Efficient Design of Thinner Broadband RAM Using Novel Formulation of Cost Function

Pooja Warhekar 1

1Indian Institute of Technology Kharagpur

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Abstract

In this manuscript, we explore the effect of cost function formulation on the design of radar absorbing material (RAM) using numerical optimization techniques. We also explore formulation for the thinner design of RAM and propose a new cost function. Our results show a significant reduction in the thickness of multilayer RAM.
Abstract—Modern military and civic applications require broadband microwave radar absorbing material (RAM). Such a RAM is multilayer and invariably increases the surface thickness of platforms (viz., aircraft, ship, vehicle) and thereby increases the overall body weight. So, all RAM design methods do some kind of optimization of layer parameters. In this paper, we study the effect of commonly used cost function formulations on obtaining global optima. We also explore formulation for thinner design of RAM and propose a new cost function. Our proposed formulation effectively works as a two-step optimizer and is able to provide thinner results without increasing the complexity of the algorithm employed for optimization. We compare our results for twelve different multilayer RAMs with the results of existing literature. The maximum and minimum thickness reduction obtained respectively are 22.9562% and 0.009%. Our results show that for a given search algorithm, the proposed formulation is more effective than other commonly used formulations to find the optimal layer sequence and minimize thickness.

Index Terms—Radar Absorbing Material (RAM), cost function, optimization, PSO, meta-heuristics.

I. INTRODUCTION

Radar absorbing materials (RAMs) have a wide range of uses in today’s military and civic industries [1]–[4]. RAMs by reducing the electromagnetic (EM) reflection of the platforms help to reduce detectability of the target. In view of this, multi-layer radar absorbing materials (MRAMS) have piqued the interest of researchers in recent decades due to the high likelihood of achieving very low reflectivity for broad operating bandwidth (BW) and angle of incidence with both transverse electric (TE) and transverse magnetic (TM) polarization. For this purpose, the surface of the platform is coated with MRAM. However, this increases thickness of the platform’s surface and as a result, weight and volume of the platform. Despite extensive research to improve RAM performance and reduce thickness, designs of thin MRAMs that operate in the desired frequency range are highly demanded [5], [6]. In the design problem of an N-layer RAM with a given material database, although there are 2N variables that need to be optimized, the optimization becomes difficult due to increase in convergence time with increase in the number of layers and other design requirements. In this regard, various single-objective meta-heuristic algorithms [7]–[12] as well as multi-objective algorithms [5], [13], [14] have been used.

The design of a multilayer RAM is generally a multi-objective problem. The commonly used objectives are to increase the operational BW and to reduce the thickness of the RAM [15]. There are different approaches to solve this problem. First, the problem is solved using multiobjective algorithms [16]. However, multiobjective solutions require a lot of resources and processing. Second, a single objective optimization algorithm is used by combining the two objectives using weighted sum method. In this method, the multiple objectives are combined using a weight factor that decides the priorities amongst the objective functions. However, the value of weight factor needs to be carefully chosen, since it plays an important role in obtaining the final solution. Therefore, weight factor should be considered as a variable for the optimization problem.

For any optimization algorithm, the formulation of the cost function plays a very important role in finding the optimal solution for the problem. However, to the best of our knowledge, the effect of the formulation of the cost function on the final design obtained using an optimization algorithm has not been studied for RAM design problem. In literature, two formulations are most commonly used. Both these formulations focus on the maximization of reflection loss in the bandwidth (BW) under consideration. However, for practical purposes, the best solution will be the thinnest RAM with the required level of performance (Reflection loss (RL)) in the desired BW. This can be achieved by designing a cost function which focuses on minimizing thickness while satisfying the performance requirement.

In this study, a novel cost function formulation that determines the optimal layer sequence and corresponding thicknesses for the MRAM design for a given RL requirement is proposed. This cost function is also independent of the weight factor. We used the Particle Swarm Optimization (PSO) algorithm [8], [17], [18] with our proposed formulation for a range of weight factor values, and compare the results with commonly used formulations (discussed in section III). The designs are also compared with the results reported in [8] to highlight the effectiveness and superiority of the suggested formulation.

The rest of this paper is organised as follows. The physical model of MRAM is given in Section II. The problem formulation along with commonly used and new cost function is given in Section III. The analysis of the cost functions and the comparative results are reported in Section IV. Finally section V concludes the paper.
II. PHYSICAL MODEL OF MULTILAYER RAM

Fig. 1 depicts the physical representation of the RAM containing N layers, on a substrate of a perfect electric conductor (PEC). An electromagnetic (EM) wave of a particular frequency is incident from air to the first interface (mathematically, layer 1) of the N-layer RAM. There are various formulation available to calculate the RL of an EM wave impinging upon a planar layered media which can be expressed as a function of constitutive parameters, thickness of each layer, the angle of incidence and polarization \([8], [19], [20]\). Following EM formulation is used to calculate the reflection coefficient of MRAM:

In Fig. 1, if angle of incidence is \(\theta^0\), then the reflection coefficient \(R_i^{TE/TM}\) at \(i^{th}\) layer for both polarizations is defined as follows:

\[
R_i^{TE/TM} = \frac{r_i^{TE/TM} + R_{i+1}^{TE/TM} e^{-2j k_{i+1} d_{i+1}}}{1 + r_i^{TE/TM} R_{i+1}^{TE/TM} e^{-2j k_{i+1} d_{i+1}}}
\]  
(1)

where, \(r_i^{TE}\) and \(r_i^{TM}\) are as follows:

\[
r_i^{TE} = \begin{cases} 
\frac{\mu_{i+1} k_i - \mu_i k_{i+1}}{\mu_{i+1} k_i + \mu_i k_{i+1}}, & i < M \\
-1, & i = M 
\end{cases}
\]  
(2)

\[
r_i^{TM} = \begin{cases} 
\frac{\epsilon_{i+1} k_i - \epsilon_i k_{i+1}}{\epsilon_{i+1} k_i + \epsilon_i k_{i+1}}, & i < M \\
1, & i = M 
\end{cases}
\]  
(3)

where, \(k_i\) is the wavenumber of the \(i^{th}\) layer and is expressed as follows:

\[
k_i = \omega \sqrt{\mu_i \epsilon_i - \mu_0 \epsilon_0 \sin^2 \theta}
\]  
(4)

\(\epsilon_i(f)\) and \(\mu_i(f)\) are frequency depended complex permittivity and permeability of \(i^{th}\) layer respectively and are expressed as:

\[
\epsilon_i(f) = \epsilon_0 [\epsilon'_i(f) - j \epsilon''_i(f)]
\]  
(5)

\[
\mu_i(f) = \mu_0 [\mu'_i(f) - j \mu''_i(f)]
\]  
(6)

In the above equations, \(\omega = 2\pi f\) is the radian frequency, \(\epsilon_0\) and \(\mu_0\) are permittivity and permeability of free space respectively and are given as:

\[
\epsilon_0 = 8.854 \times 10^{-12} \text{F/m} \quad \mu_0 = 4\pi \times 10^{-7} \text{H/m}
\]  
(7)

As the last layer of MRAM is PEC, the reflection coefficient of last layer is taken as 1 for TE polarization and -1 for TM polarization. The total RL \(R_0^{TE}\) and \(R_0^{TM}\) for the MRAM are recursively calculated using (1).

III. PROBLEM FORMULATION

In the multi-objective optimization problem of MRAM design, the objectives are considered as maximizing the operational BW and, minimizing the MRAM thickness. The BW of operation is the frequency range within which the RL of the MARM is below the required threshold. In literature, There are various methods to formulate the cost function for the multi-objective problem of MRAM design. In this study, we consider the two most commonly used formulations.

A. Formulation 1

In this formulation, the thickness of the MRAM is considered by summing the thickness of all the layers while the RL is taken into account by considering sum of RL at all frequency points \([13], [15], [20]\). The cost function is formulated by combining two conflicting objectives \((OF_a\) and \(OF_b)\) using a weighted sum approach.

\[
Cost_{RC_{TM/TE}} = \sum_{f= f_{min}}^{f_{max}} \left( R_0^{TM/TE} \right)
\]  
(8)

\[
Cost_T = \sum_{layer = 1}^{N} d_i
\]  
(9)

\[
CF_1 = \alpha OF_a + (1 - \alpha) OF_b
\]  
(10)

\[
OF_a = \frac{Cost_{RC_{TM/TE}}}{A}
\]  
(11)

\[
OF_b = \frac{Cost_T}{C}
\]  
(12)

where, \(A\) and \(C\) are normalization factors and referred as

\[
A = \text{No. of freq points}
\]  
(13)

\[
C = \text{Max No. of layers} \times \text{Max Layer Thickness}
\]  
(14)

Here, \(OF_a\) focuses on the maximization of the reflection loss in the desired BW and the \(OF_b\) focuses on the minimization of the thickness. The final value of cost function \(CF_1\) depends on \(OF_a\) and \(OF_b\), as well as the weighting factor \(\alpha\) which decides the priority between \(OF_a\) and \(OF_b\).

B. Formulation 2

In this formulation, the thickness of the MRAM is again considered by summing the thickness of all the layers while the RL is taken into account by considering the sum of maximum RL at given angle for all the frequencies \([7], [8], [10]\). Similar to \(CF_1\), the cost function \(CF_2\) is formulated by combining the two aforementioned objective functions \(OF_c\) and \(OF_d\).

\[
OF_c = 20 \log_{10}(\max(|R_0^{TE/TM}|))
\]  
(15)

\[
OF_d = \sum_{layer = 1}^{N} d_i
\]  
(16)

\[
CF_2 = \alpha OF_c + (1 - \alpha) OF_d
\]  
(17)

Here, \(OF_c\) focuses on the minimization of maximum RL provided by the RAM in desired frequency range while \(OF_d\) focuses on the reduction of total thickness.
<table>
<thead>
<tr>
<th>Material in database</th>
<th>Frequency range (GHz)</th>
<th>Integer number code for PSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>CB 0 wt%</td>
<td>8-12.5</td>
<td>1</td>
</tr>
<tr>
<td>CB 1 wt%</td>
<td>8-12.5</td>
<td>2</td>
</tr>
<tr>
<td>CB 2 wt%</td>
<td>8-12.5</td>
<td>3</td>
</tr>
<tr>
<td>CB 4 wt%</td>
<td>8-12.5</td>
<td>4</td>
</tr>
</tbody>
</table>

C. New formulation

In the above formulations, \( OF_a \) and \( OF_c \) focuses on the maximization of RL in the desired BW. For RAM applications, generally, in the desired BW, the RL must be below a required threshold and the thickness should be as low as possible. In practice, the actual value of the RL may not be important as long as it is below a predefined threshold. Therefore, based on this information we propose a new formulation as follows:

\[
OF_1 = \frac{\sum_{f_{max}}^{f_{max}} (RC_{dB} > RL_{des})}{A}
\]

where, \( RC_{dB} \) is the RL \((R_0^{TE/TM})\) in dB, \( RL_{des} \) is the desired level of RL (dB) in the BW under consideration. \( OF_2 \) is defined as:

\[
OF_2 = \frac{\sum_{layer = 1}^{N} d_i}{C}
\]

\[
CF_{new} = \alpha OF_1 + (1 - \alpha) OF_2
\]

the cost function is defined same as (10) and (17), where, unlike (10) and (17) the \( \alpha \) is fixed at 0.9.

Here, \( OF_1 \) focuses on minimizing the number of frequency points at which the RL is greater than the predefined threshold. As the RL criteria is satisfied, the summation in (18) goes to zero and therefore contribution from \( OF_1 \) goes to zero. After this point only \( OF_2 \) will contribute to \( CF_{new} \). Equation (20) becomes independent of \( \alpha \) and algorithm then focuses only on minimising the thickness of the MRAM. With \( CF_{new} \), the algorithm effectively works as a two-step optimization problem because once the RL criteria is fulfilled, the algorithm automatically switches to optimizing thickness of MRAM.

IV. RESULTS AND ANALYSIS

A. Comparison of formulations of cost function

To demonstrate the effectiveness of the proposed formulation, we optimized three different databases using formulation 1, formulation 2, and proposed formulation for same set of design requirements. The databases are listed in Tables I, II and III along with the integer number code for optimization algorithm (PSO [17], [18]). For all designs, \( RL_{des} \) was set to -10 dB. For database I and database II, operational BW of 8-12.5 GHz is considered whereas for database III an operational BW of 8 - 18 GHz is considered.

The first database (Table I) is composed of epoxy and carbon black (CB) composites. The measured permittivity of the composites is shown in Fig. 2. The permeability of the composites is assumed to be 1 [21]. The second database (Table II) is the most used in the literature for comparing performance of various optimization algorithms. These materials include frequency-dependent lossless dielectric, lossy dielectric, lossy magnetic, and relaxation magnetic materials. The third database (Table III) is of composites whose parameters are published in the literature [15], [20]. This database is composed of materials fabricated using epoxy as matrix and multiwalled carbon nanotube (MWCNT) and carbon nanofiber (CNF) as filler.

We simulate PSO algorithm over these three database for normal incidence, moreover, 20 independent trials are carried out for each case. And, the comparative results are enlisted in Table IV. The reflection coefficient versus frequency curves for \( \alpha = 0.9, 0.5 \) are shown in Fig. 3. As can be observed from Table IV, for all the databases, when \( \alpha = 1 \) all the three formulations provide thickest designs. This is because, for \( \alpha = 1 \), none of the formulations take thickness into consideration. For other values of \( \alpha \) significant decrease in the thickness can be observed for all the cases.

For cost function \((CF_1)\), with Database I, as the value of
α decreases form 0.9 to 0.5, the thickness of the MRAM decreases from 39 mm to 21.4 mm. With the decrease in value of α, the priority of the cost function shifts towards the minimization of the thickness. However, the obtained solution does not satisfy the BW requirement when α is equal to 0.5 (shown in Fig. 3(a)). In a similar way, we carried out the optimization for Database II and Database III using cost function (CF1). For Database II as α decreases from 0.9 to 0.5 the optimized thickness decreases from 7.2 mm to 0.5 mm while satisfying BW requirement (shown in Fig. 3(b)). Whereas, for Database III, as α decreases from 0.9 to 0.5, the thickness decreases from 8 mm to 2.9 mm. However, CF1 does not satisfy BW requirement for any value of α (shown in Fig. 3(c)).

For cost function (CF2), with Database I, as the value of α decreases form 0.9 to 0.5, the thickness of the MRAM decreases from 47.8 mm to 3.7 mm. However, the obtained solution does not satisfy the BW requirement when α is equal to 0.7 and 0.5 (shown in Fig. 3(a)). In a similar way, we carried out the optimization for Database II and Database III using cost function (CF2). For Database II as α decreases from 0.9 to 0.5 the optimized thickness decreases from 13.1 mm to 0.5 mm while satisfying BW requirement (shown in Fig. 3(b)). Whereas, for Database III, as α decreases from 0.9 to 0.5, the thickness decreases from 8.5 mm to 3.1 mm. However, CF2 does not satisfy BW requirement for α equal to 0.7 and 0.5 (shown in Fig. 3(c)). Therefore, for CF1 and CF2, it is observed that, the obtained optimized results depend on the value of weight factor α. In addition to that, BW requirement may not be satisfied for all the scenarios.

For cost function (CFnew), with Database I, as the value of α decreases form 0.9 to 0.5, the thickness of the MRAM of 28.6 mm is obtained for all values of α. In addition to this, the obtained solution satisfies the BW requirement for all the values of α (shown in Fig. 3(a)). In a similar way, we carried out the optimization for Database II and Database III using cost function (CFnew). For Database II as α decreases from 0.9 to 0.5, a constant optimized thickness of 0.4 mm is obtained while satisfying BW requirement for all the values of α (shown in Fig. 3(b)). For Database III, as α decreases from 0.9 to 0.5, a constant thickness of 7.1 mm is obtained while satisfying the BW requirement for all the values of α (shown in Fig. 3(c)). Therefore, for CFnew, it is observed that, the obtained optimized results does not depend on the value of weight factor α. In addition to this, the BW requirement is satisfied for all the scenarios.

From the above comparison, it can be concluded that the proposed cost function provides constant thickness for all the values of α under consideration. Thus the cost function formulation can be considered independent of the value of α. In addition to that, for all the databases, the proposed cost function provides the thinnest MRAM design which can satisfy the BW requirement.

B. Comparison of MRAM design with existing literature

To further test the effectiveness of the proposed formulation, we compare the results obtained for the optimization of MRAM design using the proposed cost function with the results reported in [8] for the same set of parameters. For the PSO implementation, the values of different search parameters such as the number of iterations, initial population, acceleration coefficient, inertial weight, the maximum number of layers, and maximum particle velocity are taken as reported in [8]. The designs are obtained at different broadband frequency ranges (viz., 2–8, 8–12, 12-18, and 2–18 GHz) for normal incidence and oblique incidence (viz. 30°, 45°, 60° and 75°). The predefined threshold $RL_{des}$ required for proposed cost function $CF_{new}$ is set to the maximum RL reported in [8] for each design. For further verification, each design is also simulated in CST. It is observed that the CST simulated results match with the obtained optimized results. The results obtained for the optimization of MRAM design with normal and oblique incidence are discussed as follows.

1) Normal incidence: We consider four different MRAM designs labeled as Model 1, Model 2, Model 3, and Model 4 corresponding to 2-8, 8–12, 12–18, and 2–18 GHz bands, respectively. The thicknesses of each design obtained at the end of the optimizations are given in Table V. For comparison, the results of MRAM designs in [8] are also presented in Table V. From Table V, it is observed that MRAM designs with the proposed cost function have the least total thickness.

The reflection coefficient versus frequency curves for Model 2 and Model 4 are illustrated in Fig. 4. As can be seen from Fig.4(a) the RL for Model 2 is below the desired $RL_{des}$ of -26.1052 dB in the entire frequency range of 8-12 GHz and
TABLE V

<table>
<thead>
<tr>
<th>Layer</th>
<th>Model 1 TE/TM</th>
<th>Model 2 TE/TM</th>
<th>Model 3 TE/TM</th>
<th>Model 4 TE/TM</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>2-18 GHz</td>
<td>2-18 GHz</td>
<td>2-18 GHz</td>
<td>2-18 GHz</td>
</tr>
<tr>
<td>Mat #</td>
<td>T (mm)</td>
<td>T (mm)</td>
<td>T (mm)</td>
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<td>0.4248</td>
<td>16</td>
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<td></td>
<td>16</td>
<td>0.2048</td>
<td>16</td>
<td>0.2838</td>
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<td>2</td>
<td>6</td>
<td>1.1458</td>
<td>6</td>
<td>0.5098</td>
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<tr>
<td></td>
<td>6</td>
<td>1.0991</td>
<td>6</td>
<td>0.3053</td>
</tr>
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<td>5</td>
<td>0.9402</td>
<td>7</td>
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<td>5</td>
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<td></td>
<td>9</td>
<td>1.3741</td>
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<td>3.4480</td>
<td>2.6959</td>
<td>2.8364</td>
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</table>

Data from [8]

TABLE VI

<table>
<thead>
<tr>
<th>Layer</th>
<th>Model 5 TE 30°°</th>
<th>Model 6 TE 30°°</th>
<th>Model 7 TE 45°°</th>
<th>Model 8 TE 45°°</th>
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<tbody>
<tr>
<td></td>
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<td>2-18 GHz</td>
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</tr>
<tr>
<td>Mat #</td>
<td>T (mm)</td>
<td>T (mm)</td>
<td>T (mm)</td>
<td>T (mm)</td>
</tr>
<tr>
<td>1</td>
<td>16</td>
<td>0.2420</td>
<td>16</td>
<td>0.3468</td>
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<tr>
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<td>0.1253</td>
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</tr>
<tr>
<td>MRC (dB)</td>
<td>-19.3131</td>
<td>-15.3940</td>
<td>-26.3193</td>
<td>-12.2444</td>
</tr>
<tr>
<td>TT (mm)</td>
<td>3.9220</td>
<td>3.8580</td>
<td>3.8213</td>
<td>3.3720</td>
</tr>
</tbody>
</table>

Data from [8]

Fig. 3. Calculated RL at normal incidence for designed structures described in Table IV (dashed line), formulation 2 (dotted line) and proposed formulation (solid line) with various values of $\alpha$ (= 0.9, 0.5) for (a) Database I (b) Database II (c) Database III.

Fig. 4. Calculated RL at oblique incidence for MRAM described in Table V compared with [8]. (a) Model 2 (b) Model 4.

Fig. 5. Calculated RL at oblique incidence for MRAM described in Table VI and VII compared with [8]. (a) Model 5 and 8 (b) Model 9 and 10.

The reflection coefficient versus frequency curves for Models 5–8 are designed for both TE and TM polarizations. For fair comparison the $RL_{des}$ is set to the maximum value reported in [8]. The designs obtained with proposed cost function formulation are shown in Table VI and Table VII. From Table VI and VII, it is observed that MRAM designs with the proposed cost function have the least total thickness. The reflection coefficient versus frequency curves for Models 5, 8, 9 and 10 are illustrated in Fig. 5.

As can be seen from Fig. 5(a) the RL for Model 5 is below the desired $RL_{des}$ of -19.3096 dB in the entire frequency range 2-18 GHz and the RL for Model 8 is below the desired $RL_{des}$ of -12.2444 dB in the entire frequency range 2-18 GHz. In addition to this, the reduction in thickness for Model 5 is 16.8587%; for Model 6 is 2.3001%; for Model 7 is 12.6104%; for Model 8 is 1.9368%. As can be seen from Fig. 5(b) the RL for Model 9 is below the desired $RL_{des}$ of -12.6104 dB in the entire frequency range 2-18 GHz and the RL for Model 10 is below the desired $RL_{des}$ of -8.5332 dB in the entire frequency range 2-18 GHz. In addition to this, the reduction in thickness for Model 5 is 16.8587%; for Model 6 is 2.3001%; for Model 7 is 12.6104%; for Model 8 is 1.9368%.

Therefore, the proposed cost function formulation can satisfy the BW requirement in the desired frequency range while providing thinner structures for oblique incidence.

V. Conclusion

In this paper, the effect of formulation of cost function on a given search algorithm is studied and a new formulation of the cost function is proposed for the optimum design of MRAM. Any algorithm with the new formulation effectively works as a two level optimiser. Therefore it can provide thinner designs for given algorithm without increasing the complexity of the algorithm. Three separate databases are optimized using two commonly used and the proposed formulation for different
values of $\alpha$. The proposed cost function is found to be independent of weight factor $\alpha$. Moreover, the results obtained with proposed cost function satisfy the BW requirement and provide thinnest MRAMs for all the databases. In addition to this, 12 MRAMs are designed and compared with results in the existing literature. Our proposed formulation effectively reduces the thickness of all the 12 models while satisfying the BW requirement. A maximum thickness reduction of 22.9562% is obtained for Model 2 and a minimum reduction of 0.009% is obtained for Model 10. The suggested idea of modifying the cost function is simple to apply to any optimization technique, and it can produce thinner MRAM designs.

### REFERENCES


