Abstract

In this letter, we consider a secondary network (SN) in which an ambient backscatter device (BD) utilizes the secondary transmitter (ST) signal to communicate its own information to the secondary destination (SD). Optimizing performance of such networks is complicated by signal reflections by the BD. It is shown in this work how the secondary transmit power and the reflection coefficient of the BD can both be jointly optimized using a simple restricted one dimensional search to satisfy the quality of service (QoS) constraints of SN as well as the primary network (PN) while maximizing performance of the backscatter link, which is termed the tertiary network (TN). Only statistical channel knowledge is used for this purpose. It is seen that careful optimization can improve spectral efficiency. Simulations validate the derived analytical expressions.
Optimizing Performance of a Backscatter-Assisted Underlay Network

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Abstract—In this letter, we consider a secondary network (SN) in which an ambient backscatter device (BD) utilizes the secondary transmitter (ST) signal to communicate its own information to the secondary destination (SD). Optimizing performance of such networks is complicated by signal reflections by the BD. It is shown in this work how the secondary transmit power and the reflection coefficient of the BD can both be jointly optimized using a simple restricted one-dimensional search to satisfy the quality of service (QoS) constraints of SN as well as the primary network (PN) while maximizing performance of the backscatter link, which is termed the tertiary network (TN). Only statistical channel knowledge is used for this purpose. It is seen that careful optimization can improve spectral efficiency. Simulations validate the derived analytical expressions.

Index Terms—Backscatter communication, reflection coefficient, throughput, underlay cognitive radio.

I. INTRODUCTION

Acute spectrum scarcity due to new data services has led to the development of a spectrum-sharing framework referred to as cognitive radio (CR) [1]. A secondary network (SN) operating without a licensed spectrum is permitted to share the spectrum of the licensed primary network (PN) in order to enhance the overall spectrum utilization efficiency [2]. Specifically in the underlay CR paradigm, which has shown significant potential, the secondary transmitter (ST) carefully manages its transmission power to maximize the performance of the secondary network (SN) while guaranteeing a desired PN performance. Backscatter communication represents another promising approach for accommodating the increasing number of machine-type devices (MTDs). Given the strict limitations on battery life for these nodes, there is a growing focus on the adoption of backscatter-type devices (BDs). Bistatic BDs in particular is well known to provide a large coverage range. However, it is the ambient type BDs (that utilize ambient radio frequency (RF) signals to communicate) that show the most promise for massive machine type communications (mMTC) [3], [4]. In [5], the authors study spectrum sharing between the ambient BD system and the PN. The throughput of the BD was optimized while ensuring that the minimum rate requirements of PN are met.

In [6], the authors consider a framework in which one source transmits information to two readers with the assistance of a secondary node to assist communication is of interest. We demonstrate that careful control of both the ST transmit power and the reflection coefficient (RC) can allow much better performance to be achieved. We also consider the realistic case when the ST-SD direct link is present, which ensures much better performance, and maximize the performance of the BD-SD link, which we refer to as the tertiary network (TN).

In this work, we assume no channel knowledge at the source, and adopt fixed-rate signalling. In underlay CRNs, due to the random nature of transmit power at the ST, signal-to-noise ratio (SNR) of the secondary network (SN) shows a large variation which degrades the Quality-of-Service (QoS) of the SN. In underlay communication too, the use of backscatter nodes to assist communication is of interest. We demonstrate that careful control of both the ST transmit power and the BD reflection coefficient (RC) can allow much better performance to be achieved. We also consider the realistic case when the ST-SD direct link is present, which ensures much better performance, and maximize the performance of the BD-SD link, which we refer to as the tertiary network (TN).

In this paper (for the first time) we derive closed-form expressions for non-outage probabilities of all three networks—PN, SN, and TN. Next, to provide a certain QoS, we make sure that the throughputs of both PN and SN are greater than a threshold value. Doing so gives us a range of RC and secondary power for which both the PN and SN achieve a minimum desired throughput. Lastly, we optimize the throughput of the TN within this valid range of RC and secondary power using a simple restricted one-dimensional search. Our key insight is that while increased interference at PN and SN due to the introduction of the BD requires careful handling, it is clear that BDs can be effectively integrated into underlay CRNs, with minimal impact on the performance of PN and SN.

Notations: $C \chi(0, \sigma^2)$ represents a 0 mean $\sigma^2$ variance complex normal distribution. The probability density function (PDF), and cumulative distribution function (CDF) of a random variable (RV) $Z$ are denoted by $f_Z(z)$, and $F_Z(z)$. The exponential integral of type 1 is denoted by $E_1[\cdot]$ [11, 3.324.1].

II. SYSTEM MODEL

As depicted in Fig. 1, we consider a network with a PN, SN, and TN. The PN consists of a primary transmitter (PT) and a primary receiver (PR), whereas the SN consists of a ST and a SD. The BD and SD constitute the TN. BD utilizes the ST signal to transmit its information to SD. Both the SN and TN share the spectrum of the PN. PN, SN, and TN transmissions cause interference to each other. We show that in spite of this,
there is an improvement in spectral efficiency by proper choice of the ST transmit powers and the RC of BD.

The channel between all the transmitting and receiving nodes is of quasi-static Rayleigh fading type. The channel from PT to PR, ST to SD, ST to BD, and BD to SD are denoted by $h_{pp}$, $h_{sd}$, $h_{sb}$, and $h_{bd}$, respectively. TN causes interference to the PN and SN, SN causes interference to the PN and TN, and the PN introduces interference to SN and TN. We denote the interference channels from ST to PR and BD to PT by $g_{sp}$ and $g_{bd}$, and the interference channels from PT to BD and PT to SD by $g_{pb}$ and $g_{pd}$, respectively. Since the channels are of Rayleigh fading type, and $h_{xy} \sim \mathcal{CN}(0, \lambda_{xy})$ ($g_{xy} \sim \mathcal{CN}(0, \lambda_{xy})$) $x \in \{p, s, b, d\}$ and $\lambda_{xy} = d_{xy}^2$ where $d_{xy}$ is the distance between the nodes $x$ and $y$, and $\alpha$ is path loss exponent (PLE).

The unit-energy symbols $x_a$ and $y_a$ intended for PR and SD are transmitted by PT and ST to PN and SD respectively with transmit powers $P_p$ and $P_s$. Denote the information rates of $x_a$ and $y_a$ by $R_p$ and $R_s$, and the target SNR thresholds by $\gamma_p = 2^{R_p} - 1$ and $\gamma_s = 2^{R_s} - 1$. Let $r_c$, $x_b$, and $R_p$, respectively denote the complex-valued RC of BD (whose magnitude takes values between 0 and 1), the unit-energy symbol transmitted by BD, and the target information rate of $x_b$ (corresponding threshold SNR $\gamma_b = 2^{R_b} - 1$).

The signal $y_b$ and $y_p$ received at the BD and PR are

$$y_b = \sqrt{P_p} h_{sb} x_b + \sqrt{P_p} g_{pb} x_p,$$

$$y_p = \sqrt{P_p} h_{pp} x_p + \sqrt{P_p} g_{sp} x_s + y_r g_{sb} x_b + n_p,$$

where $n_p \sim \mathcal{CN}(0, N_0)$ is the additive white Gaussian noise (AWGN) sample at PR. As in all work on CR, PR is assumed to decode $x_p$ treating the signal from ST and BD symbol as noise. Substituting $y_b$ into (2), the signal-to-interference-plus-noise-ratio (SINR) $\Gamma^p_p$ at PR to decode $x_p$ is

$$\Gamma^p_p = \frac{P_p |h_{pp}|^2}{P_p |g_{sp}|^2 + P_p |g_{pb}|^2 |r_c|^2 + P_p |h_{sb}|^2 |g_{sb}|^2 |r_c|^2 + N_0}.$$  

(3)

The signal $y_d$ received at SD, which is a combination of signals coming from ST, BD and PT is

$$y_d = \sqrt{P_d} h_{sd} x_d + y_r h_{bd} x_b + \sqrt{P_d} g_{pd} x_p + n_d,$$

where $n_d \sim \mathcal{CN}(0, N_0)$ is AWGN sample at SD. Substituting $y_b$ into (4), the SINR at SD to decode $x_b$ as

$$\Gamma^b_d = \frac{P_p |h_{bd}|^2}{P_p |g_{pb}|^2 |r_c|^2 |h_{bd}|^2 + P_p |g_{pd}|^2 + N_0}. 
$$  

(5)

After decoding of $x_b$ successive interference cancellation (SIC) is used to decode $x_p$. The SINR $\Gamma^p_d$ to decode $x_p$ is

$$\Gamma^p_d = \frac{P_p |h_{bd}|^2 |r_c|^2}{P_p |g_{pb}|^2 |r_c|^2 |h_{bd}|^2 + P_p |g_{pd}|^2 + N_0}.$$  

(6)

### III. PERFORMANCE ANALYSIS

In this section, we derive a valid range of $P_s$ and $r_c$ to maximize the TN performance while ensuring a certain PN and SN QoS. To do so, first, we derive expressions for the primary and secondary non-outage probabilities. For ease of exposition, we use $|h_{pp}|^2 = X, \lambda_{pp} = \lambda_x, |h_{sd}|^2 = Y, \lambda_{sd} = \lambda_y, |h_{sb}|^2 = Z, \lambda_{sb} = \lambda_c, |h_{bd}|^2 = U, \lambda_{bd} = \lambda_w, |g_{sp}|^2 = V, \lambda_{sp} = \lambda_v, |g_{sb}|^2 = W, \lambda_{sb} = \lambda_w, |g_{pd}|^2 = A, \lambda_{pd} = \lambda_b, |g_{pd}|^2 = B$ and $\lambda_{pd} = \lambda_b$. The PDF $f_X(x)$ and complimentary CDF (CCDF) $F_X(x)$ of the exponential RV $X$ are $f_X(x) = \lambda_x e^{-\lambda_x x}$ and $F_X(x) = e^{-\lambda_x x}$, respectively.

PN is said to be in outage if PR fails to decode $x_p$, and SN is in outage when SD fails to decode $x_b$. The outage probabilities $P_o^p$ and $P_o^b$ of PN and SN are

$$P_o^p = \Pr \{ \Gamma^p_p < \gamma_p \} \quad \text{and} \quad P_o^b = \Pr \{ \Gamma^b_d < \gamma_b \}.$$  

(7)

Note that SD uses SIC. It first decodes $x_b$, and then $x_p$. TN is in outage when SD is unable to decode $x_b$ ($\Gamma^b_d < \gamma_b$), or when it decodes $x_b$, but not $x_p$. Mathematically, the outage probability $P_o^b$ of TN is

$$P_o^b = \Pr \{ \Gamma^b_d < \gamma_b \} + \Pr \{ \Gamma^b_d \geq \gamma_b, \Gamma^p_d < \gamma_b \}.$$  

(8)

In the following, we derive expressions for $P_o^p$, $P_o^b$, and $P_o^b$. Denote by $\bar{P}_o^p$ and $\bar{P}_o^b$ the desired outage QoS of PN and SN.

#### A. Evaluation of Non-outage probability of PN

A closed-form expression for non-outage $P_o^p = 1 - \bar{P}_o^p$ is presented in the following theorem.

**Theorem 1.** The closed-form expression for $\bar{P}_o^p$ is given by

$$\bar{P}_o^p = \frac{C_1 C_2}{C_4 (P_s + C_1) (P_s - C_2)|r_c|^2} \exp \left( \frac{-\lambda_{pp} \gamma_p N_0}{C_b} \right) \left\{ \mathbb{E} \left[ \frac{C_2}{C_4 |r_c|^2} \right] \right\} - \exp \left( \frac{1}{C_4 |r_c|^2} \right) \mathbb{E} \left[ \frac{1}{C_4 |r_c|^2} \right] ,$$

(9)

where $C_1 = \frac{\lambda_s P_p}{\gamma_s R_s}$, $C_2 = \frac{\lambda_p P_s}{\gamma_p R_s}$ and $C_4 = \frac{\lambda_b P_s}{\gamma_b R_b}$.

Proof. Proof is presented in Appendix A.

It is extremely challenging to obtain the valid range of $P_s$ and $r_c$ from (9). Therefore, we approximate it using relation (12) 5.1.19., as follows

$$P_o^p \approx \frac{C_1 C_2}{(P_s + C_1) (P_s - C_2)} e^{-\lambda_{pp} \gamma_p N_0} \left\{ \frac{P_s}{C_b |r_c|^2 + C_2 - 1} \right\},$$

(10)

#### Lemma 1. Ensuring a PN outage QoS of $\bar{P}_o^p$ or lower implies constraints on the ranges of $r_c$ and $P_s$. If $P_s < C_2$, any $r_c$ satisfying $0 < |r_c|^2 < 1$ can be used. When

Due to separate decoding of symbols and no cooperation between SN and TN, backscattered secondary signal is considered as interference.

By taking the valid range of $r_c$ and $P_s$ into account a certain QoS at PN is ensured. For this reason, we focus on the secondary network performance.
the following constraint applies
\[
|r_c|^2 \leq \frac{\left( P_s - C_2 \right) \left\{ 1 - C_2 C_3 (P_s + C_1) \right\}}{C_3 P_s \left\{ 1 + C_3 (P_s + C_1) (P_s - C_2) \right\}}
\]
(11)
where \( C_3 = \left( 1 - P_o^p \right) e^{\frac{\lambda a \rho}{\lambda o\bar{r}}} \). \( P_s > \chi_1 \) violates the primary QoS.

**Proof.** To ensure a certain PN performance, we impose \( P_o^p \geq (1 - P_o^p) \). Substituting for \( P_o^p \) from (10), performing mathematical rearrangements and noting that \( (P_s - C_2) \) can take both positive and negative values, the valid range of \(|r_c|^2\), and \( P_2 \) can be obtained as in Lemma [1]. As \(|r_c|^2\) can not be negative, \( \chi_2 \) must be positive, which implies an upper bound \( P_s \leq \chi_1 \).

Clearly, the primary constraints imply limits on the range of valid \( P_s \) and \( r_c \). We see in what follows that the secondary QoS constraints also limit the range of these values.

**Lemma 2.** A high SNR expression for \( P_o^p \) is given by
\[
P_o^p \approx 1 - \frac{\lambda a \rho_0}{\lambda_0} \left[ \frac{1}{1 + D_1 (P_s - C_2)} \right] \left( \frac{D_1}{r_c^2} \right),
\]
where \( D_1 = \frac{\lambda a \rho_0}{\lambda_0} \). \( P_s \) and \( D_1 \) is \( \lambda a \rho_0 \). \( D_2 = \lambda a \rho_0 \).

**Proof.** The proof follows that used in Theorem [1] and is therefore omitted due to paucity of space.

Obtaining the range of \( r_c \) from (13) is quite intractable. Therefore, we express it using relation [12] 5.1.19, as follows
\[
\tilde{P}_o^p \approx \frac{P_o^p}{(P_s + D_1) (P_s - C_2)} \exp \left( \frac{D_1}{r_c^2} \right) \left( \frac{D_3}{r_c^2} \right).
\]
**Lemma 3.** Ensuring a SN outage QoS of \( P_o^p \) or lower implies constraints on the range of \( r_c \), and \( P_s \). When \( P_s \geq C_2 \),
\[
|r_c|^2 \leq \left( \frac{P_s - C_2}{D_3} \right) \left[ P_o^p \left( \frac{1}{D_1} - \frac{1}{D_3} \right) + \frac{1}{D_2} \right] \left( \frac{D_1}{r_c^2} \right).
\]
Since \(|r_c|^2\) is a positive quantity, the term \( \psi \) in (15) must be positive. This gives us the following constraint on \( P_o^p \)
\[
P_s \geq \frac{\left( 1 - P_o^p \right) D_3 + D_2}{2P_o^p} + \sqrt{\left( \frac{D_1 - D_2}{2} \right)^2 + 4D_2 (D_1 P_o^p (1 - P_o^p))}.
\]
**Proof.** Using linear approximation to the exponential term in (13) and then using \( \tilde{P}_o^p \geq (1 - \tilde{P}_o^p) \) to ensure a certain performance of the SN followed by mathematical manipulation, the valid range of \( r_c \) can be obtained as in Lemma [3].

\*The valid range of \( P_s \) and \(|r_c|^2\) (not the entire range) puts a constraint on the SN transmit power and guarantees the desired performance of the PN.

Using \( C_2 = \frac{\lambda a \rho_0}{\lambda_0 p} = \lambda a \rho_0 \), the constraint \( P_s \geq C_2 \) can be further expressed as \( \lambda_{ab} \geq \lambda_{ab} \).

**Remark 1.** To ensure a certain non-outage probability at SD \( (P_o^p \geq (1 - P_o^p)) \), the valid range of \( P_s \) depends on the ratio of the distance between PT and BD, and that between ST and BD \( \approx \lambda_{ab} \). \( \lambda_{ab} P_o^p \Rightarrow \left( \frac{D_a}{D_b} \right) \geq P_o^p \).

**Remark 2.** Combining the constraint \( P_s \geq C_2 \) with the constraint on \( P_s \), we get \( P_s \geq \max \{ C_2, \chi_2 \} \) which ensures a valid range of \( r_c \) while guaranteeing the desired performance of the SN \( (P_o^p \geq (1 - P_o^p)) \).

From Lemma [1] and Lemma [3] we can conclude that using \( P_s \) within a certain range i.e. \( \max \{ C_2, \chi_2 \} \leq P_s \leq \chi_1 \) ensures positive values of \( r_c \) and also guarantees a desired performance of both PN and SN.

**Lemma 4.** A high SNR expression for \( P_o^p \) is given by
\[
P_o^p \approx 1 - \frac{\lambda a \rho_0}{\lambda_0} \left[ \frac{1}{1 + D_1 (P_s - C_2)} \right] \left( \frac{D_1}{r_c^2} \right) \left( \frac{D_3}{r_c^2} \right).
\]
**Proof.** Substituting \( D_1 \) and \( C_2 \) in (14) and using \( P_s = P_o^p \), higher values of \( P_s \) and \( D_3 \), (18) can be readily obtained.

**Remark 3.** It is clear from Lemma [2] and [4] that both \( \tilde{P}_o^p \) and \( P_o^p \) saturate (becomes independent of transmit powers \( P_s \) and \( P_o^p \)) in high PN/SN SNR region and thus the desired requirement of both PN and SN can be met.

**Remark 4.** It clear from Lemma [2] and Lemma [2] that the decreasing values of \(|r_c|^2\) improves the outage performance of both PN and SN.

**C. Evaluation of Non-outage probability of TN**
A closed-form expression for \( P_o^p \) is presented in the following theorem.

**Theorem 3.** The closed-form expression of \( P_o^p \) is given by
\[
P_o^p \approx 1 - \frac{\lambda a \rho_0}{\lambda_0} \left[ \frac{1}{1 + D_1 (P_s - C_2)} \right] \left( \frac{D_1}{r_c^2} \right) \left( \frac{D_3}{r_c^2} \right).
\]
**Proof.** Using linear approximation to the exponential term in (13) and then using \( \tilde{P}_o^p \geq (1 - \tilde{P}_o^p) \) to ensure a certain performance of the SN followed by mathematical manipulation, the valid range of \( r_c \) can be obtained as in Lemma [3].

IV. **TN Throughput Optimization**
The PN, SN and TN throughputs are \( \tau_p = (1 - P_o^p) R_p = \tilde{P}_o^p R_p \), \( \tau_s = (1 - P_o^p) R_s = \tilde{P}_o^p R_s \), and \( \tau_s = (1 - P_o^p) R_p = \tilde{P}_o^p R_p \). In this paper we aim to maximize \( \tau_p \) while ensuring that \( \tau_p \geq \tau_s = (1 - P_o^p) R_s \), \( \tau_s \geq \tau_p = (1 - P_o^p) R_p \), and \( \tau_s \geq \tau_p = (1 - P_o^p) R_p \). From Lemma [1] and Lemma [3] it can be seen that max \( \{ C_2, \chi_2 \} \leq
Fig. 2. Demonstration of (a) $P^b_0$ versus $P_s$ (b) $\tau_p$ versus $r_c$ and (c) $P_s$ versus $r_c$.

$P_s \leq \chi_1$ is needed to ensure PN and SN performance. It follows from Remark 5 that a search over $r_c$ is not required - the largest feasible value $|r_c|_{\text{lim}} = \min(\psi, \chi_2, 1)$ is used. The optimization reduces to a one-dimensional search to find the optimum $P_s^{\text{opt}}$.

$$P_s^{\text{opt}} = \max_{\tau_b(b), r_c(2)} \tau_b(b, r_c(2))_{|C_2, 2| \leq P_s \leq \chi_1}.$$ (19)

Clearly, the optimization is reduced to one-dimensional search.

V. SIMULATION RESULTS

In this section, we utilize the obtained analytical outcomes to conduct a comprehensive numerical assessment using Monte Carlo simulations of the performance of three distinct networks, namely PN, SN, and TN (we average over $10^6$ realizations). For the sake of simplicity, yet without sacrificing generality, we assume that the noise power is normalized to unity ($N_0 = 1$). Unless mentioned otherwise, the other considered system parameters are as follows: $P_s = 17$ dB, $\alpha = 3$, target rates $R_p = R_c = R = 1.75$ bps/Hz, $d_{pp} = 1.2$, $d_{sb} = 2.1$, $d_{sp} = 4.2$, $d_{sj} = 4$ and $d_{pd} = 6$.

From Fig. 2a we observe that as $P_s$ increases, $P^b_0$ increases because the interference from SN onto the PR increases. $P^p_0$ and $P^b_0$ decrease with an increase in $P_s$ due to the fact that the signal power increases for both networks. A certain desired performance needs to be met for both PN and SN (respectively denoted by $P^p_0$ and $P^b_0$), which limits the outage probabilities of respective networks, i.e., $P^p_0 < P^p_0$ for PN and $P^b_0 < P^b_0$ for SN. This limits the $P_s$ range, as indicated in the plot. This plot also depicts the comparison between two cases- when $r_c = 0$ (i.e., no BD case) and when $r_c = 0.3$. It can be observed that the presence of BD improves the performance of TN at a certain cost to SN performance, while PN performance remains almost the same.

In Fig. 2a as $r_c$ increases, $\tau_p$ decreases because the interference from SN at PR increases. $\tau_s$ also decreases because the interference at SD due to BD increases with an increase in $r_c$. $\tau_b$ increases with increase in $r_c$.

In Fig. 2a the location of BD is changed while keeping $d_{sb} = d_{bd}$ constant (at 4.1 m for this plot). $\tau_p$ depends only on the channel gain between ST and BD (denoted by $|h_{sb}|^2$), which becomes weak as we increase $d_{sb}$. Hence $\tau_p$ increases because the interference at PR from SN reduces. $\tau_s$ and $\tau_b$ have a dependence on gain of both the channels involved, i.e., $|h_{sb}|^2$ (channel gain between ST and BD) and $|h_{bd}|^2$ (channel gain between BD and SD), so the corresponding throughputs should reach the maximum only when the product of both the channel gains is large (towards the center of the plot). If either of them is a weak channel, their product is small and the throughput is less (this is evident at the extremes of the $x$-axis).

In Fig. 3 the maximum tertiary throughput $P^\text{max}_s$ is plotted versus $R_s = R_b = R$ in the valid range of $P_s$ and $r_c$ (that guarantees desired throughput for PN and SN: $\tau_p \geq \tau_p$ and $\tau_s \geq \tau_s$). There exists a certain target rate for which we can achieve the optimum tertiary throughput $P^\text{opt}_s$ (the corresponding information rate is denoted by $R_s^{\text{opt}}$). $P^\text{opt}_s$ reduces as we increase either $\tau_p$ or $\tau_s$. This is due to the fact that the valid range of $P_s$ and $r_c$ reduces. However, we observe from the plot that there is a strong dependence of $P^\text{opt}_s$ on the desired performance of the SN, and a weak dependence on the desired performance of the BD.

VI. CONCLUSION

In this paper, we consider a network in which a backscatter device (BD) reflects a signal from a secondary cognitive transmitter to communicate to a secondary destination. Reflection from BD adds to the interference at the primary receiver. We show that by careful control of the secondary transmit power and the channel gain between ST and BD, it is possible to maximize the throughput of the backscatter link while ensuring a certain desired performance of the primary and the secondary networks.
APPENDIX A

Proof of Theorems 4 and 5 Using $\Gamma_p^a$ from 3, and performing mathematical rearrangements, we have

$$\bar{P}_p^a = \Pr \left[ X \geq \gamma_p (P_s V + P_p r_c |r_c|^2 A W + P_r |r_c|^2 Z W + N_o) \right].$$

(20)

Using the CCDF of RV $X$, $\bar{P}_p^a|_{X,V,A,Z,W}$ can be obtained as

$$\bar{P}_p^a|_{X,V,A,Z,W} = \exp \left( -\frac{\lambda_2 \gamma_p (P_s V + P_p |r_c|^2 A W + P_r |r_c|^2 Z W + N_o)}{P_p} \right).$$

Averaging the above over the RVs $V,A,Z$ and $W$, $\bar{P}_p^a$ we obtain

$$\bar{P}_p^a = e^{-\frac{\lambda_2 \gamma_p N_o}{P_p}} \times \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} e^{-\frac{\lambda_2 \gamma_p (P_s V + P_p |r_c|^2 A W + P_r |r_c|^2 Z W + N_o)}{P_p}} \times f_V(v) f_A(a) f_Z(z) f_W(w) \, dw \, dz \, da \, dv$$

(21)

On solving the inner integrals, the above can be expressed as

$$\bar{P}_p^a = e^{-\frac{\lambda_2 \gamma_p N_o}{P_p}} \times \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} e^{-\frac{\lambda_2 \gamma_p (P_s V + P_p |r_c|^2 A W + P_r |r_c|^2 Z W + N_o)}{P_p}} \times f_V(v) f_A(a) f_Z(z) f_W(w) \, dw \, dz \, da \, dv$$

(22)

Further, using relation $\int_0^{\infty} \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} e^{-\lambda_2 \gamma_p (P_s V + P_p |r_c|^2 A W + P_r |r_c|^2 Z W + N_o)} \times f_V(v) f_A(a) f_Z(z) f_W(w) \, dw \, dz \, da \, dv = e^{\lambda_2 \gamma_p N_o} [ab]_{11}^{3.352.4}$, $\bar{P}_p^a$ can be expressed as in 9.

APPENDIX B

Proof of Theorem 3 Using $\Gamma_h^a$ and $\Gamma_h^b$ from 5 and 6 respectively, $\bar{P}_h^b$ can be expressed as

$$\bar{P}_h^b = \Pr \left[ Y \geq \gamma_h (P_s |r_c|^2 A U + P_r |r_c|^2 U + P_r B + N_o) \right].$$

(24)

Using the CCDF of RV $Y$, the above can be obtained as

$$\bar{P}_h^b|_{Y,U,A,B,Z} \geq \gamma_h = e^{-\frac{D a}{\gamma_h}} \times \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} e^{-\frac{\lambda_h \gamma_h N_o}{P_h}} \times f_Y(y) f_U(u) f_A(a) f_B(b) \, dy \, du \, da \, db$$

(25)

Averaging the above over the PDF of RVs, we obtin

$$\bar{P}_h^b = e^{-\frac{D a}{\gamma_h}} \times \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} e^{-\frac{\lambda_h \gamma_h N_o}{P_h}} \times f_Y(y) f_U(u) f_A(a) f_B(b) \, dy \, du \, da \, db$$

(26)

On solving the integrals involved in the above, we obtain

$$\bar{P}_h^b = e^{-\frac{D a}{\gamma_h}} \times \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} e^{-\frac{\lambda_h \gamma_h N_o}{P_h}} \times f_Y(y) f_U(u) f_A(a) f_B(b) \, dy \, du \, da \, db$$

(27)

It is challenging to solve the above. Therefore, we use a linear approximation to the exponential term to express $\bar{P}_h^b$ as

$$\bar{P}_h^b = \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} e^{-\frac{\lambda_h \gamma_h N_o}{P_h}} \times f_Y(y) f_U(u) f_A(a) f_B(b) \, dy \, du \, da \, db$$

(28)

After solving for each individual term, we get

$$\bar{P}_h^b = D_2 \gamma_h^b \left[ \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} e^{-\frac{\lambda_h \gamma_h N_o}{P_h}} \times f_Y(y) f_U(u) f_A(a) f_B(b) \, dy \, du \, da \, db \right]$$

(29)

Further, using relation $\int_0^{\infty} \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} e^{-\lambda_2 \gamma_p (P_s V + P_p |r_c|^2 A W + P_r |r_c|^2 Z W + N_o)} \times f_V(v) f_A(a) f_Z(z) f_W(w) \, dw \, dz \, da \, dv = e^{\lambda_2 \gamma_p N_o} [ab]_{11}^{3.352.4}$, $\bar{P}_p^a$ is obtained as in 17.

REFERENCES


