Performance of a New Dynamic TS Protocol with Intelligent Battery Management in a Full-Duplex Relay Network

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Keywords: Dynamic time-switching, dynamic battery energy, energy harvesting, full-duplex, simultaneous wireless information and power transfer, throughput.

I. INTRODUCTION

In the years to come, technologies for intelligent manufacturing, e-robotics, and smart cities will be deployed, where millions of interconnected internet of things (IoT) devices will be served at high data

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rates [2]. To meet the requirements for high spectral efficiency (SE), ultra-high reliability, low latency, and reduced power consumption, IoT nodes are required to have large battery lifetimes [3]. The SE can be approximately doubled by using the full-duplex (FD) communication nodes (FD nodes allows simultaneous transmission and reception in the same time/frequency block) compared to convectional half-duplex (HD) communication nodes. However, the applicability of the FD communication nodes is limited due to incurred self-interference (SI) from the transmit antenna to the receive antenna. Fortunately, recent advancements in active and passive cancellation techniques has ensured the mitigation of this strong self-interference (SI) to the residual levels, which in turn offers the practical utilization of the FD relays [4], [5].

Communication devices are often assumed to be battery-powered, with strict requirements on battery lifetimes. It is therefore undesirable for a node to drain its own battery for the purpose of relaying. To prevent frequent replacement of the batteries, the use of energy harvesting (EH) for the relaying is advantageous since it prolongs the battery lifetime of the communication nodes [6]. However, the harvested energy from RF is typically small, making links with such nodes unreliable. There is clearly a great practical requirement to explore efficient strategies in the context of wireless energy transfer (WET). EH has emerged as a promising technology for deploying self-sustaining green machine-type devices. However, the harvested energy is random and may sometimes not be enough to meet these communication nodes’ strict quality of service (QoS) requirements. To meet these requirements, some suggested solutions are 1) use of energy buffers to store the excess harvested energy in a harvest-store-use framework [7], and 2) augmenting the harvested energy with a controlled amount of battery energy in order to achieve the desired QoS while prolonging the relay’s battery lifetime [8]. The latter approach is readily feasible with today’s technology and is the focus of this paper. It is referred to as the battery-assisted EH framework.

A. Related Works

The possibility of transferring both information and energy together has spurred research on simultaneous wireless information and power transfer (SWIPT). SWIPT can result in notable gains in terms of energy consumption, time delay, SE, and interference management [9]. The harvested energy is often stored in super-capacitors, though re-chargeable batteries can also be used [10]. For green communication, self-sustaining EH nodes based on the SWIPT principle have emerged as an appealing alternative to avoid frequent battery replacements [11]. For enabling SWIPT, two EH protocols, namely time-switching (TS) and power-splitting (PS), has been widely studied in the literature [12]. In the TS protocol for EH, the total signalling interval is apportioned into two phases 1) the EH phase (used for harvesting the energy from incoming symbol using EH circuit) 2) the information transmission (IT) phase (used for forwarding the information symbol using energy harvested in the EH phase), whereas
in PS protocol for EH, the incoming symbol is divided into two portions out of which one is used for EH and other is utilized for IT. The TS protocol has the advantage of low-complexity circuits, while PS requires more complex circuitry \[13\]. In this context, most of the available literature focuses on linear EH. However, the use of nonlinear devices like diodes and transistors in EH circuitry cause the output power to saturate at higher values of input energy \[14\]. The nonlinear EH offers several critical challenges such as higher computational complexity and early saturation floors even with advanced SI cancellation due to the saturation threshold of the nonlinear EH circuits.

In \[15\], the performance of a two-hop network with TS Eh protocol has been derived in terms of outage probability (OP) and throughput considering nonlinear EH at decode-and-forward (DF) FD relay. Further in \[16\], the polarization enabled SI cancellation technique has been used to determine the throughput of amplify-and-forward (AF) FD relay network. In \[17\], a comparison was made between the outage performance of HD and FD relaying modes, and the utilization of self-recycled energy was highlighted in the context of FD relaying. A battery-assisted HD framework has been studied in \[18\], where authors have derived the throughput and OP for selective DF and incremental relaying with nonlinear EH. In \[19\], the impact of residual SI and hardware impairments has been studied on the system outage performance for SWIPT enabled FD two-way DF relay network. The OP and throughput performance for cooperative FD non-orthogonal multiple access have been analyzed in \[20\]. The throughput performance with PS-EH protocol has been investigated and the system capacity was maximized by optimally choosing the PS parameter \[21\]. In \[22, 23\], the cooperative FD NOMA with PS EH and TS EH protocol have been studied and the optimal use of PS/TS parameter was proposed for enhanced performance. A detailed comparison of our work with existing literature on nonlinear EH based FD relaying is provided in Table I.

Table I: Comparison of existing literature and our work.

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B. Motivations and Contributions

The precise battery management strategy used by the battery-assisted EH node determines both the throughput performance and energy consumption. In [8], for the idealistic linear EH case, the static TS protocol was analyzed, where a fixed amount of battery energy was added to the harvested energy in every symbol interval. We refer to this as the fixed battery energy (FBE) scheme. Notice that it has the advantage of simplicity but fails to utilize the battery energy efficiently. Therefore, in this work, we propose a novel intelligent battery management scheme referred to as the dynamic battery energy (DBE) scheme, wherein the battery energy can be judiciously drawn to guarantee the desired QoS. Although it requires a more complex circuity, it can be more energy efficient. In this paper, we consider both the FBE and DBE battery management schemes in a battery-assisted FD relay framework and proposes a new dynamic TS EH protocol that utilizes the instantaneous channel state information (CSI). The instantaneous CSI can help self-sustaining devices in prolonging their battery lifetime by allowing the opportunistic utilization of the device’s battery energy based on the amount of the harvested energy. While static TS requires knowledge of only average channel gains and is easier to implement, dynamic TS has the advantage of superior performance. Dynamic TS can help in intelligent battery management by lowering the average battery energy consumption while increasing the throughput at the same time.

To the best of the authors’ knowledge, the CSI availability-based EH protocols have never been discussed in the literature for networks with FD relays. In such a scenario, the EH parameter is determined by utilizing the CSI, which offers critical challenges in determining the range of EH parameters to ensure non-outage. Further, the smart use of battery energy is essential with the FD EH node, as for very high transmit power the residual SI can result in degraded performance. To address these issues, in this work, we emphasize that the node-level energy considerations are practically very important but have received no attention in the literature. This work aims to fill this important research gap. In this paper, a dual-hop cooperative battery-assisted FD relay framework is investigated with saturation-based nonlinear EH. The two intelligent battery management schemes, namely FBE and DBE, are explored with static TS and dynamic TS schemes. The major contributions of this paper are highlighted as follows:

1) We investigate the performance of a two-hop FD relay network where the harvested energy at the relay is augmented with a limited amount of relays’ battery energy to ensure reliable communication between the source and the destination. Considering the static TS EH protocol

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1 For both FBE and DBE battery management schemes, only the source-to-relay channel knowledge is required at the relay. Thus, the proposed DBE scheme does not increase the system overhead in any manner.

2 In practical systems which encounter slow fading, the instantaneous CSI can be estimated on a short-term basis using a reverse training mechanism, wherein a known pilot sequence is transmitted. Thus, the channel information can be estimated using the combined knowledge of the received and transmitted signals.
as a benchmark, the network performance is analyzed with two battery management schemes, namely FBE and DBE.

2) A new dynamic TS EH protocol based on instantaneous CSI is proposed to enhance the performance while simultaneously lowering the average energy consumption. In the dynamic TS EH protocol, the optimal choice of EH parameter is derived for both FBE and DBE battery management schemes.

3) Utilizing the derived range of TS parameters, the closed-form expressions for the throughput are derived considering all four possible combinations, 1) static TS EH protocol with FBE scheme, 2) static TS EH protocol with DBE scheme, 3) dynamic TS EH protocol with FBE scheme, and 4) dynamic TS EH protocol with DBE scheme. Among these four, the proposed dynamic TS EH protocol provides a percentage throughput gain of almost 50% and 25% over the static TS EH protocol for FBE and DBE schemes, respectively.

4) For the case of the DBE scheme, the closed-form expressions for the average energy consumption are derived for both static and dynamic TS EH protocols. Further, it is shown that the DBE scheme with dynamic TS EH protocol results in lower battery energy consumption compared to the static TS EH protocol with the DBE scheme.

5) It is also shown that by jointly optimizing the battery energy and the TS parameter, the throughput for both static TS and dynamic TS EH protocols can be maximized for both FBE and DBE schemes. Finally, it is demonstrated that for a fixed rate requirement, the dynamic TS EH protocol with the DBE scheme results in battery energy savings of more than 50% compared to that of static TS with the DBE scheme. While with the dynamic TS, the battery energy requirement is lowered by approximately 30% with the DBE scheme compared to the FBE scheme.

The remainder of this paper is structured as follows. Section II describes the system model as well as the battery management schemes. Section III presents the throughput analysis for both the FBE and DBE battery management schemes for static TS and dynamic TS EH protocols. In Section IV, for both static TS and dynamic TS EH protocols, closed-form expressions for the average battery energy consumption. Section V presents the throughput maximization problem. Finally, numerical results are presented in Section VI to validate the accuracy of analytical expressions, and Section VIII presents concluding remarks.

Notation: $f_X(x)$ and $F_X^c(x)$ respectively denote the probability density function (PDF) and complementary cumulative distribution function (CCDF) of a random variable $X$. $\mathbb{E}[]$ and $|x|$ respectively denote the expectation operator and the absolute value of $x$. $\mathcal{C}\mathcal{N}(0, \sigma^2)$ represents a zero-mean complex Gaussian distribution with variance $\sigma^2$, and $\mathcal{E}_1(x) = \int_1^\infty t^{-1} \exp(-xt)dt$ denotes the exponential integral of type 1. $\Gamma(v, \mu u) = \mu^v \int_{u}^{\infty} x^{v-1} \exp(-\mu x)dx$ [24, 3.381.1], denotes the upper incomplete Gamma function.
Fig. 1: System Model

II. SYSTEM MODEL

As illustrated in Fig. 1, we consider a cooperative network consisting of a source node (S), a battery-assisted EH relay node (R) and destination node (D). S is equipped with L transmit antennas and communicates with R over a direct link, whereas R communicates to D in the decode-and-forward (DF) relaying mode. R is equipped with two separate antennas, one for transmission and one for reception, while D is equipped with a single receive antenna. R utilizes the TS EH protocol for harvesting energy from S. The direct S-D link does not exist due to severe shadowing. The EH circuit mounted at R is assumed to exhibit nonlinear characteristics, i.e., the harvested energy is a non-linear function of input energy and saturates when input energy exceeds a threshold level [14].

Quasi-static Rayleigh fading channels are assumed between each pair of nodes. \( g_{ab} \sim \mathcal{CN}(0, \lambda_{ab}^{-1}) \) denotes the channel coefficient between nodes a and b where \( a \in \{s,r\} \) and \( b \in \{r,d\} \). Here, \( \lambda_{ab} = d_{ab}^\theta \), where \( \theta \) is the path-loss exponent. In particular, the loop-back interference channel at the FD relay follows the complex Gaussian distribution \( g_{rr} \sim \mathcal{CN}(0, \lambda_{rr}^{-1}) \). Fortunately, with analog and digital SI cancellation techniques, this SI can be suppressed to residual levels [5]. However, at higher transmit powers of the FD node, this residual SI can be a primary limiting factor, and the effect of this residual SI cannot, therefore, be completely ignored. For this reason, in this work, we assume that the SI suppression is imperfect but of high quality. In such a scenario, we can justify the residual SI channel gain \( |g_{rr}|^2 \) by its mean value \( \lambda_{rr} \), as \( \mathbb{E}(|g_{rr}|^2) \) is very small [25]. The throughput and EE performance

3The battery-assisted FD relay allows us to augment the harvested energy with a quantum of the battery energy to prolong the battery life and ensure reliable communication between the source and the destination node. As illustrated in Fig. 1, the FD relay is equipped with two separate antennas, one for transmission and the other for reception. The received signal at the relay gets split into two parts by the TS-based splitting circuit. Further, the relay consists of the EH unit for harvesting the energy from the incoming signal in the EH phase and the IT unit which decodes information from the received symbol and simultaneously transmits the previously decoded symbols using the transmit antenna. The energy available for the transmission of the information from the relay node is the sum of a controlled amount of the relay’s battery energy and the harvested energy. This controlled amount of battery energy can be fetched from the battery based on the availability of the channel knowledge in two different ways: 1) if only statistical properties of the channel are known, then in each signalling interval a fixed amount of the battery energy \( Q_b \) augments the harvested energy, and 2) when based on CSI availability provided by the central control unit, harvested energy is augmented with as little as battery energy in order to achieve the desired QoS. The availability of channel knowledge provided by the central control unit not only allows efficient utilization of battery energy but also helps in maximizing the system throughput for both the FBE and DBE battery management schemes discussed above.
of the EH schemes depend on the battery management technique used. CSI required for dynamic TS and DBE can be acquired in a very simple manner using pilots. A pilot transmitted by S enables estimation of $g_{sr}$ and that transmitted by R enables estimation of $g_{rd}$ (followed by feedback of estimates of $|g_{rd}|^2$ to R). Using these estimates, R determines the dynamic TS parameter to be used by the energy harvesting circuit. The dynamic battery energy value to be used is calculated and used at R in the DBE case. Note that the energy required for pilot transmission is negligible as compared to that required for information transmission and is therefore ignored.

We assume that a fraction $\alpha$ of the overall signalling interval $T$ is used for EH and the remaining $1 - \alpha$ fraction is used for information transmission with $\alpha \in \{0, 1\}$. In this paper, we consider both static and dynamic TS protocols as mentioned earlier. While static TS is easier to implement, dynamic TS EH protocol has the advantage of superior performance. The former is determined based on knowledge of only average channel gains, while the latter depends on the instantaneous CSI of the S-R and R-D links. In this paper, we demonstrate that energy management at the battery-assisted relay significantly influences throughput and average energy consumption. We consider the FBE and the DBE schemes as discussed earlier. Together with the choice of the static or dynamic TS EH protocol, there are four cases to be considered. In what follows, we briefly discuss the signalling in each of these cases.

### A. Static TS EH Protocol with FBE Scheme

We consider a practical nonlinear EH model [26] which captures the saturation characteristics of the practical EH circuits. S uses the maximal ratio transmission to transmit information to R. In the EH phase, the received signal at R is given by $y_{r,eh} = \sqrt{Q_s} \psi^H g_{sr} x_s(k) + n_{r,a}$ where $g_{sr} = [g_{s1}, g_{s2}, \ldots, g_{sL}]$, $\psi = g_{sr}/||g_{sr}||$ represent the beam forming vector, and $n_{r,a} \sim CN(0, \sigma_{r,a}^2)$ is the additive antenna noise. After neglecting the insignificant antenna noise [27], the input energy available for EH per symbol interval is given by

$$Q_{in} = Q_s ||g_{sr}||^2.$$  

(1)

At R, the energy harvested per symbol during EH phase can be expressed as

$$Q_h = \begin{cases} \eta Q_s ||g_{sr}||^2, & Q_{in} \leq Q_{sat} \\ \eta Q_{sat}, & Q_{in} > Q_{sat} \end{cases}$$  

(2)

where $\eta$ denotes the energy conversion efficiency and $Q_{sat}$ represents the energy saturation threshold. A quantum of battery energy $Q_b$ per symbol interval augments the harvested energy, which implies

\footnote{The nonlinear EH models capture more accurately the behavior of practical circuits. In the literature on EH, two different models are widely adopted for nonlinear EH. The first one, proposed in [26], is based on a nonlinear sigmoidal function, while the second one is a piecewise nonlinear EH model [14]. Both of the available models are based on the use of curve-fitting tools. Therefore, neither of these models has a general superiority over the other. Thus, either choice is suitable for modeling the nonlinear EH. Since the piecewise nonlinear EH model allows for tractable analysis compared to the sigmoidal function-based model, the latter is used in this paper as in [14], [18] (with similarly chosen values for the saturation threshold).}
that \((1 - \alpha)Q_r = \alpha Q_h + Q_b\). Now, the transmit energy at R which can be expressed as

\[
Q_r = \begin{cases} 
\frac{\eta \alpha Q_s |g_{sr}|^2 + Q_b}{1 - \alpha}, & Q_{in} \leq Q_{sat} \\
\frac{\eta \alpha Q_{sat} + Q_b}{1 - \alpha}, & Q_{in} > Q_{sat}.
\end{cases}
\]

(3)

Note that linear EH is a special case that follows when \(Q_{sat} \to \infty\).

In the IT phase, both S and R transmit concurrently. S transmits unit energy symbols \(x_s(k)\) with symbol energy \(Q_s\) and symbol rate \(R_s\). \(P_s = Q_sR_s\) denotes the transmit power of S. The FD relay decodes \(x_s(k)\) and then transmits the decoded symbol \(x_s(k - \delta)\) to D, where \(\delta \geq 1\) represents the processing delay. The sampled matched filter output at R is given by

\[
y_r(k) = \sqrt{Q_s} \psi^H g_{sr} x_s(k) + \sqrt{Q_r} g_{rr} x_s(k - \delta) + n_r(k),
\]

(4)

where \(Q_r\) is the available transmit energy at R, and \(n_r \sim \mathcal{C}\mathcal{N}(0, \sigma^2)\) represents the overall additive noise (sum of antenna noise and RF to baseband processing noise) at R. Note that advancements in passive and active cancellation techniques has allowed the SI mitigation up to 80 - 110 dB [5]. In this work, we assume that SI suppression is imperfect but of high quality. In such a scenario, the replacement of \(|g_{rr}|^2\) by the mean value \(\lambda_{rr}\) is justifies due to the fact that \(g_{rr}\) due to the fact that \(E[|g_{rr}|^2]\) is very small [22].

The signal to interference-plus-noise ratio (SINR) at R can be expressed as

\[
\Gamma_{FBE}^r = \begin{cases} 
\frac{(1 - \alpha)\rho_s |g_{sr}|^2}{\eta \alpha \rho_s |g_{sr}|^2 \lambda_{rr} + \rho_b \lambda_{rr} + (1 - \alpha)}, & Q_{in} \leq Q_{sat} \\
\frac{(1 - \alpha)\rho_s |g_{sr}|^2}{\eta \alpha \rho_{sat} \lambda_{rr} + \rho_b \lambda_{rr} + (1 - \alpha)}, & Q_{in} > Q_{sat},
\end{cases}
\]

(5)

where \(\rho_s = Q_s/\sigma^2\), \(\rho_b = Q_b/\sigma^2\) and \(\rho_{sat} = Q_{sat}/\sigma^2\). The received signal at D is expressed as

\[
y_d(k) = \sqrt{Q_r} g_{rd} x_s(k - \delta) + n_d(k),
\]

(6)

where \(n_d \sim \mathcal{C}\mathcal{N}(0, \sigma^2)\) is the additive baseband antenna noise at D. Using (5), the signal-to-noise ratio (SNR) at D is

\[
\Gamma_{FBE}^d = \begin{cases} 
|g_{rd}|^2 (\eta \alpha \rho_s |g_{sr}|^2 + \rho_b), & Q_{in} \leq Q_{sat} \\
|g_{rd}|^2 (\eta \alpha \rho_{sat} + \rho_b), & Q_{in} > Q_{sat}.
\end{cases}
\]

(7)

In what follows, we discuss how \(\alpha\) and \(Q_b\) can be chosen to maximize throughput.

B. Static TS EH Protocol with DBE Scheme

In this case, the relay always transmits using energy \(Q_d\) per symbol. We elaborate on choice of \(Q_d\) later in this paper. Energy is drawn from the battery only when the harvested energy \(Q_h\) is insufficient.
The quantum $Q_b$ of energy drawn from the battery can be expressed as

$$Q_b = \max\{(1 - \alpha)Q_d - \alpha Q_h, 0\},$$  

(8)

where $Q_d$ is the fixed transmit energy available at $R$. Substituting (2) into the above, the battery energy drawn in DBE mode can be expressed as

$$Q_b = \begin{cases} 
\max(Q_d(1 - \alpha) - \eta_\alpha Q_s |g_{sr}|^2, 0) & Q_{in} \leq Q_{sat} \\
\max(Q_d(1 - \alpha) - \alpha Q_{sat}, 0) & Q_{in} > Q_{sat}.
\end{cases}$$  

(9)

Using a fixed transmit energy $Q_d$ at $R$, the SINR to decode $x_s(k)$ at $R$ can be expressed as

$$\Gamma_{DBE}^R = \frac{\rho_s |g_{sr}|^2 (1 - \alpha)}{\rho_d |g_{rr}|^2 + (1 - \alpha)}.$$  

(10)

where $\rho_d = \frac{Q_d}{\sigma^2}$. Since $R$ forwards the decoded symbol to $D$, the SNR at $D$ can be expressed as

$$\Gamma_{DBE}^D = \frac{\rho_d |g_{rd}|^2}{(1 - \alpha)}.$$  

(11)

C. Dynamic TS EH Protocol with FBE Scheme

In contrast to static TS EH protocol, in dynamic TS EH protocol, the TS parameter is dynamically determined based on instantaneous channel gains. Energy efficiency gains can therefore be expected. Note that $R$ can now exploit $R-D$ channel knowledge and transmit only when decoding will be successful at $D$. This also leads to energy savings. Let $\rho_{th} = 2^{R_t} - 1$ denote the SNR threshold corresponding to symbol information rate $R_t$ defined in bits per channel use (bpcu). Successful S-to-D communication is possible only if both $R$ and $D$ decode the incoming information symbols correctly. Therefore, the TS parameter needs to satisfy the minimum QoS requirements $\Gamma_{DBE}^R \geq \rho_{th}$ and $\Gamma_{DBE}^D \geq \rho_{th}$. Using (5), the qualified $\alpha$ that satisfies the QoS requirement $\Gamma_{DBE}^R \geq \rho_{th}$ can be characterized by

$$\alpha \leq \alpha_{FBE}^1,$$  

(12)

where

$$\alpha_{FBE}^1 = \begin{cases} 
\frac{(\rho_s |g_{sr}|^2 - \rho_{th}) - \rho_{th} \rho_b \lambda_{rr}}{(\rho_s |g_{sr}|^2 - \rho_{th}) + \eta \rho_{th} \rho_b \lambda_{rr}} & Q_{in} \leq Q_{sat} \\
\frac{(\rho_s |g_{sr}|^2 - \rho_{th}) - \rho_{th} \rho_b \lambda_{rr}}{(\rho_s |g_{sr}|^2 - \rho_{th}) + \eta \rho_{th} \rho_{sat} \lambda_{rr}} & Q_{in} > Q_{sat}
\end{cases}.$$  

(13)

subject to $\rho_{th} (1 + \rho_b \lambda_{rr}) < \rho_s |g_{sr}|^2$.

(14)

In (12), it can be clearly observed that $\alpha_{FBE}^1 < 1$ is always true, however, $\alpha_{FBE}^1 > 0$ yields constraint (14). Similarly, the qualified $\alpha$ that satisfies the QoS requirement $\Gamma_{DBE}^D \geq \rho_{th}$ can be characterized by

$$\alpha \geq \max\{0, \alpha_{FBE}^2\},$$  

(15)
$\alpha_{FBE}^2 = \begin{cases} \frac{\rho_{\text{th}} - \rho_b |g_{rd}|^2}{\eta \rho_s |g_{sr}|^2 |g_{rd}|^2 + \rho_{\text{th}}}, & Q_{\text{in}} \leq Q_{\text{sat}} \\ \frac{\rho_{\text{sat}} - \rho_b |g_{rd}|^2}{\eta \rho_s |g_{rd}|^2 + \rho_{\text{in}}}, & Q_{\text{in}} > Q_{\text{sat}} \end{cases}$ \quad (16)

subject to $\rho_b |g_{rd}|^2 < \rho_{\text{th}}$. \quad (17)

In (15), it can be observed that $\alpha_{FBE}^2 < 1$ is always holds, however, $\alpha_{FBE}^2 > 0$ yields constraint (17).

D. Dynamic TS EH Protocol with DBE Scheme

In the case of dynamic TS EH protocol with DBE scheme, both the battery energy $Q_b < Q_d$ and the TS parameter is also determined dynamically based on instantaneous channel gains. It is apparent from (10) and (11) that the qualified $\alpha$ values that satisfy the minimum QoS requirements $\Gamma_{r}^{DBE} \geq \rho_{\text{th}}$ at R and $\Gamma_{d}^{DBE} \geq \rho_{\text{th}}$ at D are given by

$$\alpha \leq \alpha_{1}^{DBE}$$ \quad (18)

and

$$\alpha \geq \max[0, \alpha_{2}^{DBE}],$$ \quad (19)

wherein

$$\alpha_{1}^{DBE} = \frac{\rho_{\text{th}} - \rho_d |g_{sr}|^2}{\rho_{\text{th}} (1 + \rho_d \lambda_{rr})}$$ \quad (20)

subject to $\rho_{\text{th}} (1 + \rho_d \lambda_{rr}) \leq \rho_s |g_{sr}|^2$, \quad (21)

and,

$$\alpha_{2}^{DBE} = \frac{\rho_{\text{th}} - \rho_d |g_{rd}|^2}{\rho_{\text{th}}}$$ \quad (22)

subject to $\rho_d |g_{rd}|^2 < \rho_{\text{th}}$, \quad (23)

respectively. Similar to the FBE scheme, in (20), it can be clearly observed that $\alpha_{1}^{DBE} < 1$ is always true, however, $\alpha_{1}^{DBE} > 0$ yields constraint (21). Further, in (22), it can be observed that $\alpha_{2}^{DBE} < 1$ is always holds, however, $\alpha_{2}^{DBE} > 0$ yields constraint (23).

III. Throughput Analysis

In this section, we present expressions for the throughput of the static and dynamic TS EH protocols with both FBE and DBE battery management schemes. Let $R_t$ represent the target information rate, so that the threshold SNR is $\rho_{\text{th}} = 2^{R_t - 1}$. For fixed-rate signalling with a target rate $R_t$, the throughput is defined as the average rate of successfully received transmissions. For a fixed minimum received SNR $\rho_{\text{th}}$ and non-outage probability $p_{\text{no}}$, the throughput is expressed as $\tau = p_{\text{no}} R_t$. The expressions for the throughput for static TS and dynamic TS protocols are presented in what follows.
A. Static TS EH Protocol with FBE Scheme

Denote the non-outage probability by \( P_{\text{no},s}^{\text{FBE}} \). It is the probability that both \( \Gamma_r^{\text{FBE}} \) and \( \Gamma_d^{\text{FBE}} \) are larger than the threshold \( \rho_{th} \). The throughput is defined as the Considering the linear and the nonlinear EH regions, the throughput for the static TS with FBE can be expressed as

\[
\tau_s^{\text{FBE}} = R_t (1 - \alpha) \frac{\Gamma_r^{\text{FBE}} > \rho_{th}, \Gamma_d^{\text{FBE}} > \rho_{th}}{P_{\text{no},s}^{\text{FBE}}} 
\]

\[
= R_t (1 - \alpha) \left( \frac{\Pr\{Q_{in} < Q_{sat}, \Gamma_r^{\text{FBE}} > \rho_{th}, \Gamma_d^{\text{FBE}} > \rho_{th}\} + \Pr\{Q_{in} > Q_{sat}, \Gamma_r^{\text{FBE}} > \rho_{th}, \Gamma_d^{\text{FBE}} > \rho_{th}\}}{P_{\text{no}1}} \right) \sum_{\rho_{th} > \rho} \left( \frac{\rho_{th}^{\lambda_{\text{sat}}+\lambda_{\text{tr}}+1} \lambda_{\text{sat}}^{\lambda_{\text{sat}}+\lambda_{\text{tr}}+1} \lambda_{\text{tr}}^{\lambda_{\text{sat}}+\lambda_{\text{tr}}+1}}{\rho_{th}^{\lambda_{\text{sat}}+\lambda_{\text{tr}}+1} \lambda_{\text{sat}}^{\lambda_{\text{sat}}+\lambda_{\text{tr}}+1} \lambda_{\text{tr}}^{\lambda_{\text{sat}}+\lambda_{\text{tr}}+1}} \exp \left( \frac{-\lambda_{\text{tr}} \rho_{th}(1 - \alpha)}{\eta \alpha \rho_{th}} \right) \right). \tag{24}
\]

where \( \Gamma_r^{\text{FBE}} \) and \( \Gamma_d^{\text{FBE}} \) are expressed in (5) and (7), respectively. The term \( P_{\text{no}1} \) and \( P_{\text{no}2} \) represent the non outage event in linear EH and saturation EH regions, respectively.

Lemma III.1. A closed-from expression for the throughput of the static TS EH protocol with FBE is given by

\[
\tau_s^{\text{FBE}} = R_t (1 - \alpha) \left[ \sum_{\rho_{th} > \rho} \left( \frac{\rho_{th}^{\lambda_{\text{sat}}+\lambda_{\text{tr}}+1} \lambda_{\text{sat}}^{\lambda_{\text{sat}}+\lambda_{\text{tr}}+1} \lambda_{\text{tr}}^{\lambda_{\text{sat}}+\lambda_{\text{tr}}+1}}{\rho_{th}^{\lambda_{\text{sat}}+\lambda_{\text{tr}}+1} \lambda_{\text{sat}}^{\lambda_{\text{sat}}+\lambda_{\text{tr}}+1} \lambda_{\text{tr}}^{\lambda_{\text{sat}}+\lambda_{\text{tr}}+1}} \exp \left( \frac{-\lambda_{\text{tr}} \rho_{th}(1 - \alpha)}{\eta \alpha \rho_{th}} \right) \right) \right] \tag{26}
\]

where

\[
P_{\text{no}1} = \sum_{\rho_{th} > \rho} \left( \frac{\rho_{th}^{\lambda_{\text{sat}}+\lambda_{\text{tr}}+1} \lambda_{\text{sat}}^{\lambda_{\text{sat}}+\lambda_{\text{tr}}+1} \lambda_{\text{tr}}^{\lambda_{\text{sat}}+\lambda_{\text{tr}}+1}}{\rho_{th}^{\lambda_{\text{sat}}+\lambda_{\text{tr}}+1} \lambda_{\text{sat}}^{\lambda_{\text{sat}}+\lambda_{\text{tr}}+1} \lambda_{\text{tr}}^{\lambda_{\text{sat}}+\lambda_{\text{tr}}+1}} \exp \left( \frac{-\lambda_{\text{tr}} \rho_{th}(1 - \alpha)}{\eta \alpha \rho_{th}} \right) \right) \tag{27}
\]

\[
P_{\text{no}2} = \sum_{\rho_{th} > \rho} \left( \frac{\rho_{th}^{\lambda_{\text{sat}}+\lambda_{\text{tr}}+1} \lambda_{\text{sat}}^{\lambda_{\text{sat}}+\lambda_{\text{tr}}+1} \lambda_{\text{tr}}^{\lambda_{\text{sat}}+\lambda_{\text{tr}}+1}}{\rho_{th}^{\lambda_{\text{sat}}+\lambda_{\text{tr}}+1} \lambda_{\text{sat}}^{\lambda_{\text{sat}}+\lambda_{\text{tr}}+1} \lambda_{\text{tr}}^{\lambda_{\text{sat}}+\lambda_{\text{tr}}+1}} \exp \left( \frac{-\lambda_{\text{tr}} \rho_{th}(1 - \alpha)}{\eta \alpha \rho_{th}} \right) \right) \tag{27}
\]

Proof. See Appendix A.

Remark 1. An expression for the throughput with linear EH for \( L = 1 \) is obtained by substituting \( \rho_{sat} \rightarrow \infty \) in (25) and after performing mathematical rearrangement is given by

\[
\tau_s^{\text{FBE}} \approx R_t (1 - \alpha) \left[ \exp \left( \frac{-\lambda_{\text{sat}} \rho_{th} [\rho_{th}(1 - \alpha)]}{\rho_{sat}(1 + \eta \alpha \rho_{th})} \right) + \frac{\lambda_{\text{sat}} \rho_{th} [\rho_{th}(1 - \alpha)]}{\eta \alpha \rho_{sat}} \exp \left( \frac{\lambda_{\text{sat}} \rho_{th} [\rho_{th}(1 - \alpha)]}{\eta \alpha \rho_{sat}} \right) \right] \tag{28}
\]

Clearly, the throughput in (28) depends on the choice of \( \alpha \) and \( \rho_{th} \). For high values of \( \rho_{th} \), the R-D link remains in non outage. However, the term \( \rho_{th} \lambda_{\text{tr}} \) also increases and leads to increased level of residual SI. This in turn can cause unsuccessful decoding of the received signal at the relay, which in
turn results in a reduction of throughput. Therefore, a judicious choice of the relay’s battery energy is crucial in such EH-based battery-assisted nodes.

B. Static TS EH Protocol with DBE Scheme

The throughput of static TS EH protocol with the DBE scheme can be expressed as

\[ \tau_{DBE} = R_t (1 - \alpha) \Pr \{ \Gamma_{DBE}^r > \rho_{th}, \Gamma_{DBE}^d > \rho_{th} \}, \]

where \( \Gamma_{DBE}^r \) and \( \Gamma_{DBE}^d \) are as in (10) and (11) respectively.

Lemma III.2. An expression for the throughput of static TS EH protocol with DBE scheme is given by

\[ \tau_{DBE} = \frac{R_t (1 - \alpha)}{(1 - \alpha)} \Gamma(L) \left( \frac{\lambda_{sr} \rho_{th} \rho_d \lambda_{rr} + (1 - \alpha)}{\rho_s (1 - \alpha)} \right) \exp \left( -\frac{\lambda_{rd} \rho_{th} (1 - \alpha)}{\rho_d} \right). \]

Proof. See Appendix [B].

Remark 2. The throughput in (30) depends on the choice of \( \alpha \) and \( \rho_d \). For \( \rho_d = 0 \), the RD link remains in outage which results in \( \tau_{DBE} = 0 \). An increase in \( \rho_d \) improves throughput, however, simultaneous increase in the term \( \rho_d \lambda_{rr} \) can cause unsuccessful decoding of the received signal at the relay, which in turn results in a reduction of throughput. Therefore, a judicious choice of the \( \rho_d \) is important.

C. Dynamic TS EH Protocol with FBE Scheme

Notice that if the TS parameter \( \alpha \) is chosen anywhere in the range \( \max\{0, \alpha_{FBE}^2\} < \alpha < \alpha_{FBE}^1 \) in realizations when \( \alpha_{FBE}^1 > \max\{0, \alpha_{FBE}^2\} \), information is successfully forwarded to D (note that the R-D transmission occurs only if R successfully decodes the incoming signal from S). In realizations when it is not possible to choose \( \alpha \) in this given range, S-D outage is inevitable. The above choice of \( \alpha \) leads to non-zero throughput. From (24), it can be seen that the throughput is directly proportional to \( (1 - \alpha) \). Thus if we choose \( \alpha = \max\{0, \alpha_{FBE}^2\} \), maximum throughput can be achieved. However, the system will remain in non outage for any choice of \( \alpha \) between \( \alpha_{FBE}^1 \) and \( \max\{0, \alpha_{FBE}^2\} \). Therefore, unlike the conventional static TS EH protocol, the throughput in case of the proposed dynamic TS EH protocol can be expressed as

\[ \tau_{FBE} = \mathbb{E} \left[ (1 - \max\{0, \alpha_{FBE}^2\}) R_t I \right], \]

where \( \mathbb{E} \) denotes the expectation operator, and \( I \) denotes an indicator function defining the non outage events, which can be expressed as
Lemma III.3. An expression for the throughput of dynamic TS EH protocol with FBE scheme is given by

\[ \tau_{d}^{\text{FBE}} = \tau_{L1} + \tau_{L2} + \tau_{S1} + \tau_{S2}, \]  

where

\[ \frac{\tau_{L1}}{R_t} = \frac{1}{\Gamma(L)} \left[ 1 - e^{-\frac{\lambda_s \rho_{th}}{\rho_b}} \right] [\Gamma(L, \lambda_s \chi_L) - \Gamma(L, \lambda_s \chi_{L4})] + \left( \frac{\rho_b / \eta - \lambda_{rd} \rho_{th}^2 \lambda_{sr}}{\rho_s \lambda_s \Gamma(L)} - \frac{\rho_b}{\eta \rho_s \lambda_s \Gamma(L)} e^{-\frac{\lambda_{rd} \rho_{th}}{\rho_b}} \right) \times \left[ \Gamma(L + 1, \lambda_s \chi_L) - \Gamma(L + 1, \chi_{L4}) \right] - \frac{\lambda_{rd} \lambda_s \chi_{L4}}{\eta \rho_s^2 \lambda_s \Gamma(L)} \left( \frac{\Gamma(L + 2, \lambda_s \chi_{L4}) - \Gamma(L + 2, \lambda_s \chi_{L}) - \lambda_{rd} \lambda_s \chi_{L}}{\lambda_s \Gamma(L)} \right) \Lambda_1, \]  

where \( \chi_L = \max(\chi_{L1}, \chi_{L2}, \chi_{L3}) \) with \( \chi_{L1} = \frac{-b_1 + \sqrt{b_1^2 - 4a_1 c_1}}{2a_1} \) wherein \( a_1 = \eta \rho_s^2, b_1 = \rho_s (\rho_b - \eta \rho_{th}), \) and \( c_1 = -\rho_b \rho_{th}. \) Further, \( \chi_{L2} = \rho_{th} / \rho_s (1 + \rho_b \lambda_{rr}), \chi_{L3} = \frac{-b_2 + \sqrt{b_2^2 - 4a_2 c_2}}{2a_2} \) with \( a_2 = \eta \rho_s^2, b_2 = \rho_s (\rho_b - \eta \rho_{th})(1 + \rho_b \lambda_{rr}), c_2 = -\rho_b \rho_{th}(1 + \rho_b \lambda_{rr}), \) and \( \lambda_{rd} = \frac{\rho_{sat}}{\rho_b} \). Also, \( \Lambda_1 = \left( \frac{\chi_{L4} - \chi_L}{2} \right) \sum_{i=1}^n w_i f_i(x_i) \) wherein \( f_i(x) = \frac{\lambda_{rd} \rho_{th}^2 \lambda_{sr} \rho_s}{\eta \rho_{s}^2 \lambda_{sr} \Gamma(L)} \left[ \left( \frac{\lambda_{rd} \rho_{th}^2 \lambda_{sr} \rho_s}{\eta \rho_{s}^2 \lambda_{sr} \Gamma(L)} \right) - \frac{\lambda_{rd} \rho_{th}}{\eta \rho_{s} \lambda_{sr} \Gamma(L)} \right] \times \left[ \frac{\lambda_{rd} \rho_{th}^2 \lambda_{sr} \rho_s}{\eta \rho_{s}^2 \lambda_{sr} \Gamma(L)} \right] - \frac{\lambda_{rd} \rho_{th}}{\eta \rho_{s} \lambda_{sr} \Gamma(L)} \right] \] 

Further, \( \tau_{L2} \) is given by

\[ \tau_{L2} = \frac{R_t}{\Gamma(L)} \left( \Gamma(\lambda_s \chi_{L1}, L) - \Gamma(\lambda_s \chi_{L4}, L) \right) \exp \left( -\frac{\lambda_{rd} \rho_{th}}{\rho_b} \right). \]  

Similar to \( \tau_{L1}, \tau_{S1} \) is expressed as

\[ \tau_{S1} = \frac{(1 + \rho_b / \eta \rho_{sat})}{\Gamma(L)} \left( \frac{\Gamma(L, \lambda_s \chi_S)}{\Gamma(L)} - \frac{\lambda_{rd} \rho_{th}^2 \lambda_{sr} \rho_s}{\lambda_{sr} \rho_{th} \rho_s \Gamma(L)} \right) \exp \left( -\frac{\lambda_{rd} \rho_{th}}{\rho_b} \right) \left( \frac{\lambda_{rd} \rho_{th}}{\rho_b} \right) \] 

\[ -\frac{\lambda_{rd} \rho_{th}}{\eta \rho_{sat}} \left( \frac{\rho_{th} + \rho_{th} \lambda_{rd}}{\eta \rho_{sat}} \right) \left( \lambda_{sr} \Gamma(L) \right) \]
where $\chi_S = \max(\chi_{S1}, \chi_{S2}, \chi_{S4})$ with $\chi_{S1} = \frac{\rho_{th}}{\rho_s}$, $\Lambda_2 = \sum_{i=1}^{n} v_i f_2(t_i)$ with $v_i = \frac{t_i}{(n+1)^2 L_{n+1}(t_i)}$ and $f_2(t) = E_1\left(\frac{\lambda_{rd} \rho_{th}^2 \lambda_{sr}}{\eta \rho_{sat}} + \frac{\lambda_{rd} \rho_{th}^2}{\eta \rho_{sat}}\right) t^{L-1} \exp(-\lambda_{sr} t)$, wherein $L_n(t) = \sum_{k=0}^{n} \left(\frac{n}{k}\right) \frac{(-1)^k t^k}{k!}$ representing the Laguerre polynomial. Finally, $\tau_{S2}$ is given by

$$\tau_{S2} = \frac{R_t}{\Gamma(L)} \exp\left(-\frac{\lambda_{rd} \rho_{th}}{\rho_d}\right) \Gamma(L, \lambda_{sr} \chi_S).$$

(38)

**Proof.** See Appendix C.  

D. Dynamic TS EH Protocol with DBE Scheme

Using (18) and (19), the throughput can be expressed as:

$$\tau_d^{DBE} = E\left[(1 - \alpha_2^{DBE})R_t \mid \alpha_2^{DBE} > 0, \alpha_1^{DBE} > \alpha_2^{DBE}, \alpha_1^{DBE} > 0\right] + E\left[(1 - 0)R_t \mid \alpha_2^{DBE} < 0, \alpha_1^{DBE} > 0\right].$$

(39)

**Lemma III.4.** An expression for the throughput of dynamic TS EH protocol with DBE scheme is given by

$$\tau_d^{DBE} = \frac{R_t \rho_d}{\Gamma(L) \lambda_{rd} \rho_{th}} \Gamma(L, \lambda_{sr} \chi_d) \left(1 - \Gamma\left(2, \frac{\lambda_{rd} \rho_{th}}{\rho_d}\right)\right) + \frac{\lambda_{rd} \rho_{th}}{\rho_d} \exp\left(-\frac{\lambda_{rd} \rho_{th}}{\rho_d}\right) \right).$$

(40)

**Proof.** See Appendix D.  

IV. AVERAGE BATTERY ENERGY CONSUMPTION

To compare the energy consumption of the FBE and DBE schemes, and to optimize the performance of both schemes, it will be useful to have analytical expressions for the average battery energy consumption.

For both static TS and dynamic TS EH protocol with FBE scheme, a fixed amount of battery energy $Q_b$ augments the harvested energy in each symbol interval. The computation of the average battery energy consumption is therefore quite trivial in these cases. In contrast to this, for the DBE schemes, the battery energy drawn is a random quantity. For ease of exposition, we denote the average value in the DBE case by $Q_b$, omitting the superscript DBE.

A. Static TS EH Protocol with DBE Scheme

For static TS with the DBE scheme, the average battery energy consumption per symbol interval is expressed by (9). In the DBE case the battery energy (less than $Q_d$) is judiciously chosen to augment the harvested energy and ensure successful decoding at $R$. Further the battery energy drawn cannot be a negative quantity. Therefore, using (9), the average battery energy required for static TS with the DBE scheme can be expressed as
In the case of DBE, the relay always transmits using energy \( Q_d \) per symbol. Energy is drawn from the battery only when the harvested energy \( Q_h \) is insufficient. Since the battery is required only if R successfully decodes the incoming symbols and forwards them towards D, the necessary conditions for \( Q_b \) utilization is \( \alpha_1^{DBE} > \max[0, \alpha_2^{DBE}] \), \( \alpha_1^{DBE} > 0 \). Therefore, for dynamic TS EH protocol with DBE scheme, the average energy drawn from the battery is given by

\[
Q_b = \mathbb{E} \left[ \max \left( 1 - \max \left[ 0, \alpha_2^{DBE} \right] \right) Q_d - \max \left[ 0, \alpha_2^{DBE} \right] Q_h, 0 \right] \quad \left| \alpha_1^{DBE} > \max[0, \alpha_2^{DBE}], \alpha_1^{DBE} > 0 \right. \tag{43}
\]
Note that the EH operates in the linear mode when $|g_{sr}|^2 < \frac{Q_{sat}}{Q_0}$, and in the nonlinear mode when $|g_{sr}|^2 > \frac{Q_{sat}}{Q_0}$. Substituting for $Q_h$ from (2) and using (20) and (22) in (43), the average battery energy of dynamic TS EH protocol with DBE scheme can be expressed as

$$Q_b = \mathbb{E}_{|g_{sr}|^2 < \frac{Q_{sat}}{Q_0}} \left[ (Q_d (1 - \alpha_2^{DBE}) - \eta \alpha_2^{DBE} |g_{sr}|^2) | \alpha_2^{DBE} > 0, \alpha_1^{DBE} > \alpha_2^{DBE}, \alpha_1^{DBE} > 0, Q_d > \frac{\eta \alpha_2^{DBE} Q_s |g_{sr}|^2}{(1 - \alpha_2^{DBE})} \right] \left. + \mathbb{E}_{|g_{sr}|^2 > \frac{Q_{sat}}{Q_0}} \left[ (Q_d (1 - \alpha_2^{DBE}) - \eta \alpha_2^{DBE} Q_{sat}) | \alpha_2^{DBE} > 0, \alpha_1^{DBE} > \alpha_2^{DBE}, \alpha_1^{DBE} > 0, Q_d > \frac{\eta \alpha_2^{DBE} Q_{sat}}{(1 - \alpha_2^{DBE})} \right] \right]$$

(44)

**Lemma IV.2.** An expression for the average battery energy of the static TS EH protocol with DBE is given by

$$Q_b = Q_{b_{L1}} + Q_{b_{L2}} + Q_{b_{S1}} + Q_{b_{S2}},$$ \hspace{1cm} (45)

where

$$Q_{b_{L1}} = \frac{\eta p_s}{\lambda_{sr}} \left[ \exp \left( \frac{\lambda_{rd} \rho_{th}}{\rho_d} \right) - \frac{\rho_d}{\lambda_{rd} \rho_{th}} + \frac{\lambda_{rd} \rho_{th}}{\rho_d} \right] \left[ \Gamma(2, \lambda_{sr} \lambda_{D}) - \Gamma(2, \lambda_{sr} \lambda_3) + \Gamma(2, \lambda_{sr} \lambda_4) - \Gamma(2, \lambda_{sr} \lambda_{L}) \right]$$

$$- \Gamma \left( \frac{\lambda_{rd} \rho_{th}}{\rho_d}, \frac{\rho_d^2}{\lambda_{rd} \rho_{th}} \right) \left[ \exp(-\lambda_{sr} \lambda_{D}) - \exp(-\lambda_{sr} \lambda_3) + \exp(-\lambda_{sr} \lambda_4) - \exp(-\lambda_{sr} \lambda_{L}) \right] + \lambda_{sr} \lambda_3 + \lambda_{sr} \lambda_4,$$ \hspace{1cm} (46)

wherein $\lambda_{D} = \rho_{th}(1 + \lambda_{th} \eta_{ch}), \lambda_{1} = \max(\lambda_{D}, \lambda_{L}),$ with $\lambda_{L} = \frac{\rho_{th}}{\lambda_{rd} \rho_{th}}$, $\lambda_{3} = \eta_{ch} \lambda_{L}$, and $\lambda_{4} = \max(\lambda_{D}, \lambda_{L})$ with $\lambda_{2} = \frac{-b_{3} + \sqrt{b_{3}^{2} - 4a_{3}c_{3}}}{2a_{3}}$ where $a_{3} = \frac{\rho_{th}}{\eta_{ch}} \rho_{ch}(1 + \rho_{ch} \lambda_{th}), c_{3} = -\rho_{d} \rho_{ch} \lambda_{th}$. Also, $\Lambda_{3} = \left( \frac{\lambda_{D} - \lambda_{L}}{2} \right) \sum u_{i} f_{3}(p_{i})$ and $\Lambda_{4} = \left( \frac{\lambda_{D} - \lambda_{L}}{2} \right) \sum z_{i} f_{3}(r_{i})$ where $f_{3}(p) = \left[ \frac{\rho_{d}}{\lambda_{rd} \rho_{th}} \eta_{ch} \rho_{ch} + \rho_{d} \right] \Gamma \left( 2, \frac{\lambda_{rd} \rho_{th}}{\rho_{d} \eta_{ch} \rho_{ch}} \right) - \eta_{ch} \rho_{d} \exp \left( -\frac{\lambda_{rd} \rho_{th}}{\rho_{d} \eta_{ch} \rho_{ch}} \right) u_{i} \left[ \frac{1}{2} - \frac{q_{i}}{2} \right]$, \hspace{1cm} (47)

with $u_{i} = \frac{1}{2} \left[ \frac{p_{i}^{2}}{\rho_{th}} \right]^{2}$, $p_{i} = \left( \frac{\lambda_{D} - \lambda_{L}}{2} \right) g_{i} + \left( \frac{\lambda_{D} - \lambda_{L}}{2} \right) z_{i} = \frac{2}{2} \left[ p_{i}^{2} \right]^{2}$, $\lambda_{5} = \frac{2}{2} \left[ \frac{\lambda_{D} - \lambda_{L}}{2} \right] s_{i} + \left( \frac{\lambda_{D} - \lambda_{L}}{2} \right) t_{i}$, Further, $Q_{b_{L2}}$ is given by

$$Q_{b_{L2}} = \frac{\rho_{d}}{\Gamma(L)} \left[ \Gamma(L, \lambda_{sr} \lambda_{D}) - \Gamma(L, \lambda_{sr} \lambda_{L}) \right] \exp \left( -\frac{\lambda_{rd} \rho_{th}}{\rho_{d}} \right).$$

The expression for $Q_{b_{S1}}$ is given by

$$Q_{b_{S1}} = \left[ \frac{\rho_{d}}{\lambda_{rd} \rho_{th}} \left( \eta_{ch} \rho_{ch} + \rho_{d} \right) \Gamma \left( 2, \frac{\rho_{d} \eta_{ch} \rho_{ch}}{\rho_{d} \eta_{ch} \rho_{ch}} \right) - \eta_{ch} \rho_{d} \exp \left( -\frac{\lambda_{rd} \rho_{th}}{\rho_{d} \eta_{ch} \rho_{ch}} \right) \right] \left[ \frac{1}{\Gamma(L)} \left[ \Gamma(L, \lambda_{sr} \lambda_{1}) - \Gamma(L, \lambda_{sr} \lambda_{5}) \right] + \frac{\lambda_{L}}{\Gamma(L)} \Lambda_{5}, \hspace{1cm} (48) \right]$$

where $\lambda_{5} = \frac{\rho_{d} \left( \eta_{ch} \rho_{ch} + \rho_{d} \right)}{\eta_{ch} \rho_{ch} + \rho_{d}}$ and $\lambda_{6} = \max(\lambda_{1}, \lambda_{5})$. Also, $\Lambda_{5} = \left( \frac{\lambda_{D} - \lambda_{L}}{2} \right) \sum i f_{5}(m_{i})$ where $f_{5}(m) = \frac{\lambda_{D} - \lambda_{L}}{2} \sum i f_{5}(m_{i})$. The expression for $Q_{b_{S2}}$ is given by

$$Q_{b_{S2}} = \left[ \frac{\rho_{d}}{\lambda_{rd} \rho_{th}} \left( \eta_{ch} \rho_{ch} + \rho_{d} \right) \Gamma \left( 2, \frac{\rho_{d} \eta_{ch} \rho_{ch}}{\rho_{d} \eta_{ch} \rho_{ch}} \right) - \eta_{ch} \rho_{d} \exp \left( -\frac{\lambda_{rd} \rho_{th}}{\rho_{d} \eta_{ch} \rho_{ch}} \right) \right] \left[ \frac{1}{\Gamma(L)} \left[ \Gamma(L, \lambda_{sr} \lambda_{1}) - \Gamma(L, \lambda_{sr} \lambda_{5}) \right] + \frac{\lambda_{L}}{\Gamma(L)} \Lambda_{5}, \hspace{1cm} (48) \right]$$
\[
\left[ \frac{r_d}{\lambda m - p_{\text{th}}} \right] \left( \eta p_{\text{sat}} + p_d \right) \Gamma \left( 2, \frac{\lambda m^2}{p_s m - p_{\text{th}}} \right) - \eta p_{\text{sat}} \exp \left( -\frac{\lambda m^2}{p_s m - p_{\text{th}}} \right) m^{L-1} \exp \left( -\frac{\chi}{L} \right) j_i + \left( \frac{\chi}{L} \right),
\]

\[
l_i = \frac{2}{(1-\alpha^2)} \left[ P_n^*(j_i) \right]^2, \text{ and } P_n(j) = \sum_{k=0}^{n} \binom{n}{k} \left( \frac{n+k}{2} \right)^k. \]

Finally, \( Q_{b_{S2}} \) can be evaluated as

\[
Q_{b_{S2}} = \frac{r_d}{\Gamma(L)} \Gamma(L, \lambda m^2) \exp \left( -\frac{\chi}{2} \right).
\]

**Proof.** See Appendix F.

**Remark 5.** The expression for the average battery energy for the linear EH case can be derived by substituting \( p_{\text{sat}} \rightarrow \infty \) into (45).

In the next section, we discuss how the choice of the TS parameter and battery energy is crucial to attaining the maximum throughput performance.

**V. THROUGHPUT MAXIMIZATION**

For static TS, from (25) (for the FBE scheme) and (30) (for the DBE scheme) it is clear that the throughput depends on the choice of \( \alpha \) and the battery energy \( Q_b \) in the FBE scheme, and on \( \alpha \) and relay transmit energy \( Q_d \) in the DBE scheme. In (42), the expression for the average battery energy consumption \( Q_b \) with DBE is derived in terms of \( Q_d \) and \( \alpha \). Therefore, for fixed \( Q_d \), the throughput of the DBE scheme for static TS is also a function of \( \alpha \) and \( Q_b \). Thus, for both FBE and DBE schemes in the case of static TS, a judicious choice of \( \alpha \) and \( Q_b \) is essential to maximize the throughput. The corresponding optimization problem can be formulated as

\[
(Q_b^*, \alpha^*) = \arg \max_{(Q_b, \alpha)} E_b^0 \quad \text{s.t.} \quad 0 < \alpha < 1,
\]

\[
Q_{b_{\min}} < Q_b < Q_{b_{\max}} \text{ or } Q_{d_{\min}} < Q_d < Q_{d_{\max}},
\]

where \( \omega \in \{ \text{FBE, DBE} \} \), \((Q_b^*, \alpha^*)\) represents the jointly optimal choice of average battery energy and TS parameter, \( Q_{b_{\min}} \) and \( Q_{b_{\max}} \) denote the minimum and maximum values of the battery energy for the FBE scheme, and \( Q_{d_{\min}} \) and \( Q_{d_{\max}} \) denote the minimum and maximum values of the fixed relay energy in the DBE scheme. Due to the highly involved mathematical expressions for throughput in (25) and (30), \((E_b^*, \alpha^*)\) cannot be determined in the closed form. However, the optimal point \((E_b^*, \alpha^*)\) can be determined using an offline numerical search.

In contrast to the static TS EH protocol, the dynamic TS EH protocol uses the optimal TS parameter for throughput calculations. Therefore the throughput (derived in (34) for the FBE scheme and (40) for the DBE scheme) depends on the choice of \( Q_b \) for the FBE scheme and \( Q_d \) for the DBE scheme. Since (45) derives the average battery energy consumption \( Q_b \) in terms of \( Q_d \) of the DBE scheme, for
fixed $Q_d$, the throughput of the DBE schemes of the dynamic TS EH protocol is also a function of $Q_b$. Hence, an optimal choice of average battery energy consumption $Q_b$ is essential for maximizing the throughput of the DBE scheme.

$$Q_b^* = \arg \max_{Q_b} \tau_d^\omega$$

$$s.t. \quad Q_{b_{\min}} < Q_b < Q_{b_{\max}} \quad or \quad Q_{d_{\min}} < Q_d < Q_{d_{\max}},$$

where $\omega \in \{\text{FBE, DBE}\}$ and $Q_b^*$ represents the optimal choice of battery that maximizes the throughput. Due to highly complicated mathematical expressions of the throughput in (34) and (40), $Q_b^*$ cannot be determined analytically. However, $Q_b^*$ can be determined using an offline one-dimensional search.

VI. NUMERICAL RESULTS

In this section, numerical results are presented to validate the analysis, investigate the behavior of the performance with varying system parameters, and to draw useful insights regarding the TS EH protocol with the FBE and DBE schemes. Unless stated otherwise, the system parameters used are as follows: $\eta = 0.5$, $\sigma^2 = -80$ dBm, $1/\lambda_{rr} = -50$ dBm, $\theta = 3$, $L = 1$, $T = 1$ ms, and $R_t = 2$ bpcu. With a symbol rate $R_s = 1$, the source transmit power is expressed as $P_s = Q_s R_s$ and the saturation threshold is set at $P_{sat} = Q_{sat} R_s = 9.2$ $\mu$W [14], [28]. The inter-node distances $d_{sr}$ and $d_{rd}$ are set as 10 meters and 6 meters, respectively. To validate the derived analytical results, Monte Carlo simulations are performed over $10^7$ independent channel realizations.

Fig. 2 shows the variation of throughputs of static TS EH protocol with FBE scheme ($\tau_{s}^{\text{FBE}}$) and dynamic TS EH protocol with FBE scheme ($\tau_{d}^{\text{FBE}}$) versus the average transmit power $P_s$ for different levels of battery energy $Q_b$ with $L = 2$. The accuracy of the derived analytical expressions for $\tau_{s}^{\text{FBE}}$ in (25) and $\tau_{d}^{\text{FBE}}$ in (34) is clearly verified by the numerical simulations using (24) and (31), respectively. Initially, for very small $P_s$, the impact of the available battery energy is insignificant for both $\tau_{s}^{\text{FBE}}$ and $\tau_{d}^{\text{FBE}}$.
and $\tau_d^{\text{FBE}}$. Since at very low transmit powers, little energy is harvested at R, and it fails to decode the incoming symbols from S, which results in zero throughput. With increasing $P_s$, more energy can be harvested at R, and a small increase in $Q_b$ results in dramatic gain in both $\tau_s^{\text{FBE}}$ and $\tau_d^{\text{FBE}}$. At very high transmit powers both $\tau_s^{\text{FBE}}$ and $\tau_d^{\text{FBE}}$ exhibit saturation (due to saturation threshold $Q_{\text{sat}}$) with the nonlinear EH. On the other hand with the linear EH, both $\tau_s^{\text{FBE}}$ and $\tau_d^{\text{FBE}}$ increase linearly with $P_s$. It is also evident that the dynamic TS EH protocol achieves higher throughput gains in comparison to the static TS EH protocol. For example, with $Q_b = 10 \mu J$ and $P_s = 20$ dBm, the throughput achieved with static TS is $\tau_s^{\text{FBE}} = 1.045$ bpcu and with dynamic TS EH protocol is $\tau_d^{\text{FBE}} = 1.528$ bpcu- clearly, the dynamic TS EH protocol provides a percentage throughput gain of 50% in comparison to the static TS EH protocol. This is because the use of instantaneous channel knowledge in dynamic TS EH protocol allows for a better choice of the TS parameter in comparison to the static TS EH protocol.

Fig. 3 shows the variation of throughputs of static TS EH protocol with DBE ($\tau_s^{\text{DBE}}$) and dynamic TS EH protocol with DBE ($\tau_d^{\text{DBE}}$) versus the average transmit power $P_s$ for different levels of fixed relay energy $Q_d$ with $L = 2$. The accuracy of the derived expressions for $\tau_s^{\text{FBE}}$ in (30) and $\tau_d^{\text{FBE}}$ in (40) is clearly verified by the numerical simulations of (29) and (39), respectively. Initially, for very small $P_s$, R is not able to decode the incoming symbols successfully, which results in very poor throughput. An increase in $P_s$ results in the successful decoding of $x_s$ with high probability, while successful decoding at D depends on the relay’s transmit energy $Q_d$. Clearly, $Q_d = 0$ signifies no transmission of symbols over the R-D link, and thus the throughput is zero. An increase in $Q_d$ allows transmission of the decoded symbols over the R-D link, which therefore ensures an increase in throughput for both $\tau_s^{\text{DBE}}$ and $\tau_d^{\text{DBE}}$. Since the input energy at R ($Q_{\text{in}} = Q_s|g_{sr}|^2$) depends on $E_s$ and is limited by the saturation threshold $E_{\text{sat}}$ of the EH circuit, throughput $\tau_s^{\text{DBE}}$ and $\tau_d^{\text{DBE}}$ exhibit a saturation effect at large $P_s$ values. Similar to the case of FBE, with DBE, the instantaneous channel knowledge in dynamic TS EH protocol allows a better choice of the TS parameter in comparison to the static TS

Fig. 4: $\tau_s^{\text{FBE}} / \tau_d^{\text{FBE}}$ vs. $P_s$ for different $L$.  
Fig. 5: $\tau_s^{\text{DBE}} / \tau_d^{\text{DBE}}$ vs. $P_s$ for different $L$.  

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EH protocol. For example, at $Q_b = 105 \mu J$ and $P_s = 20$ dBm, the throughput achieved with static TS is $\tau_{s\text{DBE}} = 1.299$ bpcu and with dynamic TS EH protocol is $\tau_{d\text{DBE}} = 1.624$ bpcu- clearly, dynamic TS EH protocol ensures a percentage throughput gain of 25% in comparison to the static TS EH protocol.

Fig. 4 and Fig. 5 plot the throughput versus $P_s$ for different number of source transmit antennas $L$ for FBE and DBE schemes, respectively. The battery energy $Q_b$ and is fixed at 5 $\mu J$ for FBE and the relay transmit energy is fixed at $Q_d = 10 \mu J$ for DBE. As expected, with increasing $L$, the throughput for both FBE and DBE schemes increases. This is because, with more number of antennas at source, more energy can be harvested at lower range power, while at higher $P_s$, the throughput is limited by the saturation threshold. We also compare the proposed scheme with the existing half-duplex counterpart [18]. It can be clearly observed that the proposed dynamic TS scheme results in superior throughput performance than that of static TS and hence completely outperforms the half-duplex counterpart.

Fig. 6 and Fig. 7 plot the throughput $\tau_{s\text{FBE}}, \tau_{d\text{FBE}}$ and $\tau_{s\text{DBE}}, \tau_{d\text{DBE}}$ versus target rate for FBE and DBE schemes, respectively. The average transmit power is fixed at $P_s = 10$ dBm. With both FBE and DBE schemes, initially, the throughput increases with increasing $R_t$ and attains a maximum value. A further increase in $R_t$ results in a decrease in throughput for both FBE and DBE schemes. This is because at higher rates the probability of success at R and D decreases. Also, linear EH allows us to attain high throughput at different battery energy levels. In contrast with nonlinear EH, the throughput is limited by the saturation threshold. It can be clearly observed that the dynamic TS EH protocol for both DBE and FBE performs better than the static TS EH protocol. In the DBE scheme, the R-D link SNR depends on the relay transmit energy $Q_d$. In the DBE schemes (both static and dynamic TS EH protocols in Fig. 7) with $Q_d = 0$, information transmission is not possible over the R-D link, while for the FBE scheme (both static and dynamic TS EH protocols in Fig. 6) with $Q_b = 0$, the information transmission depends on the amount of the harvested energy at R. For a fixed transmit power and
available battery energy, dynamic TS (for both FBE and DBE schemes) supports the higher target rates in comparison to that of static TS EH protocol.

Fig. 8 depicts the variation of the throughput of the FBE scheme (for both static and dynamic TS EH protocols) versus the saturation threshold \( P_{\text{sat}} \) for different values of the average battery energy \( Q_b \) with \( P_s = 10 \) dBm and \( R_t = 2 \) bpcu. Initially, when \( P_{\text{sat}} = 0 \), no harvested energy is available for the transmission and throughput depends on the amount of available battery energy. With an increase in \( P_{\text{sat}} \), the harvested energy augments to the relay’s battery energy and a significant improvement in the throughput is observed. Finally, for large values of \( P_{\text{sat}} \), the throughput with nonlinear EH approaches that with linear EH (\( P_{\text{sat}} = \infty \), for linear EH). The performance of static TS EH protocol is compared to that of dynamic TS EH protocol for different \( Q_b \) values. Interestingly, we observe that for higher \( Q_b \) values, the performance of dynamic TS EH protocol with nonlinear EH approaches that of linear EH at very high levels of \( P_{\text{sat}} \) - this is because, for large values of \( Q_b \), RSI also increases, which causes a reduction in SINR at \( R \), causing a small reduction in throughput. As expected, the dynamic TS EH protocol performs significantly better than the static TS EH protocol.

Fig. 9 plots the throughputs \( \tau_{\text{FBE}}^s \), \( \tau_{\text{FBE}}^d \), \( \tau_{\text{DBE}}^s \), and \( \tau_{\text{DBE}}^d \) versus RSI levels for different values of battery energy \( Q_b \) and fixed relay energy \( Q_d \) (in case of FBE and DBE schemes, respectively). The other system parameters are set at \( P_s = 10 \) dBm, and \( R_t = 4 \) bpcu. It can be observed that in the lower RSI range eg. from \(-60 \) dBm to \(-40 \) dBm, the throughput of all the schemes shows no variation with RSI. However, a sudden dip in the throughput performance can be seen at higher RSI levels. Notice that for very small \( Q_b \) values the effect of RSI is less as compared to that for higher values of \( Q_b \). This is because the additional battery energy provides a performance gain by allowing efficient information transmission over R-D link, while at the same time increasing the RSI at \( R \).

In Fig. 10 we plot the average battery energy consumption of static and dynamic TS EH protocols...
with DBE versus average source transmit power for different saturation levels. The system parameters are set at $Q_d = 50 \mu J$ and $R_t = 5$ bpcu. The accuracy of analytical expressions for the average battery energy derived in (42) and (45) are validated through the numerical results from (41) and (43) for static TS EH protocol with DBE and dynamic TS EH protocol with DBE, respectively. Initially for very low $P_s$ values, no energy and information transmission takes place over the S-R link, thus, $R$ does not transmit the information symbols to $D$, and the battery energy is not used. With an increase in $P_s$, for a fixed target rate requirement, the average battery energy consumption increases. In the case of nonlinear EH, with increasing $P_s$, the amount of harvested energy is limited by the saturation threshold, and consequently, the average battery energy consumption depends significantly on $P_{sat}$. For lower values of $P_{sat}$, the average battery energy consumption is almost constant after attaining a maximum value. As $P_{sat}$ increases, the average battery consumption decreases with increasing $P_s$. For linear EH where $P_{sat} = \infty$, at very high transmit powers, the average battery energy consumption is zero as the harvested energy is enough for successful information transmission. Further, comparing the cases of static and dynamic TS EH protocols show that the dynamic TS EH protocol provides enormous energy savings in comparison to the static TS EH protocol. For example, at $P_s = 10\ dBm$, $R_t = 5$ bpcu and $Q_d = 50\ \mu J$, the average battery energy consumption of the dynamic TS EH protocol is $26\ \mu J$ while of the static TS EH protocol is $40\ \mu J$ - a percentage energy saving of $53\ %$ is achieved with dynamic TS EH protocol.

In Fig. 11 we depict the variation of the throughput versus average battery energy consumption of all four schemes. The system parameters are set at $P_s = 7\ dBm$, $R_t = 6$ bpcu and $1/\lambda_{rt} = -55\ dBm$. For both FBE and DBE schemes with static TS, an optimum pair $(Q_b^*, \alpha^*)$ exists that maximizes the throughput. In contrast to this for dynamic TS EH protocol, an optimum value of average battery energy $Q_b^*$ exists at which the throughput is maximized. For $Q_b > Q_b^*$, the throughput of all four
schemes is limited due to the residual SI. A comparison of the FBE scheme and the DBE scheme’s performance curves with static TS EH protocol as well as dynamic TS EH protocol shows that the DBE scheme results in significant battery energy savings in comparison to FBE. For example for static TS, the throughput is maximized at an energy consumption of 2130 $\mu$J with the FBE scheme and 1760 $\mu$J with the DBE scheme - thus the DBE scheme provides an energy saving of 26.7%. On the other hand for dynamic TS EH protocol, the throughput is maximized at an energy consumption of 1277 $\mu$J with the FBE scheme and 876.7 $\mu$J with the DBE scheme - thus the DBE scheme provides an energy-saving of 31.3% over the FBE scheme. Clearly, the DBE scheme is more energy efficient than the FBE scheme for both static and dynamic TS EH protocols.

VII. CONCLUSIONS

In this paper, a new instantaneous CSI based dynamic TS EH protocol was proposed with intelligent battery management at the relay. The two battery management schemes, namely FBE and DBE, were considered with static and dynamic TS EH protocols, and performance was analyzed. The closed-form expressions for the throughput were derived considering all four possibilities, i.e., static TS EH protocol with FBE scheme, static TS EH protocol with DBE scheme, dynamic TS EH protocol with FBE scheme, and dynamic TS EH protocol with DBE scheme, respectively. With both FBE and DBE schemes, the dynamic TS EH protocol resulted in substantial gains in throughput compared to the static TS EH protocol. Further, for the DBE scheme (where the supplementary battery energy is supplemented only if the harvested energy is not enough) the closed-form expressions were derived for the average battery energy consumption in the case of both static TS and dynamic TS EH protocols. The derived results established that DBE scheme shows significant battery energy savings compared to the FBE scheme (in each signalling interval, a fixed amount of battery energy is augmented to the harvested energy). It was also demonstrated that the dynamic TS EH protocol with DBE scheme results in higher throughput and enormous relay battery energy saving in comparison to the static TS EH protocol. This paper demonstrated how node-level energy considerations determine the performance of the overall network. Overall, this paper provided several useful insights and directions for efficient utilization of the battery-assisted energy harvesting nodes, which will be helpful in realizing the goal of a longer battery lifetime of green communication nodes in future generation networks.

APPENDIX A

Proof of Lemma III.1: Substituting for $\Gamma^{FBE}_r$ from (5) and $\Gamma^{FBE}_d$ from (7) into (24), and first solving for $P_{\text{no}1}$ we get

$$P_{\text{no}1} = \text{Pr}\left\{ \chi \leq X \leq \chi_{1,4}, Y \geq \frac{\rho_{\text{th}} (1 - \alpha)}{\eta \alpha \rho_s X + \rho_b} \right\} = \int \int_{\chi \leq \frac{\chi_{d,4}}{\eta \alpha \rho_s X + \rho_b}} \lambda_{\text{sr}} \exp(-\lambda_{\text{sr}} x) \lambda_{\text{rd}} \exp(-\lambda_{\text{rd}} y) \, dx \, dy. \quad (52)$$
where \( \chi = \frac{\rho_{th} [\rho_b \lambda_T + (1 - \alpha)]}{\rho_s [1 - \alpha (1 + \eta \rho_s \lambda_T)]} \). Using the fact that random variables \( X \) and \( Y \) are independent, we first average (52) over the PDF of \( Y \) conditioned on \( X \) to obtain

\[
P_{n_0|X} = \int_{\xi}^{\infty} \lambda_r d \exp(-\lambda_r y) dy = \exp\left(-\lambda_r \frac{\rho_{th} (1 - \alpha)}{\eta \rho_s X + \rho_b}\right).
\]

Further, applying some algebraic manipulations on (53) and averaging over the PDF of \( X \), we obtain

\[
P_{n_0} = \frac{\chi_{L_d}}{\Gamma(L)} \int_{x}^{\infty} \frac{\chi_{L_d}}{\Gamma(L)} \exp\left(-\lambda_s x\right) x^{L-1} \lambda_r d \exp(-\lambda_r y) dy,
\]

where \( \xi = \max\left(\chi_{L_d}, \frac{\rho_{th} (1 - \alpha)}{\eta \rho_s \lambda_s + \rho_b}\right) \). After solving the above, we obtain a closed-form expression for \( P_{n_0} \) as

\[
P_{n_0} = \frac{1}{\Gamma(L)} \Gamma(L, \lambda_s \xi) \exp\left(-\lambda_s \frac{\rho_{th} (1 - \alpha)}{\eta \rho_s \lambda_s + \rho_b}\right).
\]

**Appendix B**

Proof of Lemma III.2: Substituting \( \Gamma_{r}^{DBE} \) and \( \Gamma_{d}^{DBE} \) from (10) and (11) into (29), we obtain

\[
P_{n_0, s}^{DBE} = \Pr\left\{ X \geq \frac{\rho_{th} [\rho_d \lambda_T + (1 - \alpha)]}{\rho_s (1 - \alpha)}, Y \geq \frac{\rho_{th} (1 - \alpha)}{\rho_d}\right\}
\]

\[
= \int_{\rho_{th} [\rho_d \lambda_T + (1 - \alpha)]}^{\infty} \frac{\chi_{L_d}}{\Gamma(L)} \exp(-\lambda_s x) dx \int_{\rho_{th} (1 - \alpha)}^{\infty} \lambda_r \exp(-\lambda_r y) dy
\]

\[
= \frac{1}{\Gamma(L)} \Gamma\left(L, \frac{\lambda_s \rho_{th} [\rho_d \lambda_T + (1 - \alpha)]}{\rho_s (1 - \alpha)}\right) \exp\left(-\rho_{th} \frac{\lambda_s (1 - \alpha)}{\rho_d}\right).
\]

After substituting \( P_{n_0, s}^{DBE} \) from (57) into (29), we obtain a closed-form expression for \( \tau_{s}^{DBE} \) given by (30).

**Appendix C**

Proof of Lemma III.3: Using the definitions of \( \alpha_2^{FBE} \) and \( \alpha_1^{FBE} \) in (33) and separating the linear and nonlinear EH regions, we have
\[
\tau_{L_{1}} = R_{t} \int_{\chi_{L}}^{\chi_{L_{4}}} \int_{\zeta_{L}}^{\chi_{L}} \left(1 - \frac{\rho_{th} - \rho_{b}y}{\eta \rho_{b} x + \rho_{th}}\right) f_{X,Y}(x,y) dx dy.
\]

Using the fact that random variables \(X\) and \(Y\) are independent and exponentially distributed (quasi-static Rayleigh fading is assumed), we first average over the PDF of \(Y\) conditioned on \(X\). After some algebraic manipulations in (59), we obtain

\[
\frac{\tau_{L_{1}|X}}{R_{t}} = \left(\exp(-\lambda_{rd} \zeta_{L}) - \exp(-\lambda_{rd} \rho_{th}/\rho_{b})\right) \left[1 + \frac{\rho_{b}}{\eta \rho_{b} x}\right] - \frac{\lambda_{rd} \rho_{b}}{\eta \rho_{b} x} \left(\frac{\rho_{th}}{\rho_{b}} + \frac{\rho_{th}}{\eta \rho_{b} x}\right) \exp\left(\frac{\rho_{th} \lambda_{rd}}{\eta \rho_{b} x}\right)\]

Using \([24], 3.352.2]\) we solve the above integral to obtain

\[
\tau_{L_{1}|X} = \left[\exp(-\lambda_{rd} \zeta_{L}) - \exp(-\lambda_{rd} \rho_{th}/\rho_{b})\right] \left[1 + \frac{\rho_{b}}{\eta \rho_{b} x}\right] - \frac{\lambda_{rd} \rho_{b}}{\eta \rho_{b} x} \left(\frac{\rho_{th}}{\rho_{b}} + \frac{\rho_{th}}{\eta \rho_{b} x}\right) \exp\left(\frac{\rho_{th} \lambda_{rd}}{\eta \rho_{b} x}\right)\]

Substituting for \(\zeta_{L}\) into (61) and then averaging above over the PDF of \(X\), we obtain

\[
\tau_{L_{1}} = \frac{\chi_{L_{4}}}{\Gamma(L)} \exp(-\lambda_{s} x) \left[\exp(-\lambda_{rd} \rho_{th}^{2} \lambda_{rr} (x + \rho_{th}/\eta \rho_{b})/\rho_{b}) - \frac{\lambda_{rd} \rho_{b}}{\eta \rho_{b} x} \left(\frac{\rho_{th}}{\rho_{b}} + \frac{\rho_{th}}{\eta \rho_{b} x}\right) \exp\left(\frac{\rho_{th} \lambda_{rd}}{\eta \rho_{b} x}\right)\right] x^{L-1} dx
\]
Further, in (58)

\[ \int_{\chi_L}^{\chi_{L+1}} \frac{\lambda_{st} \rho_b}{\Gamma(L)} e^{-\lambda_{st} x} \left[ 1 + \frac{\rho_b}{\eta \rho_s x} \right] e^{-\frac{\lambda_{rd} \rho_b}{\eta \rho_s x}} \rho_b e^{-\frac{\lambda_{rd} \rho_b}{\eta \rho_s x}} \eta L \left( \rho_{th} + \frac{\rho_{th}}{\eta \rho_s x} \right) \lambda_{rd} \rho_b \eta \rho_s x \right] \chi_{L-1} \, dx. \]  

(62)

For higher values of \( \rho_s \), we use the linear approximation to the exponential term \( \exp(\Phi) \approx (1 + \Phi) \). Further, neglecting the insignificant higher order terms of \( 1/\rho_s \) and using some algebraic manipulations, we obtain

\[
\frac{\tau_{L_1}}{R_t} = \lambda_{st} \int_{\chi_L}^{\chi_{L+1}} \left( 1 + \frac{\rho_b}{\eta} - \frac{\lambda_{rd} \rho_b}{\eta \rho_s x} \right) e^{-\lambda_{st} x} \, dx - \lambda_{st} \int_{\chi_L}^{\chi_{L+1}} \left( \frac{\rho_{th}}{\rho_b} + \frac{\rho_{th}}{\eta \rho_s x} \right) \exp \left( \frac{\lambda_{rd} \rho_{th}}{\eta \rho_s x} \right) \eta \rho_s x \, dx
\]

\[
\times \mathcal{E}_1 \left( \frac{\lambda_{rd} \rho_{th} \rho_b \eta x}{\rho_s (x + \frac{\rho_b - \rho_{th}}{\eta \rho_s})} \right) + \frac{\lambda_{rd} \rho_{th}}{\eta \rho_s x} e^{-\lambda_{st} x} \, dx - \exp \left( \frac{\lambda_{rd} \rho_{th}}{\rho_b} \right) \int_{\chi_L}^{\chi_{L+1}} e^{-\lambda_{st} x} \, dx - \lambda_{st} \rho_{th} \exp \left( \frac{\lambda_{rd} \rho_{th}}{\rho_b} \right) \eta \rho_s x \, dx.
\]  

(63)

In (63), we solve the integral terms using [24, 3.381.3] and [29, 25.4.30] to obtain \( \frac{\tau_{L_1}}{R_t} \) as in (55). Further, in (58) \( \tau_{L_2} \) can be expressed as

\[
\tau_{L_2} = \mathbb{E} \left[ R_t \bigg| Y > \frac{\rho_{th}}{\rho_b}, \chi_{L_2} \leq X \leq \chi_{L_4} \right] = R_t \int_{\chi_{L_2}}^{\chi_{L_4}} \frac{\lambda_{st} \rho_b}{\Gamma(L)} \chi_{L-1} \exp(-\lambda_{st} x) \, dx \int_{\chi_{L_2}}^{\infty} \lambda_{rd} \exp(-\lambda_{rd} y) \, dy
\]

\[
= \frac{R_t}{\Gamma(L)} \left( \Gamma(\lambda_{st} \chi_{L_2}, L) + \Gamma(\lambda_{st} \chi_{L_4}, L) \right) \exp \left( -\lambda_{rd} \frac{\rho_{th}}{\rho_b} \right). \]  

(64)

Similar to \( \tau_{L_1} \) and \( \tau_{L_2} \), the expressions for \( \tau_{S_1} \) and \( \tau_{S_2} \) are derived and given by (37) and (38), respectively (derivations for \( \tau_{S_1} \) and \( \tau_{S_2} \) are omitted due to paucity of space).

**Appendix D**

**Proof of Lemma III.4:** Substituting for \( \alpha_1^{DBE} \) and \( \alpha_2^{DBE} \) from (18) and (19) into (39), we obtain

\[
\tau_{d_1}^{DBE} = \mathbb{E} \left[ R_t \bigg| Y > \frac{\rho_{th}}{\rho_b}, \chi_{D} < Y < \frac{\rho_{th}}{\rho_b}, X > \chi_{D} \right] + \mathbb{E} \left[ R_t \bigg| Y > \frac{\rho_{th}}{\rho_b}, X > \chi_{D} \right]. \]

(65)

where \( \zeta_D = \frac{\rho_{th}^2 \lambda_{st}}{\rho_b X - \rho_{th}}, \chi_D = \frac{\rho_{th}(1 + \rho_{th} \lambda_{st})}{\rho_X} \). Now, we first solve for \( \tau_{d_1}^{DBE} \) by averaging over the PDFs of random variables \( X \) and \( Y \) as

\[
\tau_{d_1}^{DBE} = R_t \int_{\chi_D}^{\chi_{D+1}} \int_{\zeta_D}^{\infty} \frac{\rho_{th} \lambda_{rd} \exp(-\lambda_{rd} y) \lambda_{st} L L \chi_{L-1} \exp(-\lambda_{st} x) \, dx \, dy.} \]

(66)

Solving the inner integral term using [24, 3.351.2], we obtain
\[ \tau_{DBE}^d = R_l \frac{\lambda_{Sr} L}{\Gamma(L) \lambda_{rd} \rho_{th} \rho_d} \int_{x_D}^{\infty} \left( 2, \frac{\lambda_{rd} \rho_{th}}{\rho_d} \right) x^{L-1} e^{-\lambda_{sr} x} dx - R_l \frac{\lambda_{Sr} L}{\Gamma(L) \lambda_{rd} \rho_{th} \rho_d} \int_{x_D}^{\infty} \left( 2, \frac{\lambda_{rd} \rho_{th}}{\rho_d} \right) x^{L-1} e^{-\lambda_{sr} x} dx. \quad (67) \]

Observing that \( \frac{\lambda_{rd} \rho_{th}}{\rho_d} \) is very small (\( \lambda_{sr}/\rho_s \) is very small), therefore neglecting the term \( \Gamma \left( 2, \frac{\lambda_{rd} \rho_{th}}{\rho_d} \right) \) in above. After algebraic simplification, we obtain

\[
\tau_{DBE}^d = R_l \frac{\lambda_{Sr} L}{\Gamma(L) \lambda_{rd} \rho_{th} \rho_d} \int_{x_D}^{\infty} x^{L-1} \exp(-\lambda_{sr} x) dx. \quad (68)
\]

Solving the above integral using [24], 3.381.3 we obtain

\[
\tau_{DBE}^d = R_l \frac{\lambda_{Sr} L}{\Gamma(L) \lambda_{rd} \rho_{th} \rho_d} \int_{x_D}^{\infty} \left( 2, \frac{\lambda_{rd} \rho_{th}}{\rho_d} \right) \Gamma(L, \lambda_{sr} x) dx. \quad (69)
\]

The term \( \tau_{DBE}^d \) in (65) is expressed as

\[
\tau_{DBE}^d = R_l \frac{\lambda_{Sr} L}{\Gamma(L) \lambda_{rd} \rho_{th} \rho_d} \int_{x_D}^{\infty} x^{L-1} \exp(-\lambda_{sr} x) dx \int_{\rho_d}^{\rho_{th}} \lambda_{rd} \exp(-\lambda_{rd} y) dy 
\]

\[
= R_l \frac{\lambda_{Sr} L}{\Gamma(L) \lambda_{rd} \rho_{th} \rho_d} \exp\left( -\lambda_{rd} \frac{\rho_{th}}{\rho_d} \right) \Gamma\left( L, \lambda_{sr} x_D \right). \quad (70)
\]

Using (69) and (70) in (65), we obtain (40).

**APPENDIX E**

*Proof of Lemma IV.1:* We substitute for \( \tau_{DBE}^d \) from (10) into (41) to obtain

\[
Q_b = \left[ \mathbb{E} \left[ Q_d \left( 1 - \alpha \right) - \eta Q_s x \right] \right]_{ \bar{\omega}_0 \leq x \leq \bar{\omega}_1 } + \left[ \mathbb{E} \left[ Q_d \left( 1 - \alpha \right) - \eta Q_s x \right] \right]_{ x > \bar{\omega}_2 }, \quad (71)
\]

where \( \bar{\omega}_0 = \frac{\rho_{th}}{\rho_d} \alpha^{-1} \), \( \bar{\omega}_1 = \min \left( \frac{Q_{sat}}{Q_s}, \frac{Q_d(1-\alpha)}{\eta Q_s} \right) \), and \( \bar{\omega}_2 = \max \left( \frac{Q_{sat}}{Q_s}, \bar{\omega}_0 \right) \). The \( Q_b \) in (71) can be expressed as

\[
\begin{align*}
Q_b = & \int_{\bar{\omega}_0}^{\bar{\omega}_1} \left[ Q_d \left( 1 - \alpha \right) - \eta Q_s x \right] \frac{\lambda_{Sr} L}{\Gamma(L)} x^{L-1} \exp(-\lambda_{sr} x) dx \\
+ & \left[ Q_d \left( 1 - \alpha \right) - \eta Q_{sat} \right] \left[ Q_d - \frac{\eta Q_{sat}}{1 - \alpha} \right] \int_{\bar{\omega}_2}^{\infty} \frac{\lambda_{Sr} L}{\Gamma(L)} x^{L-1} \exp(-\lambda_{sr} x) dx.
\end{align*}
\]

Solving the above integrals using [24], 3.381.3, we obtain an exact expression for average battery energy \( Q_b \) which is given by (42).

**APPENDIX F**

*Proof of Lemma IV.2:* Substituting for \( \alpha_1^{DBE} \) and \( \alpha_2^{DBE} \) from (18) and (19) into (44), we obtain (73) shown at the top of the next page wherein \( \zeta_1 = \frac{\rho_{th}}{\rho_d} \frac{\lambda_{sr}}{\alpha} \), \( \zeta_2 = \frac{\rho_{th} \eta \lambda_{sr}}{\rho_d} \), \( \zeta_3 = \frac{\rho_{th} \eta \lambda_{sr}}{\rho_d} \), \( \lambda_D = \frac{\rho_{th}}{\rho_d} \alpha^{-1} \), and \( \chi_1 = \max \left( \chi_{D, \lambda_{L4}} \right) \) with \( \chi_{L4} = \frac{Q_{sat}}{\rho_s} \). Now in (73), we first consider \( Q_{bl,1} \). Depending on \( \zeta_1 \) and \( \zeta_2 \), \( Q_{bl,1} \) can be expressed as
\[ Q_b = \mathbb{E} \left[ \frac{\rho_d Y}{\rho_{th}} (\eta \rho_s X + \rho_d) - \eta \rho_s X \middle| \max(\zeta_1, \zeta_2) \leq Y \leq \frac{\rho_{th}}{\rho_d} x, x \leq x_{L_4} \right] + \mathbb{E} \left[ Q_{bl,1} \right] + \mathbb{E} \left[ Q_{bl,2} \right] \]

\[ Q_{bl,1} = \mathbb{E} \left[ \frac{\rho_d Y}{\rho_{th}} (\eta \rho_s X + \rho_d) - \eta \rho_s X \middle| \max(\zeta_1, \zeta_3) \leq Y \leq \frac{\rho_{th}}{\rho_d} x > x_1 \right] \]

\[ Q_{bl,2} = \mathbb{E} \left[ \frac{\rho_d Y}{\rho_{th}} (\eta \rho_s X + \rho_d) - \eta \rho_s X \middle| \max(\zeta_1, \zeta_4) \leq Y \leq \frac{\rho_{th}}{\rho_d} x > x_1 \right] \]

where \( \chi_3 = \min(\chi_{L_4}, \chi_2) \), \( \chi_4 = \max(\chi_{D}, \chi_2) \), \( \chi_2 = \frac{-b_3 + \sqrt{b_3^2 - 4\rho_s^2 c_3}}{2\rho_s} \) (derived using \( \frac{\rho_{th}}{\rho_b} > \zeta_L \)) with \( a_3 = \frac{\eta \rho_s^2, b_3 = -\eta \rho_{th} (1 + \rho_d \lambda_{tr}), c_3 = -\rho_{th} \rho_d \lambda_{tr} \). From (74) averaging \( Q_{bl,11} \) over the PDF of the random variable \( Y \) conditioned on \( X \), we obtain

\[ Q_{bl,11} = \mathbb{E} \left[ \frac{\rho_d Y}{\rho_{th}} (\eta \rho_s X + \rho_d) \right] \]

Using [24.3352.2] to solve the above integrals, we obtain

\[ Q_{bl,11} = \frac{\rho_d}{\lambda_{rd} \rho_{th}} (\eta \rho_s x + \rho_d) \left( \Gamma \left( 2, \frac{\lambda_{rd} \rho_{th}^2 \lambda_{tr}}{\rho_s x - \rho_{th}} \right) - \Gamma \left( 2, \frac{\lambda_{rd} \rho_{th}^2}{\rho_s x - \rho_{th}} \right) \right) - \eta \rho_s X \left( \exp \left( -\frac{\lambda_{rd} \rho_{th}^2 \lambda_{tr}}{\rho_s x - \rho_{th}} \right) - \exp \left( -\frac{\lambda_{rd} \rho_{th}^2}{\rho_s x - \rho_{th}} \right) \right). \]

Averaging (76) over the PDF of \( X \) and solving the integrals using [24.3381.3] and [29.254.30], we obtain

\[ Q_{bl,1} = \frac{\eta \rho_s}{\lambda_{sr}} \Gamma(L) \left[ \exp \left( -\frac{\lambda_{rd} \rho_{th}}{\rho_d} \right) - \frac{\rho_d}{\lambda_{rd} \rho_{th}} \Gamma \left( 2, \frac{\lambda_{rd} \rho_{th}}{\rho_d} \right) \right] \left[ \Gamma(L + 1, \lambda_{sr} \chi_{D}) - \Gamma(L + 1, \lambda_{sr} \chi_3) \right] \]

\[ - \frac{\rho_d^2}{\lambda_{rd} \rho_{th} \Gamma(L)} \left[ \Gamma(L, \lambda_{sr} \chi_{D}) - \Gamma(L, \lambda_{sr} \chi_3) \right] + \frac{\Lambda_3}{\Gamma(L)} \Lambda_3, \]

where \( \Lambda_3 = \left( \frac{x_{L_4} - x_{D}}{2} \right) \sum_{i=1}^{n} u_i f_3(p_i) \) with \( f_3(p) = \frac{\rho_d}{\lambda_{rd} \rho_{th}} (\eta \rho_s p + \rho_d) \left( \frac{2}{\lambda_{rd} \rho_{th}^2 \lambda_{tr}} \right) \rho_s p - \exp \left( -\frac{\lambda_{rd} \rho_{th}^2 \lambda_{tr}}{\rho_s p - \rho_{th}} \right) \right) \)

\[ p^{L-1} \exp(-\lambda_{sr} p), u_i = \frac{2}{1 - q_i} \left( P_n(q_i) \right)^2, \]

\[ \left( \frac{\lambda_{D} - \lambda_{D}}{2} \right) q_i + \left( \frac{\lambda_{D} + \lambda_{D}}{2} \right), \text{ and } P_n(q) = \sum_{k=0}^{n} \left( \frac{n}{k} \right) \left( \frac{n-k}{k} \right) \left( \frac{a-1}{2} \right)^k \]

denotes the Legendre polynomial. Similarly, \( Q_{bl,12} \) can be expressed as

\[ Q_{bl,12} = \frac{\eta \rho_s}{\lambda_{sr}} \Gamma(L) \left[ \exp \left( -\frac{\lambda_{rd} \rho_{th}}{\rho_d} \right) - \frac{\rho_d}{\lambda_{rd} \rho_{th}} \Gamma \left( 2, \frac{\lambda_{rd} \rho_{th}}{\rho_d} \right) \right] \left[ \Gamma(L + 1, \lambda_{sr} \chi_{D}) - \Gamma(L + 1, \lambda_{sr} \chi_{L_4}) \right] \]
p_s(\frac{L_1 - \chi_d}{2}) \sum_{i=1}^{\infty} z_i f_4(r_i), \quad z_i = \frac{2}{(1 - r_i^2)} [p_d'(s_i)]^2, \quad r_i = \left( \frac{L_1 - \chi_d}{2} \right) s_i + \left( \frac{L_2 - \chi_d}{2} \right), \quad f_4(r) = \left[ \frac{p_d}{\lambda_{rd} p_d} (\eta_{rd} r + \rho_d) \right]^{L_4}(2, \frac{\lambda_{rd} \eta_{rd} r}{\rho_d + \rho_d}) \eta_{rd} r - \exp \left( - \frac{\lambda_{rd} \eta_{rd} r}{\rho_d + \rho_d} \right), \quad L_1 \leq \chi_d \leq L_4 \leq L_2.

(78)

where \( \Lambda_4 = \left( \frac{\lambda_{rd} - \chi_d}{2} \right) \sum_{i=1}^{\infty} z_i f_4(r_i), \) \( z_i = \frac{2}{(1 - r_i^2)} [p_d'(s_i)]^2, \) \( r_i = \left( \frac{\lambda_{rd} - \chi_d}{2} \right) s_i + \left( \frac{\lambda_{rd} - \chi_d}{2} \right), \) \( f_4(r) = \left[ \frac{p_d}{\lambda_{rd} p_d} (\eta_{rd} r + \rho_d) \right]^{L_4}(2, \frac{\lambda_{rd} \eta_{rd} r}{\rho_d + \rho_d}) \eta_{rd} r - \exp \left( - \frac{\lambda_{rd} \eta_{rd} r}{\rho_d + \rho_d} \right) \) represents the Legendre polynomial. Using (77) and (78) into (74), the expression for \( Q_{bl_1} \) is expressed in (46). Further, solving for \( Q_{bl_2} \) from (73), we obtain

\[
Q_{bl_2} = \mathbb{E} \left[ Q_d \bigg| Y > \frac{P_{th}}{\rho_d}, \chi_D \leq X \leq \chi_{L_4} \right] = \frac{\rho_d}{\Gamma(L)} \int_{\chi_D}^{\chi_{L_4}} \lambda_{rd} x^{L-1} \exp(-\lambda_{rd} x) dx \int_{\chi_{L_4}}^{\infty} \lambda_{rd} \exp(-\lambda_{rd} y) dy

= \frac{\rho_d}{\Gamma(L)} \left[ \Gamma(L, \lambda_{rd} \chi_D) - \Gamma(L, \lambda_{rd} \chi_{L_4}) \right] \exp \left( - \lambda_{rd} \frac{P_{th}}{\rho_d} \right). \quad (79)
\]

Similar to \( Q_{bl_1} \) and \( Q_{bl_2} \), the expressions for \( Q_{bs_1} \) and \( Q_{bs_2} \) are derived and given by (48) and (49), respectively (derivations for \( Q_{bs_1} \) and \( Q_{bs_2} \) are omitted due to paucity of space).

REFERENCES


