- Modeling the RIS metasurface as PA allows to provide its “orthogonalization” analysis, based on the “universal” eigenbasis \{PSWF\}, in the framework of the incoherent paradigm for transmission system design.

- The PSWF virtual trajectories formed at the Tx and Rx terminals as a part of the phenomenological GKCM MIMO channel model must be encoded in the same way, as it was performed for the RIS radiation patterns, while digitalizing the RIS metasurface.

- Considering, that the number of the PSWF artificial radiated patterns is always **rather limited**, the implementation of the RIS with the controller algorithm (4.5) looks computationally simple, flexible and effective: though opportunistic for practice.

- It is time for the statement that, it was initially clear for the author, that the above presented material, as well as that in [2, 3, etc.], is somehow “unusual” for many readers, but there is a strong hope, that the interested reader, comparing it with the material in [9-11, etc.], will easily figure out that it is “somehow similar” to [9-11] with only one difference: the analysis must be “shifted” to the space of the “virtual concepts” of radiations, beams, propagation issues, etc. by applying the orthogonalization approach, which simply must be figured out as it is very similar to the Fourier series!.

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