Heterogeneous Unknown Multi-Agent Systems of Different Relative Degrees: A Distributed Optimal Coordination Design

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Abstract

This paper addresses the distributed optimal coordination problem for a network of heterogeneous agents. To the best of our knowledge, few works where the control design requires no information regarding agents dynamics can be found in the literature. Compared to these works in which agents are only allowed to be of relative degree one, this paper extends the results for networks that are mixtures of agents with different relative degrees. The main goal is that all agents’ outputs globally converge to the minimizer of the global cost function. We introduce a novel distributed two-layer control policy to do so. The top layer searches for the minimizer and generates reference signals for the bottom layer. The top layer is identical for all agents. The bottom layer provides each agent with an adaptive controller, enabling the agent to track its associated reference signal. Local cost functions are assumed to be strictly convex and have smooth gradients. The proposed control policy is fully distributed, meaning agents only depend on their own and their neighbors’ information to reach a consensus on the global minimizer. Numerical simulations validate the theoretical results.
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Index Terms—Multiagent Systems, Cooperative Systems, Distributed Control, Optimization, Uncertain Systems, Adaptive Control

I. INTRODUCTION

Multi-Agent Systems (MAS) have been utilized recently in a wide variety of different applications such as robotics[1], power systems[2], sensor networks[3], air traffic control[4], and so on. Many different distributed control protocols have been developed so far to ensure agents reach consensus, which is the most fundamental objective in MASs, despite various real-world constraints such as time-varying communication topology, communication delays, limited bandwidth, and saturation in agents’ actuators. However, observing some sense of optimality is an essential portion of control goals for most real-world applications. Optimality in MAS is interpreted in several ways, which causes several distinct fields of study to emerge, such as Distributed Optimization Problems (DOP)[5], Cooperative Optimal Control Problems[6], Differential Games[7], and Differential Graphical Games[8].

Distributed Optimal Coordination

In DOP, agents aim to calculate the minimizer of a global cost function, which is the summation of the agents local cost function. Attaining this goal must be done through a distributed algorithm. This means that the algorithm must be designed so that each agent only requires information regarding its local cost function and the information transmitted by its neighbors to implement the algorithm. DOP has been investigated for a long time. Some pioneering works in this field of study can be found in the context of parallel and distributed computation [9], [10]. This decade has witnessed a surge in researchers’ interest in DOP owing to its growing renewed applications in communication networks, machine learning, and sensor networks, to name a few[11]. Thus, several significant papers have been published recently proposing various combinations of consensus-based and gradient-based approaches to deal with DOP [12]–[15].

The emergence of Cyber-Physical Systems (CPSs), where computation/communication (cyber) parts and dynamic/control (physical) parts are integrated to perform desired tasks, has extended the notion of DOP. Many real-world CPSs, such as power systems, a team of robots, UAVs or UGVs, and sensor networks, can be categorized as MASs consisting of physical dynamic agents. Investigating DOP for dynamic MASs is often termed Distributed Optimal Coordination (DOC) by researchers in the control community. There exists a marked difference between DOC and conventional DOP. When it comes to studying DOC, the designer must take into account the agents dynamics, which is not the case in conventional DOPs. Regarding DOC, computing the minimizer is only a portion of an optimization algorithm role and the algorithm must control the state of agents, which are physical variables, in such a way that they achieve consensus on the minimizer. Consequently, the traditional algorithms for DOP [12]–[15] are not viable for DOC.

Generally speaking, existing algorithms for DOC can be categorized into 1) double-layer open-loop and 2) integrated closed-loop classes. The first class takes advantage of a traditional DOP algorithm as its top layer, often called the cyber layer, computing the minimizer and generating a reference trajectory for the bottom layer. In the bottom layer, often referred to as the physical layer, each agent is equipped with a controller enabling the agent to track the trajectory provided by

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the top layer and finally reach the optimal point [16]. Although each of these two layers has been broadly investigated independently, combining them for DOC is a daunting task because they become dependent on each other in this case. The difficulty stems from the fact that the dynamics of agents only permit them to track particular trajectories, which means the top layer is only allowed to generate tractable reference signals. In contrast, in this class, the bottom layer does not provide the top one with any feedback [17]. Despite the first class, in the integrated closed-loop structure agents are not provided with reference trajectories; instead, the control inputs, which are constructed utilizing the MAS states or outputs, explore the optimal point and steer agents toward this point simultaneously. This method has been widely employed in the literature for various dynamic MASs, including but not limited to single/double integrator MASs [18]–[20], high-ordered MAS [21]–[23], Euler-Lagrangian interconnected systems [24]–[26], nonlinear MAS [27], [28].

**Motivation and Contribution**

In many industrial applications, a network of mixed agents of heterogeneous dynamics must interact with each other to realize global tasks. Furthermore, heterogeneous MASs possess more capability to perform complicated missions; thus, these systems might perform more satisfactorily when working in environments including considerable diversities [29]. Take the example of a network of UGVs and UAVs cooperating to explore an environment encompassing terrestrial and aerial areas [30]. UGVs are often considered second-order systems, and the UAVs are often modeled as higher-order systems [31].

Several remarkable works investigating the DOC problem for heterogenous MASs can be found in the literature [17], [32]–[35]. Authors in [34] proposed a solution assuming the gradients of local cost functions follow a specific form. In [17], a more general structure is considered for the gradients of local cost functions. Instead, agent system matrices must feature a set of algebraic conditions. Authors in [32] relaxed these conditions. Furthermore, their DOC algorithm ensures exponential convergence. In [35], the authors presumed the same condition on agents dynamics as [17]. They developed a distributed control protocol to address the resource allocation application under inequality constraints. The main contribution of [33] is to remove the restricting conditions on the agents dynamics. To do so, the authors exploited the double-layer open-loop structure (first class) to develop their optimization algorithm in contrast with algorithms presented in [17], [32], [34], [35] utilizing the integrated closed-loop structure (second class). They derived conditions ensuring the reference trajectories generated by the top layer are tractable by the physical agents.

However, the papers mentioned so far do not address the DOC problem when there exist some uncertainties that deplete the effectiveness of their results in practice because a considerable portion of real-world applications is subjected to uncertainties or contains some unknown parameters. Recently, some research has been conducted to address this issue [36]–[39]. Authors in [36] offered an adaptive distributed optimization algorithm to deal with the case in which the Lipschitz constants and network connectivity are unknown. In [39], authors considered a DOC problem where the gradients of the local cost functions are unknown. They addressed this problem for a network of agents with Euler-Lagrange dynamics. Authors in [38] considered a MAS composed of nonlinear agents whose dynamics can be represented by parametric strict-feedback form. They addressed the same problem for this MAS as the one in [39]. To do so, they proposed an adaptive backstepping distributed optimization algorithm. However, the agents dynamics are assumed to be known in these three works.

The DOC problem for Euler-Lagrange MASs in which the inertial parameters are unknown is studied in [37]. In [16], authors considered agents with uncertain high-order nonlinear dynamics. They designed a double-layer DOC algorithm based on the high-gain approach for this network. Their proposed algorithm needs the uncertain part to be constrained by a linear growth condition which greatly restricts the generality of the algorithm. Furthermore, the algorithm can only guarantee that the agents converge to a small vicinity of the global minimizer.

This paper considers an LTI heterogeneous MAS composed of a mixture of agents with different relative degrees. We introduce a novel distributed control policy taking advantage of a double-layer open-loop structure. The top layer is a distributed double integrator optimizer. In the bottom layer, each agent is provided with an adaptive controller enabling the agent to track the reference trajectory despite the unknown dynamics of the agent. It should be noted that LaSalle’s Invariance Principle, Lyapunov theorem, and Barbalet lemma play vital roles in proving our design’s global convergence and stability.

**Our contributions to improving the existing results can be listed as follows:**

- Despite the results in [17], [32]–[35], which consider heterogeneous MASs without uncertainties or unknown parameters, this paper takes into account the heterogeneous unknown agents case.
- Different from [36], [38], [39], in which the considered uncertainties are not associated with the agents dynamics, this paper deals with the case where the dynamics of agents are entirely unknown. In [37], agents are Euler-Lagrange, not encompassing LTI systems considered in this work. In addition, in [37], only one unknown parameter exists in the agents models, while in this work, all system matrices are assumed to be unknown.
- The agents dynamics considered in [16] does not encompass general LTI form. Furthermore, the results only provide approximate consensus. However, our presented results can be applied to more general form of LTI MAS and also ensures asymptotically consensus on the minimizer.

**Paper Organization**

The remainder of this work is arranged as follows. In Section II the preliminaries are presented. Section III includes the formulation of the DOC problem. Section IV offers a novel distributed adaptive control policy and elaborates on its convergence and stability properties. Section V provides and discusses numerical examples to verify the correctness of the theoretical results. Finally, Section VI is devoted to the conclusion.
II. PREMIMARIES

Notation

diag(m_1, m_2, ..., m_k) ∈ ℜ^{k×k} denotes a diagonal matrix in which ith diagonal entry is equal to m_i. If M_i ∈ ℜ^{p_i×q_i},

diag(M_1, M_2, ..., M_k) ∈ ℜ^{∑(p_i×q_i)×∑(k×q_i)} is a block diagonal matrix whose ith entry equals to M_i. col(b_1, b_2, ..., b_k), where b_i ∈ ℜ^{p_i×1} i = 1, 2, ..., k, collects k real vector in a new vector. Y > 0 (or Y ≥ 0) means square matrix Y is positive (or semi-positive) definite. For a differentiable function h: ℜ^{p×1} → ℜ, ∀h stands for its gradient.

Basics on Graph Theory

The communication network between agents is described by a graph G(V, E, A) where each of its nodes is associated with an agent. V = {v_1, v_2, ..., v_N} is the set of the graph’s nodes in which N denotes the number of agents in the network. The communication links between nodes are included in the set E ⊆ V × V where (v_i, v_j) ∈ E shows that ith agent directly receives information from jth node. In this paper, the communication graph G is assumed to be undirected and connected. The formal means if (v_i, v_j) ∈ E, then (v_j, v_i) ∈ E and based on the latter, at least one communication path exists between any pairs of agents. The adjacency matrix associated with graph G is denoted by the square matrix A = [a_{ij}] ∈ ℜ^{N×N} in which a_{ij} = 1 when (v_i, v_j) ∈ E, otherwise a_{ij} = 0. Moreover, self-loops do not exist in G meaning that a_{ii} = 0. The associated Laplacian matrix is presented by the matrix L = [l_{ij}] ∈ ℜ^{N×N}. In this matrix, l_{ij} = -a_{ij} for i ≠ j, and l_{ii} = ∑_{j=1}^{N} a_{ij}. Since the communication graph is undirected and connected, L has N - 1 eigenvalues with positive real parts and one zero eigenvalue which its associated eigenvector is 1_N satisfying 1_N L = 0.

Basics on Optimization Theory

Definition 1: Differentiable function h: ℜ^{p×1} → ℜ is strictly convex if and only if (∇h(x) − ∇h(y))^T (x − y) > 0 [21].

Lemma 1: If h(y) is twice differentiable function i.e. ∇^2h(y) exists and ∇^2h(y) > 0, then h(y) is strictly convex [21].

Lemma 2: y^* is the unique minimizer for the strictly convex function h(y) if and only if the following equation holds [21].

\[ ∇h(y^*) = 0 \] (1)

III. PROBLEM STATEMENT

A MAS of N agents sharing their information through a communication network is considered as follows.

\[ \dot{x}_{p_i} = A_{p_i} x_{p_i}(t) + b_{p_i} u_i(t) \]
\[ y_{p_i}(t) = c_{p_i}^T x_{p_i}(t) \]
\[ i \in \{1,2, ..., N\} \]

Where \( x_{p_i} ∈ ℜ^{n_i×1} \) is ith agent state vector, \( u_i ∈ ℜ \) is the control input, and \( y_{p_i} ∈ ℜ \) is the output. The system matrix \( A_{p_i} ∈ ℜ^{n_i×n_i} \), input vector \( b_{p_i} ∈ ℜ^{n_i×1} \), and output vector \( c_{p_i} ∈ ℜ^{n_i×1} \) describe the ith agent state-space model. The ith agent’s transfer function is given by:

\[ \frac{L[y_p(t)]}{L[u_i(t)]} = G_{pi}(s) = \frac{k_p}{s^{n_i} + b_i s^{n_i-1} + ... + b_0} \] (3)

The state-space model and the coefficients \( a_j, b_i \), and the high-frequency gain \( k_{pi} \) are unknown for all agents.

Assumption 1: \( s \operatorname{gn}(k_{pi}) \) is known.

Assumption 2: The relative degree of each agent transfer function i.e. \( n_i^* = n_i - w_i \) are assumed known and also \( n_i^* = 1, 2 \).

Assumption 3: The zeros of each agent transfer function are located in the open left-half plane.

Assumption 4: The communication graph between agents \( G \) is assumed undirected and connected.

This paper’s main goal is to propose a solution for the following DOC problem.

Problem 1: Design a distributed control protocol \( u_i, i ∈ \{1,2, ..., N\} \) for the MAS (2) guarantees output consensus on the optimal solution of the following optimization problem.

\[ \min_{i=1}^{N} \sum_{i=1}^{N} f_i(y_{p_i}) \quad y_i ∈ ℜ \] (4)
\[ s.t. y_{p_k} = y_{p_j} \quad for \ ∀k, j ∈ \{1,2, ..., N\} \]

Where \( f_i: ℜ → ℜ \) stands for ith agent local cost function.

The term “distributed” refers to the case that the output of each agent is only affected by its own information and its neighbors’ information in the network through its control protocol.

We make the following assumption on agents’ cost functions.

Assumption 5: local cost functions \( f_i \) is twice differentiable and \( \nabla f_i > 0 \). Furthermore, there exists \( \mu ∈ ℜ^+ \) such that \( |\nabla^2 f_i| ≤ \mu \).

Remark 1: According to Lemma 1 and Lemma 2, Assumption 5 ensures \( y^* \) minimizes the global cost function if and only if the following equation holds. Furthermore, \( y^* \) would be unique.

\[ \sum_{i=1}^{N} \nabla f_i(y^*) = 0 \] (5)

IV. MAIN RESULTS

In this section, a distributed control protocol steering MAS (2) toward output consensus on \( y^* \) is introduced inspired by [21]. Some definitions and lemma must be presented first.

Let us define the following two auxiliary systems.

\[ \dot{\psi}_{1i} = \Pi^1 \dot{\psi}_{2i}(t) + l_i u_i(t) \]
\[ \dot{\psi}_{2i} = \Pi^2 \dot{\psi}_{2i}(t) + l_i y_{p_i}(t) \] (6)
Where $\psi_{i1}, \psi_{i2} \in R^{n_i - 1}$. $\Pi_i$ is Hurwitz and $(\Pi_i, l_i)$ is controllable. In addition, the following reference MAS is required:

$$
\begin{align*}
\dot{q}_i(t) &= \rho_i \\
\dot{\rho}_i &= r_i \\
\dot{v}_i &= \sum_{j=1}^{N} \alpha_{ij} \left( (q_i - q_j) + (\rho_i - \rho_j) \right) \\
r_i &\triangleq -\sigma_{\Delta_1}(\rho_i) - \\
\sigma_{\Delta_2} &\left( \sum_{j=1}^{N} \alpha_{ij} \left( (q_i - q_j) + (\rho_i - \rho_j) \right) \right) - \\
\sigma_{\Delta_3} &\left( v_i + \kappa\nabla f_i(q_i) + \rho_i \right) - \sigma_{\Delta_4} \left( \kappa V^2 f_i(q_i) \rho_i \right)
\end{align*}
$$

Where $q_i, \rho_i, v_i \in R$. $\sigma_{\Delta}(s): R \rightarrow R$ denotes saturation function with threshold $\Delta > 0$, meaning $\sigma_{\Delta}(s) = sgn(s) \min(|s|, \Delta)$. Furthermore, $\Delta_1, \Delta_2, \text{and } \Delta_3$ satisfy the following inequalities.

$$
\begin{align*}
\Delta_1 &> N(\Delta_2 + \Delta_3 + \Delta_4) & (a) \\
\Delta_2 &> N(\Delta_3 + \Delta_4) & (b) \\
\kappa &\leq \frac{\Delta_4}{\mu_\Delta_1} & (c)
\end{align*}
$$

Now, we are ready to propose our main results.

**Theorem 1:** In unknown SISO heterogeneous MAS (2) composed of agents with $n^* = 1, 2$, if Assumption 1-3 hold, following adaptive distributed control policy, solves the DOC Problem 1.

$$
\begin{align*}
u_i &= \eta_i^T \psi_i + \epsilon_i \\
\psi_i &= \left[ \eta_i \right] \text{T} \quad \text{for agents with } n^* = 1, \eta_i \text{, and } r_i \text{ is defined as follows:} \\
\epsilon_i &\equiv 0 \\
\eta_i &\triangleq -sgn(k_{pi}) \sum_{i} (y_{pi} - q_i) \psi_i \\
r_i &\triangleq -\delta_i r_i + \bar{r}_i
\end{align*}
$$

For agents with $n^* = 2, \epsilon_i, \eta_i$, and $r_i$ is introduced as follows:

$$
\begin{align*}
\epsilon_i &\equiv 0 \\
\eta_i &\triangleq -sgn(k_{pi}) \sum_{i} (y_{pi} - q_i) \bar{\psi}_i \\
r_i &= \bar{r}_i
\end{align*}
$$

In both cases, $\Sigma_i$ is a positive definite matrix and $\bar{r}_i$ is defined as:

$$
\bar{r}_i \triangleq (\alpha_i \delta_i) q_i + (\alpha_i + \delta_i) \rho_i + r_i 
$$

Where $\alpha_i$ and $\delta_i$ are positive scalers. Furthermore

$$
\dot{\bar{\psi}}_i = -\delta_i \bar{\psi}_i + \psi_i 
$$

**Proof:**

The proof includes four main steps. In the first step, it is indicated that the equilibrium point of reference MAS (7), $(q_i^*, \rho_i^*, v_i^*)$, solve the minimization problem (4), i.e. $q_i^* = y^*$, where $y^*$ is solution for (4). Next, using LaSalle’s Invariance Principle, we prove that the reference MAS system (7) achieves a consensus on its equilibrium point, i.e. $\lim_{t \rightarrow \infty} q_i(t) = y^*$.

Third, the proof of boundedness of $\varepsilon_i = y_{pi} - q_i$ is provided. Finally, taking advantage of Barbalet Lemma we demonstrate that $\lim_{t \rightarrow \infty} \varepsilon_i = 0$ meaning that $y_{pi}$ converges to $y^*$.

**Step 1:** The compact form for the reference MAS (7) is

$$
\begin{align*}
\dot{q} &= \rho \\
\dot{\rho} &= -\sigma_{\Delta_1}(\rho) - \sigma_{\Delta_2}(L(q + \rho)) - \\
\sigma_{\Delta_3} &\left( v + \kappa\nabla F(q) + \rho \right) - \sigma_{\Delta_4} \left( \kappa V^2 F(q) \rho \right) \\
\dot{v} &= L(q + \rho) \\
\dot{\psi} &= 0
\end{align*}
$$

Where $\psi = \text{col}(q_1, q_2, \ldots, q_N)$.

**Step 2:** This section is devoted to proving the asymptotic stability of the equilibrium point of the reference MAS (7).

By defining $m \triangleq [v^T \ Q^T \ \rho^T]^T$, we can write the compact form of (7) as follows:

$$
\begin{align*}
\dot{m} &= \left[ 0 \ L \ L \right] m + \left[ 0 \right] r \\
T &= \left[ I_N \ 0 \ I_N \right]
\end{align*}
$$

Applying the transformation (22) on (21) derives the following state space equation.
\[
\hat{\bar{m}} = \begin{bmatrix} 0 & L & 0 \\ 0 & 0 & I_N \\ 0 & 0 & 0 \end{bmatrix} \bar{m} + \begin{bmatrix} I_N \\ I_N \end{bmatrix} r 
\]

(23)

in other words

\[
\begin{align*}
\hat{\bar{q}} &= \bar{\rho} + r \\
\hat{\bar{\rho}} &= r \\
\hat{\bar{v}} &= L\bar{q} \\
r &= -\sigma_{\Delta_1}(\bar{\rho}) - \sigma_{\Delta_2}(L\bar{q}) \\
-\sigma_{\Delta_3}(\bar{v} + \kappa\nabla F(q)) &= -\sigma_{\Delta_4}(\kappa\nabla^2 F(q)\bar{\rho})
\end{align*}
\]

(24)

First, we concentrate on \(\bar{\rho}\) and propose the following Lyapunov candidate to investigate the stability properties of \(\bar{\rho}\).

\[
V_{\bar{\rho}} = \frac{1}{2} \bar{\rho}^T \bar{\rho}
\]

(25)

Taking time derivative from (25) results in the following equation.

\[
\begin{align*}
\dot{V}_{\bar{\rho}} &= -\bar{\rho}^T \sigma_{\Delta_1}(\bar{\rho}) - \bar{\rho}^T \sigma_{\Delta_2}(L\bar{q}) \\
&\quad -\bar{\rho}^T \sigma_{\Delta_3}(\bar{v} + \kappa\nabla F(q)) - \bar{\rho}^T \sigma_{\Delta_4}(\kappa\nabla^2 F(q)\bar{\rho})
\end{align*}
\]

Next, we define the following set.

\[
Q_{\bar{\rho}} = \{\bar{\rho} \in \mathbb{R}^N \| \bar{\rho} \|_\infty \leq \Delta_1\}
\]

(27)

Thus for \(\bar{\rho} \notin Q_{\bar{\rho}}\), the following inequality holds.

\[
\dot{V}_{\bar{\rho}} \leq -\Delta_1^2 + N\Delta_2\Delta_3 + N\Delta_3\Delta_4
\]

(28)

From (8), the following result is obtained.

\[
\dot{V}_{\bar{\rho}} < 0
\]

(29)

which means \(\bar{\rho}\) enters \(Q_{\bar{\rho}}\) and remains in this set.

According to Assumption 5 and (8), the following result holds

\[
\|\kappa\nabla^2 F(q)\bar{\rho}\|_\infty \leq \kappa \mu \Delta_1 \leq \Delta_4
\]

(30)

This means \(\sigma_{\Delta_4}(\kappa\nabla^2 F(q)\bar{\rho})\) becomes equal to \(\kappa\nabla^2 F(q)\bar{\rho}\) while \(\bar{\rho}\) enters \(Q_{\bar{\rho}}\). Thus, the following dynamics will govern \(\bar{q}\) after a while.

\[
\begin{align*}
\hat{q} &= -\sigma_{\Delta_2}(L\bar{q}) - \sigma_{\Delta_3}(\bar{v} + \kappa\nabla F(q)) - \kappa\nabla^2 F(q)\bar{\rho}
\end{align*}
\]

(31)

Next, we aim to investigate the stability properties of \(L\bar{q}\). The following Lyapunov candidate is considered to do so.

\[
V_q = \frac{1}{2} \bar{q}^T L\bar{q}
\]

(32)

It is evident that \(V_q \geq 0\) and \(V_q = 0\) if and only if \(L\bar{q} = 0\) owing to the fact that the communication graph is connected. The time derivative of (32) along its trajectories results in the following equation:

\[
\dot{V}_q = -(L\bar{q})^T \sigma_{\Delta_2}(L\bar{q}) - (L\bar{q})^T \sigma_{\Delta_3}(\bar{v} + \kappa\nabla F(q)) \\
&\quad - (L\bar{q})^T \kappa\nabla^2 F(q)\bar{\rho}
\]

(33)

We define the following set.

\[
Q_q = \{\bar{q} \in \mathbb{R}^N \| \|L\bar{q}\|_\infty \leq \Delta_2\}
\]

(34)

Thus for \(\bar{q} \notin Q_q\), the following inequality holds.

\[
\dot{V}_q \leq -\Delta_2^2 + N\Delta_2\Delta_3 + N\Delta_3\Delta_4
\]

(35)

Based on (8), the following result is obtained.

\[
\dot{V}_q < 0
\]

(36)

According to this result, \(\bar{q}\) will converge to \(Q_q\) and remain in this set, which means \(\sigma_{\Delta_2}(L\bar{q}) = L\bar{q}\). Subsequently, (24) can be rewritten as follows:

\[
\begin{align*}
\hat{\bar{q}} &= \bar{\rho} + r \\
\hat{\bar{\rho}} &= r \\
\hat{\bar{v}} &= L\bar{q} \\
r &= -\bar{\rho} - L\bar{q} - \sigma_{\Delta_3}(\bar{v} + \kappa\nabla F(q)) - \kappa\nabla^2 F(q)\bar{\rho}
\end{align*}
\]

(37)

By applying the state transformation \(m = T^{-1}\bar{m}\) to (37), (14) can be reformulated as follows.

\[
\begin{align*}
\dot{\bar{q}} &= \rho \\
\dot{\bar{\rho}} &= -L(q + \rho) - \sigma_{\Delta_3}(v + \kappa\nabla F(q) + \rho) \\
&\quad -\kappa\nabla^2 F(q)\rho \\
\dot{\bar{v}} &= L(q + \rho) \\
v(0) &= 0
\end{align*}
\]

Next, we propose the following Lyapunov candidate to analyze the asymptotic stability of (38).

\[
V_p = \frac{1}{2} \rho^T \rho + \frac{1}{2} q^T Lq + \psi(v + \kappa\nabla F(q) + \rho)
\]

(39)

Where \(\psi(s) : R^m \rightarrow R \) and \(\psi(s) = \sum_{i=1}^m \int_0^s \sigma_\Delta(t) dt\). Thus, \(V_p(q, \rho, v) \geq 0\) and \(V_p(q, \rho, v) = 0\) holds if and only if \((q, \rho, v) = (q^*, \rho^*, v^*)\). The following equation holds.

\[
\begin{align*}
\dot{V}_p &= -\rho^T \left( -\rho - L(q + \rho) - \sigma_{\Delta_3}(v + \kappa\nabla F(q) + \rho) \\
&\quad -\kappa\nabla^2 F(q)\rho + \rho^T Lq \\
&\quad -\sigma_{\Delta_3}(v + \kappa\nabla F(q) + \rho) \left( \rho + \sigma_{\Delta_3}(v + \kappa\nabla F(q) + \rho) \right) \right) \\
&\quad = -\|\rho + \sigma_{\Delta_3}(v + \kappa\nabla F(q) + \rho)\|^2 - \rho^T L\rho - \kappa\rho^T \nabla^2 F(q)\rho
\end{align*}
\]

(40)

Since the communication graph is connected, we have

\[
\dot{V}_p \leq -\|\rho + \sigma_{\Delta_3}(v + \kappa\nabla F(q) + \rho)\|^2 - \kappa\rho^T \nabla^2 F(q)\rho
\]

(41)

Let define \(Q_p = \{(q, \rho, v) \in R^{3N} \| V_p \leq 0\}\). According to Assumption 5, \(\nabla^2 F(q) > 0\). As a result:

\[
Q_p = \mathbb{R}^{3N}
\]

(42)

Moreover, we define \(R = \{(q, \rho, v) \in R^{3N} \| \rho = 0 \) and \(v + \kappa\nabla F(q) = 0\}\)

(43)

Next, we aim to explore \(R\) for its largest invariant set. To do so, we take the derivative of \(v + \kappa\nabla F(q)\) and make it equal to zero, as follows

\[
-\kappa\nabla^2 F(q)\rho - L(q + \rho) = 0
\]

(44)

From (43) and (44) we derive

\[
\dot{q} = 0 \\
Lq = 0
\]

(45)

which means the largest invariant set in \(R\) is as follows:
\[ M_i = \{(q^i, \rho^i, \nu^i)\} \]  

Equations (43) and (46) satisfy the conditions under which LaSalle’s Invariance Principle can be applied. Using this principle and (42), we can conclude that \((q^i, \rho^i, \nu^i)\) is asymptotically stable. Utilizing this conclusion and (20), the following result is reached:

\[ \lim_{t \to \infty} q_i(t) = y^*, \text{ } i = 1, 2, \ldots, N \]  

**Step 3:** The dynamics of \(q_i\) and \(\rho_i\) in the reference MAS (7) can be rewritten as follows using (12).

\[ \dot{q}_i = \rho_i \]

\[ \dot{\rho}_i = -(a_i + \delta_i)q_i - (a_i + \delta_i)\rho_i + \bar{r}_i \]

Thus the following transfer function is derived:

\[ \frac{L(q_i(t))}{L(\bar{r}_i(t))} = \frac{1}{(s + a_i)(s + \delta_i)} \]  

\(n^* = 1\) case: 

In this case, \(\bar{r}_i = -(\delta_i + \gamma_i)\bar{r}_i + \bar{r}_i\) from (10). Thus, one can define the reference transfer function for \(i\) th agent with \(n^* = 1\) based on (49):

\[ G_i(s) = \frac{L(q_i(t))}{L(\bar{r}_i(t))} = \frac{1}{s + a_i} \]  

Since \(n^* = 1\), if \(A_{pi}, B_{pi}\), and \(c_{pi}\) were known, the distributed control policy \(u_i^* = (\eta_i^*)^T \psi_i(t), \eta_i^* \triangleq \left[ \frac{1}{k_{pi}} (\eta_{2i}^T) \eta_{2i}^T \right] \) would be found such that the input-output behavior of the MAS (2) can be expressed by \(y_{pi}(t) = G_{i1}(s)\bar{r}_i(I)\). Thus, the closed-loop system for \(i\) th agent can be expressed as:

\[ \dot{X}_i(t) = \mathcal{A}_i X_i(t) + \frac{1}{k_{pi}} B_i \bar{r}_i(t) \]

\[ y_{pi}(t) = c_i^T X_i(t) \]  

Where \(X_i = [\psi_i(t) \psi_{2i}(t)]^T\) and:

\[ \mathcal{A}_i = \begin{bmatrix} A_{pi} + b_{pi} \eta_{3i} c_i^T & b_{pi}(\eta_i^2)^T & b_{pi}(\eta_i^4)^T \\ l_i \eta_{3i} c_i^T & \Pi_i + l_i(\eta_i^2)^T l_i(\eta_i^4)^T & \end{bmatrix} \]

\[ B_i = \begin{bmatrix} b_{pi}^T & l_i^T & 0 \end{bmatrix} \] 

\[ c_i^T = \begin{bmatrix} 0 & 0 \end{bmatrix} \]  

And the following equation holds:

\[ \frac{1}{k_{pi}} c_i^T s I - \mathcal{A}_i)^{-1} B_i = G_i(s) \]  

By introducing the control coefficient error \(\beta_i = \eta_i - \eta_i^*\), (51) can be rewritten as:

\[ u_i = \beta_i^T \psi_i(t) + (\eta_i^*)^T \psi_i(t) \]  

So the closed-loop system for \(i\) th agent can be described as:

\[ \dot{X}_i(t) = \mathcal{A}_i X_i(t) + \frac{1}{k_{pi}} B_i r_i(t) \]

\[ y_{pi}(t) = c_i^T X_i(t) \]  

Furthermore, due to the fact that \((\mathcal{A}_i, B_i, C_i^T)\) is a realization of \(G_i(s)\), the reference MAS (48) can be described by the following equation:

\[ \dot{X}_i(t) = \mathcal{A}_i X_i(t) + \frac{1}{k_{pi}} B_i r_i(t) \]

\[ q_i(t) = C_i^T X_i(t) \]  

Next, we define \(e_i = X_i - \chi_i\) and \(E_i = y_{pi} - q_i\). Considering (56) and (57), the differential equation for \(e_i\) is achieved as:

\[ \dot{e}_i(t) = \mathcal{A}_i e_i(t) + B_i \beta_i^T \psi_i(t) \]

\[ \dot{E}_i(t) = \frac{1}{k_{pi}} c_i^T \mathcal{X}_i(t) \]  

The transfer function from \(\beta_i^T \psi_i(t)\) to \(\dot{e}_i(t)\) is \(G_i(s) = \frac{1}{s + p_i}\) which is SPR. Thus, according to Meyer–Kalman–Yakubovich lemma, matrices \(P_1 > 0\) and \(Q_1 > 0\) exist such that:

\[ \mathcal{A}_i^T P_1 + P_1 \mathcal{A}_i = -Q_1 \]

\[ P_1 B_i = \frac{1}{k_{pi}} c_i^T \]  

Now, we propose the following Lyapunov candidate to investigate the stability of (58):

\[ V_i(t) = e_i^T(t)P_1 e_i(t) + \frac{1}{|k_{pi}|} \beta_i^T(t) \Sigma_i^{-1} \beta_i(t) \]  

The time derivative of (60), along with (9) and (58) is calculated as follows:

\[ \dot{V}_i(t) = e_i^T(t)(\mathcal{A}_i^T P_1 + P_1 \mathcal{A}_i)e_i(t) \]

\[ + 2 e_i^T(t) P_1 B_i \beta_i^T(t) \psi_i(t) \]

\[ + \frac{2}{|k_{pi}|} \beta_i^T(t) \Sigma_i^{-1} \beta_i(t) \]  

It follows from (59) and (61):

\[ \dot{V}_i(t) = -e_i^T(t)Q_i e_i(t) \]

\[ + 2 sgn(k_{pi}) e_i^T(t)c_i \beta_i^T(t) \psi_i(t) \]

\[ + \frac{2}{|k_{pi}|} \beta_i^T(t) \Sigma_i^{-1} \beta_i(t) \]  

It can be easily verified that \(\dot{\beta}_i(t) = \eta_i(t)\). Substituting \(\eta_i(t)\) from (9) into (62) obtains the following equation:
\[ V_i(t) = -e_i^T(t)Q_i e_i(t) + \frac{2sgn(k_{p_i})}{|k_{p_i}|} \dot{E}_i(t) \beta_i^T \psi_i(t) - \frac{2sgn(k_{p_i})}{|k_{p_i}|} E_i(t) \beta_i^T(t) \psi_i(t) = -e_i^T(t)Q_i e_i(t) \]

In other words \( V_i(t) \leq 0 \). Thus, the following results are concluded:

\[ V_i(t), e_i(t), E_i(t), \eta_i(t) \in L_\infty \]

Next, we define \( e_i = X_i - \chi_i \) and \( E_i = y_{pi} - q_i \). Considering (56) and (57), the differential equation for \( e_i \) is achieved as:

\[ \dot{e}_i(t) = A_i e_i(t) + B_i (u_i - (\eta_i)^T \psi_i(t)) \]

Thus (71) can be rewritten as:

\[ \dot{e}_i(t) = \frac{1}{k_{p_i}} C_i^T X_i(t) \]

The transfer function from \( u_i - (\eta_i)^T \psi_i(t) \) to \( e_i(t) \) is

\[ G_i(s) = \frac{1}{s + \beta_i} \]

Equations (13) and (72) result in \( L(\eta_i(t)) = (s + \beta_i)L(\tilde{u}(t)) \) respectively. Thus (74) can be rewritten as:

\[ \dot{\tilde{u}}_i(t) = \tilde{u}_i(t) = \frac{1}{k_{p_i}} C_i^T \chi_i(t) \]

By defining \( \beta_i = \eta_i - \tilde{\eta}_i \):

\[ \dot{e}_i(t) = A_i e_i(t) + B_i (s + \beta_i) \beta_i^T(t) \psi_i(t) \]

Equations (13) and (72) result in \( L(\eta_i(t)) = (s + \beta_i)L(\tilde{u}(t)) \) respectively. Thus (74) can be rewritten as:

\[ \dot{e}_i(t) = \frac{1}{k_{p_i}} C_i^T X_i(t) \]

The transfer function from \( e_i(t) \) to \( \beta_i^T(t) \psi_i(t) \) is obtained as:

\[ G_i(s) = \frac{1}{s + \beta_i} \]

We define \( \tilde{e}_i = e_i - B_i \beta_i^T(t) \psi_i(t) \). By substituting \( \tilde{e}_i \) into (75), the following dynamics is derived:
Based on (76), the transfer function from $\bar{e}_i(t)$ to $\beta_i^2(t)\psi_i(t)$ is
\[ \frac{1}{(s+\alpha_i)} \Rightarrow \frac{1}{k_{p_i}}C_i^T(sI-A_i)^{-1}\bar{B}_i = \frac{1}{k_{p_i}}C_i \] which is SPR. This means Meyer–Kalman–Yakubovich lemma can be utilized for (78). According to this lemma matrices $P_i > 0$ and $Q_i > 0$ exist such that:
\[ A_i^T P_i + P_i A_i = -Q_i \]
\[ P_i \bar{B}_i = \frac{1}{k_{p_i}} C_i \] (79)

The following Lyapunov candidate is proposed to investigate the stability of (78):
\[ V_i(t) = \bar{e}_i^T(t)P_i\bar{e}_i(t) + \frac{1}{[k_{p_i}]} \beta_i^2(t)\Sigma_i^{-1}\beta_i(t) \] (80)

The time derivative of (80) along with the trajectories of system (78) is obtained as follows:
\[ \dot{V}_i(t) = \bar{e}_i^T(t)\left(A_i^T P_i + P_i A_i\right)\bar{e}_i(t) + 2\bar{e}_i^T(t)P_i\bar{B}_i\beta_i(t)\dot{\psi}_i(t) + \frac{2}{[k_{p_i}]} \beta_i^2(t)\Sigma_i^{-1}\dot{\beta}_i(t) \] (81)

Using (79), (61) can be rewritten as:
\[ \dot{V}_i(t) = -\bar{e}_i^T(t)Q_i\bar{e}_i(t) + 2\text{sgn}(k_{p_i}) \bar{e}_i^T(t)C_i\beta_i(t)\dot{\psi}_i(t) + \frac{2}{[k_{p_i}]} \beta_i^2(t)\Sigma_i^{-1}\dot{\beta}_i(t) \] (82)

Since $\beta_i = \eta_i - \eta_i^*$, the following equation is derived using (11)
\[ \dot{\beta}_i = -\text{sgn}(k_{p_i})\Sigma_i(y_i - q_i)\dot{\psi}_i \] (83)

Substituting (83) into (82) results in:
\[ \dot{V}_i(t) = -\bar{e}_i^T(t)Q_i\bar{e}_i(t) \] (84)

It is obvious that $\dot{V}_i(t) \leq 0$. Thus, the following conclusion holds:
\[ V_i(t), \hat{e}_i(t), E_i(t), \eta_i(t) \in L_\infty \] (85)

**Step 4:**

In both $n^* = 1$ and $n^* = 2$ cases $\tilde{n}_i = (a_i \delta_i)q_i + (a_i + \delta_i)\rho_i + r_i$. As it can be observed in (7), $r_i$ is bounded owing to the definition of the saturation function $\sigma_2(.)$. Furthermore, in Step 2, it is shown that $\lim_{t \to \infty} q_i(t) = q^*$, $\lim_{t \to \infty} \rho_i(t) = 0$ assuring that $q_i(t), \rho_i(t) \in L_\infty$. Accordingly, the following result is derived:
\[ \tilde{n}_i(t) \in L_\infty \] (86)

We would rather separate the remainder of the proof in Step 4 into $n^* = 1$ and $n^* = 2$ cases for the sake of clarity.

Case $n^* = 1$:

As it can be observed in (10), $\bar{r}_i(t)$ is the input of a BIBO system whose output is $r_i(t)$. Thus $r_i(t)$ is bounded too:
\[ r_i(t) \in L_\infty \] (87)

Moreover, it is shown in step 3 that $A_i$ is Hurwitz. This fact and (87) together confirm that $r_i(t)$ is a bounded input for an LTI system with Hurwitz eigenvalues. As a result, the following holds:
\[ x_i(t) \in L_\infty \] (88)

Using (64) and (88), the following conclusion is obtained:
\[ x_i(t) \in L_\infty \] (89)

$x_i(t)$ includes $x_{p_i}, \psi_1$ and $\psi_2$, thus:
\[ x_{p_i}, y_{p_i}, \psi_1, \psi_2 \in L_\infty \] (90)

From (87) and (90), the following is derived
\[ \psi_i(t) \in L_\infty \] (91)

From (64), (90), (91) and our previous results confirming that the reference MAS converges to its equilibrium point, we come to the conclusion that all signals in unknown SISO heterogeneous MAS (2) under the proposed control policy in (9) are bounded.

Integrating both sides of (63) achieves the following equation:
\[ -V_i(t) + V_i(0) = \int_0^t e_i^T(t)Q_i e_i(t) \, dt \] (92)

We have already shown that $V_i(t) \in L_\infty$, thus $\int_0^t e_i^T(t)Q_i e_i(t) \, dt \in L_\infty$ which means:
\[ e_i(t) \in L_2 \] (93)

In addition, all signals on the right-hand side of the dynamics for $e_i(t)$ provided in (58) are bounded. Thus, the following result is derived.
\[ \dot{e}_i(t) \in L_\infty \] (94)

The results in (64), (93), and (94) are summarized as follows:
\[ e_i(t) \in L_\infty \cap L_2 \quad \dot{e}_i(t) \in L_\infty \] (95)

Consequently, taking advantage of Barbalet’s lemma, we can conclude:
\[ \lim_{t \to \infty} e_i(t) = 0 \] (96)

Which in turn:
\[ \lim_{t \to \infty} E_i(t) = 0 \] (97)

In other words, $\lim_{t \to \infty} y_{p_i}(t) = \lim_{t \to \infty} q_i(t)$. Also from step 2, $\lim_{t \to \infty} q_i(t) = y^*$ thus $\lim_{t \to \infty} y_{p_i}(t) = y^*$ is concluded.

Case $n^* = 2$:

Based on (12) $r_i = \tilde{n}_i$, thus according to the argument provided in the beginning of this step, the following conclusion is achieved
\[ r_i(t) \in L_\infty \] (98)

In (70) $\mathcal{A}_i$ is Hurwitz. This fact along with the boundedness of $r_i(t)$ result in the following:
\( \chi(t) \in \mathcal{L}_\infty \)

\( \psi_i(t) \) can be expressed as follows owing to the fact that \( u_i = G^{-1}_{pi}(s)y_{pi} \).

\[
\bar{\psi}_i = \frac{1}{s + \bar{\theta}} \left[ (sI - \Pi_i)^{-1}l_iG^{-1}_{pi}(s)y_{pi} \right]
\]

(100)

According to Assumption 2, Assumption 3, and (6), we can deduce that each element in \( \bar{\psi}_i \) is the output of a proper stable transfer function whose input is \( y_{pi} \) or \( r_i \). We have already proved that \( E_i(t) \in \mathcal{L}_\infty \), thus \( y_{pi} \in \mathcal{L}_\infty \). Moreover \( r_i(t) \in \mathcal{L}_\infty \) from (98). As a result, the following is derived:

\[
\bar{\psi}_i(t) \in \mathcal{L}_\infty
\]

(101)

We have defined \( \bar{e}_i \equiv e_i - \bar{B}_i \beta_i^T(t)\bar{\psi}_i(t) \). \( \beta_i \), \( \bar{e}_i \) and \( \bar{\psi}_i \) are bounded from (85) and (101). Hence, we can conclude:

\[
e_i(t) \in \mathcal{L}_\infty
\]

(102)

Using (99) and (102), the following conclusion is obtained:

\[
\chi_i(t) \in \mathcal{L}_\infty
\]

(103)

\( \chi_i(t) \) includes \( x_{pi}, \psi_{1i} \) and \( \psi_{2i} \), thus:

\[
x_{pi}, \psi_{1i}, \psi_{2i} \in \mathcal{L}_\infty
\]

(104)

From (85), (101), (104) and our previous results confirming that the reference MAS converges to its equilibrium point, we come to the conclusion that all signals in unknown SISO heterogeneous MAS (2) under the proposed control policy in (9) are bounded.

By integrating both sides of (84), one can achieve the following equation.

\[
-V_i(t) + V_i(0) = \int_0^t \dot{\bar{e}}_i^T(t)Q_i\dot{\bar{e}}_i(t)\,dt
\]

(105)

We have proved that \( V_i(t) \in \mathcal{L}_\infty \) which means \( \int_0^t \dot{\bar{e}}_i^T(t)Q_i\dot{\bar{e}}_i(t)\,dt \in \mathcal{L}_\infty \). In other words, the following holds.

\[
\dot{\bar{e}}_i(t) \in \mathcal{L}_2
\]

(106)

In addition, all signals on the right-side hand of the dynamics for \( \dot{\bar{e}}_i(t) \) provided in (78) are bounded. Thus, the following result is derived.

\[
\dot{\bar{e}}_i(t) \in \mathcal{L}_\infty
\]

(107)

The results in (85), (106) and (107) are summarized as follows:

\[
\bar{e}_i(t) \in \mathcal{L}_\infty \cap \mathcal{L}_2 \quad \dot{\bar{e}}_i(t) \in \mathcal{L}_\infty
\]

(108)

As a result, using Barboulet lemma, we can deduce:

\[
\lim_{t \to \infty} \dot{\bar{e}}_i(t) = 0
\]

(109)

Which means

\[
\lim_{t \to \infty} E_i(t) = 0
\]

(110)

In other words, \( \lim_{t \to \infty} y_{pi}(t) = \lim_{t \to \infty} q_i(t) \). Also from step 2, \( \lim_{t \to \infty} q_i(t) = y^* \) thus \( \lim_{t \to \infty} y_{pi}(t) = y^* \) holds.

V. ILLUSTRATIVE EXAMPLE

A network of four unknown heterogenous SISO agents is considered. In this network, the first and third agents are of \( n^* = 1 \) where \( A_{p1} = [4 - 3; 1 0] \), \( B_{p1} = [1; 0] \), \( C_{p1} = [1; 1] \) and \( A_{p3} = [6 - 9; 1 0] \), \( B_{p3} = [1; 0] \), \( C_{p3} = [1; 3] \). The second and fourth agents are of \( n^* = 2 \) where \( A_{p2} = [0 1; -5 - 6] \), \( B_{p2} = [0; 1] \), \( C_{p2} = [1; 0] \) and \( A_{p4} = [0 1; -6 - 5.5] \), \( B_{p4} = [0; 1] \), \( C_{p4} = [1; 0] \). These values are only utilized in simulation of MAS outputs and are not used in developing the control input. Furthermore, \( \Delta_1 = 4050 \), \( \Delta_2 = 810 \), \( \Delta_3 = 100 \), \( \Delta_4 = 100 \) and \( \kappa = 0.012 \), which satisfy (8). The design parameters \( a \) and \( \theta \) equal \(-2\) and \(-3\) respectively. The local cost functions are determined as follows:

\[
\begin{align*}
&f_1 = (y + 5)^2 \quad f_2 = (y - 10)^2 \\
&f_3 = (y - 2)^2 \quad f_4 = (y + 13)^2
\end{align*}
\]

(111)

The optimal point for \( \min \sum_{i=1}^4 f_i(y) \) is \( y^* = -1.5 \). Regarding the connected communication graph between agents, the associated Laplacian matrix is as follows.

\[
L = \begin{bmatrix}
2 & -1 & 0 & -1 \\
-1 & 2 & -1 & 0 \\
0 & -1 & 2 & -1 \\
-1 & 0 & -1 & 2
\end{bmatrix}
\]

(112)

Fig. 1 shows the outputs of all agents converge to the solution of the global cost function. The boundedness of the closed-loop system must be validated too. According to Fig. 2 all MAS states are bounded. Fig. 3 to Fig. 8 depict that other states, including reference MAS states \( q_i \), \( \rho_i \) and \( v_i \), auxiliary states \( \psi_{1i} \) and \( \psi_{2i} \), and the time-variant control gain \( \eta_i \) are bounded too. Moreover, it can be observed in Fig. 4 that \( \rho_i \) converges to zero as expected.
Fig. 3. Evolution of the reference MAS first state $q_i$

Fig. 4. Evolution of the reference MAS second state $\dot{q}_i$

Fig. 5. Evolution of the reference MAS third state $v_i$

Fig. 6. Evolution of the auxiliary state $\psi_{2i}$

Fig. 7. Evolution of the auxiliary state $\psi_{2i}$

Fig. 8. Evolution of the norm of the time-variant control gains $\eta_i$

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**VI. CONCLUSION**

This paper studied global optimal coordination problems for a network of LTI agents where system dynamics are completely unknown. In the case that agents relative degrees are allowed to be different from each other, a novel double-layer adaptive distributed optimization algorithm, which enables the agents to achieve consensus on the minimizer of the global cost function, was introduced. Each agent is only required to be aware of its own local cost function and the information of its neighbors, which is shared on a connected communication graph to implement this algorithm. This means the proposed optimization algorithm is fully distributed. Our future research may be directed toward the case of more general dynamics, switching digraphs, and saturation in actuators.

**REFERENCES**


