Applicability Extension and Calculation Acceleration of Pattern-Multiplication Principle in Far-Field Analysis of Conformal Arrays

li zhao 1, You-Feng Cheng 2, Cheng Liao 2, Fan Peng 2, Guo-Feng Gao 2, and Xiao Ding 2

1Institute of Electromagnetics
2Affiliation not available

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Abstract

This paper proposes a near-field Euler rotation-based technique to transform the far-field calculation of arbitrary conformal arrays into the pattern-multiplication form. Additionally, the calculation is greatly accelerated by a three-dimensional virtual equivalent source expansion (3-D VESE) technique and a layer-wise two-dimensional fast Fourier transform (2-D FFT) technique. Computational complexity analysis and several numerical examples validate the advantages of the proposed solution in terms of the efficiency and accuracy.
Applicability Extension and Calculation Acceleration of Pattern-Multiplication Principle in Far-Field Analysis of Conformal Arrays

Li Zhao, You-Feng Cheng, Member, IEEE, Cheng Liao, Member, IEEE, Fan Peng, Guo-Feng Gao, and Xiao Ding, Senior Member, IEEE

Abstract—Due to the beam-pointing and polarization diversities of carrier-mounted elements, it is widely recognized that the principle of pattern multiplication is not applicable to conformal arrays, particularly when coupling effects are considered. This paper proposes a near-field Euler rotation-based technique to transform the far-field calculation of arbitrary conformal arrays into the pattern-multiplication form. Specifically, this technique involves rotating and transforming the electric and magnetic field vectors, along with the position vectors on the near-field sampling surface in the local coordinate system (LCS), to a unified sampling surface encompassing all elements in the global coordinate system (GCS). Based on the Schelkunoff’s equivalence principle, the equivalent electric and magnetic currents, which are referred to as equivalent sources, are determined. Consequently, the far fields of the conformal array can be regarded as equivalent to those of arrays comprised of these equivalent sources, with the radiation fields of each equivalent-source array conforming to the pattern-multiplication form. Furthermore, the equivalent-source arrays are transformed into three-dimensional (3-D) uniform arrays through a virtual expansion operation. To expedite the far-field calculation, the technique incorporates the layer-wise two-dimensional fast Fourier transform (2-D FFT) technique. This is achieved by analyzing the relationship between the array factor of these 3-D arrays and the layer-wise 2-D FFT. Finally, several numerical examples, which involve cylindrical and hemispherical-conical arrays and take the mutual coupling (MC) effects into consideration, are provided to validate the advantages of the proposed technique in terms of calculation efficiency and accuracy.

Index Terms—Conformal array, fast Fourier transform (FFT), near-field Euler rotation, principle of pattern multiplication.

I. INTRODUCTION

CONFORMAL arrays have great potential for applications in airborne and naval communication systems due to their wide beam coverage and excellent aerodynamic performance [1]. It is well known that radiation beams and polarizations of conformal elements are dependent on the platform carrier, making the principle of pattern multiplication unsuitable for evaluating the far fields of conformal arrays [2]. That means some efficient solutions, such as the three-dimensional nonuniform fast Fourier transform (3-D NuFFT) technique for the fast calculation of the array factors of conformal arrays [3], are not applicable since the array factor cannot characterize the array features. Initially, the numerical algorithm-based full-wave simulations are widely used for the accurate far-field calculation [4], [5], [6], [7]. However, these solutions are quite time-consuming. Afterwards, some analytical solutions, such as methods based on vector plane wave spectrum (MVPWS) and stationary phase (MSP), have been developed and reported by researchers in the general field of theoretical and computational electrodynamics and photonics [8], [9], [10]. Nevertheless, the analyses of conformal arrays with these analytical methods would make the computational effort heavy when mutual coupling (MC) effects are considered. Subsequently, researchers have proposed some semi-analytical and semi-numerical algorithms, such as the surface integral equation-active element pattern (AEP) method [11], the AEP method based on antenna current Green’s function (ACGF) [12], and the ACGF technique based on the method of moments (MOM) [13]. These methods have the common feature of considering not only the element anisotropy of conformal elements but also the MC effects. Besides, they aim to achieve both high accuracy and improved computational speed. The common drawback of these methods is that they still require significant improvements in terms of the calculation efficiency.

To achieve the fast far-field computation of conformal arrays, Euler rotation techniques have been introduced to eliminate the element anisotropy. In the literature, the existing Euler rotation-based methods, aimed at reducing computational complexity, include the far-field coordinate transformation technique (FFCTT) [14], [15], [16], [17], [18], the pattern-angle indexing technique (PAIT) [19], [20], and the joint multidimensional vector clustering technique [21], [22]. Besides, MC effects can also be considered by integrating the AEPs of small subarrays into the above solutions [23].

The FFCTT, which transforms the element patterns from a local coordinate system (LCS) to a global coordinate system (GCS) by coordinate transformation and Euler rotation operations, is the most common solution. This method enables the calculation of the radiation characteristics of antenna elements loaded on arbitrary curved surfaces [14], [15], thereby facilitating far-field analysis of the entire conformal array. However, this method is only applicable when the analytical expression of the element pattern is known. In addition, since

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L. Zhao, Y.-F. Cheng, C. Liao and F. Peng are with the Institute of Electromagnetics, Southwest Jiaotong University (SWJTU), Chengdu 610031, China (e-mail: L.Zhao@my.swjtu.edu.cn; juwencheng@swjtu.edu.cn; c.liao@swjtu.edu.cn; pfan2023@my.swjtu.edu.cn).

G. F. Gao and X. Ding are with the Institute of Applied Physics, University of Electronic Science and Technology of China (UESTC), Chengdu 610054, China (e-mail: 15008443175@163.com, xding@uestc.edu.cn).
it involves solving plenty of transcendental equations during the computation, this method requires complex and time-consuming operations for large-scale antenna arrays.

The PAIT firstly samples the far-field pattern of the element in the LCS and defining it as the reference data. Then, through the Euler rotation, the observation angles of elements in the GCS are transformed into ones which are indexed with the sampling angles in the reference data. In this situation, the corresponding pattern data of these angles can be matched. This method can also be applicable to arbitrary carriers and seen as the reverse operation of the FFCTT. In [19], this method was utilized to analyze and synthesize the far-field patterns of a cylindrical array. The drawback of this solution is that it requires a sufficiently fine sampling interval. Otherwise, it becomes challenging to achieve high-precision indexing between observation and sampling angles, which undoubtedly increases the computational cost of this method.

The joint multidimensional vector clustering technique achieves the far-field calculation by projecting conformal elements onto a plane. The projected position and pattern of each element can be obtained by the Euler rotation. In [22], this solution is successfully employed to evaluate the far-field polarization pattern of a 3-D conformal array. Similar to the FFCTT, it also relies on the availability of predicted radiation patterns for the elements.

In summary, proposing and validating an efficient method for calculating the far-field pattern of conformal arrays, which ensures sufficient accuracy compared to full-wave simulations, holds significant potential for practical applications. In light of the aforementioned research background, this paper proposes a novel approach based on a near-field Euler rotation technique (NFERT) which can transform the far-field calculation of arbitrary conformal arrays into the pattern-multiplication form. In this situation, the relationship between the array factors of equivalent sources (electric and magnetic currents) and the layer-wise two-dimensional FFT (2-D FFT) is further analyzed. Furthermore, the layer-wise 2-D FFT technique is applied to accelerate the far-field calculation. Several numerical examples are presented to demonstrate the effectiveness and advantages of the proposed method. The paper is organized as follows. The applicability extension of the principle of pattern multiplication to arbitrary conformal arrays is formulated in Section II. Calculation acceleration of equivalent-source arrays based on 2-D layer-wised FFT will be analyzed in Section III. Numerical examples, which include four array layouts and three types of carriers, are presented in Section IV. Finally, Section V presents some meaningful conclusions.

II. APPLICABILITY EXTENSION OF THE PRINCIPLE OF PATTERN MULTIPLICATION

The proposed far-field calculation method for conformal arrays is based on NFERT and near-to-far-field transformation. Initially, it is required to obtain the near-field data (E- and H-fields) of the elements. Generally, one can choose a closed surface $S^{\text{L}}$ to enclose the analyzed element in the LCS to meet the sampling requirements, where the superscript ‘L’ represents the LCS. As shown in Fig. 1(a), without loss of generality, a hexahedral box can be chosen as the near-field closed surface. Once $S^{\text{L}}$ is selected and the corresponding sampling points on the surface are determined, a series of discrete near-field data $(\mathbf{E}^{\text{L}}, \mathbf{H}^{\text{L}})$ and spatial position data $(\mathbf{P}^{\text{L}}, \mathbf{n}^{\text{L}})$ can be obtained by using full-wave simulation or through measurement. Here, $\mathbf{E}^{\text{L}}$ and $\mathbf{H}^{\text{L}}$ represent the tangential electric and magnetic fields at the sampling points, $\mathbf{P}^{\text{L}}$ represents the position vector of the sampling point, and $\mathbf{n}^{\text{L}}$ represents the outward normal vector of $S^{\text{L}}$.

The Euler rotation is employed to transform the above data from the LCS to the GCS. Taking the $n$th sampling point...
$P_L(x_n^L, y_n^L, z_n^L)$ on the top surface of $S^L$ as the example, the transformed near-field data and spatial position data can be expressed as

\[
\begin{bmatrix}
E_{x_n^L}^G \\
E_{y_n^L}^G \\
E_{z_n^L}^G 
\end{bmatrix} = R_x R_y R_z \begin{bmatrix}
E_{x_n}^L \\
E_{y_n}^L \\
E_{z_n}^L
\end{bmatrix},
\quad
\begin{bmatrix}
H_{x_n^L}^G \\
H_{y_n^L}^G \\
H_{z_n^L}^G
\end{bmatrix} = R_x R_y R_z \begin{bmatrix}
H_{x_n}^L \\
H_{y_n}^L \\
H_{z_n}^L
\end{bmatrix}
\]  \tag{1}

\[
\begin{bmatrix}
x_n^G \\
y_n^G \\
z_n^G
\end{bmatrix} = R_x R_y R_z \begin{bmatrix}
x_n^L \\
y_n^L \\
z_n^L
\end{bmatrix},
\quad
\begin{bmatrix}
x_n^L \\
y_n^L \\
z_n^L
\end{bmatrix} = R_x R_y R_z \begin{bmatrix}
x_n^G \\
y_n^G \\
z_n^G
\end{bmatrix}
\]  \tag{2}

$R_x$, $R_y$, and $R_z$ represent the Euler rotations when the point $P_n^L$ rotates around $x^G$, $y^G$, and $z^G$-axes by specific angles ($\alpha$, $\beta$, and $\gamma$), respectively [24, 25]. The aforementioned Euler rotation matrices and angle definitions are based on the right-hand coordinate system. After these rotations, the position vector $P_n^L$ and the outward normal vector $n^L((0, 0, n_z^L)$ are represented as $P_n^G(x_n^G, y_n^G, z_n^G)$ and $n^G(n_x^G, n_y^G, n_z^G)$ in the GCS, respectively. In terms of the $E$- and $H$-fields, $E_n^L(E_{x_n^L}^L, E_{y_n^L}^L, 0)$ and $H_n^L(H_{x_n^L}^L, H_{y_n^L}^L, 0)$ at the point $P_n^L$ only contain components along $x^L$- and $y^L$-axes. By comparison, the transformed $E_n^G(E_{x_n^G}^G, E_{y_n^G}^G, E_{z_n^G}^G)$ and $H_n^G(H_{x_n^G}^G, H_{y_n^G}^G, H_{z_n^G}^G)$ have three components along $x^G$, $y^G$, and $z^G$-axes at the point $P_n^G$ after these rotations. Here the superscript ‘G’ represents the GCS. Following the same principle, the same rotation operations can be applied to other five surfaces of $S^G$. Finally, $S^L$ in the LCS is transformed into a new closed surface $S^G$ in the GCS, namely a set of the point $P_n^G$.

As shown in Fig. 1(b), based on Schelkunoff’s equivalence principle [26], at a designated sampling point of the top surface of $S^G$, the near-field data can be equivalently processed to determine the corresponding equivalent electric and magnetic currents of each element. These equivalent electric and magnetic currents at all sampling points of $S^G$ can be seen as equivalent sources of the whole conformal array. Different from array element, each kind of equivalent sources generates the identical radiation. That means the radiation anisotropy can be eliminated when realistic radiation sources are replaced by the equivalent sources, which will be analyzed below.

It is worthwhile to be note that, MC effects in the conformal array can be taken into consideration by incorporating them into the amplitudes and phases of the equivalent sources generated by the active element near fields (AENFs). Unlike the AEP, the AENF is defined as the radiation near fields on a closed sampling surface which only surrounds the analyzed element. The AENFs can be obtained by some solutions, such as the subarray extrapolation strategy [27] and the MC compensation matrix (MCCM) technique [28], [29], [30], [31], [32]. Besides, all elements are in the unique array environment which contains the edge effects. The subarray extrapolation-based AENFs is also able to reasonably take the edge effects into consideration [33].

After the equivalent sources are determined, the far fields of the conformal array can be further evaluated. To this hence, a pair of vector potential functions is defined. Here the top surface of $S^G$ is also taken as an example, the vector potential functions are calculated as

\[
A_{\theta} = \sum_{n=1}^{N} \Delta x_n^L \Delta y_n^L e^{j k P_n^G e_{x_n^G} (j \varphi_n^G \cos \theta^G \cos \varphi^G + j \varphi_n^G \sin \theta^G)}
\]

\[
A_{\varphi} = \sum_{n=1}^{N} \Delta x_n^L \Delta y_n^L e^{j k P_n^G e_{y_n^G} (j \varphi_n^G \cos \theta^G \cos \varphi^G + j \varphi_n^G \sin \theta^G)}
\]

\[
F_{\theta} = \sum_{n=1}^{N} \Delta x_n^L \Delta y_n^L e^{j k P_n^G e_{z_n^G} (j \varphi_n^G \cos \theta^G \cos \varphi^G + j \varphi_n^G \sin \theta^G)}
\]

\[
F_{\varphi} = \sum_{n=1}^{N} \Delta x_n^L \Delta y_n^L e^{j k P_n^G e_{z_n^G} (j \varphi_n^G \cos \theta^G \cos \varphi^G + j \varphi_n^G \sin \theta^G)}
\]

where $N$ represents the number of sampling points, $\Delta x_n^L$ and $\Delta y_n^L$ are the sampling intervals along the $x^L$- and $y^L$-directions on the top surface, respectively. $J_{x_n^G}^G$, $J_{y_n^G}^G$, $M_{x_n^G}^G$, $M_{y_n^G}^G$, and $M_{z_n^G}^G$ are surface electric and magnetic currents components on the nth sampling point $P_n^G$ along the $x^G$, $y^G$, and $z^G$-directions, respectively. $\theta^G$ and $\varphi^G$ are the observation angles in the GCS, $k = 2\pi/\lambda$ is the wavenumber, where $\lambda$ is the wavelength at the center frequency $f_0$ in free space. $e_{x_n^G}$ is a unit vector along the observation direction ($\theta^G, \varphi^G$).

For a hexahedral box, the electric and magnetic currents in (3) can be expanded to ones on all six surfaces. In this case, these currents can be treated as the equivalent radiation sources of aperture fields, and thus the equivalent sources on all six surfaces of the hexahedron are gridded and regarded as two kinds of six virtual planar antenna arrays. Note that, each element of the conformal array has its own equivalent-source arrays on their sampling surfaces. Taking the electric current components ($J_{x_n}^{GA}$) along the $x^G$-axis direction on the top surfaces of all array elements as the example, the vector potential $A_{\theta_{x}}^{J_{n}}$ contributed by $J_{x_n}^{GA}$ can be expressed as the following multiplication form:

\[
A_{\theta_{x}}^{J_{n}} = A_{\theta_{x}}^{J_{n=0}} A F_{x_{n}}^{J_{x}}
\]  \tag{4}

where $A_{\theta_{x}}^{J_{n=0}}$ and $A F_{x_{n}}^{J_{x}}$ represents the element vector potential and the array factor, respectively. Based on the hypotheses of $J_{x_n}^G = J_{x_n}^{GA} F_{x_n}^{J_{x}}$ and $P_n^G = P_n^G + P_0^G$, the right terms in (4) can be further formulated as

\[
A_{\theta_{x}}^{J_{n}} = \Delta x_n^L \Delta y_n^L J_{x_n}^{G} \cos \theta^G \cos \varphi^G e^{j k (x_n^G u + y_n^G v + z_n^G w)}
\]  \tag{5}

\[
A_{\theta_{x}}^{F_{x_{n}}^{J_{x}}} (u, v) = \sum_{n=1}^{N} J_{x_n}^{G} e^{j k (x_n^G u + y_n^G v + z_n^G w)}
\]  \tag{6}

where $u = \sin \theta^G \cos \varphi^G$, $v = \sin \theta^G \sin \varphi^G$, $w = \cos \theta^G$. Similarly, the vector potentials $F_{\theta_{x}}^{J_{n}}$, $F_{\varphi_{x}}^{J_{n}}$, and $F_{\theta_{x}}^{J_{n=0}}$ contributed by $J_{x_n}^{GA}$ can also be calculated, given by

\[
\begin{cases}
A_{\theta_{x}}^{J_{n}} = A_{\theta_{x}}^{J_{n=0}} A F_{x_{n}}^{J_{x}} \\
F_{\theta_{x}}^{J_{n}} = F_{\theta_{x}}^{J_{n=0}} A F_{x_{n}}^{J_{x}} \\
F_{\varphi_{x}}^{J_{n}} = F_{\varphi_{x}}^{J_{n=0}} A F_{x_{n}}^{J_{x}}
\end{cases}
\]  \tag{7}

where

\[
A_{\theta_{x}}^{J_{n=0}} = -\Delta x_n^L \Delta y_n^L J_{x_n}^{G} \sin \varphi^G e^{j k (x_n^G u + y_n^G v + z_n^G w)}
\]

\[
F_{\theta_{x}}^{J_{n=0}} = 0, F_{\varphi_{x}}^{J_{n=0}} = 0
\]  \tag{8}
Afterwards, the electric fields of the equivalent-source array $\mathbf{J}_x^{GA}$ on the top surface of $S^G$ can be calculated based on these vector potential functions, which is

$$
\begin{align*}
E_{g}^{x} &= -\frac{j k e^{-j k R_0}}{4 \pi R_0} \left( F_{g}^{J_x} + Z_0 A_{0}^{J_x} \right) \\
E_{\varphi}^{x} &= \frac{j k e^{-j k R_0}}{4 \pi R_0} \left( F_{g}^{J_x} - Z_0 A_{0}^{J_x} \right)
\end{align*}
$$

(9)

where $R_0$ represents the distance from the coordinate origin to the observation point, $Z_0$ is the wave impedance of free space. Furthermore, by substituting (4) and (7) into (9), one can obtain

$$
E_{g}^{x} = E_{g}^{J_{x0}} AF_{J_x}^{x0}, \\
E_{\varphi}^{x} = E_{\varphi}^{J_{x0}} AF_{J_x}^{x0}
$$

(10)

where $E_{g}^{J_{x0}}$ and $E_{\varphi}^{J_{x0}}$ are the radiation field of $J_x^{GA}$. Obviously, (10) explains that the far fields of the equivalent sources obey the pattern-multiplication form.

By repeating the above steps, radiation fields driven by all vector potentials of $J_x^{GA}$, $J_x^{SA}$, $M_x^{MA}$, $M_x^{GA}$, and $M_x^{SA}$ on all six surfaces of $S^G$ can be further realized.

### III. Calculation Acceleration of Equivalent-Source Arrays

#### A. Virtual Equivalent Source Expansion

According to (6), the calculation of array factors of these equivalent sources involves heavy superposition operations, which dramatically limits the computational efficiency. Besides, it can be found that, if the equivalent source arrays are transformed into virtual uniform ones, (6) forms a Fourier series that relates the equivalent source to its array factor through a discrete inverse Fourier transform. In this situation, the calculation can be accelerated.

Inspired by the concept of virtual AEP [34, 35], these equivalent sources on the near-field sampling surfaces are virtually expanded. Similarly, for convenience of description, $J_x^{GA}$ shown in Fig. 1(b) is taken as an example. As shown in Fig. 2, $J_x^{GA}$ is expanded as a virtual uniform 3-D array. Note that, the excitation (amplitudes and phases) of the virtual array is determined to make the virtual array and $J_x^{GA}$ have the same array factor. In order to compute the excitation $E_x^{J_x}$ of the virtual array by the excitation $E_x^{GA}$ of $J_x^{GA}$, a matrix $T$ is incorporated to construct the relationship between $E_x^{GA}$ and $E_x^{J_x}$, which is

$$
E_x^{J_x} = TE_x^{GA}.
$$

(11)

The matrix $T$ can be obtained by solving a matrix equation via the least-square solution, which has been detailly analyzed in [35].

Then, spatial phase factor in (6) satisfies

$$
eck(x_{n\lambda}^{G} u + y_{n\lambda}^{G} v + z_{n\lambda}^{G} w) = \sum_{q_{m}=-q_0}^{q_0} \sum_{q_{w}}^{q_0} \sum_{q_{w}}^{q_0} \sum_{l_{q_{m}l_{q_{w}}}l_{q_{w}}} \sum_{l_{q_{m}l_{q_{w}}}} e^{j k x_{n\lambda}^{G} u} e^{j k y_{n\lambda}^{G} v} e^{j k z_{n\lambda}^{G} w} /
$$

$$
\times e^{j k x_{n\lambda}^{G} u} e^{j k y_{n\lambda}^{G} v} e^{j k z_{n\lambda}^{G} w} /
$$

(12)

where $x_{n\lambda} = 2 x_{n\lambda}^{G} / \lambda, y_{n\lambda} = 2 y_{n\lambda}^{G} / \lambda, z_{n\lambda} = 2 z_{n\lambda}^{G} / \lambda, p = [(q + 2) / r], A_x = \text{ceil}(\max(x_{n\lambda}) - \min(x_{n\lambda})) + p, A_y = \text{ceil}(\max(y_{n\lambda}) - \min(y_{n\lambda})) + p, A_z = \text{ceil}(\max(z_{n\lambda}) - \min(z_{n\lambda})) + p, A_u = u A_x / 2, A_v = v A_y / 2, A_w = w A_z / 2, L_x = [r A_x], L_y = [r A_y], and L_z = [r A_z]. Here, \text{ceil}(\cdot)$ rounds the value to the nearest integer towards infinity. $\lfloor \cdot \rfloor$ rounds the value to the nearest integer. $p, q, r$ and $\tau$ are expansion parameters. $u$ and $v$ initially are uniformly discretized with $Q$ points, respectively.

The uniform array expanded by $J_x^{GA}$ can be seen from Fig. 2, where $d_x = A_x \lambda / (2 L_x), d_y = A_y \lambda / (2 L_y), d_z = A_z \lambda / (2 L_z), L_x, L_y$, and $L_z$ are the inter spacings and element numbers of the uniform arrays along $x^G$, $y^G$, and $z^G$ directions, respectively.

#### B. Calculation Acceleration via Layer-Wise 2-D FFT

The main purpose of this subsection is to compute the array factor of the achieved virtual uniform 3-D array shown in Fig. 2. The array factor can be solved efficiently by the 3-D NuFFT which is presented in [3]. However, the layer number along the $z$-axis direction of the virtual array is generally quite small. To realize a faster calculation compared to the 3-D NuFFT, here a layer-wise 2-D FFT technique is proposed and adopted.

The array factor of the $l_z$-th layer of the virtual array, which has a complex expansion matrix of $E_x^{J_x}$, can be expressed as

$$
A F_x^{J_x}(u, v) = e^{j k l_z d_w} \sum_{l_z=0}^{L_x-1} \sum_{l_y=0}^{L_y-1} E_x^{J_x}(l_z, l_y) e^{j k (l_x d_w + l_y d_v)}.
$$

(13)

Specifically, by sampling with $u = f \lambda / (F d_x)$ and $v = g \lambda / (G d_y)$, where $f = -F / 2, \cdots, F / 2 - 1 (F \geq L_x)$ and $g = -G / 2, \cdots, G / 2 - 1 (G \geq L_y)$, (13) can be rewritten as

$$
A F_x^{J_x}(f, g) = e^{j k l_z d_w k} \sum_{l_z=0}^{L_x-1} \sum_{l_y=0}^{L_y-1} E_x^{J_x}(l_z, l_y) e^{j k l_x f} e^{j k l_y g}
$$

(14)

where $w_k = (1 - u^2 - v^2)^{1/2}$. In this case, the $l_z$-th layer of $E_x^{J_x}$ satisfies

$$
A F_x^{J_x}(l_z) = F G F^{-1}(E_x^{J_x}) e^{j k l_z d_w w_k}
$$

(15)

where the sign $F^{-1}$ is the inverse FFT operator.
by applying the 2-D FFT with $F \times G$-point sequence [36], [37], [38], [39]. Based on (15), the total array factor of the virtual array can be then calculated by

$$AF^{J_x} = FG \sum_{k_z=0}^{L_z-1} F^{-1} \left( E_{J_x}^{k_z} e^{j k_x d_x w_k} \right).$$  

(16)

It is worth noting that, as described before, $AF^{J_x}$ calculated by (16) equals to the array factor of the equivalent source $J_x^{GA}$. Therefore, the array pattern of all equivalent sources $J_x^{GA}$ can be obtained by using the principle of pattern multiplication shown in (10). In addition, array patterns of the equivalent sources $J_y^{GA}$, $J_z^{GA}$, $M_x^{GA}$, $M_y^{GA}$, and $M_z^{GA}$ can be calculated similarly. Finally, the far-field pattern of the conformal array can be computed as the vector superposition of the array patterns of these equivalent sources, namely

$$E_{\theta} = E_{\theta}^{J_x} + E_{\theta}^{J_y} + E_{\theta}^{J_z} + E_{\theta}^{M_x} + E_{\theta}^{M_y} + E_{\theta}^{M_z}$$
$$E_{\phi} = E_{\phi}^{J_x} + E_{\phi}^{J_y} + E_{\phi}^{J_z} + E_{\phi}^{M_x} + E_{\phi}^{M_y} + E_{\phi}^{M_z}$$

(17)

$$PAT_{\text{total}} = \sqrt{|E_{\theta}|^2 + |E_{\phi}|^2}$$

(18)

where $E_{\theta}^{J_x}$, $E_{\theta}^{J_y}$, $E_{\theta}^{J_z}$, $E_{\theta}^{M_x}$, $E_{\theta}^{M_y}$, $E_{\theta}^{M_z}$, $E_{\phi}^{J_x}$, $E_{\phi}^{J_y}$, $E_{\phi}^{J_z}$, $E_{\phi}^{M_x}$, $E_{\phi}^{M_y}$, $E_{\phi}^{M_z}$, and $E_{\phi}^{M_z}$ represent the vectorial pattern of the equivalent sources $J_x^{GA}$, $J_y^{GA}$, $J_z^{GA}$, $M_x^{GA}$, $M_y^{GA}$, and $M_z^{GA}$, respectively.

C. Computational Complexity Analysis

The time complexity of the proposed method, which mainly consists of NFERT, implementation of the Schelkunoff's equivalence principle, virtual equivalent source expansion (VESE), and layer-wise 2-D FFT, is analyzed and compared with several other methods in this part. In general, the numbers of sampling points $F$ and $G$ can be set as the same (viz., $F = G = M$) in the layer-wise 2-D FFT. Here the element number of the conformal array is denoted as $K$. Consequently, the time complexity of the proposed method is

$$O(T_{\text{pro}}) = O \left( 6q^3 NK \right) + O \left( 6qFG \log_2 (FG) \right) = O \left( (NK + O \left( M^3 \log_2 (M) \right) \right).$$

(19)

Note that, the number ‘6’ in the first term of the right part represents the six surfaces of the global sampling surface, and that in the second term of the right part represents the six kinds of equivalent sources.

If the layer-wise 2-D FFT technique is not adopted, the time complexity of the pure NFERT method will be changed as

$$O(T_{\text{NFERT}}) = O \left( 6K_1 N_A \right) + O \left( 6FGN_1 K \right) = O \left( (NK + M^2 K) \right).$$

(20)

Fig. 3(a) and 3(b) presents the time complexities of the above two approaches. By comparison, with the increases of $M$ and $N$, the proposed solution has the significant advantage.

In addition, if the layer-wise 2-D FFT technique is replaced by the 3-D NuFFT presented in [3], the numbers of sampling points $F$, $G$, and $H$ will remain the same (viz., $F = G = H = M$). In this case, the computational complexity of the NFERT+3D-NuFFT method can be calculated as

$$O \left( T_{\text{NFERT+3D}} \right) = O \left( 6q^3 NK \right) + O \left( 6FG \log_2 (FG) \right) = O \left( (NK + O \left( M^3 \log_2 (M) \right) \right).$$

(21)

Fig. 3(c) plots the calculated results of (21). By comparing the results shown in Fig. 3(b) and 3(c), the complexity of the pure NFERT method is about ten times of that of the NFERT+3-D NuFFT solution. However, the proposed solution has an order of magnitude advantage over the NFERT+3-D NuFFT approach.

On the other hand, the time complexity of the traditional far-field summation-based solutions, such as the PAIT method, is written as

$$O(T_{\text{PAIT}}) = O \left( 3KN_A \right) + O \left( 2KFG \right) = O \left( K \cdot (N_A + M^2) \right)$$

(22)

where $N_A$ denotes the number of discrete sampling points of the element patterns. The calculated results of (22) are drawn in Fig. 3(d). By comparison, this far-field summation-based solution has the highest computation complexity.

IV. NUMERICAL EXAMPLES

In this section, numerical examples of conformal arrays based on microstrip elements are used to validate the advantages of proposed method in terms of the efficiency and accuracy. Regarding to the calculation accuracy, the mean square error (MSE) is introduced for the evaluation, which is expressed as

$$\text{MSE} = \frac{1}{FG} \sum_{f=1}^{F} \sum_{g=1}^{G} \left| PAT_{\text{cal.}}(f,g) - PAT_{\text{sim.}}(f,g) \right|$$

(23)

where $PAT_{\text{cal.}}$ and $PAT_{\text{sim.}}$ represent the calculated and simulated patterns, respectively. The simulated patterns are obtained by a commercial full-wave simulator. All the calculations in this paper are performed on a computer server with an AMD EPYC 7502 32-Core Processor with 256-GB RAM.
These examples use the same microstrip element whose geometry is shown in Fig. 4(a). The radiation patch is printed on a dielectric substrate (Rogers RT5880) with a relative permittivity of $\varepsilon_r = 2.2$. Detailed physical parameters are: $L_1 = 83.6$ mm, $W_1 = 73.5$ mm, $L_0 = 50.13$ mm, $W_0 = 39.44$ mm, $h = 1.58$ mm, and $P(x,y) = (0,9.03)$ mm. Fig. 4(b) shows the simulated reflection coefficient of the designed patch antenna, which indicates that the operating frequency is about 2.45 GHz. Fig. 4(c) depicts the simulated E- and H-plane ($\varphi^G = 90^\circ$ and $\varphi^G = 0^\circ$) radiation patterns of the patch antenna at the operating frequency.

### A. 13-Element Broadside Array Mounted on an Arc Carrier

As depicted in Fig. 5, the first example is a 13-element array which is conformed to an arc carrier. The radius of the carrier is set as $R_1 = 8\lambda_0$, where $\lambda_0$ is the free-space wavelength at the center operation frequency. The patch antennas are distributed with a uniform interval of $d_1 = 0.8\lambda_0$ along the circumferential direction (the angular interval is $d_\theta = 5.73^\circ$).

By applying the NFERT and the VESE, a series of equivalent source arrays and their virtual expanded 3-D arrays are obtained. Specifically, the nearest distance between the closed surface and the enclosed element/subarray in the LCS is set as $d_{\min} = 0.25\lambda_0$. The sampling distances along $x^L$-, $y^L$-, and $z^L$-directions all are chosen as $\Delta x^L = \Delta y^L = \Delta z^L = 0.04\lambda_0$. Furthermore, AENFs on $(19 \times 19 + 19 \times 13 + 19 \times 13) \times 2 \times 7 = 11970$ sampling points are extracted in the full-wave simulator. In the VESE, $r = 1.2$, $q = 8$, $p = 8$, $Q = 41$, and $F = G = 512$ are chosen.

It is worth noting that, as shown in Fig. 5, the MC effects are incorporated into the amplitudes and phases of the equivalent sources by subarray extrapolation strategy. In general, only a few neighboring array elements have significant influence on the E-field of the specific one. Therefore, the E-field can be approximated as that of the radiation element which is placed in a subarray with the same conformal carrier and array layout of the whole array. In this situation, the size of the subarray can be determined by using a truncated value $M_T$ which assures the sufficient precision to consider the MC effects. The value is defined as $M_T = \text{celld}(d_{\text{MC}}/d_1)$, where $d_{\text{MC}}$ is the distance between the considered center element and the most external one of the subarray. According to engineering experience reported in [40], the distance is generally set as $d_{\text{MC}} \geq 2.5\lambda_0$. As shown in Fig. 5, according to simulation-based comparison, the MC effects can be taken into full account when $M_T$ is chosen as 3. In this case, the subarray is set as a seven-element one in which the elements are denoted as 1 $\sim$ 7. That means the AENFs of 1 $\sim$ 3 and 11 $\sim$ 13 of the whole array can be approximated as those of 1 $\sim$ 3 and 5 $\sim$ 7 of the subarray, respectively. Also, the AENFs of 4 $\sim$ 10 can be approximated as those of 4 of the subarray.

Afterwards, the far-field pattern of the conformal array is calculated based on the layer-wise 2-D FFT techniques. Uniform amplitudes and progressive phases are loaded to form a broadside beam. Also taking $\mathbf{f}^{\text{GA}}$ as the example, the equivalent-source array and its virtual 3-D uniform array are shown in Fig. 6. It is seen that the nonuniformly spaced equivalent-source array is expanded into a $12 \times 34 \times 14$-element virtual uniform array with the inter-element spacings of $d_x = 0.42\lambda_0$, $d_y = 0.41\lambda_0$, and $d_z = 0.43\lambda_0$.

Besides, for further comparison, the far-field transform-based solution presented in [19] is also used for the calculation. AEPs of $46 \times 91 \times 7 = 29302$ observation angles with angular sampling interval of $\Delta \theta^L = \Delta \varphi^L = 4^\circ$ are extracted for the PAIT technique, where the number of sampling points of AEP is close to that of the above AENF.

Simulated and calculated normalized 3-D far-field patterns are plotted in Fig. 7. For clear observation, their normalized 2-D patterns at $\theta = 0$ cutting plane are depicted in Fig. 8. It is seen that results of the NFERT-based solutions agree well with the simulated ones. In the sidelobe regions, there exists some inconsistency between the simulated and PAIT-based results. Detailed performance comparisons in terms of the number of sampling points, MSE, and CPU time are listed in Table I. Compared to the PAIT solution, the proposed approach
requires a smaller number of sampling points and particularly less computational time, which validates the calculation efficiency of the proposed solution. Besides, the proposed method has the lowest MSE. That means the proposed solution has an improved calculation accuracy.

B. 169-Element Arc Cylinder-Mounted Array with Rectangular Grid

As shown in Fig. 9(a), another example is a 169-element microstrip array conformed to an arc cylinder with a radius of $8\lambda_0$. The array of Fig. 5 is significantly extended with 13 rows of elements along the $x^G$-axis direction. The element spacings are $d_x = 0.8\lambda_0$ both in the circumferential (the angular interval is $d_\theta = 5.73^\circ$) and the generatrix directions. A subarray presented in Fig. 9(b) is adopted to extract the AENFs/AEPs of edge, adjacent edge, adjacent interior, and interior elements through full-wave simulation. This subarray consists of 49 radiating elements and the other parameters are the same as those in the entire array. In order to reduce the number of sampling points, as illustrated in Fig. 9(c), the mapping strategy between the subarray and entire array is used to calculate the AENFs/AEPs of the entire array. Through this way, AENFs with $1710 \times 49 = 83790$ sampling points and AEPs with $1891 \times 49 = 92659$ sampling points are extracted. Similarly, the coupling between array elements is incorporated into the complex excitations of of the equivalent sources. Also, taking $J^G_{\text{EA}}$ as the example, as depicted in Fig. 10, the equivalent-source array is expanded into a $42 \times 42 \times 18$-element virtual uniform array with the spacings of $d_x = d_y = d_z = 0.36\lambda_0$.

Simulated and calculated 3-D patterns with uniform amplitudes and progressive phases, which is used to steer the main beam tilting toward the broadside direction, are illustrated in Figs. 11. Also, Fig. 12(a) and 12(b) present the 2-D normalized patterns at $u = 0$ and $v = 0$ cutting planes, respectively. It can be found that a good agreement is achieved between these results. Besides, as listed in Table II, the proposed solution also has the lowest MSE. These comparisons validate the calculation accuracy of the proposed approach. Table II also shows the efficiency features in terms of the number of sampling points and CPU times. The proposed method is quite

![Image](image_url)

**Fig. 6.** Schematic of the virtual expansion of the equivalent-source array of the 13-element broadside array.

![Image](image_url)

**Fig. 7.** 3-D patterns of the 13-element broadside array obtained by (a) Full-wave simulation, (b) the proposed solution, (c) the pure NFERT solution, (d) the NFERT+3-D NuFFT solution, and (e) the PAIT solution.

![Image](image_url)

**Fig. 8.** Simulated and calculated 2-D normalized results of the 13-element broadside array at the $u = 0$ plane.

<table>
<thead>
<tr>
<th>Method</th>
<th>No. of sampling points</th>
<th>MSE (dB)</th>
<th>CPU Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full-wave Simulation</td>
<td>–</td>
<td>–</td>
<td>3080</td>
</tr>
<tr>
<td>PAIT</td>
<td>29302</td>
<td>3.84</td>
<td>443</td>
</tr>
<tr>
<td>Pure NFERT</td>
<td>11970</td>
<td>0.92</td>
<td>443</td>
</tr>
<tr>
<td>NFERT+3-D NuFFT</td>
<td>11970</td>
<td>0.90</td>
<td>443</td>
</tr>
<tr>
<td>Proposed</td>
<td>11970</td>
<td>0.89</td>
<td>443</td>
</tr>
</tbody>
</table>

**Table I**

**Performance Comparison of the 13-Element Broadside Array**
rapider than others, especially compared to the PAIT solution. In addition, three arrays with different element intervals \(d_g\) mounted on a same arc cylinder (shown in Fig. 9) are studied. The impacts of \(d_g\) on calculated results are listed in Table III. It found that, the coupling level has little influence on the computation accuracy and efficiency when the AENFs are obtained correctly. Note that, since the increase of \(d_g\) will enlarge the size of the subarray, the simulation burden also gets heavy and thus the simulation time rises.

C. 169-Element Arc Cylinder-Mounted Array with Triangular Grid

An arc cylinder-mounted array with triangular grid is investigated and depicted in Fig. 13(a). The array elements are arranged in 13 rows and 26 columns. The intervals between adjacent rows and adjacent columns are set as \(d_g\) and \(0.5d_g\), respectively. Here, \(d_g\) is chosen as \(0.8\lambda_0\). It is worth noting that the triangular-grid array has the same type and number of antenna elements with those of the square-grid one shown in Fig. 9. Similarly, the AENFs are evaluated by the subarray extrapolation strategy. The subarray is plotted in Fig. 13(b). Detailly, the AENFs of the elements in interior rows and interior columns are obtained by the \((4, 8)\)th element of the subarray shown in Fig. 13(c). Regarding to the elements in the edge, adjacent edge, and adjacent interior rows/columns, their AENFs are approximated as those of the corresponding elements of the subarray. The mapping relationship between the whole array and the subarray is illustrated in Fig. 13(c).
The calculation results are also listed in Table III. Under the same sampling condition, the proposed method has the MSE of 3.53 dB and computational time of 4.72 s.

In summary, when the proposed solution is utilized to arbitrary conformal arrays with various degrees of inter-element MC, it is able to realize efficient and accurate evaluation of far-filed patterns. Therefore, these numerical examples and their computation results validate the robustness of the AENFs-based method.

D. 103-Element Hemispherical-Conical Array (HCA) With Its Main Beam Steering to the Direction of \( (\theta_0^{G} = 45^\circ, \varphi_0^{G} = 0^\circ) \)

The last example presents the calculation and analysis of a conformal array with its 103 microstrip elements arranged on a HCA carrier. As shown in Fig. 14, the hemispherical part of the HCA, which has a radius of \( R_s = 2.2\lambda_0 \) and the element number of \( N_s = 50 \), is composed of five concentric rings along the \( z^G \)-direction. These elements are uniformly distributed along the \( \theta^G \)-direction from \( \theta_1 = 0^\circ \) to \( \theta_2 = 80^\circ \).

In terms of the conical part, elements form three concentric rings with their radii of \( R_{c1} = 2.4\lambda_0 \), \( R_{c2} = 2.5\lambda_0 \), and \( R_{c3} = 2.6\lambda_0 \), respectively. The total number of elements mounted on the conical part is \( N_c = 53 \). The three rings have their element numbers of 15, 18, and 20, respectively.

In each ring shown in Fig. 14(a), the array environments of partial elements are not identical but close. That means the AENFs of elements in each ring exist a little difference. However, considering the tradeoff between the calculation efficiency and accuracy, all the elements in each ring share the same AENFs in the calculation. Specifically, as shown in Fig. 14(a), radiation elements 2 ∼ 5, 6 ∼ 15, 16 ∼ 30, 31 ∼ 50, 51 ∼ 65, 66 ∼ 83, and 84 ∼ 103 share the AENFs/AEPs of the elements 2, 6, 16, 31, 51, 66, and 84, respectively. Similarly, the AENFs/AEPs of the elements 2, 6, 16, 31, 51, 66, and 84 are also obtained by the subarray extrapolation strategy. In this way, AENFs with \( 1710 \times 8 = 13680 \) sampling points and
AEPs with $1891 \times 8 = 15128$ sampling points are extracted. Again, taking $J_{x}^{G}A$ as the example, as shown in Fig. 15, the equivalent-source array is expanded into a $25 \times 25 \times 22$-element virtual uniform array with element spacings of $d_{x} = d_{y} = 0.42\lambda_{0}$ and $d_{z} = 0.41\lambda_{0}$. Note that, in the calculation, uniform amplitudes and progressive phases are implemented to make the main beam be steered to the direction of $(\theta_{G}^{0} = 45^\circ, \phi_{G}^{0} = 0^\circ)$.

Figs. 16 and 17 depict the simulated and calculated 3-D and 2-D normalized patterns. A good agreement is also achieved between the simulated and calculated results in the main-lobe region shown in Fig. 17. In the side-lobe regions, there exists some discrepancies between the simulated and calculated results due to the inevitable partial neglect of the MC variation. Note that, compared with other techniques listed in Table IV, the proposed method also maintains the advancement in terms of the calculation accuracy.

V. CONCLUSION

The main contribution of this paper is that the principle of pattern multiplication is extended and used for the far-field calculation of conformal arrays by the near-field Euler rotation-based technique. Additionally, the calculation is greatly accelerated by the 3-D VESE and the layer-wise 2-D FFT technique. By rotating the near-field vectorial data and position vectors on the sampling surfaces from the LCS to the GCS, the element anisotropy of the radiation characteristics is eliminated. The equivalent electric and magnetic currents (equivalent sources) of the rotated antenna element are obtained based on the Schelkunoff’s equivalence principle. Therefore, the far fields of conformal arrays can be calculated as the multiplication of the element factor and the array factor of the equivalent sources. Furthermore, the array factor is equivalent to that of an expanded virtual 3-D uniform array so that it satisfies the form of the 3-D FFT. Therefore, the calculation of the array factor is accelerated by the proposed
layer-wise 2-D FFT technique. Note that, the concept of the AENF based on the subarray extrapolation technique is developed in this paper, which is able to take the MC and edge effects into consideration. Finally, several numerical examples are provided to validate the advantages of the proposed method in terms of calculation efficiency and accuracy. Compared with other solutions, it is obvious that the computational time can be greatly reduced within very small MSE by applying the proposed method for the far-field calculation of conformal arrays.

REFERENCES


