Attenuation Correction of Airborne Gamma Ray Spectrometric Data Using Digital Terrain Models

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Abstract

Attenuation correction of airborne gamma-ray spectrometric data using digital terrain models. Gamma-rays are partially absorbed in air on their way from Earth’s surface to the sensor package in the aircraft, thus attenuated following an exponential law, coming from a footprint area with a few hundred metres diameter. This effect shall be removed as far as flying height deviates from some nominal flying height. Instead of assuming a flat surface, this is done for the true travel distance of the gamma-rays using digital terrain models and an arbitrarily fine sector system around the sensor package.
Attenuation Correction of Airborne Gamma Ray Spectrometric Data Using Digital Terrain Models

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ABSTRACT
Airborne measurements of gamma-ray radiation are typically corrected to a nominal height above ground. While the detector system receives radiation from all directions within a radius of a few hundred meters, the traditional correction method considers only the distance to the ground directly below the aircraft and assumes level terrain. In more rugged terrain, the topography should be considered in more detail. The method described in this paper uses the concept of a virtual division of the terrain below the aircraft into regions called tesserae. The count rate of photons reaching the detector from each tessera is calculated for a given surface radiation intensity. Summing this up for all tesserae to which the detector system is exposed and comparing this with the count rate from a flat surface at nominal flying height gives a correction value for the actual count rates independent of flying height. This approach is easy to implement and can be performed routinely for any gamma-ray survey. The method shows identical results to the traditional method whenever the footprint area is level while giving improved results for rugged terrain. It also allows for a detailed directional sensitivity model of the detector system.

INTRODUCTION
Airborne gamma-ray surveys are a very important tool for geological mapping, to control our environment with respect to potential contamination with radioactive elements or to help the mining industry find minerals which are associated with enhanced levels of radioactivity. The measurement in these surveys is the energy and quantity of gamma-rays, i.e. photons of the electromagnetic radiation spectrum of very high energy. These photons are mainly generated by radioactive decay of three naturally occurring decay chains which are linked to the abundance of potassium, uranium and thorium.

A number of processing steps are carried out to convert the measurement to an estimate of the abundance of a particular element. This is normally done following the publications of the International Atomic Energy Agency (1991, 2003). Major processing steps correct for the influence of cosmic radiation and aircraft background as well as effects of airborne radon, removal of mutual influence on radiation levels from different decay chains with overlapping energies in the radiometric spectrum (stripping) and correcting for changing count rates depending on different flying heights, before converting the count rates into (equivalent) concentrations of radioactive elements.

Only gamma-rays coming from the surface and near the surface of the earth may reach the detector system as gamma-rays originating from deeper sources are attenuated before leaving the soil. On their way to the detector system in the aircraft, the gamma-rays may interact with particles of the atmosphere, thus it is possible that a gamma-ray loses energy on its way or does not reach the aircraft at all. The probability of interaction with an air particle between the source of the photon and the aircraft depends on the distance to the ground source and the air density. Thus the count rate measured at the aircraft for any radiation from the surface follows an inverse exponential law with the distance to the detector system. Surface areas beyond a certain distance, in the presence of an attenuating medium, contribute to the count rate insignificantly or not at all. The area with dominating influence on the count rate is called the footprint of the detector system.

The following sections will show in brief the conventional height correction, a short summary of previously developed methods for tackling the problems which occur in rugged topography and then a description of a new
method of height correction that is well suited for rugged topography.

**CONVENTIONAL CORRECTION FOR HEIGHT ATTENUATION**

Following the document of the [International Atomic Energy Agency](https://www.iaea.org) (1991), count rates of gamma-rays are attenuated by passing through air towards the detector and can closely be approximated by an exponential law of the form

\[ N_h = N_0 e^{-\mu h}, \]  

where \( N_h \) is the count rate at the height \( h \) above ground level, corrected to the equivalent height at standard temperature and pressure (STP), \( N_0 \) is the fully corrected and stripped count rate at ground level, \( \mu \) is the attenuation coefficient. Count rate or simply counts refers to the number of photons per second. The coefficient \( \mu \) is established using a flight over a test range flown at different altitudes. Based on the accuracy and noise level, this model has been quite successful for reducing count rates attained at varying altitudes to a nominal flying height in not too rugged topography within the normal range used for calibration.

The attenuation coefficient \( \mu \) depends on the air density. In the traditional approach where only one distance factor is involved, namely height, a correction was applied to the height value for the so called effective or equivalent height as in equation [1]. According to [International Atomic Energy Agency](https://www.iaea.org) (1991) applying the correction to the height gives

\[ h = h_0 \frac{273.15}{T + 273.15} \times \frac{p}{1013.25}, \]  

where \( h_0 \) is the observed height, \( h \) is the equivalent height at STP (effective height), \( T \) is the air temperature in °C, and \( p \) is the barometric pressure in mbar.

An attenuation correction calculated using these equations is only a good approximation for a flat area and a relatively small range of flying height. The attenuation coefficients must be calibrated for each system and height range, taking into account the angular sensitivity distribution of the detector system and the relative change of the footprint area with height. In rugged topography, this can lead to considerable errors since the flying height may over- or underestimate the effective distance to the footprint area.

**HEIGHT CORRECTION IN RUGGED TOPOGRAPHY**

One approach of attenuation correction for rugged topography was shown in [Schwarz et al.](https://www.sciencedirect.com) (1992) with the first equation in their paper referenced from [Kogan et al.](https://www.sciencedirect.com) (1969). They used a geometry shown in Figure 1 with a rod consisting of radioactive matter behind the surface of an infinitesimal surface element giving a radiation intensity at the detector of

\[ dJ_r = \frac{q}{\mu_s} \times \frac{ds \cos \theta}{4\pi R^2_a} e^{-\mu_a R_a}, \]  

with the volumetric radioactivity \( q \) of the soil within the rod, \( \mu_s, \mu_a \) are the attenuation coefficients of soil and air \((m^{-1})\), \( R_a \) is the distance to the detector, \( ds \) is the horizontal cross-section of the rod at the distance \( R_a \) and \( \theta \) is the angle of inclination to the vertical.

**Figure 1: A rod of radioactive matter, facing detector.**

[Schwarz et al.](https://www.sciencedirect.com) (1992) developed their procedure for the attenuation correction in the Swiss Alps, considering the topography in detail throughout each footprint. Integration of equation [3] is done over each element of a digital terrain model within the footprint giving the radiation intensity at the detector system. Multiplying this value with the sensitivity of the detector system at the particular direction gives the count rate measured due to that surface element. This calculation added up for all surface elements within the footprint in real topography and compared to the same calculation for a flat surface at nominal flying height determines the required correction factor.

A more recent approach developed by [Druker](https://www.sciencedirect.com) (2012) includes establishing a relationship between strips along line of flight covering a high percentage of the footprint and solves for the radioactivity by inversion in order to developing grids of radio-element concentrations. This approach has the considerable advantage that homogeneous radionuclide distribution does not need to be assumed over the field of view (footprint) of the detector system. However similarly to [Schwarz et al.](https://www.sciencedirect.com) (1992), it requires calculation of the relationship between each surface element within the footprint area and the detector system.

Solving this problem through evaluation of the relative effect of each surface element within the footprint leads to mathematically difficult terms and requires intensive data processing. In the case of more rugged terrain, it will also be necessary to determine whether any surface element is hidden by areas closer to the aircraft as its radiation would never reach the detector system.

While the procedure of [Schwarz et al.](https://www.sciencedirect.com) (1992) was a very important development setting the standard for this purpose, it has not been widely adopted and it is not easily applied as a routine process.
FROM HALF SPACE TO TESSERAE

A procedure to account for the topography throughout the whole footprint instead of assuming a flat half space is developed in this paper, starting with a model of the directional sensitivities of the detector system in any desired detail. It is shown to achieve better performance in rugged terrain while providing results identical to the conventional half space method whenever the footprint area is level. Key to this procedure is the realization that, apart from attenuation and a given radio-element concentration, the count rate of a surface element depends only on the solid angle of a surface element with respect to the detector system.

Instead of determining coefficients from an attenuation test flight on a calibration range, linear attenuation coefficients can be used from known physical values as listed in Table 1. The test flight would then be obsolete for the purpose of deriving attenuation coefficients, though still required to ascertain the system sensitivities for converting count rates into (equivalent) radio-element concentrations.

In equation 1, there would be no difference if the air density correction of equation 2 was applied to the attenuation coefficient $\mu$ instead of $h$. This correction for $\mu$ will be applied for every measurement and only true distances instead of effective height will be used here. The physical size of the detector system is considered negligible compared to the source-sensor distances involved.

The first fraction $\frac{\cos \theta}{\mu a^2}$ of equation 3 defines the source density. To calculate the height correction value, it is assumed that the source density is uniform over the footprint of each measurement. If this is not the case, the correction value might be biased, over- or underestimating the source-sensor distance slightly. For a smooth distribution of source density, this bias should be very small. This assumption is required in the same way as in the procedure of Schwarz et al. (1992). There are other distributions that would also lead to unbiased correction values, e.g. where the source distributions depend only on azimuth on a level surface. In both cases, the source density distribution could be used as a weight function for a second iteration to give further improved corrections.

It is important to see from equation 3 that the second fraction is the definition of the solid angle. It is trivial to determine the solid angle $\Omega$ for areas confined by angles $\theta_1$, $\theta_2$ and azimuth $\phi_1$, $\phi_2$. Integration of equation 3 over this area using the mean value theorem gives the radiation intensity $J_t$ due to sector $t$ by

$$J_t = \int_{S_t} \frac{q_i}{\mu a^2} \frac{\cos \theta}{4 \pi R_a^2} e^{-\mu a R_a} \, ds$$

$$= q_i \Omega_a e^{-\mu a R_a}$$

with $\min(R_a) < R_a < \max(R_a)$, source density $q_s = \frac{2}{\pi R_a^2}$ and surface area $S_t$ of the small area corresponding to the solid angle $\Omega_t$ with

$$\Omega_t = \int_{S_t} \frac{\cos \theta}{4 \pi R_a^2} \, ds$$

$$= (\cos(\theta_1) - \cos(\theta_2)) (\phi_2 - \phi_1).$$

Figure 1 is being used to illustrate an infinitesimal small rod with horizontal intersection $ds$. It shows actually a small surface area $S_t$ between circles for $\theta_1 = 30^\circ$ and $\theta_2 = 35^\circ$ projected onto a level surface between azimuth values $\phi_1 = 210^\circ$ and $\phi_2 = 215^\circ$. $S_t$ is covering a solid angle $\Omega_t$ with respect to the centre of the detector system.

The small areas on the ground corresponding to small sectors around the detector system can be called tesserae. To cover the footprint in less rugged terrain, only the lower half of the detector system needs to be considered, but in mountainous areas, sectors for the upper half can be added. The sectors can be compared to areas of the southern globe divided into sectors by lines of latitude and longitude. Only the central tessera immediately below the detector system is circular, while every other tessera has four corners and is the projection of the sectors around the detector system onto the Earth’s surface. It should be noted that the detector system is not divided into sectors, the only division is to the incoming gamma-rays from the respective directions which may interact with the complete crystal volume.

The distance $R_a$ for a level surface is given by

$$R_a = h/cos(\theta)$$

and equation 5 lends itself to numerical integration to any level of accuracy. With $\Omega_t$ from equation 7 the distance for this equation applied to the full tessera is

$$\bar{R}_a = -\ln \left( \frac{J_t}{q_s \Omega_t} \right) / \mu,$$

independent of $q_s$ which is also a factor in $J_t$. The angle $\theta_r$ for the correct distance to the level surface is then from equations 8 and 9

$$\theta_r = \arccos(h/R_a).$$

For a given system of sectors covering the detector system, the radiation intensity due to each tessera can be calculated using equation 5 for each sector. To arrive at a count rate for the detector system, the sensitivity for each sector, i.e. the probability that an arriving photon actually is counted, must be considered. For various reasons, especially mutual shielding, the sensitivity is generally not isotropic. Photons from each tessera hit the detector system from a certain angle with a sensitivity $w_s$. The total count rate for some measurement over level half space therefore is

$$w_h = q_s \sum_i w_s(i) \Omega_i e^{-\mu d_s(i)},$$

where $i$ is the identifier for any sector with its tessera, $w_s(i)$ the relative sensor sensitivity of the sector $i$, $\Omega_i$ the
solid angle of the sector \( i \), \( \rho_s \) is the source density and \( d_h(i) \) is the distance to the tessera \( i \) at the point where the direction vector intersect the surface at angle \( \theta_i(i) \).

Replacing the level surface with a terrain grid, the distance \( d_t \) to the intercepts of the directional vectors with the surface could be closer or more distant and the total count rate would become

\[
w_t = \rho_s \sum_i w_s(i) \Omega_i e^{-\mu d_t(i)}.
\]

(12)

Correcting the measured count rate \( N_{raw} \) for reduced count rate \( N_{corr} \) at nominal height is then done using

\[
N_{corr} = N_{raw} \frac{w_h}{w_t}.
\]

(13)

The correction factor is independent of the source density. The sensitivities \( w_s \) and also the solid angles \( \Omega_i \) would not change the correction factor when multiplied by some non-zero factor. The direction vectors at angles \( \theta_r \) intersect level tesserae at the correct distance for equation 5 at nominal flying height. They intersect the tesserae generally very close to their centres and the potentially introduced error is expected to be very small. If a tessera is sloped, the effect on the count rate is small as this would be a rotation only at the point of intercept, subareas with reduced distance to some degree compensated by subareas with increased distance.

Tesserae at higher angles in \( \theta \) tend to cover larger areas of the terrain and representing the distance by just one point of intersection seems to limit the accuracy. However, these points are generally distant and few photons reach the detector system from these sectors, reducing any error effect. This can be tested using a system of much smaller sectors. Some tesserae are annotated in the south east in Figure 10 showing the root points where the direction vectors intersect the surface for one footprint.

For more rugged terrain, the sectors should also cover higher angles in \( \theta \) even though no contribution would come from these sectors on a level surface. The direction vectors for sectors ranging into areas with \( \theta > 90^\circ \) will just be centred in \( \phi \) and \( \theta \).

MODEL CALCULATIONS

Calculations of count rates for 2D terrain models were done using the linear attenuation coefficients for uranium (Table 1). To determine attenuation coefficients for the half space model, count rates for different heights of the detector system were calculated, corresponding to count rates from a calibration flight over a test range using a detector system from these sectors, reducing any error from these points are generally distant and few photons reach the detector system from these sectors, the potentially introduced error is expected to be very small. If a tessera is sloped, the effect on the count rate is small as this would be a rotation only at the point of intercept, subareas with reduced distance to some degree compensated by subareas with increased distance.

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Figure 2 shows the calculations with identical sensitivities in all directions (isotropic) and for the actual evaluated system. The hexagons and triangles are the logarithm of count rate vs. height with a linear slope defining the negative attenuation coefficient shown as circles and diamonds. The slopes are assigned to the end of each interval of count rates shown with the scale on the right. This figure allows two important observations: The slope is not at all constant, making equation 1 a rather poor approximation. The other observation is that the attenuation for the half space model also depends on the sensitivity distribution with \( \theta \). Due to the mutual shielding from all crystals of a larger detector system, the sensitivity for gamma-rays from higher angles in \( \theta \) is smaller and thus the system is vertically focused. This results in smaller attenuation coefficients for the half space model, closer to the values of linear attenuation coefficients. The dependency of the sensitivities on \( \theta \) can thus be chosen that the attenuation coefficients for the half space model match the results from a calibration flight. These attenuation coefficients are used here to simulate the conventional attenuation corrections.

For a first estimate of system sensitivity with \( \theta \), the combined cross-section of all crystals could be considered, e.g. having 16 crystals in a 4 \( \times \) 4 configuration, there are 16 crystals visible from below and only four crystals from the sides and the system sensitivity then is proportional to \( 16 \times 0.8 \cos(\theta) + 4 \times \sin(\theta) \) with 0.8 as the probability to be counted hitting the crystals vertically. Smaller deviations here only have small effects for the end results.

Calculations of count rates were done for a model shown in Figure 3 with a terrain surface shown as a brown line below with a value set to 1.0 and decreasing sensitivity at higher angles of \( \theta \) due to mutual shielding, ending for the evaluated system at \( w_s = 0.4 \) at highest angles towards \( \theta = 90^\circ \). All sensitivities could be multiplied by about 0.8 for more realistic count rates at given model parameters but constant factors are not important in this context. The total count efficiency for 2.62 MeV photons is given in Billings and Hovgaard (1999) as 78% along the shorter axis of typical NaI crystals.
falling vertically at $x = 0$ from a height of 150 m to 0 with an upward slope from $x = 500$ m to a height of 100 m at $x = 700$ m. Four level flight lines above the terrain are shown as green lines. The sensitivities of the actual survey detector system were used with a source density giving a count rate of 1000 at nominal flying height at 100 m (STP) over level half space. The count rates expected in the detector system are shown in Figures 4 to 7 as red lines with diamonds. Green lines with triangles show the reduced count rates applying the traditional method using the vertical height only with an attenuation factor of $\mu = 0.00917$ m$^{-1}$ as derived from Figure 2 using the altitude interval between 70 m and 80 m. The blue line with circles shows values at nominal height of 100 m above half space at given source density and attenuation.

Count rates at the lowest flight line are considerably higher, being always close to the source area (Figure 4). The numbers are particularly high at the ends where the detector system is exposed to the vertical surface on the left or getting closer to the sloped surface on the right. In the central area, most of the footprint area of the detector system is a flat surface 50 m below. As can be expected, the traditional height correction reduces the data in the central region to the value for the nominal flying height. The increased radiation on the left is not corrected as the flying height remains unchanged while on the right side, the flying height is decreasing leading to a much better reduction of the data. The observations of Figure 5 are very similar, only that the flying height is identical to the nominal height for the most part and the data there remains unchanged.

Figure 6 clearly shows some limitations for the conventional method for height correction. Only Figure 7 covers the full extent of the terrain model with nominal flying height on the right at a count rate of 1000. More interesting is the point at the vertical fall of the terrain at $x = 0$. Half of the footprint is level 50 m below, the other half is (mostly) level 200 m below while the vertical surface is not contributing at all. The traditional height correction is based on only 50 m effective height up to this point, jumping to 200 m effective height, highly over-correcting the data. In this model, the raw count rate of the detector is 999. The calculations for level surfaces at 50 m and 200 m over full half space would give count rates of 1585 and 442 respectively, averaged 1013.5 counts. This is quite accurate at such topography.

Figure 8 illustrates the results of a trough or gorge model. In this model, the flying height is 30 m, rather than the nominal 100 m thus the count rate is accordingly high at almost 2000 instead of 1000. At the relatively narrow trough, the altitude above ground is 150 m and the count rate gets almost as low as 1000 but stays higher because the flanks of the trough contribute counts. Traditional corrections are simulated with $\mu = 0.00971$ m$^{-1}$, using
Figure 8: Trough model, upper part topography (brown) and flight line (green) and lower part with count rates.

Figure 9: Ridge model, upper part topography (brown) and flight line (green) and lower part with count rates.

the altitude interval of Figure 2 between 50 m and 60 m. Figure 9 shows a similar situation above a ridge. In both models, the traditional height correction gives good results when the footprint is over level terrain but fails otherwise. It is interesting to see the small local minimum right above the centre of the ridge with small increases in count rates on both sides. At the centre point, the detector system is close to a small surface area and the flanks of the ridge are exactly in line not contributing to the count rate at all while all other surface areas are at rather large distance. At the neighbouring points on either side, the detectors are still exposed to the small surface area at a small distance but also get some radiation from closest areas of the flanks. Traditional corrections are simulated with \( \mu = 0.00838 \text{ m}^{-1} \), using the altitude interval of Figure 2 between 120 m and 130 m.

FIELD APPLICATION

Some field examples have been calculated. Figure 10 shows all the root points where the direction vector towards the centre of each tessera hits the surface for one single measurement of the aircraft flying over a gorge making up a footprint. All root points belonging to one detector zone, i.e. of common angle \( \theta \), are connected by a white line. On flat terrain, these white lines would map perfect circles around the measurement point. Some tesserae have been outlined in the south east of the footprint. The total distance between the detector and each tessera is given by

\[
d = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}
\]
Gamma-ray attenuation correction for rugged terrain

(a) Topography with a narrow topographic feature.

(b) Potassium counts with new method.

(c) Potassium counts showing correlation with topography using conventional method.

(d) Potassium counts, conventional half space method minus new topographic method.

Figure 11: Grids over a gorge, effective height and nominal flying height of 125 m in SW-NE direction.

Figure 12: Profile 1133 at 125 m effective height running SE-NW.
with the horizontal displacements $\Delta x$ and $\Delta y$ determined from the map and $\Delta z$ the difference between the flying height at 1125 m for this sample and the ground surface heights, which range from 900 m to 1040 m in the area of the detector footprint. The surface height just beneath the measuring point is 909 m.

Generally the result of the new approach for processing show a smoothing effect on the attenuation correction which was also reported by Schwarz et al. (1992). Thus a map of anomalies does not change its general appearance. This is different if there is a relatively narrow gorge through otherwise smooth terrain. Figure 11a shows some terrain in British Columbia (Canada), north-west of Kamloops. The area shown extends over 10 km in E-W and 8 km in N-S direction.

The relatively stable channel for potassium counts for processing has been shown to demonstrate the effect of the new method. Figure 11b shows an enhanced and active level of potassium without giving a clear indication of the course of the gorge. This is not the case in Figure 11c with corrections done in the conventional way and becomes even clearer in Figure 11d showing the difference of both methods. The high amplitude in the difference compared to the counts shown in Figure 11a illustrates that this is a significant improvement.

Figure 11 illustrates a profile over a gorge near the south edge of the section of Figure 11 at $y \approx 5682 000$ m. The upper half with the scale on the left shows, in brown, the terrain and in blue, the path of the aircraft following a drape surface. The green line with the scale on the right shows the clearance above ground. The data processed using the traditional method for attenuation correction clearly shows places where the high clearance leads to over-correction in the count rate compared to the new method.

**DISCUSSION**

The traditional attenuation correction considers the full half space with a solid angle of $2\pi$ using the distance from the aircraft to the central point of the area covered by the footprint. Since this distance grossly underestimates the distance to most of the radiation sources, the attenuation coefficients are chosen to compensate for this shortcoming as derived from test flights. A considerable improvement can be achieved by subdividing the footprint into smaller pieces leading to the system of sectors described in this paper. Their solid angle is easily calculated and the contribution for a given source density from each sector depends on the distance to the area the sector covers and the relative sensitivity of the detector system with respect to its direction. Key to this procedure was the realization that the incoming count rate at a given source density is dependent only on the solid angle if the attenuation is not considered.

Developing this sector system, some bias in the distances to the tesserae was considered acceptable as long as these distances can be considered as proportionate, e.g. choosing always the shortest or perhaps farthest distance to the tessera, as some internal re-calibration process would mostly compensate for any minor bias. Likewise it was assumed that even a very coarse system of sectors would be a considerable improvement to remove effects from the ruggedness of the terrain. With more elaboration, a system was developed without bias on distances to the source areas for level footprints at nominal height and the direct use of the linear attenuation coefficients may be advisable. Using the linear attenuation coefficients directly avoids the often problematic determination of the attenuation coefficient for uranium where the influence from airborne radon is difficult to quantify.

The internal re-calibration process is a section in the program which optionally calculates attenuation coefficients for the new sector system using the attenuation coefficients from the calibration test flights above a level surface. In an iterative procedure starting from some estimated attenuation coefficient, values are determined giving exactly identical attenuation corrections for this sector system as using the half space method at two chosen heights within the calibration range.

It is interesting to note the change of the attenuation coefficients from the internal re-calibration step, as they should be close to the linear coefficients which can be found listed e.g. in Hubbel and Seltzer (1991) from the US National Institute of Standards and Technology. Some example values are listed in Table 1 with $\mu_0$ calculated from $\alpha$ assuming an air density of 1.2 kg/m$^3$ with values for the three elements of interest added by interpolation. Table 2 gives the re-calculation for the sector model with values from a recent project.

The sensitivity of the whole detector system is dependent on the direction. It may be reduced by shielding. It increases with area and depth of crystal volume facing incoming gamma-rays from a particular direction. Theo-

<table>
<thead>
<tr>
<th>Element</th>
<th>E [MeV]</th>
<th>$\alpha$ [cm$^2$/g]</th>
<th>$\mu_0$ [m$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>1.46</td>
<td>0.008383</td>
<td>0.006045</td>
</tr>
<tr>
<td>U</td>
<td>1.765</td>
<td>0.007560</td>
<td>0.005325</td>
</tr>
<tr>
<td>Th</td>
<td>2.614</td>
<td>0.006642</td>
<td>0.004536</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>0.006862</td>
<td>0.004724</td>
</tr>
</tbody>
</table>

Table 1: Linear attenuation coefficients

<table>
<thead>
<tr>
<th>Element</th>
<th>E [Mev]</th>
<th>half space [m$^{-1}$]</th>
<th>topo [m$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
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<tr>
<td>Total</td>
<td></td>
<td>0.006862</td>
<td>0.004724</td>
</tr>
</tbody>
</table>

Table 2: Re-calibrated attenuation coefficients
retical considerations as shown by Billings and Hovgaard (1999) or practical tests might help. Eventually a dependency on angle \( \theta \) only may be sufficient. From the mutual shielding of many crystals at common height on the aircraft floor, the sensitivities tend to decrease considerably towards \( \theta = 90^\circ \). As shown in Figures 2, the traditional attenuation coefficients for a full half space are also dependent on the sensitivity distribution with respect to \( \theta \) which can be used to determine a sensitivity model matching the attenuation found from the traditional calibration flight.

Any direction vector that does not reach the surface within some upper horizontal limit, e.g. 2000 m, will return a very high value as its distance, in which case any radiation from this sector will be virtually zero or may be skipped. A footprint covering a radius in the order of this limit contains virtually 100\% of all counts as can also be seen from Grasty et al. (1979). This limit can be set to a high value in order to be certain not to insert any influence of the choice of this cut-off value, without a high penalty for number crunching. Some model calculations were also done to investigate the size of the footprint, especially the percentage of a total half space versus footprint radius using numerical integration of equation [5]. This matched exactly the results from Grasty et al. (1979) giving some independent check for this method for the level case.

This method is also free of the problem of hidden surfaces. Hidden surfaces are simply invisible from the position of the aircraft and any direction vector will intersect with the nearest surface element. The program starts vertically below the aircraft and works upward at the azimuth of the direction vector crossing rows or columns of a terrain grid, whichever is crossed more often, until the angle \( \theta \) is higher than the one of the direction vector. This way, points adjacent to the intercept are found and a locally refined terrain position is used to get the distance.

Another consideration is the movement of the aircraft while it is collecting counts. Here it is assumed that the topology for the average position is typical for the counting period, generally one second. This seems to be an acceptable assumption for drape flying, where the relative change of distance to the surrounding surface is generally reduced compared to level flights. This is verified by looking at the correction factor shown in dark blue in the lower part of Figure 12 showing the integrated effect of the terrain within the footprint every second along the flight. This is the duration of collecting photons for each sample and the smoothness allows the use of the central point for the whole duration.

A sensitivity model dependent on \( \theta \) only is read from the parameter file. This procedure is also designed to apply a set of weight coefficients dependent on azimuth with respect to the aircraft to consider the detailed directional sensitivity of the whole detector system. This will require the attitude data of the aircraft in order to apply a rotation to all direction vectors of sector positions at each measurement, or as a minimum for each flight direction.

Another interesting aspect may be that the use of so called upward looking crystals can be improved. Their shielding in very rugged terrain can be incorporated in the sensitivity model for the main "downward looking" detector system. Using the same system of sectors for processing efficiency, the relative impact from ground sources into the upward looking crystals can be estimated with much higher accuracy, perhaps even further improved by using the attitude data of the aircraft.

To calculate the correction factors, as shown in Figure 12 the program only needs besides the aircraft coordinates, the target height, the temperature and barometric height or air pressure for air density and the definition of the sensor sector system, e.g. nine lines with upper limit of \( \theta \), relative sensitivity of the zone and number of sections for a full circle. From this, the correction factor can be calculated for each given linear attenuation coefficient \( \mu_0 \). The count rate, the actual data, is here not required, these correction factors could be delivered as a service. It is also possible to calculate the correction factor which the conventional system would have delivered and thus, the proper correction could be applied retroactively, provided that the final data was not manipulated to hide the adverse terrain effects.

CONCLUSION

Gamma-ray data from rugged terrain will show artefacts when conventional height corrections have been applied. This is true for every terrain feature being smaller in at least one dimension than the footprint size of the gamma-ray detector system, in particular for elongated features like ridges or gorges and dry river beds. The new method described in this paper can effectively remove terrain artefacts with any desired accuracy without a heavy burden in number crunching and is easily implemented. It needs only a digital terrain model and a definition of an arrangement of sectors around the detector system.

Gamma-ray data have been processed using this new method for several recent SGL airborne survey projects. The problem of hidden surfaces is avoided in this method, making it suitable for extremely mountainous areas. Utilizing attitude information of the aircraft, the system of directional vectors probing the distance to the terrain of each sector could be rotated accordingly for each point of measurement, thus removing manoeuvrer effects. This could also substantially improve the usefulness of upward looking crystals.

The new method can make use of the attenuation coefficients from a conventional calibration flight with an internal re-calibration of these coefficients by matching those of the conventional method in flat areas but better results can be expected from using linear attenuation coefficients as published in textbooks of physics.
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