Standardized Kalman Filtering for Time Serial Source Localization of Simultaneous Subcortical and Cortical Brain Activity

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Abstract

Objective: Introduce standardized Kalman filtering (SKF) as a new spatiotemporal source localization method for accurate localization and time evolution tracking of brain activity. The method can be used with the detailed head model of thousands of source points without the need for high-performance computing.

Methods: Description of the standardization methodology relying on the Kalman filtering (KF), referred to as SKF. Localization and evolution tracking were studied using fully controlled simulation studies and real somatosensory evoked potentials while utilizing an MRI-based personalized head model. SKF was compared with standardized low-resolution brain electromagnetic tomography (sLORETA) and the KF approach without standardization.

Results: Within the studied signal-to-noise ratios (25, 15, 5 dB), SKF distinguished cortical and subcortical contributors in all of them. sLORETA tracked both at 25 and 15 dB, but subcortical was suppressed. KF tracked only the cortical activity. SKF localized originators accordingly while sLORETA failed in some cases and KF mislocalized the cortical activity it estimated.

Conclusions: The numerical results suggest that SKF inherits the advantages of both methods: estimation accuracy of sLORETA and traceability of KF while producing focal estimations. Localization results of the SEP data are following the literature.

Significance: The proposed standardized methodology can help study time-evolving brain activities and localize landmarks when there is no prior knowledge of evolution or the activity is located in deep brain structures.

Keywords: Electroencephalography (EEG), Inverse methods, Source localization, Kalman filtering, Standardization, Spatiotemporal
often used which results in deteriorating the spatial resolution of the source reconstruction (Rullmann et al., 2009).

An alternative path to capture the source activity course with high spatiotemporal resolution is to use Bayesian filtering. Considering linear Gaussian state-space modeling, this leads to the well-known Kalman filter (KF) (Kalman and Bucy, 1961). Kalman filter has been previously used to reconstruct simulated and real brain activity distributions from EEG recordings using a regular grid-based low-resolution brain model (Galka et al., 2004; Barton et al., 2009), and later coupled with LORETA method forming DynLORETA (Yamashita et al., 2004), where an approximation of the Kalman filter is used. It is also used to reconstruct cortical components of somatosensory evoked potentials from magnetoencephalography (MEG) recordings (Long et al., 2011) and in combination with EEG/MEG in localization of epileptic spikes (Hamid et al., 2013) using high-resolution models. The regional version of spatiotemporal Kalman filtering is used to localize deep brain activity correctly (Hamid et al., 2021), where the nearest-neighbor coupling is used to simplify the computation. However, the fully-coupled state-space Kalman filtering approaches suffer from depth-biased estimations, meaning that they favor reconstructions in superficial brain locations while failing to recover deep activity, similar to the well-known minimum norm estimate (MNE) (Hämäläinen and Ilmoniemi, 1994). To overcome the depth bias of MNE, Standardized low-resolution brain electromagnetic tomography (sLORETA) has been developed (Pascual-Marqui, 2002). The idea is to modify the minimum norm estimation so that the estimated variables are standardized current densities, which were found to have high localization accuracy (Sekihara et al., 2005; Pascual-Marqui, 2007) and high measurement noise robustness (Saha et al., 2015; Dümpelmann et al., 2012). Various types of Bayesian methods have been reported to have the aforementioned positive properties when coupled with standardization (Schimpf and Liu, 2008; Lahtinen et al., 2024) which leads us to expect that a standardized KF could also achieve these properties. However, as the Kalman filter is a time-dependent recursive Bayesian estimation technique, the standardization weights are time-dependent unlike in the time-invariant estimation technique sLORETA.

In this work, we propose a standardized Kalman filtering. As the first step, we give probability-based interpretation for sLORETA. Then, we introduce a novel sLORETA-type standardization to the dynamical EEG source imaging problem, namely to the Kalman filtering, where the state-space output of the Kalman filter is modified so that the estimated source reconstruction is standardized in order to reduce the depth bias. After that, we conduct experiments with synthetic and real non-invasive EEG recordings of somatosensory evoked potentials (SEP) showing the difference of spatial sLORETA, Kalman filtering (KF), and standardized Kalman filtering (SKF) estimation. We have selected SEP data because these data have previously been well-studied, have rather well-known originator locations (Emerso and Pedley, 1984; Babiloni et al., 2000; Valeriani et al., 2000; Buchner et al., 1995), and because deeper thalamic sources and superficial sources in SI contribute to the data. We want to pinpoint that reconstruction of weak deep activity is a difficult task that has grown in interest in recent years (Rezaei et al., 2021).

In the result section, we demonstrate the difference between spatial sLORETA, Kalman filtering (KF), and standardized Kalman filtering (SKF) estimation. Our results show that the usage of the Kalman filter smoothes the estimation while the standardization improves the localization. In effect, the combination of the standardization and dynamical filtering was found to improve the localization and ability to track dynamical changes in the system containing cortical and subcortical activity better than these approaches separately. Moreover, in our computer simulations, we show that the subcortical components of simulated SEP data are correctly localized only with a standardized Kalman filter. Finally, we test our algorithm with real SEP. Our findings reveal that SKF produces a more coherent and focal reconstruction of the underlying activity compared to the two other methods.

2. Methods

2.1. Bioelectromagnetic forward problem

The observation model in EEG source imaging (Hallez et al., 2007) is given by the time-varying linear system

\[ y_t = Lx_t + r_t, \]  

where \( y_t \in \mathbb{R}^m \) denotes the recorded scalp potentials at time step \( t \), \( L \in \mathbb{R}^{m \times m} \) is the finite element method-based lead field matrix (Bauer et al., 2015; Miinalainen et al., 2019), \( x_t \in \mathbb{R}^n \) is called reconstruction that gives the coefficients for the basis function representation of the discretized primary current field, and \( r_t \in \mathbb{R}^m \) denotes the measurement noise that is assumed to follow a zero-mean Gaussian distribution \( N(0, R_t) \), where \( R_t \) is the time-varying covariance matrix.

2.2. Bayesian MNE and sLORETA

Minimum norm estimate (MNE) (Hämäläinen and Ilmoniemi, 1994) can be interpreted within the Bayesian framework when one introduces the measurement noise covariance matrix \( R \) and prior source covariance matrix \( P \) instead of a regularization term (Wipf and Nagarajan, 2009). By utilizing Bayes’ rule

\[ p(x \mid y_t) \propto p(y_t \mid x)p(x), \]  

where \( p(x) \) is the time-invariant prior and \( p(y_t \mid x) \) is the likelihood for measurements at time step \( t \). When we assume Gaussian likelihood

\[ p(y_t \mid x) = N(Lx_t, R) \propto \exp\left( -\frac{1}{2}(y_t - Lx_t)^T R^{-1}(y_t - Lx_t) \right), \]  

where \( R \) is the time-varying covariance matrix.
and Gaussian prior of \( x \) as
\[
p(x) = N(0, P) \propto \left(-\frac{1}{2} x^T P^{-1} x\right).
\] (4)
The posterior distribution of Bayesian MNE is a Gaussian and reads
\[
p(x | y) \propto \exp\left(-\frac{1}{2} (y - Lx)^T R^{-1} (y - Lx)\right) \\
\times \exp\left(-\frac{1}{2} x^T P^{-1} x\right)
\] (5)
since the maximum of the distribution, so-called maximum a posteriori (MAP) estimate \( \arg \max p(x | y) \) or equivalently \( \arg \max p(x) \ln p(x | y) \), with respect to the brain activity reconstruction \( x \) constitutes the time-invariant solution that is equivalent to MNE that is
\[
\hat{x} = PL^T \left( LPL^T + R \right)^{-1} y.
\] (6)

Standardized low-resolution brain electromagnetic tomography (sLORETA) is a method utilizing probabilistic source estimation that is obtained by standardizing the minimum norm estimate. (Cohen et al., 2014)

Standardization of MNE aims to equalize the variability of \( \hat{x} \) components corresponding to each source location so that no particular source location is favored (Pascual-Marqui, 2007), which is achieved by post-hoc weighting the MAP estimate. Additionally, Pascual-Marqui states in the Generalization section in (Pascual-Marqui, 2002) that the standardization can be taken only from a reconstruction vector whose components are uncorrelated and standardized, in the Gaussian sense with respect to the prior. For this reason, a new optimization problem is defined with variable \( \mu := P^{-1/2} x \) and lead field operator \( H = LP^{1/2} \) in the derivation of the weighting. For this new variable, we will have the prior \( p(\mu) \propto \exp\left(-\frac{1}{2} \mu^T \mu\right) \) and likelihood \( p(y | \mu) \propto \exp\left(-\frac{1}{2} (y - H\mu)^T R^{-1} (y - H\mu)\right) \).

Now, the Backus-Gilbert kernel, that is in this case resolution matrix \( R \), from which the standardization weights are formed, is the following
\[
R = H^T (HH^T + R)^{-1} H.
\] (7)
The sLORETA estimate represented in the form given in (Hauk et al., 2011) reads
\[
\hat{x} = W_0 \hat{\mu} = \text{Diag}(R)^{-1/2} \hat{\mu} \\
= \text{Diag}\left(H^T (HH^T + R)^{-1} H\right)^{-1/2} \hat{\mu} \\
= \text{Diag}\left(p^{1/2} L^T (LPL^T + R)^{-1} Lp^{1/2}\right)^{-1/2} P^{-1/2} \hat{x}.
\] (8)
The current power estimation representation given by Pascual-Marqui can be derived following the Appendix A.1.

2.3. Standardized Kalman filter

In Kalman filtering, as in any Bayesian filtering approach, the aim is to find the marginal posterior distribution of the state \( x \), from noisy measurements \( \{y_1,...,y_t\} = y_{1:t} \). This means in the Bayesian sense that we need to compute the posterior of the current state given all measurements up to time step \( t \), i.e., \( p(x_t | y_{1:t}) \). These posteriors can be obtained recursively, and therefore using lighter computations, if we model the dynamics via the Markov process (Doob, 1953).

In that case, the Chapman-Kolmogorov equation gives the prediction step of the form
\[
p(x_t | y_{1:t-1}) = \int p(x_t | x_{t-1}) p(x_{t-1} | y_{1:t-1}) \, dx_{t-1},
\] (9)
where \( p(x_{t-1} | y_{1:t-1}) \) is the posterior of the previous state and \( p(x_t | x_{t-1}) \) is the transition probability model. Now the posterior \( p(x_t | y_{1:t}) \) can be computed by Bayes’ rule as
\[
p(x_t | y_{1:t}) \propto p(y_t | x_t) p(x_t | y_{1:t-1}),
\] (10)
where \( p(y_t | x_t) \) is the likelihood and where the predictive distribution \( p(x_t | y_{1:t-1}) \) can be considered as the prior information in Bayesian notion.

Let us have a linear dynamical model of brain activity
\[
x_t = A_t x_{t-1} + q_t,
\] (11)
with any linear state transition model \( A_t \in \mathbb{R}^{n \times n} \) and noise vector \( q_t \sim N(0, Q_t) \). In addition, we assume the initial state \( x_0 \) following Gaussian. This yields the Gaussian predictive distribution \( N(x_{y_{t-1}}, P_{y_{t-1}}) \) and posterior \( N(x_{q_{t}}, P_{q_{t}}) \) that we are computing recursively using Kalman filtering. Now, the output of Kalman filtering is the MAP estimate (posterior mean)
\[
x_{y_{t}} = \arg \max_{x} \left[ \ln(p(y_t | x_t)) + \ln(p(x_t | y_{1:t-1})) \right],
\] (12)
having the following form
\[
x_{q_{t}} = x_{q_{t-1}} + P_{q_{t-1}} L^T \left( LP_{q_{t-1}} L^T + R \right)^{-1} (y_t - LX_{q_{t-1}}). \] (13)

Due to the symmetry of this sLORETA weighting matrix and the square root of a prior covariance matrix, there is a linear transformation between any Gaussian distributed reconstruction and its standardized counterpart. This includes the dynamic model, i.e., the evolution model and the Gaussian initial state, of the Kalman filter. The differentiator between the time-invariant standardization of sLORETA and the standardization of the Kalman filter is that the weighting, as transformation, must be time-dependent. That is because the Kalman posterior of the current state depends on the previous one. Moreover, as we show in the Appendix B, linear and bijective (invertible) time-dependent transformation of the dynamic model of states leads to a similarly transformed posterior state. Here we define the dynamical standardization matrix \( W_t \) at time step \( t \) for the MAP estimate of the Kalman filter (posterior mean) given the states up to \( t - 1 \) following the reasoning in Section 2.2.

By connecting the expression of Kalman filtering MAP estimate for prior source covariances \( P_{y_{t-1}} \) with the expression
of the weight given in (8), we get the following formulation of the time-varying weighting matrix

\[ W_t = \text{Diag} \left( P_{\theta t-1}^{1/2} L^T (L P_{\theta t-1} L^T + R_t)^{-1} L P_{\theta t-1}^{1/2} \right)^{-1/2} \]  

(14)

and the standardized MAP estimate can be obtained as

\[ z_{\theta t} = W_t P_{\theta t-1}^{-1/2} x_{\theta t} , \]  

(15)

which current density format along with the derivation of the dynamical weighting matrix using Bayesian evidence are presented in Appendix A.2. With this over-carried transformation of the posterior states and the dynamical standardization matrix \( W_t \), there is the following connection between standardized and regular posteriors of the Kalman filter

\[ N(z_t, \hat{P}_{\theta t}) = N \left( W_t P_{\theta t-1}^{-1/2} x_t, W_t P_{\theta t-1}^{-1/2} P_{\theta t-1} P_{\theta t-1} P_{\theta t-1} P_{\theta t-1} W_t^T \right) . \]  

(16)

A concrete benefit from the presented identity is that the standardization weighting can be applied at the end of each update step, thereby, decreasing the computational burden we would otherwise have in the transformation of the model and auxiliary matrices of the Kalman filter algorithm.

By coupling standardization weighting with the Kalman filtering algorithm (Kalman and Bucy, 1961; Sarkka, 2013), we propose the following recursive algorithm for standardized Kalman filtering:

The prediction step is executed as follows

\[ x_{\theta t-1} = A_t x_{t-1 | \theta t-1} \]

\[ P_{\theta t-1} = A_t P_{t-1 | \theta t-1} A_t^T + Q_t \]

and the update step reads as

\[ S_t = L P_{\theta t-1} L^T + R_t \]

\[ K_t = P_{\theta t-1} L^T S_t^{-1} \]

\[ x_{\theta t} = x_{\theta t-1} + K_t (y_t - Lx_{\theta t-1}) \]

\[ P_{\theta t} = P_{\theta t-1} - K_t S_t K_t^T \]

\[ W_t = \text{Diag} \left( P_{\theta t-1}^{1/2} K_t S_t K_t^T P_{\theta t-1}^{-1/2} \right)^{-1/2} \]

\[ z_{\theta t} = W_t P_{\theta t-1}^{-1/2} x_{\theta t} . \]

The second last step introduces the time-dependent post-hoc weights and vector \( z_{\theta t} \) gives the standardized Kalman estimation at time step \( t \). Estimation is obtained first by normalizing \( x_{\theta t} \) with respect to prior and then by taking the standardization \( W_t \) as the procedure is described in (Pascual-Marqui, 2002) for independent sources.

3. Experiments

3.1. MRI-based segmentation and source space for the head model

We modeled the head as an 18-compartment volume conductor constructed using openly available T1-weighted MRI data from a healthy subject. The tissue conductivities were set, i.e., 0.14, 0.33, 0.0064, 1.79, 0.33 and 0.33 S/m for the white and grey matter, skull, cerebrospinal fluid (CSF), subcortical structures, and skin, respectively, based on the studies of Dannhauer et al. (2011) and Shahid et al. (2014). Of the remaining compartments, subcortical nuclei were associated with the conductivity of the grey matter and the ventricles with that of CSF according to e.g. (Rezaei et al., 2021) and the citations therewithin. The tissue compartments were segmented via FreeSurfer software\(^1\) with the

\(^1\)https://surfer.nmr.mgh.harvard.edu/
functions of the SPM12 package (Ashburner et al., 2014). The head model with 1 mm resolution and the finite element method (FEM) based forward solution were obtained using the Matlab-based Zeffiro Interface toolbox (He et al., 2019). The source space of the forward model contains 10,000 source locations which, for example, is Brainstorm software package’s standard source count.

3.2. Median nerve SEP simulations

We developed simulated EEG recordings of median nerve SEP data using 9 dipolar originators based on their description given in the literature to validate our approach. We modeled the time interval of 14-30 ms post-stimulus, where the first three components (P14 and P16) are sub-cortical located in the brainstem and thalamus following P20/N20, P22/N22 and P30/N30 components with simultaneous cortical and sub-cortical neural activity.

The SEP activity follows the medial lemniscus pathway and travels to thalamocortical volley (Noël et al., 1996) which starts from the dorsal column and eventually reaches the somatosensory cortex (SI) where the first cortical responses are expected to be detected. The earliest component P14 is expected to originate from the medial lemniscus (Noël et al., 1996). Based on the findings of Buchner et al. (Buchner et al., 1995), P16 dipoles are placed in the ventral thalamus and the lower part of the brainstem (Hsieh et al., 1996). The P20/N20 component occurring at 20 ms is modeled by two dipoles, one placed at Brodmann area 3b and another in the thalamus. Götz et al. have found simultaneous subcortical activity, particularly at ventral posterolateral (VPL) thalamus (Haueisen et al., 2007) suggesting that the thalamus can be active at the same time in this phase (Götz et al., 2014). The peak of the 22 ms component is located either in Brodmann area 1 or 4 (Buchner et al., 1995) from which we have chosen the first one. Similarly to the 20 ms events, the thalamus has been found to be active during the maximum peak of the P22/N22 (Papadelis et al., 2011). At 30 ms, the peak of the cortical generator is placed in the somatomotor area, and the sub-cortical dipole at the ventral thalamus (Cebolla and Chéron, 2015).

The exact dipole locations and orientations are presented in Figure 1. The amplitude of dipolar activities are all set to be equally 10 nA m as suggested for a simulation (Hämäläinen et al., 1993; Goldenholz et al., 2009). The time evolution of each component is modeled as a Gaussian pulse with 7 ms duration. We apply a 5 kHz sampling rate in the data generation.

In the simulations, we compare the performance of the Kalman filter (KF), standardized Kalman filter (SKF), and sLORETA to cover the effect of dynamical filtering and the standardization.

In simulation (I) we are focusing on the trackability and smoothness of the reconstructed time evolution of the overlapping pair of thalamic and cortical sulcal sources occurring at 20 and 22 ms, respectively, to investigate the dependence of cortical and deep activity in reconstructions presented in the Figure 1(C). Ideally, the pulses should be recovered as independent tracks. The simulation is performed using signal-to-noise ratios (SNR) of 25, 15, and 5 dB that are chosen to be particularly low to highlight the differences. Here we define SNR (dB) as the 10-base logarithm of the ratio between squared signal and noise amplitudes. For each noise level, 50 measurement data samples were gathered and then reconstructed. The results are displayed as time-dependent curves of averaged activity that are normalized by the maximum value obtained at 25 dB.

In the follow-up simulation (II), we reconstruct the whole time series of 9 sources and display the reconstructions of the component originators at their peaks, where SNR is the highest. The data was created using 25 dB SNR. The simulated EEG recordings are presented in Figure 1(B).

3.3. Experiment using real SEP data

The dataset used in the experiment was obtained from a 49-year-old right-handed male from whom MRI data was used to construct the head model. The subject had no history of psychiatric or neurological disorders and had given written informed consent before the experiment. The institution’s ethical review board (Ethik Kommission der Ärztakammer Westfalen–Lippe und der WWU) approved all experimental procedures on 02.02.2018 (Ref. No. 2014-156-f-S). The dataset contains a defaced head model and montage-averaged EEG recordings. MRI dataset, from which the head model was constructed, was measured by MAGNETOM Prisma scanner 3.0 T (Release D13, Siemens Medical Solutions, Erlangen, Germany) with T1 and T2-weighting (T1W/T2W) fast gradient-echo pulse sequence. SEP measurements were performed using 80 AgCl sintered ring electrodes (EASYCAP GmbH, Herrsching, Germany) with 74 EEG channels in the standard 10–10 system. A notch filter was applied in order to remove the interference caused by harmonics of the 50 Hz power line frequency and the 60 Hz of the monitor from which the subject watched a video during the measurement as a means to reduce otherwise prominent alpha-activity. A sampling rate of 1200 Hz and an online low pass filter at 300 Hz were used. A total of 1200 stimuli were recorded for averaging, following the guidelines for spinal and subcortical SEPs (Cruccu et al., 2008). We are obtaining the reconstructions at the time spots of the major SEP components, the activity corresponding to the 14-30 ms post-stimulus peaks P14, P16, P20/N20, P22/N22, and P30/N30 similar to the simulated experiment described in the previous section 3.2. The dataset is openly available (Piastra et al., 2020).

3.4. Model parameter selections

In this study, we use a simple random walk of the reconstruction as the evolution model of KF that reads

\[
x_t = x_{t-1} + q_t, \quad (17)
\]
where \( q \) obeys Gaussian \( \mathcal{N}(0, Q_1) \). In practice, we let the current density change in Gaussian distributed increments over time at every calculation node of the head model. To the best of our knowledge, there is no physical model that describes the evolution or propagation of brain activity, therefore, the basic random walk model is a reasonable choice and a common one in situations like this. To select the initial prior covariance, we use the prior estimation method based on the concept of total variation over the present head model (Rezaei et al., 2020), where the activity on individual source locations are assumed to be identically distributed and independent. Thus, the evolution covariance matrix \( Q_t \) is a diagonal matrix with equal variances \( \theta \). Prior covariance is initialized equally for both Kalman filter implementations to ensure a fair comparison. Applying the same technique to evolution prior, the evolution is set to be identically and independently distributed with variances \( q \). Assuming that the expected \( L_2 \)-norm of activity change is equal in every source location, we get the estimation for evolution variance

\[
q = \sigma_t^2(L_t^2)^{10^{-60}/20}/f,
\]

where \( \sigma_t(L_t) \) denotes the largest singular value of the lead field, \( f \) is the sampling frequency of the measurements, and \( \rho \) is a free parameter relating prior variance and evolution of prior variance in decibels. The value was set to be \( 34 \) dB by estimating the variance of the change rate of MNE and sLORETA reconstructions. Initial measurement noise covariance is estimated from the period before the first evoked spike.

4. Results

4.1. Median nerve SEP simulations

The simulation (I) concerns the temporally overlapping thalamo-cortical pair of sources at 20 and 22 ms. The results show an inability to detect the deep source for the basic Kalman filter as seen in the blue deep activity strength curve that stays zero during the whole time series on the first row of Figure 2 for each noise level case 25, 15, and 5 dB from left to right, respectively. On one hand, the Kalman filter tracks the beginning of the cortical activity (green curve) with only slight deviations on the postcentral gyrus at 25 dB (the first on the left in the figure) as the lighter coloring represents the standard deviation of activity tracks at each time point. On the other hand, the damping of the activity is slower than it should be as we can see comparing the track to Figure 1(C). sLORETA is able to detect both sources with only minor deviations at 15 and 25 dB. However, the strength of deep activity is suppressed and non-independent. In an independent scenario, the estimated strength of the peaked deep activity would be equal to the cortical activity, and we would not see increased activity on the thalamus when cortical activity appears. In the case of Kalman coupled with the standardization, we obtain the highest activity strength on the deep structure among the compared methods. Also with SKF, small deviations can be observed in the results of 15 and 25 dB.

In simulation (II), presented in Figures 3, all reconstructions display the cortical activity at the correct location for tangential sources at 20 and 30 ms. The basic Kalman slightly mislocalizes the radially oriented gyral source at 22 ms. Subcortical sources at 14 and 16 ms produce a faint cortical projection for sLORETA, which yielded overall less focal results than with filtering approaches while SKF was the most focal. While KF does not detect any activity in deep structures, its standardized counterpart localizes the activities correctly in the medulla-pontine junction at 14 ms, the bottom of the brainstem at 16 ms, and in left thalamus at 20, 22, and 30 ms.

4.2. Experiment with real SEP data

In Figure 4, for SKF and sLORETA, the reconstruction peak on the brainstem is located at pons for the 14 ms component and at the bottom of the brainstem at 16 ms. In line with the simulations, deep activities are not detected by KF. In addition, KF localizes the cortical activity to the superior parietal lobule while SKF and sLORETA have the cortical maximum on the central sulcus at 20, 22, and 30 ms. Of the two, SKF has a slightly more focal reconstruction, i.e., a smaller spread of activity distribution. Reconstruction maxima of the thalamic component of P20/N20 are obtained on the brainstem at different location with SKF than sLORETA. Both of the reconstruction peaks extend to the left thalamus. The reconstructions of subcortical activity at 22 ms found via SKF and sLORETA are similar, and their maxima locate at the left thalamus. A small discrepancy can again be observed in SKF and sLORETA reconstructions of the subcortical activity at 30 ms as the SKF peak is located at the left thalamus and the peak of sLORETA is at the bottom of the brainstem.

5. Discussion and conclusions

In this paper, we have introduced a way to couple the standardization of sLORETA with the Kalman filter in a mathematically rigorous manner. Our formulation allows the utilization of the entire source space without the use of the nearest neighborhood space reduction (Yamashita et al., 2004; Hamid et al., 2021), simplified mesh (Galka et al., 2004), or rank-reduced model (Ou et al., 2009), and of a common fine spatial resolution, e.g., a space of 10,000 sources, without the need for high-performance computing (Long et al., 2011). Because of this, highly accurate source localization is possible. Based on the two numerical simulations considering the traceability of time evolution and localizability of activity peaks of synthetic median nerve SEP originators, in comparison to sLORETA and Kalman filter (KF), the standardized Kalman filter (SKF) inherits the advantages of both methods while seeming to provide a greater focality of the reconstruction as its own ability. The results show how the KF without any aid cannot recover deep activity from EEG recordings. This is no surprise since KF is closely related to MNE which is notorious for its ineptitude in recovering far-field sources (Lucka et al., 2012). The low-resolution aspect
Figure 2: Mean and the standard deviation of tracked brain activity time evolutions of deep (blue) and cortical (green) components in simulation (I) under different noise levels (25, 15, 5 dB from left to right). The area covered in a lighter coloring represents the standard deviation over samples and the darker green curve represents the sample mean. The data contains 50 noise realizations. True tracks are shown in Figure 1 (C).

Figure 3: Left: Topographic plots and cortical part of the reconstructions on peak time points of simulated SEP data for all three methods. Reconstructions are scaled from 0 to 1 normalizing by the highest reconstruction magnitude over the whole time series. Right: Sub-cortical part of the reconstructions on thalamus and brainstem.

of sLORETA manifested itself in reconstructions of deep activities: other than reconstruction of 20 ms are so widespread that the activity is hard to be localized in sub-levels of thalamus and brainstem. As is the case with the standardized MNE (sLORETA), standardization makes the subcortical activity detectable also with the standardized Kalman filter. Also, robustness to measurement noise obtained with standardization in earlier studies (Saha et al., 2015; Dümpelmann et al., 2012) is present in our simulation (I) considering trackability, as only slight deviations from the mean are obtainable. Moreover, the benefits of dynamical modeling can be seen when comparing the track of SKF to sLORETA’s ones: SKF is the only method compared that can retain the time evolution of subcortical and cortical activity at 5 dB. Reconstructions obtained with real SEP data seem clearer compared to the simulated cases which may be due to a high number of averaged montages in real data compared to 25 dB noise used in the simulation (II) because averaging reduces the standard error of the mean. Locations of cortical and subcortical peaks of sLORETA and SKF are mostly according to the literature reviewed in the section 3.2, but 22 ms cortical peak at the central sulcus and extending over its walls is inconclusive since the said originator is located at Brodmann areas 2 and 4 for different patients. Interestingly, the basic Kalman filter mislocalizes the cortical activity. A probable explanation could be that the earlier subcortical components get reconstructed evenly on the cortex because of KF’s depth bias which interferes with the estimation of cortical dynamics causing false localization. At 20 ms, the subcortical reconstruction of SKF and sLORETA are partly spread over the VPL thalamus, peaking in different parts of the brainstem. The deviation in peak locations, as compared to the ex-
Figure 4: **Left:** Topographic plots and cortical part of the reconstructions obtained from real SEP peak time points for all three methods. **Right:** Sub-cortical part of the reconstructions on thalamus and brainstem. The reconstructed activity strength is scaled.

Expected thalamic activity, might follow from the given prior, which affects the reconstruction in weakly distinguishable areas most. Due to the standardization, SKF and sLORETA might produce a slightly too deep originator for the gyral 22 ms component, which is present in each reconstruction found in the simulated experiment (II). With 30 ms SKF’s peak is located in agreement with the literature while the sLORETA peak is at the bottom of the brainstem.

To summarize, the results suggest that SKF can track, i.e., follow the time evolution of the activity, and localize accurately cortical and subcortical originators of somatosensory evoked potentials. The experiment setup is not particularly simple for a localization algorithm as the far-field component produces only weak EEG signals. Therefore, the simultaneous cortical and subcortical activity obscures especially the subcortical signals. Here we demonstrated that SKF can also endure a significant amount of additional measurement noise. This is characteristic for methods that apply standardization. (Pascual-Marqui, 2002; Schimpf and Liu, 2008; Lahtinen et al., 2024). Results obtained with a healthy patient follow the literature. However, due to the known variation in placement of the 22 ms originator and widespread reconstruction, we cannot state anything about the localization accuracy of this component. This might be due to the specific noise situation in this N-of-1 trial. One important future goal is thus the application of SKF in a SEP group study for the reconstruction of the P22 component.

In the study of connections between brain regions, including deep ones, and dynamic behavior in both functional and dysfunctional cases, good localization and tracking of activity are required. Due to the f ocality and high noise robustness of SKF, the applicability of the method to recover epileptic sources could be investigated as a potential future work.

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**Appendix A. The current power representation of sLORETA and standardized Kalman filter and derivation of weighting using Bayesian evidence**

**Appendix A.1. Standardized low-resolution brain electromagnetic tomography**

Using the denotation from Eq. (8), the current power estimation, denoted here as the inner product of sLORETA re-
construction $\hat{z}$ with itself at $d$-dimensional index set $I$, reads
\[
\langle \hat{z}_I, z_I \rangle_{\text{LORETA}} = \hat{\mu}_{II}^{T} [P_I^{-1}]_{II}^{T} \hat{\mu}_I
\]
\[
= \hat{\mu}_{II}^{T} [1 + (LPL + R)^{-1}]_{II}^{-1} P_I^{-1} \hat{\mu}_I
\]
where the dimension $d$ indicates the degree of orientational freedom of a dipole.

The weights can be derived alternatively by Bayesian evidence, where the measurements are considered as a random variable generated from the given prior causing the MAP estimator also to be a random variable. Let us consider the following marginal likelihood constituting from a Gaussian measurement noise $r$ following marginal likelihood constituting from a Gaussian freedom of a dipole.

In practice, this means that covariances of $\langle \cdot \rangle_{\text{MAP}}$ interpreted as a Gaussian random variable for which it can now be seen that standardization as post-hoc weighting is analogous to the location-wise standardization of MAP interpreted as a Gaussian random variable for which (\cdot)LORETA induces a Mahalanobis distance (Mahalanobis, 1936). In practice, this means that covariances of $\hat{z}_I$ are equal for each source location, i.e.,
\[
\text{cov} [\hat{z}_I] = \text{cov} [P_I^{-1/2} \hat{\mu}_I] = [R_I^{-1}]_{II}^{T} \text{cov} [\hat{\mu}]_{II}^{-1} = I.
\]

**Appendix A.2. Standardized Kalman filtering**

In the case of the Kalman filter, The marginal likelihood is
\[
p(Lx + r) = \int N(Lx, R) \times N(x_{t-1}, P_{t-1}) dx_t
\]
\[
= N \left( x_{t-1}, LP_{t-1} + R_{t-1} \right)
\]
where $x_{t-1}$ is fixed. Substituting this to the place of $\hat{y}_t$ in MAP from Eq. (13):
\[
\hat{x}_{\varphi}(\hat{y}_t) = x_{t-1} + K_t(\hat{y}_t - Lx_{t-1}),
\]
we get
\[
\hat{x}_{\varphi}(Lx_t + r_t) \sim N \left( x_{t-1}, K_tS_tK_t^T \right),
\]
where $S_t = LP_{t-1}L_t^T + R_t$, and similarly as in the sLORETA case,
\[
P_{t-1/2}^\varphi \hat{x}_{\varphi} \sim N \left( P_{t-1/2}^\varphi x_{t-1}, P_{t-1/2}^\varphi (LP_{t-1}L_t^T + R_t)^{-1} LP_{t-1/2}^\varphi \right),
\]
which gives the following representation for the block-diagonal weighting matrix
\[
[W_{\varphi}]_{II} = \left[ \frac{P_{t-1/2}^\varphi (LP_{t-1}L_t^T + R_t)^{-1} LP_{t-1/2}^\varphi}{P_{t-1/2}^\varphi K_tS_tK_t^T P_{t-1/2}^\varphi} \right]_{II}
\]
and the current power estimation is then
\[
\langle \hat{x}_{\varphi,t}, \hat{z}_{\varphi,t} \rangle = \hat{\mu}_{II}^{T} \left[ \frac{P_{t-1/2}^\varphi K_tS_tK_t^T P_{t-1/2}^\varphi}{P_{t-1/2}^\varphi} \right]_{II} \hat{\mu}_{II},
\]

where $\hat{\mu}_{II} = [P_{t-1/2}^\varphi x_{t-1}]_I$ for some index set $I$.

**Appendix B. Linear transformation on Kalman filter’s estimation**

In the case of a time-dependent linear and bijective transformation of the state vector $z_t = M_t x_t$, the dynamical model should be written as
\[
z_t = M_t (A_t x_{t-1} + q_t) = M_t A_t M_{t-1}^{-1} z_{t-1} + M_t q_t.
\]

When the prior is distributed as $x_{t-1} \sim N(A_t x_{t-1-1}, P_{t-1-1})$, we get
\[
z_t \mid y_{1:t-1} \sim N( M_t A_t x_{t-1}, M_t P_{t-1-1} M_t^T )
\]
by the linear transformation property of Gaussian distributions. Using the distribution above, we can write the joint distribution
\[
\begin{bmatrix}
\hat{z}_t \\
y_t
\end{bmatrix}
\sim N \left( \left[ z_{t-1} \right]_{1:t-1} , \left[ \Sigma \right]_{1:t-1} \right)
\]
where
\[
\Sigma = \begin{bmatrix}
M_t P_{t-1-1} M_t^T & M_t P_{t-1-1} L_{t-1}^T \\
L_{t-1} P_{t-1-1} M_t^T & L_{t-1} P_{t-1-1} L_{t-1}^T + R_{t-1} - 1
\end{bmatrix}
\]
Next, using the Bayesian rule, we obtain
\[
p(z_t \mid y_t) = \frac{p(y_t \mid z_t) p(z_t \mid y_{1:t-1})}{p(y_t)}
\]
\[
= N \left( z_{t-1}, M_t P_{t-1} M_t^T \right),
\]
where $z_{t-1} = z_{t-1} + M_t K_t (y_t - L x_{t-1})$
\[
M_t P_{t-1} M_t^T = M_t P_{t-1-1} M_t^T - M_t K_t S_t K_t^T M_t^T.
\]

Now we can see that the linear transformation $M_t$ has carried from the prior estimate to the posterior state, i.e., $z_t \mid y_t = M_t x_t \mid y_t$ for all $t = 1, \cdots, T$.  

9

Piastra et al., M.C., 2020. The WWU DUNEuro reference data set for combined EEG/MEG source analysis. doi:10.5281/zenodo.3890381. The research related to this dataset was supported by the German Research Foundation (DFG) through project WO1425/7-1 and the EU project ChildBrain (Marie Curie Innovative Training Networks, grant agreement 641652).


