Correction and Addendum for Consistent Optical and Electrical Noise Figure?

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Abstract

The minimum noise figure of an electrical amplifier is $F_e=1$. E. Desurvire’s traditional optical noise $F_{\text{opt}}$ of an optical amplifier has the minimum value $F_{\text{opt}}=2$. If $F_{\text{opt}}$ is a noise figure then power, gain and $F_e$ need to be redefined. $F_{\text{opt}}$ is in conflict with physics and $F_e$. The correct optical noise figure $F_{\text{opt},IQ}$, observable in coherent I&Q receivers, has the minimum $F_{\text{opt},IQ}=1$ and is compatible with $F_e$. In the derivation of the consistent unified noise figure $F_{\text{opt}}$ for all frequencies, from $F_e$ and $F_{\text{opt},IQ}$, thermal noise energy is needed. Its usual simplified expression $kT$ is now replaced by Nyquist’s correct result. This holds also in a unified homodyne noise figure $F_I$, against which H. Haus unified noise figure $F_{\text{haus}}$ is discussed.
Correction and Addendum for “Consistent Optical and Electrical Noise Figure”

Reinhold Noe

Abstract—The minimum noise figure of an electrical amplifier is \( F_e = 1 \). E. Desurvire’s traditional optical noise \( F_{pnf} \) of an optical amplifier has the minimum value \( F_{pnf} = 2 \). If \( F_{pnf} \) is a noise figure then power, gain and \( F_e \) need to be redefined. \( F_{pnf} \) is in conflict with physics and \( F_e \). The correct optical noise figure \( F_{o,IQ} \), observable in coherent I&Q receivers, has the minimum \( F_{o,IQ} = 1 \) and is compatible with \( F_e \). In the derivation of the consistent unified noise figure \( F_{IQ} \) for all frequencies, from \( F_e \) and \( F_{o,IQ} \), thermal noise energy is needed. Its usual simplified expression \( kT \) is now replaced by Nyquist’s correct result. This holds also in a unified homodyne noise figure \( F_i \), against which H. Haus’ unified noise figure \( F_{inh} \) is discussed.

Index Terms—Noise figure, Optical amplifiers, Optical fiber communication

I. INTRODUCTION

Equation, figure and reference numbering of the original paper [17] is continued here. The correct optical I&Q noise figure \( F_{o,IQ} \) as the 1:1 equivalent of the electrical noise figure \( F_e \) has been derived in [17]. Nothing needs to be changed there regarding optical noise figure (NF). The same is true for the optical homodyne noise figure \( NF_{o,I} \).

This paper adds to the description of the unified NF \( F_{IQ} \) for all frequencies. Thermal noise energy must be corrected from its usual simplified expression \( kT \) to Nyquist’s accurate expression, in order to “avoid the UV catastrophe” (Section III). For sake of completeness, the unified NF \( F_{fus} \) [6] of pioneer H. Haus is compared against the corrected unified homodyne NF \( F_I \) (Section IV). But we start with a discussion:

II. DISCUSSION OF NOISE FIGURE ISSUES

A caution about lab jargon: When NF is given as a factor and not in dB then in reality the noise factor (= SNR quotient) is meant. “noise figure” = (10 dB) log10(“noise factor”).

The reason for deriving \( F_{o,IQ} \) [17] is illustrated in Fig. 5. The insertion of a photodiode into the signal path [3] as a kind of extra power meter defies NF definition (linear channel, 2 available quadratures, minimum amplifier NF equals 1) [18]. The resulting traditional optical NF of E. Desurvire [3], called \( F_{pnf} \) in [6], is in conflict with ~150 years of science:

Measure electrical noise figure \( F_e \)

through connection or amplifier

get square of mean power

Measure traditional optical noise figure \( F_{pnf} \)

get variance of mean power

Fig. 5: Measurement of electrical and of traditional optical noise figure. LPF/BPF/HPF = lowpass/bandpass/highpass filter

High-frequency engineers would reject the idea of inserting an extra squaring power meter into the linear signal path. And so should optical engineers. But the photodiode needed for \( F_{pnf} \) definition acts as a squarer and power meter.

Subsequently the needed “power” = “\( P \)” ~ \( I^2 \) ~ \( P^2 \), i.e. electrical power of a photocurrent \( I \) flowing through a load resistor, is proportional to the square of the optical power \( P \). It holds \( P \sim |E|^2 \) where \( E \) is the optical field. This means “\( P \)” ~ \( |E|^4 \)!

By definition,

\[
F = \frac{SNR_{in}}{SNR_{out}} = \frac{P_{signal,in}}{P_{noise,in}} \cdot \frac{P_{signal,out}}{P_{noise,out}} = \frac{G_{noise}}{G_{signal}} \tag{65}
\]

where “out” means “with device” and “in” means “without device”. If we set \( F = F_{pnf} \) and derive this NF using “\( P \)” ~ \( I^2 \) then we find that any amplifier with gain \( G = G_{signal} \) has a supposed gain \( G_{signal} = G^2 \)!

Let’s see where \( F_{pnf} \) brings us. NF definition must not depend on detector type or frequency \( f \). The photodiode can be conducted, and with Novoptel GmbH, Helmerner Weg 2, 33100 Paderborn, Germany (e-mail: info@novoptel.com).
Thermal power detectors can be criticized. This gets zero point fluctuations \(1\), see eqn. (13) of \[3\] and \(F_{\text{opt}}\) with \(F_{\text{pfn}} = F_{\text{e}}^2\) ! We have two competing NF for the same amplifier at the same \(f\). \(F_{\text{pfn}}\) claims \(F_{\text{e}}\) to be wrong, and vice versa.

All this is direct consequence of calling \(F_{\text{pfn}}\) a NF. So, either \(F_{\text{pfn}}\) is a NF, or basic physics \(P \sim |U|^2\) and \(G\) and \(F_{\text{e}}\) are correct. We know the latter holds.

All the same, \(F_{\text{pfn}}\) [3] has great historic merits in the development of optical communication.

\(F_{\text{pfn}} = 2\) is found for an ideal optical amplifier. This irritates because the sensitivity of an ideal optical receiver with 2 available quadratures is not degraded by an ideal optical preamplifier.

While H. Haus [6] exposed \(F_{\text{pfn}}\) to violate NF definition, the proposed solutions \(F_{\text{fas}}\) (very rare case, optical homodyne NF for 1 quadrature, other than \(F_{\text{e}}\) for 2 quadratures), \(F_{\text{ase}}\) (no NF because it is not the SNR degradation factor in any optical receiver) also have a minimum value of 2 for an ideal amplifier, other than \(F_{\text{e}}\). Usually the value 2 (instead of 1) is explained by claiming that optical amplifiers be special.

But standard optical amplifiers such as EDFAs are not special. They amplify 2 quadratures and add Gaussian amplitude (field) noise in the 2 quadratures [17], like standard electrical amplifiers. Also not special is optical detection noise (photon/particle aspect manifest) [19] as opposed to electrical source noise (thermal origin). Special in the light of detection noise are true homodyne detection and direct detection because these keep only 1 degree-of-freedom or quadrature and suppress the other. For an ideal optical amplifier one heuristically finds [18] the minimum

\[
F_{\text{opt,min}} = \frac{\text{number of available quadratures in amplifier}}{\text{number of available quadratures in receiver}} \tag{66}
\]

where \(F_{\text{opt}}\) is any optical NF. Even though optical direct detection is nonlinear, also \(F_{\text{pfn}}\) obeys (66) with 1 available quadrature in receiver. \(F_{\text{ase}}\) is not covered by (66), given that \(F_{\text{ase}}\) is not the NF in any known optical receiver. Clearly it makes sense to have the same number of available quadratures in amplifier and receiver, and to choose this number equal to 2 like in the electrical case \(F_{\text{e}}\). This is confirmed by the correct NF \(F_{\text{o},\text{IQ}} \geq 1\) of a standard optical amplifier [17].

Instead of claiming \(F_{\text{o},\text{min}} = 2\) to be normal \(F_{\text{pfn}}\), \(F_{\text{fas}} = F_{\text{o},\text{I}}\), also \(F_{\text{ase}}\), all with standard optical amplifier) one could with the same right claim \(F_{\text{opt,min}} = 1/2\) to be normal (degenerate parametric optical preamplifier with \(F_{\text{o},\text{IQ}} = 1/2\) blocks one quadrature and increases I&Q receiver sensitivity to that of a true homodyne receiver). Both is mathematically correct, but none should be considered as the normal case.

I have been criticized for deriving \(F_{\text{o},\text{IQ}}, F_{\text{o},\text{I}}\) [17] in semiclassical description. There, a photocurrent \(I\) has a one-sided noise power spectral density (PSD) \(2eI\) due to a Poisson distribution of photoelectrons. Indeed one can alternately assume zero point fluctuations with energy \(hf/2\) per mode. They give rise to the same noise PSD \(2eI\) in a photocurrent \(I\). Fig. 6 is similar to Fig. 1, but the splitters are replaced by 2\times2 couplers and each of the 4 inputs gets zero point fluctuations. Interferences of zero point fluctuations from the LO coupler inputs with the LO signal eventually cancel upon photocurrent subtraction. Interferences of zero point fluctuations with the received signal are negligible since the LO is strong. Interferences of zero point fluctuations in the \(E_{\text{RX1,2}}\) from the signal coupler inputs with the LO signal add upon photocurrent subtraction. Using zero point fluctuations, exactly the known \(F_{\text{o},\text{IQ}}\) and \(F_{\text{o},\text{I}}\) are obtained in the end. Likewise, \(F_{\text{pfn}}\) can be calculated with either semiclassical shot noise or zero point fluctuations.

![Fig. 6: Coherent I&Q receiver with polarization matching](image)

III. CORRECT THERMAL NOISE AT ALL FREQUENCIES

In the derivation of a unified NF \(F_{\text{fas}}\) [6] the mean value of detectable thermal photons per mode was given (in other, equivalent nomenclature) as \(\langle n_g \rangle = kT/(hf)\); see eqn. (13) of [6]. Mean thermal noise energy or thermal PSD is hence \(\langle n_g \rangle hf = kT\). In [17] I have adopted this and have derived the unified I&Q NF \(F_{\text{IQ}}\), using the optical I&Q NF \(F_{\text{o},\text{IQ}}\). But in unified noise figures the term \(kT\) needs to be corrected at high frequencies! Total thermal power is finite. It is expedient to write

\[
kT = \frac{hf}{e(hf/(kT) - 1)} \quad \text{thermal power spectral density} \tag{67}
\]
The right hand side is eqn. (7) in [20], by Nyquist. Only for \( hf \ll kT \) it approaches \( kT \). The left hand side \( k' \) is defined such that where \( k \) was written in the derivation of \( F_{IQ} \) this is now replaced by \( k' \). \( k' \) is a frequency- and temperature-dependent function which approaches the Boltzmann constant \( k \) in the case \( hf \ll kT \), and 0 in the case \( hf \gg kT \).

In agreement with \( kT \) (67), total noise energy or spectral density per mode is \( hf/2 + kT \). Here \( hf/2 \) stands for zero point fluctuations which we can alternately express by shot noise.

Now consider electrical NF measurement. Since a real power detector is thermally noisy the SNR degradation factor underestimates the true \( F_e \). In particular, a decent amplifier with gain \( G \) in front of a very, very noisy detector will even improve the SNR. To get rid of thermal detector noise one puts the source at two different temperatures and measures noises with the power detector. Linear extrapolation of the measured noises to \( T = 0 \) K yields the own thermal noise of the power detector. It is subtracted in all NF calculations. This way the calculated SNRs and NF become higher. In practice, \( T = 0 \) K cannot be reached, and maybe the power detector wouldn’t even work at \( T = 0 \) K. But this does not matter. The NF is simply the quotient of SNRs that one would achieve if the power detector had no thermal noise.

The same principle must be applied for the unified \( F_{IQ} \). In practice, thermal noise occurs also at the unused input 2 (Fig. 6). It must not enter into the NF equations. To this purpose we assume an ideal cooled absorber with temperature 0 K at input 2. Thermal noise in the electronics behind the photodiodes is likewise eliminated because we have assumed \( P_{LO} \rightarrow \infty \). A few control measurements allow isolating and removing these thermal noises in a practical receiver.

There is yet another idealization: In the responsivity \( R = \frac{ne}{hf} \), the efficiency \( \eta \) has been set as \( \eta = 1 \). This maximizes the SNR. The same \( \eta = 1 \) is also used in the traditional \( F_{pnf} \).

The corrected equations ( \( k' \) instead of \( k \) ) are:

\[
SNR_{IQ,out} = \frac{I_d^2}{\sigma_e^2 + \sigma_{I_d}^2 + \sigma_{I_s}^2}
\]

\[
= \frac{R^2GP_eP_{LO}}{G^2F_e'k'TB_o/2 + G^2F_e'GhB_o/2 + eRPLB_e}
\]

\[
= \frac{GP_e}{F_e'k'TB_o/2 + \mu GbB_o/2 + hf_e}
\]

\[
= \frac{P_{I_f}}{F_e'k'T/2 + F_{IQ,\mu}hf/2} = \frac{P_{I_f}}{k'(T + T_{ex})/2 + (\mu + 1/G)hf/2}
\]

\[
SNR_{IQ,in} = \frac{P_{I_f}}{k'T/2 + hf/2},
\]

\[
F_{IQ} = \frac{SNR_{IQ,in}}{SNR_{IQ,out}} = \frac{G(P_{I_f}k'T + F_{IQ,\mu}hf)}{k'T + hf}
\]

\[
= \frac{k'(T + T_{ex}) + (\mu + 1/G)hf}{k'T + hf}(A = \frac{k'T}{k'T + hf})
\]

The change \( k' \) instead of \( k \) affects also Table I, Figs. 2, 3 and other places in [17]. The same change is needed in [18]. We recognize thermal source noise \( k'T \), thermal noise \( k'T_{ex} \) added in amplifier, spontaneous emission field noise \( \mu hf \) added in amplifier and shot noise \( hf/G \) in detector, all input-referred and per mode. For an amplifier it is not important to know the individual contributions of \( F_e \), \( F_{IQ,\mu} \); only the resulting \( F_{IQ} \) counts.

The crossover condition \( hf = k'T \) of equally strong thermal and quantum noises yields \( hf = kT \ln 2 \). This requires \( f = 194 \) THz / 28 THz / 4.3 THz / 1.1 THz / 58 GHz at \( T = 13400 \) K / 1940 K / 300 K / 77 K / 4 K, respectively. In [21], a 66 GHz electronic circuit operates at 4 K. Quantum noise plays a role here. Cryo and space electronics in the mm wave range and possible future THz applications need \( F_{IQ} \). The same would hold for an extremely hot attenuator or amplifier at the CO2 laser frequency 28 THz.

IV. Unified Homodyne Noise Figure

This is investigated in order to complete the picture in the context of \( F_{fas} \) in [6]. According to eqn. (35) of [17], the signal power \( P_S \) in a true homodyne receiver appears multiplied by \( 4R^2GP_{LO} \). The same holds for the in-phase part \( F_e k'TB_o/2 \) of the total received thermal noise power \( F_e k'TB_o \). We add the product to the denominator of (35) and obtain

\[
SNR_{I,out} = \frac{4R^2GP_eP_{LO}}{2GP_e k'TB_o/2 + 2GP_e GbB_o/2 + 2eRPLB_e}
\]

\[
= \frac{2P_{I_f}}{F_e k'T + F_{IQ,\mu}hf/2} = \frac{2P_{I_f}}{k'(T + T_{ex}) + (\mu + 1/G)hf/2}
\]

In the absence of noise and gain this becomes

\[
SNR_{I,in} = \frac{2P_{I_f}}{k'T + hf/2}
\]

We get the homodyne / in-phase / single-quadrature NF
\[ F_I = \frac{\text{SNR}_{I,in}}{\text{SNR}_{I,out}} = \frac{F_e kT + F_{o,I} hf/2}{kT + hf/2} \]

\[ = k(T + T_{ex}) + \left(2\mu + (1/G)hf/2\right) \]

\[ = A_I + \left(1 - A_I\right)G + \left(1 - A_I\right)T_{ex}/T + \left(1 - A_I\right)2\mu) \equiv A_I + \left(1 - A_I\right)G + \left(1 - A_I\right)T_{ex}/T + \left(1 - A_I\right)2\mu) \]

(70)

Note that in the electrical domain, single-quadrature or homodyne analysis simply means that the other quadrature is suppressed. This can be done by downconversion to baseband in a multiplier/mixer, or by degenerate parametric amplification. The electrical homodyne NF \( F_{e,I} \) equals the electrical I&Q NF, \( F_{e,I} = F_e = F_{e,Q} \) because there is (thermal) source noise. In (68)-(70) one could write \( F_{e,I} \) instead of \( F_e \).

\( F_I \) will probably not be needed in the electrical domain (mm waves at low temperatures) because single-quadrature electric amplifiers have no noise advantage over standard electrical amplifiers. \( F_I \) could be applied for an extremely hot device and a homodyne receiver at the CO2 laser frequency.

For \( hf >> kT \), H. Haus’ unified NF \( F_{fas} \), eqn. (18) in [6], becomes \( F_{o,I} \). This means \( F_{fas} \) is a homodyne NF at optical frequencies. \( F_{fas} \) is very similar to \( F_I \). Differences are:

- \( F_{fas} \) contains \( kT \) in \( \langle n_g \rangle \) instead of the correct \( kT \).
- No clear number of quadratures is defined for \( F_{fas} \). Since

- \( F_{fas} \) should generalize the familiar \( F_e \) one is left to assume that in \( F_{fas} \) there is 1 quadrature at optical \( f \), 1...2 quadratures at intermediate/thermal \( f \) and the usual 2 quadratures at electrical \( f \). Such transition is of course not possible.
- In \( F_{fas} \) it is defined \( T_{ex} = 0 \) (which is allowed), and all added thermal noise is assumed to be contained in a sufficiently large spontaneous emission factor \( n_{sp} \). For a pure attenuator one must guess and set \( n_{sp} = -kT/(hf) \). Otherwise the needed \( F_{fas} = 1/G \) is not reached. For comparison, \( F_I = 1/G \) is easily derived from the known \( F_{e,I} = F_e = F_{o,I} = 1/G \) and \( n_{sp} = 0 \).

REFERENCES


[18] R. Noe, "Noise Figure and Homodyne Noise Figure" Photonic Networks; 24th ITG-Symposium, Leipzig, Germany, 09-10 May 2023, pp. 85-91

[19] R. Noe, "Do Propagating Lightwaves Contain Photons?" Photonic Networks; 24th ITG-Symposium, Leipzig, Germany, 09-10 May 2023, pp. 113-121


[21] Y. Zhang, X. Jin, W. Liang, P. Sakalas and M. Schröter, "66 GHz 11.5 mW Low-power SiGe Frequency Quadrupler Operating at 300 K and 4 K," 2022 14th German Microwave Conference (GeMiC), Ulm, Germany, 2022, pp. 100-103.
Optical and Unified Noise Figure, and Homodyne Noise Figure

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Overview

- Motivation
- Noise figures in linear/coherent optical receivers
  - In-phase and quadrature noise figure = the noise figure
  - In-phase (homodyne) noise figure = special case
- Comparison of noise figures
- Consistent unified noise figure
- Summary
How to determine noise and gain properties of amplifier

Motivation

signal source → amplifier → BPF → (.)² → LPF

Standard electrical measurement

signal source → amplifier → BPF → (.)² → LPF → HPF → (.)² → LPF

gain

$G = e^{(a-b)t} = e^{(a-b)z/v_g}$

mean number of detectable output noise photons per mode

$\mu = n_{sp}(G - 1)$

mean output noise energy per mode

$P_{n, out} = \mu hf = G \tilde{\mu} hf$

spontaneous emission factor

$n_{sp} = \frac{a}{a-b}$

Is the inserted extra power meter helpful?

Probably not. ;-) Now this is a photodiode and we are talking about optical signals!
**F_e, gain, loss, power must be redefined if F_{pnf} is valid NF!**

\[ F = \frac{\text{SNR}_{in}}{\text{SNR}_{out}} = \frac{P_{s, in} P_{n, out}}{P_{n, in} P_{s, out}} = \text{noise gain} \]

\[ G^2 \langle n \rangle^2 = G^2 \] is the signal „gain“! \( \iff \) No more (optical) dB are allowed; „gain“ must be given in „electrical“ dB! \( \iff \) Fiber loss @ 1550 nm is no longer 0.2 dB/km; „loss“ must be stated as 0.4 dB/km! \( \iff \)

Thermal power meter can replace photodiode and allows going to low \( f \): \( \iff \) Any electrical or optical amplifier with 20 dB gain has 40 dB „gain“! \( \iff \) \( F_{pnf} = F_e^2 \) \( \iff \) All „powers“ (optical, electrical, thermal, mechanical) must be \( \sim \) squared powers, since all powers can be converted into thermal power and compared! \( \iff \) „power“ is not linear! \( \iff \) work = \( \sqrt{\text{power}} \cdot \text{time} \)

Unit definitions must not depend on measurement method or \( f \)! \( F_{pnf} \) implies:

\[ \text{SNR}_{pnf, in} = \frac{\langle n \rangle^2}{\langle n \rangle} \]

\[ \text{SNR}_{pnf, out} = \frac{G^2 \langle n \rangle^2}{G \langle n \rangle + n_{sp} (2(G - 1)G \langle n \rangle + ...)} \]
Electrical noise figure (NF) is standardized since many decades.

Traditional optical noise figure $F_{pnf}$ was defined in 1990ies, for optical direct detection receivers (DD RX). Problematic aspects, in conflict with electrical NF:

- Optical signals have in-phase and quadrature components, like electrical signals. But an optical DD RX suppresses phase information.
- "Power" in signal-to-noise (SNR) ratio calculation is $\sim$ square of photocurrent in optical DD RX. Photocurrent is $\sim$ optical power $\sim$ square of field amplitude. SNR "power" is $\sim$ 4th power of field amplitude $\sim$ square of power.

Conflict with $\sim$150 years of science: $P = U^2/R$, not $P \sim U^4$.

$\Rightarrow$ NF = 2 for ideal optical amplifier, whereas NF = 1 for ideal electrical amplifier.

Noise happens on a field basis. Looking at the power is insufficient!

Ideal DD RX for intensity modulation without / with ideal optical amplifier needs $10 / 38$ photoelectrons/bit for bit error ratio $= 10^{-9}$. Ideal DD RX for differential phase shift keying: $20 / 20$ photoelectrons/bit. Where is NF = 2?

Optical: Nonlinear DD RX; non-Gaussian noise; amplifier NF depends on power and bandwidths. Electrical: Linear RX; Gaussian noise; constant NF.

Unification of all prior optical NF with electrical NF is inconsistent, contradictory.
Fields in coherent optical I&Q receiver

\[ E_{RX} = \sqrt{G\left(\sqrt{P_S} + (v_1 + jv_2)\sqrt{P_n/2}\right)}e_1e^{j\omega t} \]

\[ E_{LO} = \sqrt{P_{LO}}e_1e^{j\omega t} \]

\[ P = |E|^2 \quad I = RP = e/(hf) \cdot P \]

Power (for simplicity) \hspace{1cm} Photocurrent

\[ \langle v_1^2 \rangle = \langle v_2^2 \rangle = \sigma^2 = 1 \]

\[ e_1 \] normalized field (polarization) vector

Optical signal is linearly downconverted to baseband. Local oscillator (LO) is a strong unmodulated laser with (essentially) the same frequency as the received signal. 2 available quadratures

Baseband I&Q receiver is not mandatory! Heterodyne receiver with image rejection filter gives the same results!
Photocurrents in coherent optical I&Q receiver ...

\[ E_{RX} = \sqrt{G} \left( \sqrt{P_S} + (v_1 + jv_2)\sqrt{P_n/2} \right) e_1 e^{j\omega t} \]

\[ E_{LO} = \sqrt{P_{LO}} e_1 e^{j\omega t} \]

\[ P := |E|^2 \quad I = RP = e/(hf) \cdot P \]

(In practice, optical frequencies of signal and unmodulated local oscillator may differ a bit, causing the complex plane of \( I_{1d} \) and \( I_{2d} \) to rotate at the difference frequency.)

\[ I_{1\pm} = R \left| \pm E_{RX}/2 + E_{LO}/2 \right|^2 \]

\[ = \frac{R}{4} \left( G(P_S + 2v_1 \sqrt{P_S P_n/2} + (v_1^2 + v_2^2)P_n/2) \pm 2(\sqrt{P_S} + v_1 \sqrt{P_n/2})\sqrt{GP_{LO} + P_{LO}} \right) \]

\[ I_{2\pm} = R \left| \pm E_{RX}/2 + j E_{LO}/2 \right|^2 \]

\[ = \frac{R}{4} \left( G(P_S + 2v_1 \sqrt{P_S P_n/2} + (v_1^2 + v_2^2)P_n/2) \pm 2v_2 \sqrt{P_n/2} \sqrt{GP_{LO} + P_{LO}} \right) \]
...and their differences and sums

\[ I_{1d} = I_{1+} - I_{1-} = R \left( \sqrt{P_S} + v_1 \sqrt{P_n/2} \right) \sqrt{GP_{LO}} \]

\[ I_{2d} = I_{2+} - I_{2-} = Rv_2 \sqrt{P_n/2} \sqrt{GP_{LO}} \]

\[ I_{1s} = I_{1+} + I_{1-} = R P_{LO} / 2 \]

\[ I_{2s} = I_{2+} + I_{2-} = R P_{LO} / 2 \]

Differences and sums of photocurrents

Neglect for \( P_{LO} \to \infty \)

\[ I_{1\pm} = R \left| \pm \frac{E_{RX}}{2} + \frac{E_{LO}}{2} \right|^2 \]

\[ = \frac{R}{4} \left( G \left( P_S + 2 v_1 \sqrt{P_S P_n/2} + \left( v_1^2 + v_2^2 \right) P_n/2 \right) \pm 2 \left( \sqrt{P_S} + v_1 \sqrt{P_n/2} \right) \sqrt{GP_{LO}} + P_{LO} \right) \]

\[ I_{2\pm} = R \left| \pm \frac{E_{RX}}{2} + j \frac{E_{LO}}{2} \right|^2 \]

\[ = \frac{R}{4} \left( G \left( P_S + 2 v_1 \sqrt{P_S P_n/2} + \left( v_1^2 + v_2^2 \right) P_n/2 \right) \pm 2 v_2 \sqrt{P_n/2} \sqrt{GP_{LO}} + P_{LO} \right) \]
SNR in coherent optical I&Q receiver

\[ I_{1d} = R\left(\sqrt{P_S} + \nu_1 \sqrt{P_n/2}\right) \sqrt{G P_{LO}} \]
\[ I_{2d} = R\nu_2 \sqrt{P_n/2} \sqrt{G P_{LO}} \]
\[ I_{1s} = R P_{LO} / 2 \]
\[ I_{2s} = R P_{LO} / 2 \]

Shot noise PSD:
\[ 2eI_{1s}, 2eI_{2s} \]
Optical bandwidth:
\[ B_o = 2 B_e = 1/\tau \]

Equivalent amplifier input noise PSD per mode:
\[ \tilde{\mu} h f = P_n / B_o \]

For SNR calculation take either noise in 1 mode or (like I do it) in 1 quadrature! (Factor 2 cancels in NF calculation.)

\[ SNR_{o,IQ,\text{out}} = \frac{\overline{I_{1d}^2}}{\sigma_{I_{1d}}^2 + \sigma_{I_{1s}}^2} = \frac{R^2 P_{LO} G P_S}{R^2 P_{LO} G \tilde{\mu} h f B_o / 2 + e R P_{LO} B_e} = \frac{P_S \tau}{(\tilde{\mu} + 1/G) h f / 2} \]
Noise figures in linear/coherent optical receivers

Optical I&Q noise figure (or heterodyne with image rej.)

\[ SNR_{o,IQ,out} = \frac{P_S \tau}{(\tilde{\mu} + 1/G)hf/2} \]

No amplifier, \( G = 1, \quad \tilde{\mu} = 0 \):

\[ SNR_{o,IQ,in} = \frac{P_S \tau}{hf/2} \]

\[ \frac{SNR_{o,IQ,in}}{SNR_{o,IQ,out}} = \]

\[ F_{o,IQ} = \tilde{\mu} + 1/G = n_{sp}(1-1/G) + 1/G \]

\[ = 1 + (n_{sp} - 1)(1-1/G) \geq 1 \]

\[ F_{o,IQ} = (F_{pnf} - 1/G)/2 + 1/G \]

\[ F_{pnf} = 2(F_{o,IQ} - 1/G) + 1/G \]

\[ F_{o,IQ} \approx F_{pnf}/2 \]

'Conversion formulas'

'\( F_{o,IQ} \) obeys the usual electrical NF definition, is SNR degradation factor; powers \( \sim \) squares of amplitudes; 2 available quadratures; linearity; ideal NF = 1; pure Gaussian noise!'
Measure optical I&Q noise figure with power meter

Optical amplifier must be loaded with extra optical signal power at other times/frequencies/polarization in order to keep $G$, $\tilde{\mu}$ constant.

Usually there are $p = 2$ polarization modes. $p = 1$ requires inserted polarizer.

$$P_1 = Gp\tilde{\mu}hfB_o + GP_S$$
$$P_2 = Gp\tilde{\mu}hfB_o$$
$$PS$$

$$G = \frac{P_1 - P_2}{PS}$$
$$\tilde{\mu} = \frac{P_2}{pGhfB_o}$$

$$F_{o,IQ} = \tilde{\mu} + 1/G$$

$F_{o,IQ}$ and all other optical NF can be determined from simple optical power measurements.
Optical I noise figure (true homodyne; special case)

In such cases, phase locking is required between signal and LO or detector!
No power splitting ⇒ In equations multiply each of $P_{LO}$, $P_S$, $P_n$, $\tilde{\mu}$, $n_{sp}$ by 2.

$$F_{o,I} = 2\tilde{\mu} + 1/G$$

$$= 1 + \left(2n_{sp} - 1\right)(1 - 1/G) \geq 1$$

$F_{o,I}$ is similar to $F_{o,IQ}$ and $F_e$, but only 1 quadrature is available.

Lowest $F_{o,I} \rightarrow 1$ for $G \rightarrow 1$. Ideal $F_{o,I} = 2$ at $G \rightarrow \infty$. Why?

Optical amplifier is not special! RX is special: 1 quadrature & detection noise!
Without optical amplifier, true homodyne RX is twice as sensitive as I&Q RX because $P_{RX}$ is not split. But with optical amplifier having $G \rightarrow \infty$, output power splitting like in the I&Q RX cannot have an SNR effect. So, behind the amplifier the homodyne RX “must” have the worse sensitivity of the I&Q RX. Amplifier halves homodyne SNR!

Phase-sensitive degenerate parametric optical amplifier passes only 1 quadrature and has ideal $F_{o,I} = 1$ and $F_{o,IQ} = 1/2$ (converts I&Q into more sensitive homodyne).
Shot noise can be derived either way (but only 1 way at a time, not 2 ways at a time):

- Semiclassical theory: Poisson distribution of photoelectrons has one-sided photocurrent power spectral density (PSD) \( 2eI \).
- Zero point fluctuations interfere with signal and cause shot noise PSD \( 2eI \).

Zero point fluctuations can explain/replace shot noise PSD \( 2eI \). Let us define field such that power is \( P := |E|^2 \). Observation time is \( \tau = 1/B_0 \).

Zero point fluctuations have mean energy \( W = P\tau \) equal to \( hf/2 \) per mode:

\[
E_0 = (u_1 + ju_2)e_1e^{j\omega t}
\]

\[
\sigma_{u1}^2 = \sigma_{u2}^2 = hf/(4\tau)
\]

Signal field: \( E_S = \sqrt{P_S}e_1e^{j\omega t} \)

Total field: \( E_S + E_0 \)

Expected number of photoelectrons:

\[
n_{S+0} = |E_S + E_0|^2 \tau/(hf) = \left( |E_S|^2 + 2 \text{Re}(E_S^*E_0) + |E_0|^2 \right) \tau/(hf) \approx \left( P_S + 2u_1\sqrt{P_S} \right) \tau/(hf)
\]

Mean:

\[
\langle n_{S+0} \rangle = \frac{P_S\tau}{hf}
\]

Variance:

\[
\sigma_{n_{S+0}}^2 = \frac{hf}{4\tau} P_S \frac{2^2 \tau^2}{h^2 f^2} = \frac{P_S\tau}{hf} = \langle n_{S+0} \rangle
\]

\[
I = RP = \frac{e}{hf} P
\]

\[
\langle I_{S+0} \rangle = \langle n_{S+0} \rangle \frac{e}{\tau}
\]

\[
\sigma_{I_{S+0}}^2 = \sigma_{n_{S+0}}^2 \frac{e^2}{\tau^2} = 2e \cdot \frac{e}{hf} \cdot \frac{P_S}{\tau^2}
\]

Noise figures in linear/coherent optical receivers
I&Q NF derived with zero point fluctuations (1)

\[ E_{RX1} = [\sqrt{G}\left(\sqrt{P_S} + (v_1 + jv_2)\sqrt{P_n/2}\right) + (u_{11} + ju_{12})]e_1e^{j\omega t} \]

\[ E_{RX2} = (u_{21} + ju_{22})e_1e^{j\omega t} \]

\[ E_{LO} = \sqrt{P_{LO}}e_1e^{j\omega t} \]

\[ I = RP = e/(hf) \cdot P \quad P = |E|^2 \]

\[ w_1 = u_{11} + u_{21} \quad w_2 = u_{12} - u_{22} \]

\[ 2\langle u_{ij}^2 \rangle = \langle w_{ij}^2 \rangle = \frac{hf}{2\tau} \quad \langle v_{ij}^2 \rangle = 1 \]

\[ I_{1\pm} = R\left|\pm(E_{RX1} + E_{RX2})/2 + E_{LO}/2\right|^2 \approx \frac{R}{4}\left(G\left(P_S + 2v_1\sqrt{P_S P_n/2} + (v_1^2 + v_2^2)P_n/2\right)\right)\pm 2\left(\sqrt{G\left(P_S + v_1\sqrt{P_n/2}\right)} + w_1\right)\sqrt{P_{LO} + P_{LO}} \]

\[ I_{2\pm} = R\left|\pm(E_{RX1} - E_{RX2})/2 + j E_{LO}/2\right|^2 \approx \frac{R}{4}\left(G\left(P_S + 2v_1\sqrt{P_S P_n/2} + (v_1^2 + v_2^2)P_n/2\right)\right)\pm 2\left(\sqrt{Gv_2\sqrt{P_n/2} + w_2}\right)\sqrt{P_{LO} + P_{LO}} \]

Zero point fluctuations occur at both signal ports. Mean power of zero point fluctuations is neglected for simplicity.
I&Q NF derived with zero point fluctuations (2)

The 2 LO ports also carry zero point fluctuations. But these cancel upon subtraction of photocurrents.

\[ I_{1d} = I_{1+} - I_{1-} \]
\[ I_{2d} = I_{2+} - I_{2-} \]

\[ I_{1d} = R \left( \sqrt{G \left( \sqrt{P_S + v_1 \sqrt{P_n / 2}} \right)} + w_1 \right) \sqrt{P_{LO}} \]
\[ I_{2d} = R \sqrt{G \left( v_2 \sqrt{P_n / 2} + w_2 \right)} \sqrt{P_{LO}} \]

\[ \langle w_k^2 \rangle = \frac{hf}{2\tau} \]
\[ \langle v_k^2 \rangle = 1 \]

\[ P_n = \tilde{\mu}hfB_o \quad B_o = 1/\tau \]

\[ SNR_{o,IQ,\text{out}} = \frac{I_{1d}^2}{\sigma_{I_{1d}}^2} = \frac{R^2 P_{LO} G_{PS}}{R^2 P_{LO} \left( GP_n / 2 + hf / (2\tau) \right)} = \frac{G_{PS}}{G\tilde{\mu}hfB_o / 2 + hfB_o / 2} = \frac{P_S\tau}{(\tilde{\mu} + 1/G)hf / 2} \]

\[ SNR_{o,IQ,\text{in}} = \frac{P_S\tau}{hf / 2} \]

\[ \frac{SNR_{o,IQ,\text{in}}}{SNR_{o,IQ,\text{out}}} = F_{o,IQ} = \tilde{\mu} + 1/G \]

Same result as when derived with Poisson photoelectron distribution.
Homodyne NF derived with zero point fluctuations

\[ E_{RX} = (\sqrt{G}(\sqrt{P_S} + (v_1 + jv_2)\sqrt{P_n/2}) + (u_1 + ju_2))e_1e^{j\omega t} \]

\[ E_{LO} = \sqrt{P_{LO}}e_1e^{j\omega t} \]

\[ I = RP = e/(h\nu) \cdot P \quad P =: \begin{vmatrix} E \end{vmatrix}^2 \]

\[ \langle u_k^2 \rangle = \frac{h\nu}{4\tau} \quad \langle v_k^2 \rangle = 1 \]

\[ I_\pm = R \begin{vmatrix} \pm E_{RX} / \sqrt{2} + E_{LO} / \sqrt{2} \end{vmatrix}^2 \]

\[ \approx \frac{R}{2} \left( G(P_S + 2v_1\sqrt{P_SP_n/2} + (v_1^2 + v_2^2)P_n/2) \pm 2\sqrt{G} \left( \sqrt{P_S} + v_1\sqrt{P_n/2} \right) + u_1 \right)\sqrt{P_{LO} + P_{LO}} \]

\[ I_d = I_+ - I_- = 2R \left( \sqrt{G} \left( \sqrt{P_S} + v_1\sqrt{P_n/2} \right) + u_1 \right)\sqrt{P_{LO}} \]

\[ \text{SNR}_{o,I,\text{out}} = \frac{I_d^2}{\sigma^2 I_d} = \frac{4R^2P_{LO}G_P}{4R^2P_{LO}(GP_n/2 + h\nu/(4\tau))} = \frac{2G_P}{2G\bar{\mu}hfB_0/2 + hfB_0/2} = \frac{2P_S\tau}{(2\bar{\mu} + 1/G)hf/2} \]

\[ \text{SNR}_{o,I,\text{in}} = \frac{2P_S\tau}{hf/2} \]

\[ \frac{\text{SNR}_{o,I,\text{in}}}{\text{SNR}_{o,I,\text{out}}} = F_{o,I} = 2\bar{\mu} + 1/G \]

Same result as when derived with Poisson photoelectron distribution.
Structure of noise figure which fulfills Friis‘ formula

\[
SNR_{in} = \frac{P_S}{N_s + N_d}
\]

\[
SNR_{out} = \frac{GP_S}{GN_s + N_d + N_a}
\]

\[
F = \frac{GN_s + N_d + N_a}{G(N_s + N_d)} = A + \frac{1-A}{G} + B
\]

Device cascade:

\[
SNR_{out} = \frac{G_1G_2P_S}{G_1G_2N_s + N_d + G_2N_a1 + N_a2}
\]

\[
F - 1 = F_1 - 1 + \frac{F_2 - 1}{G_1}
\]

Complete induction yields Friis‘ formula:

\[
F - 1 = \sum_{i=1}^{n} \frac{F_i - 1}{\prod_{k=1}^{i-1} G_k}
\]

It holds for all noise figures which can be written like this, including \(F_e, F_{ase}, F_{pnf} = F_{fas} = F_{o,I}, F_{o,IQ}\)!
Comparison of noise figures

**Optical noise figures [dB] vs. gain [dB]**

Only $F_{o,IQ}$ behaves like $F_e$ (1 for amplifier with $n_{sp} = 1$; $1/G$ for attenuator).

Is not the SNR degradation factor in any optical receiver!

$F_{o,IQ} = n_{sp}(1 - 1/G) + 1/G$

$F_{pnf} = 2n_{sp}(1 - 1/G) + 1/G = F_{fas} = F_{o,I}$

Assumes source noise

$F_{ase} = n_{sp}(1 - 1/G) + 1$
## Properties of noise figures

<table>
<thead>
<tr>
<th>Type of noise figure $F$</th>
<th>SNR degradation factor</th>
<th>Linear</th>
<th>Available quadratures</th>
<th>$F$ of ideal ampl. $G \to \infty$</th>
<th>$F$ of atten., $G &lt; 1$</th>
<th>$M$ of ampl.</th>
<th>Input-referred energy per mode, $kT_{ex}$ or $\tilde{\mu}h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_e$</td>
<td>yes</td>
<td>yes</td>
<td>2</td>
<td>1</td>
<td>$1/G$</td>
<td>$\geq 0$</td>
<td>$kT(F - 1)$</td>
</tr>
<tr>
<td>$F_{o,IQ} = n_{sp}(1 - 1/G) + 1/G$</td>
<td>yes</td>
<td>yes</td>
<td>2</td>
<td>1</td>
<td>$1/G$</td>
<td>$n_{sp} - 1 \geq 0$</td>
<td>$hf(F - 1/G)$</td>
</tr>
<tr>
<td>$F_{pfnf} = F_{fas} = F_{o,I}$</td>
<td>yes</td>
<td>not</td>
<td>1</td>
<td>2</td>
<td>$1/G$</td>
<td>$2n_{sp} - 1 \geq 1$</td>
<td>$hf(F - 1/G)/2$</td>
</tr>
<tr>
<td>$F_{ase} = 1 + n_{sp}(1 - 1/G)$</td>
<td>no</td>
<td>yes</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>$n_{sp} \geq 1$</td>
<td>$hf(F - 1)$</td>
</tr>
</tbody>
</table>

Only $F_{o,IQ}$ matches conceptually with $F_e$!

Note: NF is lab jargon. Precisely, $F$ is the noise factor and $(10 \text{ dB}) \cdot \log_{10}(F)$ is the noise figure.

For lowest NF of a cascade, order amplifiers according to ascending noise measure $M$.

$$M = \frac{F - 1}{1 - 1/G}$$
Comparison of noise figures

**Ideal optical amplifier noise figure at large gain is ... ?**

\[
\text{ideal optical } NF = \frac{\text{number of available quadratures in amplifier}}{\text{number of available quadratures in receiver}}
\]

<table>
<thead>
<tr>
<th>quadratures</th>
<th>1</th>
<th>2</th>
<th>Common answer since mid 1990ies:</th>
</tr>
</thead>
<tbody>
<tr>
<td>optical amplifier</td>
<td>phase-sensitive</td>
<td>phase-insensitive</td>
<td>( F_{pnf} = 2 = F_{fas} = F_{o,I} )</td>
</tr>
<tr>
<td>optical receiver</td>
<td>direct detection (or homodyne)</td>
<td>I&amp;Q, or heterodyne with image rejection</td>
<td>But with the same logic one could answer: ( F_{o,IQ} = 1/2 )</td>
</tr>
</tbody>
</table>

Other cases are considered as special.

It makes most sense to pair amplifiers and receivers with same number of available quadratures:

- optical amplifier: phase-sensitive
- optical receiver: direct detection (or homodyne)

Nonlinear! Can it yield a NF?

- \( F_{o,I} = 1 \)
- \( F_{o,IQ} = 1 \) (like \( F_e = 1 \))

User must provide phase reference! RX can also contain phase-sensitive amplifier!

By far most frequent optical + electrical scenario today!
One cannot say one NF \( F_e \) is for electrical detectors and another \( F_{pnf} \) is for quantum detectors (photodiodes), because one might become able to build both detector types for the same \( f \) (low THz region?): This would oppose unequal NF for same usage of same amplifier at same \( f \)!

NF must be detector-independent!

The term “noise figure” without additions suggests the properties of \( F_e \), i.e. SNR degradation factor in linear system with 2 quadratures (and preferably Gaussian noise).

⇒ Term “optical noise figure” seems fit only for \( F_{o,IQ} = \tilde{\mu} + 1/G \).

To avoid misinterpretation, \( F_{pnf} = 2\tilde{\mu} + 1/G \) could be called “high-power optical \( \chi^2 \) (chi-square) noise estimator”, “photoelectron number fluctuation indicator”, ...

Likewise, \( F_{o,I} (= F_{fas} (= F_{pnf}) ) \) can be called “optical 1-quadrature NF” (= in-phase).

If SNR is defined with only in-phase noise then the electrical 1-quadrature NF \( F_{e,I} \) equals \( F_e \). I have combined \( F_{e,I} \) with \( F_{o,I} \) to form a 1-quadrature NF \( F_I \).

Result is similar to a corrected \( F_{fas} \). But number of quadratures in \( F_{fas} \) is not given and one is left to assume that in the electrical domain \( F_{fas} \) is for 2 quadratures. \( 1 \neq 2 \)!

An interpretation difference is that in \( F_{fas} \) added thermal noise is considered not separately, but as caused by spontaneous emission (set \( T_{ex} = 0 \) and take a high \( \tilde{\mu} \), with \( \tilde{\mu} \rightarrow \infty \) for \( f \rightarrow 0 \)). In a phase-sensitive amplifier, ideal \( F_{o,I} = F_{fas} = 1 \).
Consistent unified noise figure

Removing avoidable receiver or power meter noise

In NF measurement the power meter or RX is always assumed to be free of avoidable noise. In practice it is not possible to cool a power meter or RX to 0 K in order to avoid its thermal noise. For this reason the intrinsic power meter noise is measured, and subtracted during NF measurement, thereby maximizing the resulting NF.

In the coherent RX we also must assume zero thermal noise. In the foregoing this has been achieved by letting $P_{LO} \to \infty$. Practically one must subtract RX thermal noise.

Shot noise of LO is unavoidable. But shot noise of received signal is avoidable by $P_{LO} \to \infty$. Practically one must subtract shot noises caused by $GP_S$ and $P_S$.

Nonideal quantum efficiency $\eta$ also reduces and falsifies measured NF. Hence we have assumed $\eta = 1$ in the responsivity $R = \eta e/(hf)$. Practically one must correct measurements such that they represent the case $\eta = 1$.

In the coherent I&Q RX the signal splitter can be viewed as a 2×2 coupler. When considering all frequencies, thermal noise enters also at the 2nd, unused coupler input. That can be avoided by cooling the termination of the 2nd coupler input to 0 K. Practically, thermal noise due to the 2nd coupler input must be subtracted.

These corrections have been implemented. They minimize RX noise and maximize measured NF. – Direct power measurements (p. 27) achieve the same and are easier!
Shot noise is represented either semiclassically by a Poisson distribution of photoelectrons or by zero point fluctuations at all inputs.

Thermal noise energy per mode approaches $kT$ only at low frequencies $f$. For all $f$ the correct expression is:

$$\frac{hf}{e^{hf/(kT)} - 1}$$

For NF measurement, always an ideal RX or power meter is assumed!

For the source with temperature $T$ we define:

$$k'T = \frac{hf}{e^{hf/(kT)} - 1}$$

Signal input 2 shall be terminated by an absorber having $T'' = 0$ K. No thermal noise enters there.

All noises interfere essentially with the strong LO fields ($P_{LO} \to \infty$).

LO interferences of noises from the optical input, $E_{RX1}$, input 2, $E_{RX2}$ ($T'' = 0$ K), and $E_{LO}$ cancel upon photocurrent subtractions. Signal inputs add.
Consistent unified noise figure

Block diagrams with thermal and optical noises

Source, amplifier (left) or attenuator (right) and detector (electrical or coherent optical), all I&Q, noisy or noiseless with equivalent added noise energies per mode. Individual devices (top) and equivalent interpretations (middle, bottom). Detector is for 2 available quadratures. If detector were for 1 quadrature, $hf$ would be replaced by $hf/2$.

Upconversion e-o is possible with an I&Q modulator (or DSB modulator + SSB filter), downconversion o-e with an I&Q RX (or heterodyne + image rejection filter).
SNR in the presence of thermal and optical noises

To derive a consistent unified NF (I&Q !) we add noises of \( F_e \) and \( F_o,IQ \) for all \( f \).

Optical and electrical gains \( G \) are identical because they manifest at same \( f \).

Thermal noise in bandwidth \( B_o = 1/\tau = 2B_e \) is \( GF_e k'TB_o \) at amplifier output.

Half of this is in phase with signal. In coherent I&Q RX it appears multiplied with \( R^2P_{LO} \), like amplified signal power \( GP_S \). Corresponding variance \( \sigma_e^2 \) is added.

\[
SNR_{IQ,out} = \frac{I_{1d}^2}{\sigma_e^2 + \sigma_{I_{1d}}^2 + \sigma_{I_{1s}}^2} = \frac{R^2P_{LO}GP_S}{R^2P_{LO}GF_e k'TB_o/2 + R^2P_{LO} \tilde{\mu}GhfB_o/2 + eRP_{LO}B_e} = \frac{GP_S}{GF_e k'TB_o/2 + \tilde{\mu}GhfB_o/2 + hfB_e}
\]

\[
P_S\tau = \frac{P_{S}\tau}{F_e k'T/2 + F_o,IQ hf/2} = \frac{P_{S}\tau}{k'(T + T_{ex})/2 + (\tilde{\mu} + 1/G)hf/2}
\]

Detector type does not matter, as long as it is usable in linear I&Q receiver:

Powers in I&Q receiver with quantum detectors

Powers in electrical I&Q receiver

Thermal source noise
Thermal amplifier noise
Spontaneous emission field noise in amplifier
Shot noise in detector
I&Q noise figure from electrical to optical frequencies

Consistent unified noise figure

$$\frac{SNR_{IQ,\text{out}}}{SNR_{IQ,\text{in}}} = \frac{PS\tau}{F_e k'T/2 + F_o,\text{IQ} hf/2} = \frac{PS\tau}{k'(T + T_{ex})/2 + (\bar{\mu} + 1/G) hf/2}$$

$$SNR_{IQ,\text{in}} = \frac{PS\tau}{k'T/2 + hf/2}$$

(obtained with $T_{ex} = 0, \bar{\mu} = 0, G = 1$)

$$SNR_{IQ,\text{in}} = SNR_{IQ,\text{out}} = F_{IQ} = \frac{F_e k'T + F_o,\text{IQ} hf}{k'T + hf} = \frac{k'(T + T_{ex}) + (\bar{\mu} + 1/G) hf}{k'T + hf}$$

$$= A + (1 - A)/G + (A T_{ex}/T + (1 - A)\bar{\mu})$$

$$A = k'T/(k'T + hf)$$

Measured $F_{IQ}$ is just observed SNR degradation in linear system with 2 quadratures. In amplifier, $F_e, F_o,\text{IQ}$ may not be known. Anyway, $k'T_{ex} + \bar{\mu}hf$ is total added noise.

In attenuator, clear separation yields the correct result: $G < 1, T_{ex} = T(1/G - 1)$,

$$n_{sp} = 0, \bar{\mu} = 0 \implies F_{IQ} = 1/G = F_e = F_o,\text{IQ}$$

At low $f$: $F_{IQ} \rightarrow F_e$. At high $f$: $F_{IQ} \rightarrow F_o,\text{IQ}$. At 13400 / 1940 / 300 / 77 / 4 K, equal $k'T = hf$ is at $f = 194 / 28 / 4.3 / 1.1 / 0.06$ THz.

Linear! Pure Gaussian noises! 2 available quadratures! Fulfills Friis’ formula!

https://ieeexplore.ieee.org/document/9783564 = 66 GHz @ 4 K
### Measure I&Q noise figure with power meter

Usually there are \( p = 2 \) polarization modes. \( p = 1 \) needs a polarizer to be inserted. \( P_0' \) is the power readout offset caused by noise generated inside the power meter.

\[
P_1 = (k'G(T + T_{ex}) + \tilde{\mu}G\hbar f)pB_o + P_0' + GP_S
\]

\[
P_2 = (k'G(T + T_{ex}) + \tilde{\mu}G\hbar f)pB_o + P_0'
\]

\[
P_3 = k'TpB_o + P_0' + P_S
\]

\[
P_4 = k'TpB_o + P_0'
\]

\[
G = \frac{P_1 - P_2}{P_3 - P_4} \quad \text{Gain}
\]

\[
k'T_{ex} + \tilde{\mu}\hbar f = \frac{1}{G} \left( \frac{P_2 - P_4}{pB_o} - k'(G - 1)T \right) \quad \text{Added noise}
\]

It doesn’t matter, and needn’t be known, in how far added noise is of thermal or quantum origin. \( F_{IQ} \) and all other NF can be determined from simple static power measurements.

\[
F_{IQ} = \frac{k'T + (k'T_{ex} + \tilde{\mu}\hbar f) + \hbar f/G}{k'T + \hbar f}
\]
SNR with 1-quadrature noises and homodyne receiver

No power splitting $\Rightarrow P_{LO}, P_S, P_n, \tilde{\mu}, n_{sp}$ must be multiplied by 2 compared to $F_{o,IQ}$ calculation. Only 1 RX input! Total thermal noise in bandwidth $B_o$ at amplifier output is $\textcolor{red}{GF_e k'TB_o}$. Half of this is in phase with the signal. In the coherent 1-quadrature (homodyne) RX it appears multiplied with $4R^2P_{LO}$, like the amplified signal power $GP_S$. RX for 1 quadrature is a special case!

$$SNR_{I,out} = \frac{\frac{4I_{1d}^2}{4\sigma_e^2 + 4\sigma_{I1d}^2 + 2\sigma_{I1s}^2}}{4R^2P_{LO}GP_S}$$

$$= \frac{4R^2P_{LO}GP_S}{4R^2P_{LO}GF_e k'TB_o/2 + 4R^2P_{LO} \tilde{\mu}GhfB_o/2 + 2eRP_{LO}B_e}$$

$$= \frac{2GP_S}{2GF_e k'TB_o/2 + 2\tilde{\mu}GhfB_o/2 + hfB_e}$$

$$= \frac{2P_S\tau}{Fe/k'T + F_{o,I}hf/2} = \frac{2P_S\tau}{k'(T + T_{ex}) + (2\tilde{\mu} + 1/G)hf/2}$$

Powers in homodyne receiver with quantum detectors

Thermal source noise
Thermal amplifier noise
Spontaneous emission field noise in amplifier
Shot noise in detector
1-quadrature / homodyne unified noise figure

$$\text{SNR}_{I,\text{out}} = \frac{2P_S \tau}{F_e k'T + F_{o,I} hf / 2} = \frac{2P_S \tau}{k'(T + T_{ex}) + (2\tilde{\mu} + 1/G) hf / 2}$$

$$\text{SNR}_{I,\text{in}} = \frac{2P_S \tau}{k'T + hf / 2}$$

(obtained with $T_{ex} = 0$, $\tilde{\mu} = 0$, $G = 1$)

$$\frac{\text{SNR}_{I,\text{in}}}{\text{SNR}_{I,\text{out}}} = F_I = \frac{F_e k'T + F_{o,I} hf / 2}{k'T + hf / 2} = \frac{k'(T + T_{ex}) + (2\tilde{\mu} + 1/G) hf / 2}{k'T + hf / 2}$$

$$= A_I + (1 - A_I)/G + (A_I T_{ex} / T + (1 - A_I)2\tilde{\mu})$$

$$A_I = k'T/(k'T + hf / 2) \neq A$$

- $F_{o,I} \neq F_{o,IQ}$ because there is detection noise!
- $F_{e,I} = F_{e,IQ} \equiv F_e$ because there is source noise!
- 1-quadrature / homodyne $F_I$ is close to $F_{fas}$ (except $k'$ and interpretation difference)!

In definition of $F_{fas}$, number of quadratures was not discussed. $F_{fas}$ is intended to be identical with the normal electrical $F_e$, which is understood to be for 2 available quadratures. So, one is left to assume that $F_{fas}$ has 2 quadratures in the electrical and 1 quadrature in the optical domain. But that is contradictory, impossible!

(set $T_{ex} = 0$ and take a high $\tilde{\mu}$, with $\tilde{\mu} \rightarrow \infty$ for $f \rightarrow 0$)
1-quadrature / homodyne unified noise figure

\[
\text{SNR}_{I,\text{out}} = \frac{2P_{S}\tau}{F_{e}k'T + F_{o,I}hf / 2} = \frac{2P_{S}\tau}{k'(T + T_{ex}) + (2\tilde{\mu} + 1/G)hf / 2}
\]

\[
\text{SNR}_{I,\text{in}} = \frac{2P_{S}\tau}{k'T + hf / 2}
\]

\[
\frac{\text{SNR}_{I,\text{in}}}{\text{SNR}_{I,\text{out}}} = F_{I} = \frac{F_{e}k'T + F_{o,I}hf / 2}{k'T + hf / 2} = \frac{k'(T + T_{ex}) + (2\tilde{\mu} + 1/G)hf / 2}{k'T + hf / 2}
\]

\[
= A_{I} + (1 - A_{I})/G + (A_{I} T_{ex}/T + (1 - A_{I})2\tilde{\mu}) \quad A_{I} = k'T/(k'T + hf / 2) \neq A
\]

\[
F_{o,I} \neq F_{o,IQ}
\]

because there is detection noise!

\[
F_{e,I} = F_{e,IQ} \equiv F_{e}
\]

because there is source noise!

1-quadrature / homodyne \( F_{I} \) is close to \( F_{\text{fas}} \) (except \( k' \) and interpretation difference)!

Attenuator: I simply say

\[
T_{ex} = T(1/G - 1), \quad n_{sp} = 0 = \tilde{\mu}, \quad F_{I} = 1/G = F_{o,I} = F_{e} \quad (= F_{e,I}).
\]

Attenuator: To get \( F_{\text{fas}} = 1/G \) (= \( F_{I} \)) I find I must

set \( T_{ex} = 0, \quad n_{sp} = -k'T/(hf), \quad \tilde{\mu} = n_{sp}(1-1/G). \)

\[
f \to \{\infty, 0\} \Rightarrow n_{sp} \to \{0, -\infty\}, \quad \tilde{\mu} \to \{0, \infty\}.
\]
## Noise figures

RX or power detector noise would cause $F_e$ to be underestimated. Therefore RX noise is always subtracted, using reference measurements. The same way, avoidable optical RX noise can and must be subtracted at high $f$. Only LO shot noise is fundamental and is kept. Photodiode efficiency must be set equal to 1. We get:

<table>
<thead>
<tr>
<th>Optical $kT &lt;&lt; hf$</th>
<th>Unified/generalized</th>
<th>Electrical $kT &gt;&gt; hf$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{\mu} = n_{sp}(1-1/G)$</td>
<td>$\tilde{\mu} = n_{sp}(1-1/G)$</td>
<td>$k'T = \frac{hf}{e^{hf/(kT)}-1}$</td>
</tr>
</tbody>
</table>

RX with 2 available quadratures (I&Q); the noise figure:

- $F_{o,IQ} = \tilde{\mu} + 1/G$
- $F_{I,Q} = \frac{k'(T+T_{ex})+(\tilde{\mu}+1/G)hf}{k'T+hf}$
- $F_e = 1 + \frac{T_{ex}}{T}$

RX with 1 available quadrature; important only in true optical homodyne systems:

- $F_{o,I} = 2\tilde{\mu} + 1/G$
- $F_I = \frac{k'(T+T_{ex})+(2\tilde{\mu}+1/G)hf/2}{k'T+hf/2}$
- $F_{e,I} = 1 + \frac{T_{ex}}{T}$

Shot noise occurs upon detection. ⇒ Optical homodyne is different. In all cases, NF of pure attenuator is $1/G$. 

Summary

- Optical $kT << hf$
- Unified/generalized
- Electrical $kT >> hf$
All prior optical and unified NF $F_{pnf}$, $F_{fas}$, $F_{ase}$ are in conflict with electrical NF $F_e$.
A "noise figure" without special name is expected to be the SNR degradation factor in a linear system with 2 available quadratures (and Gaussian noise?!), like $F_e$.

The only optical NF which fulfills this is the optical I&Q NF $F_{o,IQ}$. It is $\geq 1$, like $F_e$.

Coherent I&Q receivers are linear field sensors. They linearize the quadratic field behavior of photodiodes. Heterodyne with image rejection is also fine.

At high gain, $F_{o,IQ} \approx F_{pnf}/2$, i.e. $\approx 3$ dB less when expressed in dB.

Electrical and optical I&Q NF are limit cases of the NF $F_{IQ}$, unified for all $f$.

Quantum noise / $F_{IQ}$ plays a role in today’s electronics at low $T = 4$ K.

The in-phase equivalent of $F_{o,IQ}$ is $F_{o,I}$, a limit case of the unified $F_{fas}$. So, $F_{fas}$ is a 1-quadrature NF and its other limit is $F_e$ for 1 quadrature, not the expected 2.

Information conveyed by the full $F_{pnf}$ (including sp-sp) of a specific DD RX can be obtained, more accurately, from $F_{o,IQ}$ (pure Gaussian noise).

Optical amplifier adds Gaussian I&Q field noise (wave aspect). Photodetection adds shot noise (particle aspect).